# Physical realization of quantum teleportation for a nonmaximal entangled state

Yoshiharu Tanaka, Masanari Asano, and Masanori Ohya
*Department of Information Sciences, Tokyo University of Science, Noda, Chiba 278-8510, Japan*
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Recently, Kossakowski and Ohya (K-O) proposed a new teleportation scheme which enables perfect teleportation even for a nonmaximal entangled state [A. Kossakowski and M. Ohya, Infinite Dimensional Analysis Quantum Probability and Related Topics **10**, 411 (2007)]. To discuss a physical realization of the K-O scheme, we propose a model based on quantum optics. In our model, we take a superposition of Schrödinger's cat states as an input state being sent from Alice to Bob, and their entangled state is generated by a photon number state through a beam splitter. When the average photon number for our input states is equal to half the number of photons into the beam splitter, our model has high fidelity.

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## I. INTRODUCTION

Quantum teleportation is one of the efficient methods to transfer quantum information, where a quantum entangled state plays an essential role. In a model proposed by Bennett *et al.* [1], the entangled state $\sigma = |\xi\rangle\langle\xi|$ is given as an Einstein-Podolsky-Rosen (EPR) pair $|\xi\rangle = 1/\sqrt{2}(|01\rangle + |10\rangle)$ in two qubits space. Alice has the first qubit of the EPR pair and Bob has the second qubit. Alice prepares an unknown quantum state $a|0\rangle + b|1\rangle$ in one qubit, and she implicates a quantum measurement on the input qubit and the first qubit of the EPR pair. The quantum measurement is a joint measurement of the observable $F = \sum_{k=1}^{4} z_k P_k$, where $z_k$ is the measurement value and $P_k$ is one of the projections $\{P_k; k = 1,2,3,4\}$;

$$P_1 = |\psi^{(-)}\rangle\langle\psi^{(-)}|, \quad P_2 = |\psi^{(+)}\rangle\langle\psi^{(+)}|, \tag{1}$$

$$P_3 = |\varphi^{(-)}\rangle\langle\varphi^{(-)}|, \quad P_4 = |\varphi^{(+)}\rangle\langle\varphi^{(+)}|, \tag{2}$$

with the Bell CONS given by

$$|\psi^{(\pm)}\rangle = \sqrt{\frac{1}{2}}(|01\rangle \pm |10\rangle), \quad |\varphi^{(\pm)}\rangle = \sqrt{\frac{1}{2}}(|00\rangle \pm |11\rangle). \tag{3}$$

Alice informs a result of the measurement to Bob via a classical channel, then Bob can recover the original input state by applying a unitary transformation to his obtained qubit. Conventional models of perfect teleportation, as that of Bennett *et al.*, are supposed that Alice and Bob share a maximal entangled state [2,3].

Recently, Kossakowski and Ohya proposed a new scheme of teleportation, in which the teleportation process is mathematically represented as a map from input to output, called a teleportation map, and perfect teleportation is possible even for a nonmaximal entangled state [4].

In this paper, we give a physical realization of the teleportation map of the K-O scheme. Our model is based on physical states in quantum optics. The input state is a superposition of Schrödinger's cat states [5,6], and the shared entangled state is generated by a photon number state and beam splitter [7]. The entangled state is nonmaximal in general. Alice measures the sum of the photon numbers and the phase difference of the photons [8,9]. Bob can recover the input state by shifting the phase [10]. These procedures are a specific case of the K-O scheme. As a result, our model has high fidelity even for a nonmaximal entangled state.

In Sec. II we explain the teleportation map of the K-O scheme. In Sec. III, we explain our model of teleportation. In Sec. IV, our model can be explained in the contexts of the K-O scheme as a specific case of the K-O scheme. In Sec. V, we estimate the fidelity between an input state and its output state in our model.

## II. SCHEME OF K-O TELEPORTATION

In this section we briefly explain the scheme of teleportation proposed by Kossakowski and Ohya. Let us take the conditions that all Hilbert space $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ are $\mathbb{C}^n$. Alice has a unknown quantum state $\rho$ on $\mathcal{H}_1$, and she was asked to teleport it to Bob. For this purpose, an entangled state $\sigma$ is prepared on $\mathcal{H}_2 \otimes \mathcal{H}_3$, and $\mathcal{H}_2$ is attached to Alice and $\mathcal{H}_3$ is to Bob. Any entangled state $\sigma$ can be given as

$$\sigma = \sum_{i,j=1}^{n} |i\rangle\langle j| \otimes \left( \sum_{s=1}^{n^2} \lambda_s f_s |i\rangle\langle j| f_s^* \right), \tag{4}$$

with $\sum_{s=1}^{n^2} \lambda_s = 1$ and $\lambda_s \geqslant 0$. Here $\{|i\rangle\}_{i=1}^{n}$ is the fixed orthonormal basis (ONB) in $\mathbb{C}^n$, and $\{f_s\}_{s=1}^{n^2}$ is an ONB in the set of all bounded operators on $\mathbb{C}^n$, which is simply denoted by $M_n$.

Alice performs the joint measurement of the observable $F$ on $\mathcal{H}_1 \otimes \mathcal{H}_2$, which is set as follows:

$$F = \sum_{k=1}^{n^2} z_k P_k \tag{5}$$

$$\equiv \sum_{k=1}^{n^2} z_k \sum_{i,j=1}^{n} g_k^* |i\rangle\langle j| g_k \otimes |i\rangle\langle j|, \tag{6}$$

where $\{g_k\}_{k=1}^{n^2}$ is another ONB in $M_n$. Then K-O defined the unnormalized teleportation map for an input state $\rho$ and the measured value $z_k$ of Alice as

$$T_k(\rho) \equiv \mathrm{tr}_{12}[(P_k \otimes I)\rho \otimes \sigma (P_k \otimes I)] \tag{7}$$

$$= \sum_{s=1}^{n^2} \lambda_s f_s g_k \rho g_k^* f_s^*, \tag{8}$$

 

where $I$ is a unity of $M_n$ and $\mathrm{tr}_{12}$ is the trace over the space $\mathcal{H}_1 \otimes \mathcal{H}_2$. It is easily seen that $T_k(\rho)$ is completely positive but not trace preserving. To consider the trace preserving map from $T_k(\rho)$, K-O considered the dual map $\tilde{T}_k(\rho)$ of $T_k(\rho)$ [i.e., $\mathrm{tr} A T_k(\rho) = \mathrm{tr}\tilde{T}_k(A)\rho$]. Indeed it is expressed as

$$\tilde{T}_k(A) = \sum_{s=1}^{n^2} \lambda_s g_k^* f_s^* A f_s g_k, \quad A \in M_n. \tag{9}$$

If and only if $\mathrm{rank}\tilde{T}_k(I) = n$ is satisfied, the dual teleportation map $\tilde{T}_k$ is normalized as

$$\tilde{\Upsilon}_k = \kappa_k^{-\frac{1}{2}} \tilde{T}_k \kappa_k^{-\frac{1}{2}}, \tag{10}$$

where

$$\kappa_k \equiv \tilde{T}_k(I) \tag{11}$$

$$= \sum_{s=1}^{n^2} \lambda_s g_k^* f_s^* f_s g_k. \tag{12}$$

The dual map $\Upsilon_k$ of $\tilde{\Upsilon}_k$ is trace preserving and it has the form

$$\Upsilon_k(\rho) = \sum_{s=1}^{n^2} \lambda_s f_s g_k \kappa_k^{-\frac{1}{2}} \rho \kappa_k^{-\frac{1}{2}} g_k^* f_s^*. \tag{13}$$

The output state in the K-O scheme is given by $\Upsilon_k(\rho)$. Note that this $\Upsilon_k(\rho)$ is linear for input $\rho$.

K-O considered a special case of the entangled state $\sigma$ defined by Eq. (4), that is,

$$\sigma = \sum_{i,j=1}^{n} |i\rangle\langle j| \otimes f|i\rangle\langle j|f^*, \tag{14}$$

where $\mathrm{tr} f^* f = 1$. The $\sigma$ is a pure entangled state so that we can express $\sigma$ by $|\xi\rangle\langle\xi|$ with a state vector

$$|\xi\rangle = \sum_{i=1}^{n} |i\rangle \otimes f|i\rangle. \tag{15}$$

The pure state $\sigma$ given by the form of Eq. (14) is maximal entangled state if and only if $f^* f = f f^* = \frac{I}{n}$. Further, under the condition of $\mathrm{rank} f = \mathrm{rank} g_k = n$, their output state is written as

$$\Upsilon_k(\rho) = f g_k \kappa_k^{-\frac{1}{2}} \rho \kappa_k^{-\frac{1}{2}} g_k^* f^*, \tag{16}$$

where

$$\kappa_k = g_k^* f^* f g_k. \tag{17}$$

Bob can recover the input state $\rho$ with the following unitary key

$$U_k = \kappa_k^{-\frac{1}{2}} g_k^* f^*. \tag{18}$$

Even if the condition of $\mathrm{rank} f = \mathrm{rank}\ g_k = n$ is satisfied, the pure state $\sigma$ is not a maximal entangled state in general. Thus, K-O teleportation does not require a maximal entangled state for teleportation.

## III. PHYSICAL MODEL FOR K-O TELEPORTATION

In this section we discuss a physical realization of the K-O teleportation scheme. To simplify our discussion, we consider the case of two-dimensional Hilbert space $\mathbb{C}^2$. Let an input state be a pure state $\rho = |\psi\rangle\langle\psi|$ with $|\psi\rangle = c_1|e_1\rangle + c_2|e_2\rangle$, where $\{|e_n\rangle; n = 1,2\}$ is an ONB of $\mathbb{C}^2$. In this case, the teleportation map given by Eq. (16) is

$$\Upsilon_k(\rho) = f g_k \kappa_k^{-\frac{1}{2}} |\psi\rangle\langle\psi| \kappa_k^{-\frac{1}{2}} g_k^* f^*, \tag{19}$$

which can be expressed as $\Upsilon_k(\rho) = |\psi'\rangle\langle\psi'|$ with a state vector as $|\psi'\rangle = f g_k \kappa_k^{-\frac{1}{2}}|\psi\rangle$. Here, if both $|e_1\rangle$ and $|e_2\rangle$ are eigenvectors of $\kappa_k$, the state vector $|\psi'\rangle$ is written as

$$|\psi'\rangle = c_1 f g_k \langle e_1|\kappa_k|e_1\rangle^{-\frac{1}{2}}|e_1\rangle + c_2 f g_k \langle e_2|\kappa_k|e_2\rangle^{-\frac{1}{2}}|e_2\rangle \tag{20}$$

$$\equiv c_1|\tilde{e}_1(k)\rangle + c_2|\tilde{e}_2(k)\rangle. \tag{21}$$

The $|\tilde{e}_i(k)\rangle (i = 1,2)$ are states which stand for $f g_k \langle e_i|\kappa_k|e_i\rangle^{-\frac{1}{2}}|e_i\rangle$. Note that the vectors of $|\tilde{e}_1(k)\rangle$ and $|\tilde{e}_2(k)\rangle$ are normalized and mutually orthogonal, that is, $\langle\tilde{e}_i(k),\tilde{e}_j(k)\rangle = \delta_{i,j}$. Hence, Bob can recover the input state $|\psi\rangle$ from $|\psi'\rangle$ with a unitary transformation $U_k = \sum_{n=1}^{2} |e_n\rangle\langle\tilde{e}_n(k)|$. Let us propose a physical model in which the output state is given as the same form as Eq. (20).

The procedures of teleportation in our model consist of following five steps.

*Step 1.* Alice prepares an input state $|\psi\rangle$ on $\mathcal{H}_1$, which is described by

$$|\psi\rangle = c_1|\alpha_{\mathrm{even}}\rangle + c_2|\alpha_{\mathrm{odd}}\rangle. \tag{22}$$

Here, $|\alpha_{\mathrm{even}}\rangle$ and $|\alpha_{\mathrm{odd}}\rangle$ are defined as

$$|\alpha_{\mathrm{even}}\rangle = \frac{1}{\sqrt{C_+}} \sum_{n=0}^{\infty} \frac{\alpha^n + (-\alpha)^n}{\sqrt{n!}} |n\rangle, \tag{23}$$

$$|\alpha_{\mathrm{odd}}\rangle = \frac{1}{\sqrt{C_-}} \sum_{n=0}^{\infty} \frac{\alpha^n - (-\alpha)^n}{\sqrt{n!}} |n\rangle, \tag{24}$$

where $C_\pm \equiv 2\exp(|\alpha|^2) \pm 2\exp(-|\alpha|^2)$ [11]. These states $|\alpha_{\mathrm{even}}\rangle$ and $|\alpha_{\mathrm{odd}}\rangle$ can be considered as Schrödinger's cat states, and the $|\psi\rangle$ can be considered as a qubit on the subspace $\mathcal{K}_1$ of $\mathcal{H}_1$ spanned by $|\alpha_{\mathrm{even}}\rangle$ and $|\alpha_{\mathrm{odd}}\rangle$.

*Step 2.* One generates an entangled state by putting a photon number state vector $|N\rangle$ into one side of a half beam splitter. The entangled state is written on $\mathcal{H}_2 \otimes \mathcal{H}_3$ as

$$|\xi\rangle = \sum_{n=0}^{N} d_n |n\rangle \otimes |N-n\rangle, \tag{25}$$

where

$$d_n = (-1)^{N-n} e^{-\mathrm{i}\phi(N-n)} \sqrt{2^{-N} \binom{N}{n}}. \tag{26}$$

The above $\phi$ is the phase difference between the reflected beam and the transmitted beam [7]. Note that $|\xi\rangle$ is a nonmaximal entangled state in general.

*Step 3.* Alice measures *the phase difference of the beams* and *the sum of the photon number* on $\mathcal{H}_1 \otimes \mathcal{H}_2$ [9]. The operator expressing the sum of the photon number is

$$\hat{N}_+ = \hat{N}_1 + \hat{N}_2 \tag{27}$$

$$\equiv \sum_{q=0}^{\infty} q \left( \sum_{t=0}^{q} |q-t\rangle\langle q-t| \otimes |t\rangle\langle t| \right), \tag{28}$$

where $\hat{N}_{1,2}$ are number operators on $\mathcal{H}_{1,2}$. The operator expressing the phase difference of the beam is

$$\hat{\Phi}_- = \sum_{m=0}^{q} \phi_m^- \left( \sum_{t=0}^{q} \left|\phi_{t+m}^{(1)}\right\rangle\left\langle\phi_{t+m}^{(1)}\right| \otimes \left|\phi_t^{(2)}\right\rangle\left\langle\phi_t^{(2)}\right| \right), \tag{29}$$

where

$$\left|\phi_m^{(k)}\right\rangle = \sum_{n=0}^{q} \frac{\exp\left(in\phi_m^{(k)}\right)}{\sqrt{q+1}} |n\rangle, \tag{30}$$

$$\phi_m^{(k)} = \phi_0^{(k)} + \frac{2\pi m}{q+1}, \tag{31}$$

are Pegg-Barnett phase state vectors on $\mathcal{H}_k$, and $\phi_m^- = \phi_0^{(1)} - \phi_0^{(2)} + \frac{2\pi m}{q+1}$ [10,12]. There exist simultaneous eigenvectors of both $\hat{N}_+$ and $\hat{\Phi}_-$, and these eigenvectors are given by

$$|q,\phi_m^-\rangle = \sum_{n=0}^{q} \frac{\exp(-in\phi_m^-)}{\sqrt{q+1}} |q-n\rangle \otimes |n\rangle, \tag{32}$$

where $q$ is an eigenvalue of $\hat{N}_+$ and $\phi_m^-$ is that of $\hat{\Phi}_-$.

*Step 4.* Alice informs the measured values of $q$ and $\phi_m^-$ to Bob by means of a classical communication.

*Step 5.* Bob can recover the input state by means of the phase-shift unitary operator defined by

$$U(\phi_m^-) = \exp(-\mathrm{i}N\phi_m^-) \sum_{n=0}^{\infty} \exp \mathrm{i}[n(\phi_m^- + \phi + \pi)]|n\rangle\langle n|. \tag{33}$$

If $N - q$ is odd, one photon is added to the output state after the above transformation [13].

## IV. REPRESENTATION WITH K-O FORMALISM

Let us rewrite $|\xi\rangle$ and $|q,\phi_m^-\rangle$ with the K-O formalism [Eqs. (25) and (32)]

$$|\xi\rangle = \sum_{n=0}^{N} |n\rangle \otimes f|n\rangle, \tag{34}$$

$$|q,\phi_m^-\rangle = \sum_{n=0}^{q} g_{q,m}^* |n\rangle \otimes |n\rangle, \tag{35}$$

where

$$f = \sum_{n=0}^{N} d_n |N-n\rangle\langle n|, \tag{36}$$

$$g_{q,m}^* = \sum_{n=0}^{q} \frac{\exp(-in\phi_m^-)}{\sqrt{q+1}} |q-n\rangle\langle n|. \tag{37}$$

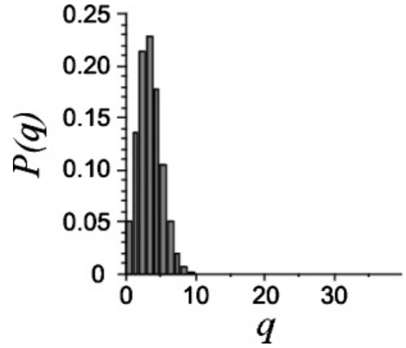


FIG. 1. Measurement probability $P(q)$ with respect to $|\alpha|^2 = 2$ and $N = 4$.

Obviously, the $g_{q,m}$ satisfy $\mathrm{tr}\,(g_{q,m}^* g_{q',m'}) = \delta_{q,q'}\delta_{m,m'}$. With the previous $f$ and $g_{q,m}$, we describe an output state in our model as

$$\left|\psi_{q,m}^{\mathrm{out}}\right\rangle = P(q,\phi_m^-)^{-\frac{1}{2}} f g_{q,m} |\psi\rangle \tag{38}$$

$$= P(q,\phi_m^-)^{-\frac{1}{2}} (c_1 f g_{q,m} |\alpha_{\mathrm{even}}\rangle + c_2 f g_{q,m} |\alpha_{\mathrm{odd}}\rangle). \tag{39}$$

The term of $P(q,\phi_m^-)$ means probability for the simultaneous measurements of $\hat{N}_+$ and $\hat{\Phi}_-$, and $P(q,\phi_m^-) = \langle\psi|\kappa_q|\psi\rangle$ where $\kappa_q = g_{q,m}^* f^* f g_{q,m}$.

Figures 1 and 2 show the probability distribution of $P(q)$ given by

$$P(q) = \sum_{m=0}^{q} P(q,\phi_m^-) \tag{40}$$

$$= (q+1)P(q,\phi_m^-). \tag{41}$$

When $N$ is increased, the peak of $P(q)$ shifts to the right. The average of $P(q)$ is achieved when $q \approx |\alpha|^2 + N/2$. In Eq. (39),

$$f g_{q,m} |\alpha_{\mathrm{even}}\rangle = [C_+ 2^N (q+1)]^{-\frac{1}{2}} \exp(\mathrm{i}N\phi_m^-) \times \sum_{n=\max(0,N-q)}^{N} \exp[-\mathrm{i}n(\phi_m^- + \phi + \pi)] \times \sqrt{\binom{N}{n}} \frac{\alpha^{q-N+n} + (-\alpha)^{q-N+n}}{\sqrt{(q-N+n)!}} |n\rangle, \tag{42}$$

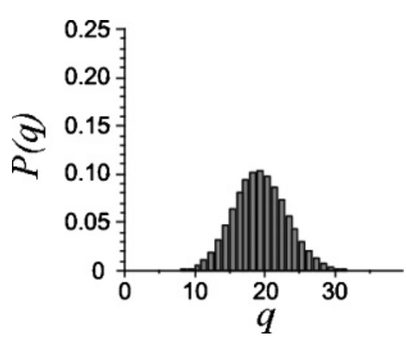


FIG. 2. Measurement probability $P(q)$ with respect to $|\alpha|^2 = 10$ and $N = 20$.

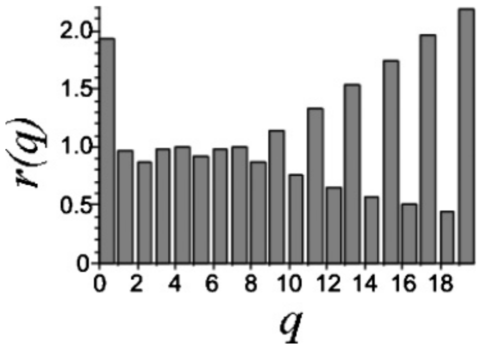


FIG. 3. Values of function $r(q)$ with respect to $q$ in the case of fixed $N = 4$, $\alpha = \sqrt{2}$, and $c_1 = c_2 = \frac{1}{\sqrt{2}}$.

and

$$fg_{q,m}|\alpha_{\text{odd}}\rangle = [C_{-}2^N(q+1)]^{-\frac{1}{2}} \times \sum_{n=\max(0,N-q)}^{N} \exp[-in(\phi_m^- + \phi + \pi)] \times \sqrt{\binom{N}{n}} \frac{\alpha^{q-N+n} - (-\alpha)^{q-N+n}}{\sqrt{(q-N+n)!}}|n\rangle, \tag{43}$$

are mutually orthogonal. Note that, when $q - N$ is an even (odd) number, the $fg_{q,m}|\alpha_{\text{even}}\rangle$ is expanded by only even (odd) number states, and the $fg_{q,m}|\alpha_{\text{odd}}\rangle$ is expanded by only odd (even) number states.

Here, let us remember the K-O state vector of Eq. (20). Importantly, the output state $|\psi_{q,m}^{\text{out}}\rangle$ has the same form with the K-O state under the condition of

$$\langle\alpha_{\text{even}}|\kappa_q|\alpha_{\text{even}}\rangle = \langle\alpha_{\text{odd}}|\kappa_q|\alpha_{\text{odd}}\rangle. \tag{44}$$

If the above condition is satisfied, we can express the $|\psi_{q,m}^{\text{out}}\rangle$ as

$$|\psi_{q,m}^{\text{out}}\rangle = c_1|\beta_1\rangle + c_2|\beta_2\rangle. \tag{45}$$

The two states $|\beta_1\rangle = fg_{q,m}\langle\alpha_{\text{even}}|\kappa_q|\alpha_{\text{even}}\rangle^{-\frac{1}{2}}|\alpha_{\text{even}}\rangle$ and $|\beta_2\rangle = fg_{q,m}\langle\alpha_{\text{odd}}|\kappa_q|\alpha_{\text{odd}}\rangle^{-\frac{1}{2}}|\alpha_{\text{odd}}\rangle$ are normalized and mutually orthogonal, that is, $\langle\beta_i,\beta_j\rangle = \delta_{i,j}$. To investigate whether Eq. (44) is satisfied or not, we estimate the values of the following function

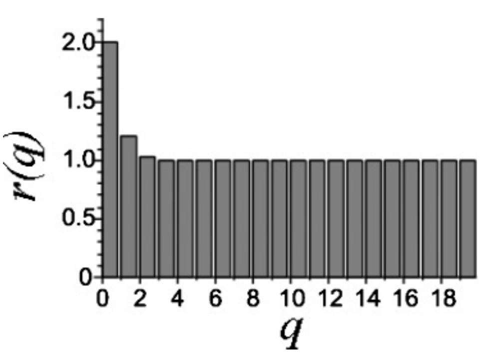


FIG. 4. Values of function $r(q)$ with respect to $q$ in the case of fixed $N = 20$, $\alpha = \sqrt{10}$, and $c_1 = c_2 = \frac{1}{\sqrt{2}}$.

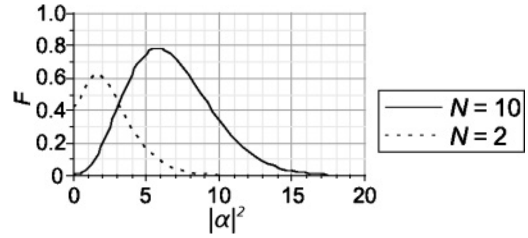


FIG. 5. Values of $F$ with respect to $|\alpha|^2$ in the case of fixed $c_1 = c_2 = \frac{1}{\sqrt{2}}$.

$$r(q,N,\alpha,c_1) = \frac{\langle\alpha_{\text{even}}|\kappa_q|\alpha_{\text{even}}\rangle}{\langle\alpha_{\text{odd}}|\kappa_q|\alpha_{\text{odd}}\rangle}. \tag{46}$$

Obviously, $r = 1$ is equivalent to Eq. (44).

Figures 3 and 4 show the value of $r$ with respect to $q$. As the value of $N$ is increased, the value of $r$ approaches 1 for almost every value of $q$. Importantly, the case that $q$ is small, that is, $q = 0$ and 1, cannot be observed due to Fig. 2 discussed previously. As a result, the output state in our model can be expressed as the form of Eq. (45) for every value of $q$, that is, teleportation in our model is perfect.

## V. FIDELITY

As we showed in a previous section, our model enables perfect teleportation. However, $|\beta_1\rangle$ and $|\beta_2\rangle$ are different from $|\alpha_{\text{even}}\rangle$ and $|\alpha_{\text{odd}}\rangle$, respectively. We estimate the fidelity between the input state and the recovered state after (Step 5). If $q - N$ is an even number, only the phase-shifting operator $U(\phi_m^-)$ applies to the output state $|\psi_{q,m}^{\text{out}}\rangle$, then the recovered state $|\psi_{q,m}^{\text{rec}}\rangle$ is obtained as

$$|\psi_{\text{even}}^{\text{rec}}\rangle \equiv U(\phi_m^-)|\psi_{q,m}^{\text{out}}\rangle. \tag{47}$$

If $q - N$ is an odd number, one photon is added to $U(\phi_m^-)|\psi_{q,m}^{\text{out}}\rangle$, and the recovered state is

$$|\psi_{\text{odd}}^{\text{rec}}\rangle \equiv \left\|a^* U(\phi_m^-)|\psi_{q,m}^{\text{out}}\rangle\right\|^{-\frac{1}{2}} a^* U(\phi_m^-)|\psi_{q,m}^{\text{out}}\rangle, \tag{48}$$

where $a^*$ is the creation operator [13]. Average fidelity $F$ is given as

$$F = F_{\text{even}} + F_{\text{odd}} \tag{49}$$

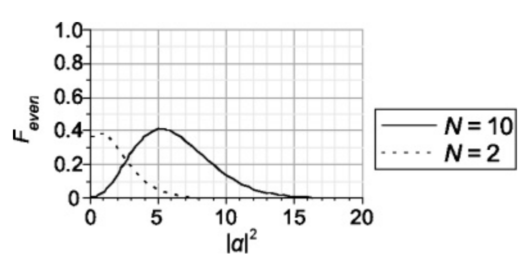


FIG. 6. Values of $F_{\text{even}}$ with respect to $|\alpha|^2$ in the case of fixed $c_1 = c_2 = \frac{1}{\sqrt{2}}$.

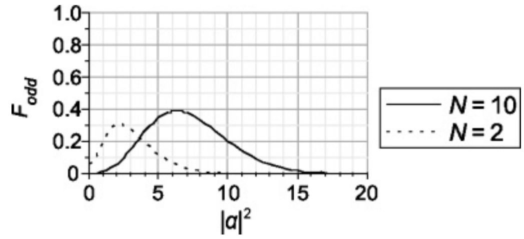


FIG. 7. Values of $F_{\rm odd}$ with respect to $|\alpha|^2$ in the case of fixed $c_1 = c_2 = \frac{1}{\sqrt{2}}$.

where

$$F_{\rm even} = \sum_{q:\rm even} \sum_{m=0}^{q} P(q,\phi_m^-) |\langle \psi | \psi_{\rm even}^{\rm rec} \rangle|^2, \tag{50}$$

$$F_{\rm odd} = \sum_{q:\rm odd} \sum_{m=0}^{q} P(q,\phi_m^-) |\langle \psi | \psi_{\rm odd}^{\rm rec} \rangle|^2. \tag{51}$$

In general, values of $F$ ($F_{\rm even/odd}$) depend on the average of photon number $|\alpha|^2$ and the coefficients $c_1$ and $c_2$ of $|\Psi\rangle$. Figures 5, 6, and 7 show the behavior of $F$ and $F_{\rm even/odd}$ to $|\alpha|^2$ for $c_1 = c_2 = 1/\sqrt{2}$. One can see that the fidelities in Fig. 5 have maximal values at $|\alpha|^2 \approx N/2$. Figure 8 shows values of $F$ to $c_1$ for a fixed $|\alpha|^2 \approx N/2$. It is expected that our teleportation gives high fidelity for any $c_1$ when $N$ is increased. In our teleportation, an output is written as $|\psi_{q,m}^{\rm out}\rangle = c_1|\beta_1\rangle + c_2|\beta_2\rangle$ with a set of the orthogonal basis $\{|\beta_1\rangle, |\beta_2\rangle\}$. The basis is different from the basis $\{|\alpha_{\rm even}\rangle, |\alpha_{\rm odd}\rangle\}$ of the input state vector $|\psi\rangle = c_1|\alpha_{\rm even}\rangle + c_2|\alpha_{\rm odd}\rangle$. Therefore, our teleportation is perfect, but the fidelity is not equal to 1. It is important that our teleportation becomes perfect even if a small number of photons are input into a beam splitter, that is, this teleportatin does not need infinite energy.

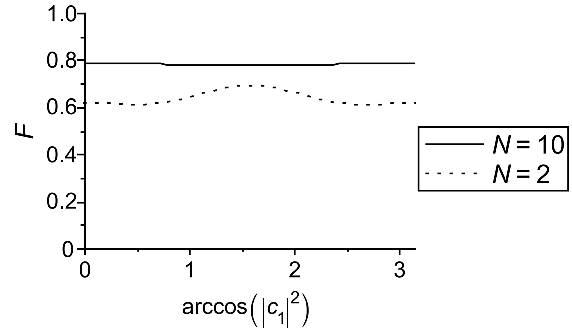


FIG. 8. Values of $F$ with respect to arccos $(|c_1|^2)$ in the case of $|\alpha|^2 = N/2$.

## VI. CONCLUSION

We have discussed the output state in a specific case of K-O teleportation. Based on this discussion, we proposed a physical model where the output state has the same form of the K-O output state. We have shown that our model makes perfect teleportation for the nonmaximal entangled state.

In our model, the input state is given as a superposition of Schrödinger's cat states. The experimental setups for generating a superposition of coherent states were discussed in [5,6]. The entangled state prepared between Alice and Bob is generated by a photon number state through a beam splitter. It should be noted that our teleportation does not need to prepare a large number of photons as shown in Fig. 8. Even if a small number of photons are used, our teleportation becomes perfect. The experiment for generating a small number of photons were discussed in [14]. In our model, Alice measures the sum of the photon number and the phase difference of beams. Bob recovers the input state from the output state by means of a phase-shift operator. The experiments for the joint measurement were presented in [15].

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