Casimir pressure in a multilayer system with a fixed total length

Z. Lenac
Department of Physics, University of Rijeka, HR-51000 Rijeka, Croatia
(Received 12 May 2010; published 27 August 2010)

We consider Casimir pressure on a slab in a configuration consisting of various dielectrics with planar symmetry. In such configurations, one usually calculates the Casimir pressure (force) on a particular slab assuming that lengths of all other slabs remain unchanged. Alternatively, one can consider a multilayer system with a fixed total length. With this restriction only, the length of each slab can eventually be changed under the Casimir pressure that will try to minimize the total Casimir energy of the system. Here we calculate the Casimir pressure on the slab in such a "constrained" configuration and compare the results with the standard approach. It turns out that, by applying different boundary conditions, one can obtain significantly different Casimir pressures on the same object. In particular, when the thicknesses of the slab and surrounding layers are on the nanometer scale, the Casimir pressure on the slab can change from strongly squeezing in the case of fixed thicknesses of surrounding slabs to strongly relaxing in the case when only the length of the total system remains fixed.

DOI: 10.1103/PhysRevA.82.022117 PACS number(s): 31.30.jh, 42.50.Ct, 12.20.−m, 73.20.Mf

I. INTRODUCTION

The Casimir effect was originally predicted for a system consisting of two parallel, ideally reflecting plates separated by a vacuum layer. Such plates attract each other due to the vacuum fluctuations of the electromagnetic field between them [1]. This effect can be considered within different concepts, and probably the most common one is to determine the difference between the ground-state energy of the field in a given configuration and in the corresponding free space (i.e., the Casimir energy) and to connect the interaction between the plates with the change of this energy. Later on, the same idea was expanded to much more complicated systems and also to the systems at nonzero temperatures [2]. The main problem with such an approach is related to the fact that the electromagnetic ground-state energy is infinite, so one has to find a way how to appropriately regularize it. In the case of two perfect mirrors mentioned before, this regularization was done by Casimir [1]. Since then, a lot of different regularization procedures were proposed resulting, in principle, in different ground-state energies of the system [3]. In conjunction with the Casimir energy, it becomes important to determine the Casimir force acting on the corresponding surfaces of the system, which obviously should not depend on the regularization procedure. Again, this can be done using different methods, with or without regulation [4]. The existence of the Casimir effect is nowadays well supported by the experimental results [5,6].

The Casimir force on a slab can be simply defined as a force that tries to change the slab thickness, thus lowering the electromagnetic energy of the system. In the multilayer configuration with the planar symmetry, the Casimir force on the surfaces of the slab j is accordingly calculated as a partial derivative of the Casimir energy with respect to the slab thickness d_j. Note that in this approach all other slab thicknesses in the system are (implicitly) assumed fixed. This concept has been used, for example, in several recent articles, concerning the Casimir force on very thin layers [7–10].

It is interesting to discuss the Casimir force on slab j in a system with the same geometry, but with different constraints implemented on the system's configuration. Particularly, we here analyze a situation where only the total length of the system (and not of particular layers) is fixed. To point out the differences between this and the standard approach, we take two examples: (i) *A metallic slab in a (vacuum) cavity surrounded by (perfect) mirrors.* In this well-known model the squeezing of the metallic slab as well as the squeezing of the vacuum space between the slab and cavity walls is energetically favorable. But if the distance between the cavity walls is fixed, both squeezing processes are obviously not possible, and one has to determine the resulting force that acts on the metallic slab trying to minimize the total Casimir energy of the system. (ii) *A thin vacuum layer surrounded by two mirrors (Casimir-like configuration).* If the mirrors are not perfect and their thicknesses are finite, we can assume that the distance between the outer surfaces of the mirrors is fixed. Then squeezing the vacuum layer between the mirrors means also relaxing the thicknesses of the mirrors. This process will obviously change the total Casimir energy and accordingly influence the Casimir force on the mirrors, as well as the attraction between them.

Essentially the same approach can be applied to similar systems of different "cavity" geometries. In that sense the fixed total length of the system means that the change of the shape of one (metallic) layer requires the (opposite) change of another (vacuum) layer within the system. The calculations would be more complicated, for example, for curved slabs or for one sphere (tube) into another, but here we are going to use simple models to demonstrate how the fixed length of the whole system can significantly influence the resulting Casimir force on each system's layer.

The article is organized as follows. In Sec. II we give a short theoretical background with emphasis on the derivation of the Casimir force or pressure on a particular layer with the specific boundary conditions. All details needed for analysis of a three-layer system are given in Sec. III. The discussion of the simplest case of a thin metallic layer in a cavity is given in Sec. III A, while another important case of a thin vacuum layer surrounded by metallic layers is discussed in Sec. III B. In both cases we analyze the Casimir pressure on the surfaces of layer j in a model where only the total length of the system is fixed and compare it with the corresponding results [9,10] obtained

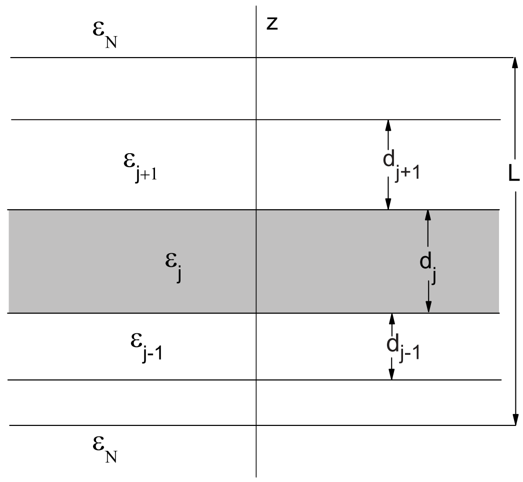

FIG. 1. Geometry of the model.

in a model that takes into account the possible change of only one layer thickness d_j. The conclusion is given in Sec. IV.

II. THEORETICAL BACKGROUND

Let us analyze a system consisting of parallel dielectric plates of thicknesses d_j $(j = 0,1,2,\ldots,N-1)$ and surrounded by an (infinite) medium $j = N$ (Fig. 1). We shall assume that all media are described by the real dielectric functions $\epsilon_j(\omega)$. This enables us to use the standard procedure in obtaining the dispersion relations of the system's electromagnetic modes. In the following, we use the same notation as in Refs. [9,10]. Particularly, we characterize an electromagnetic mode of frequency ω by the wave vector $\mathbf{k}$ parallel to the system surfaces, the perpendicular wave vector β_j in a layer j, $\beta_j(\omega,k) = [\epsilon_j(\omega)\omega^2/c^2 - k^2]^{1/2}$, and the polarization index $q = p$ or $q = s$, denoting the p and s polarization of the mode, respectively.

We start by calculating the Casimir energy E_C of the system using the surface mode summation method [11]. Surface modes are defined as modes whose field vanishes in the outer medium, and the Casimir energy is derived as the summation only over the surface mode frequencies $\omega_n^q(k)$:

$$E_C = \sum_{\boldsymbol{k}} \sum_{q=p,s} \frac{\hbar}{2} \sum_n \omega_n^q(k). \tag{1}$$

Index n enumerates all surface modes with given $(q,\boldsymbol{k})$.

In our system the electromagnetic field in each layer j behaves as $\sim\exp(\pm i\beta_j z)$, where z is the perpendicular coordinate. Then, with respect to a layer j, the dispersion relation of the surface modes can be written in the appropriate form [10,11]

$$Q_j^q(\omega,k) \equiv R_j^q(\omega,k) e^{-2\alpha_j d_j} = 1, \tag{2}$$

where $\alpha_j = -i\beta_j$ and $R_j^q(\omega,k)$ is the average reflectivity of the layer boundaries which does not depend on the layer thickness d_j. Note that the factor $e^{-2\alpha_j d_j}$ ($\mathrm{Re}\alpha_j > 0$) in the $d_j \to \infty$ limit leads to $\lim_{d_j\to\infty} Q_j^q(\omega,k) = 0$, which violates the dispersion relation (2); that is, the frequencies $\omega_n^q(k,d_j \to \infty)$ are poles of the term $R_j^q(\omega,k)$.

The contribution from the frequencies of all surface modes is infinite, so one has to introduce a regularization term in order to make the Casimir energy (1) finite. Putting all that together, it is appropriate to choose the regularization energy as the energy of the same system but with an infinite thickness of the jth layer:

$$E_C^j = \sum_{\boldsymbol{k}} \sum_{q=p,s} \frac{\hbar}{2} \sum_n \left[\omega_n^q(k) - \omega_n^q(k,d_j \to \infty)\right]. \tag{3}$$

Such defined Casimir energy is finite, but also principally different for each $j = (0,1,2,\ldots,N-1)$.

Since the energy difference does not depend on the regularization factor and since this factor does not depend on d_j, we can write the infinitesimal change of the Casimir energy $E_C(d_0,d_1,d_2,\ldots,d_{N-1})$ as

$$dE_C = \sum_{j=0}^{N-1} \left(\frac{\partial E_C}{\partial d_j}\right) d(d_j) = \sum_{j=0}^{N-1} \left(\frac{\partial E_C^j}{\partial d_j}\right) d(d_j). \tag{4}$$

Each partial derivative of E_C^j is finite and can be associated with the corresponding Casimir force F_C^j.

As pointed out, we consider different forces on the plate j with respect to various boundary conditions. In order to avoid any possible confusion regarding the definition of the Casimir force, we rather discuss the Casimir pressure on the slab; that is,

$$P_C^j = -\frac{1}{A}\frac{\partial E_C^j}{\partial d_j} \tag{5}$$

is interpreted as the Casimir pressure acting on surfaces A of the slab j due to the change of the total Casimir energy. The sign is chosen so that the negative or positive Casimir pressure tries to squeeze or relax the slab, respectively.

After transforming the summation over $\mathbf{k}$ into the corresponding integration and using the argument theorem to change the summation over n into the integration over ω [11], the Casimir energy (3) becomes

$$E_C^j = \frac{\hbar}{2}\frac{A}{4\pi^2 i}\int_0^\infty dkk \int_{-i\infty}^{i\infty} d\omega \sum_{q=p,s} \ln\left[1 - Q_j^q(\omega,k)\right]. \tag{6}$$

To obtain the Casimir pressure in the Lifshitz-like form we introduce the new variables (ξ,p) through $\xi = -i\omega, k^2 = (\xi^2/c^2)(p^2-1)$ [12]. Equations (5) and (6) then give

$$P_C^j = -\frac{\hbar}{2\pi^2 c^2}\int_1^\infty dpp \int_0^\infty d\xi \xi^2 \alpha_j \sum_{q=p,s} \frac{Q_j^q(i\xi,p)}{1 - Q_j^q(i\xi,p)}. \tag{7}$$

In terms of integration parameters we find $\alpha_j = (\xi/c)[p^2 - 1 + \epsilon_j(i\xi)]^{1/2}$, so that the term $e^{-2\alpha_j d_j}$ in Q_j^q ensures the convergence when integrating along the (p,ξ) axis. This term also provides an obvious result for the Casimir pressure on a very thick slab:

$$\lim_{d_j\to\infty} P_C^j = 0. \tag{8}$$

An important contribution to the Casimir pressure in many configurations is related to surface polaritons (SP), the modes that are evanescent in all layers. The frequencies of SP modes are solutions of dispersion relation (2) with all α_j being real. A dielectric multilayer can support only a finite number of SP modes and all of them are p polarized. Accordingly, the SP contribution to the Casimir energy, E_S^j, can be calculated directly from Eq. (3) if we only interchange the summation indexes: $(q,n) \to \sigma$, where σ enumerates all SP modes. The renormalized energy E_S^j is finite and in many cases it gives a good approximation of the total Casimir energy E_C^j [9,10]. The pressure on the slab j resulting only from the SP modes is defined as

$$P_S^j = -\frac{1}{A}\frac{\partial E_S^j}{\partial d_j} = -\frac{\hbar}{4\pi}\int_0^\infty dkk \sum_\sigma \frac{\partial \omega_\sigma(k)}{\partial d_j}. \tag{9}$$

From now on we assume that only the total space between the outer ($j = N$) plates remains fixed; that is, $L = (d_0 + d_1 + d_2 + \cdots + d_{N-1}) = \text{const.}$ (Fig. 1). In that sense the thickness of the outer medium is taken as infinite, and $P_C^N = 0$ [Eq. (8)]. We denote the small change (displacement) of the layer thickness d_j by δd_j. Then the displacements of all layer thicknesses are connected through $\delta L = \sum_j \delta d_j = 0$. Let us concentrate on the layer $j = J$ and its displacement δd_J. We define all other displacements δd_j by

$$\delta d_j = -p_j\, \delta d_J, \quad \text{with} \quad p_J = -1, \quad \sum_{j\neq J} p_j = 1.$$

The coefficients p_j follow from the requirement that the change of the total Casimir energy δE_C due do the displacements δd_j takes its minimum value:

$$\frac{1}{A}\delta E_C = \left(\sum_j p_j P_C^j\right)\delta d_J = \text{min}. \tag{10}$$

Obviously $\delta d_J < 0$ ($\delta d_J > 0$) refers to squeezing (relaxing) of the slab J. Now we can calculate the total Casimir pressure P_C^{TJ} on the $j = J$ slab as

$$P_C^{TJ} = -\frac{1}{A}\frac{\delta E_C}{\delta(d_J)} = P_C^J - \sum_{j\neq J} p_j P_C^j. \tag{11}$$

The interpretation of this result is plausible: The Casimir pressure P_C^{TJ} tries to change (diminish or enlarge) d_J, but also other d_j thicknesses, in order to minimize the electromagnetic energy in the system which total length is fixed. It means that, for example, the pressure $P_C^J < 0$ could be squeezing, but if we find $\sum_{j\neq J} p_j P_C^j < P_C^J$, the total Casimir pressure on the slab $j = J$ will in fact try to relax it ($P_C^{TJ} > 0$).

In all the preceding considerations the Casimir energy is calculated from the ground-state electromagnetic energy of a given configuration. Assuming the $T = 0$ stationary state, that configuration is obviously determined by the ground-state energy of the whole system, with all interactions included, and the Casimir energy is typically only a small part of it. Therefore, to obtain real δd_j (or p_j) values one would have to take into account the total Hamiltonian of the system (i.e., the corresponding stretching coefficients of the slabs). However, in a suitably chosen setup, which we discuss in the next sections, one can take the present results to correctly describe the influence of the Casimir pressure on the whole system and to draw some interesting conclusions.

III. THREE-LAYER SYSTEM

In this section we shall discuss various systems, but all of them are described within a model of three layers ($j = 0,1,2$) surrounded by an external medium $N = 3$ (Fig. 1). The dispersion relation for the surface modes in such a system can be derived in a standard way, assuming $\alpha_3^2 > 0$. Following Ref. [10] we can put it in the required form (2) $Q_1^q = 1$, with

$$Q_1^q = r_{01}^{qd} r_{21}^{qd} e^{-2\alpha_1 d_1}. \tag{12}$$

The reflection coefficients r_{j1}^{qd} are obtained from the Fresnel reflection coefficients r_{jl}^q for the $j - l$ interface

$$r_{jl}^p = \frac{\epsilon_l\alpha_j - \epsilon_j\alpha_l}{\epsilon_l\alpha_j + \epsilon_j\alpha_l}, \quad r_{jl}^s = \frac{\alpha_j - \alpha_l}{\alpha_j + \alpha_l}, \tag{13}$$

with the replacement

$$\alpha_j \to \alpha_j^{qd} \equiv \alpha_j \frac{\left(1 - r_{j3}^q e^{-2\alpha_j d_j}\right)}{\left(1 + r_{j3}^q e^{-2\alpha_j d_j}\right)}, \quad j = (0,2).$$

Analogous results can be obtained for Q_0^q and Q_2^q. By transforming the dispersion relation $Q_1^q = 1$ (12) into $Q_j^q = 1$, we find

$$Q_j^q = r_{1j}^{qd} r_{3j}^q e^{-2\alpha_j d_j}, \quad j = (0,2). \tag{14}$$

The corresponding reflection coefficients r_{1j}^{qd} are given by Eq. (13) with the replacement, for given ($j = 0, l = 2$) or ($j = 2, l = 0$),

$$\alpha_1 \to \alpha_1^{ql} \equiv \alpha_1 \frac{\left(1 + r_{l1}^{qd} e^{-2\alpha_1 d_1}\right)}{\left(1 - r_{l1}^{qd} e^{-2\alpha_1 d_1}\right)}.$$

Casimir pressures P_C^j on layers $j = (0,1,2)$ as well as the total Casimir pressure, for example, on the slab $j = 1$,

$$P_C^{T1} = P_C^1 - p_0 P_C^0 - p_2 P_C^2, \quad p_0 + p_2 = 1, \tag{15}$$

can now be calculated from Eqs. (7)–(14).

As we have pointed out, we are analyzing our system in the stationary state at $T = 0$ while taking into account only the Casimir energy. Relative displacements of the slabs $p_j = -\delta_j/\delta_1$, $j = (0,2)$ are therefore calculated with two important restrictions: (i) We neglect the mechanical interaction of the two plates in contact, which can be approved only if the stretching coefficient of one of the plates is negligible, for example, if this is a vacuum plate. In the three-layer system this can be achieved in some important cases, discussed in the next sections. (ii) The Casimir ground-state energy is calculated for a given configuration which is the result of all interactions in the system. Therefore one can obtain for the Casimir pressures, for example, $P_C^0 > 0$ and $P_C^2 < 0$, which would, if only the Casimir energy is taken into account, move the slab $j = 1$ in the $0 \to 2$ direction. It is not allowed in the real stationary state, so we assume that the displacement δ_1 is simply compensated by the displacements (δ_0,δ_2), that is, $0 \leqslant (p_0, p_2) \leqslant 1$.

In the specific case $|\epsilon_3| \gg |\epsilon_j|$, $|\epsilon_1| \gg |\epsilon_j|$, the dielectrics $j = (0,2)$ are surrounded by highly reflecting media, so that the pressures P_C^j are determined by $Q_j^q = e^{-2\alpha_j d_j}$. In the case $\epsilon_j = 1$ we obtain the well-known Casimir result [1] for the pressure that tries to squeeze the vacuum layer j surrounded by perfect mirrors:

$$P_C^c = -\frac{\hbar c \pi^2}{240\ d_j^4}. \tag{16}$$

A. Thin metallic slab in a cavity

In our article [10], we have analyzed the Casimir pressure (force) on a d_1 thick metallic slab surrounded by dielectric slabs of finite thicknesses d_0 and d_2 and immersed in an outer medium $N = 3$. These calculations are based on the assumption that the thicknesses (d_0, d_2) are fixed; that is, squeezing or relaxing of d_1 will not affect (d_0, d_2). Now we want to discuss the total Casimir pressure P_C^{T1}, Eq. (15), on the metallic slab in the same configuration, but assuming that only the total length of the system $L = (d_0 + d_1 + d_2)$ remains fixed.

Let us describe the metallic slab $j = 1$ by the standard dielectric function

$$\epsilon_1(\omega) = 1 - \frac{\omega_P^2}{\omega^2}, \tag{17}$$

where ω_P is the electron-plasma frequency, and take the plasma wavelength $\lambda_P = 2\pi c/\omega_P$ as the characteristic length to scale the system. Since we are analyzing pressures on different layers, the normalization pressure is taken as a value that depends only on ω_P: $P_P = -\hbar\omega_P^4/c^3$. With a typical value $\omega_P = 10$ eV we find $\lambda_P = 124$ nm and $P_P = -2.11 \times 10^5\ N/m^2$.

In the following calculations the metallic plate is separated from the outer medium by the vacuum layers ($\epsilon_0 = \epsilon_2 = 1$). In order to point out the influence of the boundary conditions on the Casimir pressure, we start with the Casimir choice and describe the outer medium as perfect mirrors ($|\epsilon_3| \to \infty$).

Let us first analyze the central (*symmetric*) position of the metallic slab ($d_0 = d_2$). Figures 2 and 3 represent the Casimir pressure on the metallic slab P_C^1 calculated assuming fixed distances of vacuum layers (d_0, d_2) and pressures on vacuum layers $P_C^0 = P_C^2$ calculated assuming fixed thickness of the metallic slab (d_1). As expected, all pressures are attractive (squeezing), but it is interesting to notice the similarities between the two pictures; that is, pressures on the vacuum layers and on the metallic slab are rather close, assuming that corresponding thicknesses coincide. Since pressure P_C^2 is much more sensitive on the variation of d_2 than pressure P_C^1, and vice versa, there is a crossing point ($P_C^1 = P_C^2$) that occurs roughly at $d_1 \approx d_2$.

Now we calculate the total Casimir pressure P_C^{T1} on a metallic slab assuming that the distance between the outer slabs L remains fixed; that is, we put a thin metallic slab in a cavity with fixed walls. Since the pressures are the same on both sides of the slab ($P_C^0 = P_C^2$), we find from Eq. (15) $P_C^{T1} = (P_C^1 - P_C^2)$ regardless of the specific values of parameters (p_0, p_2).

From the previous considerations it follows that, if the vacuum layers are chosen to satisfy $d_0 \approx d_2 \approx d_1$, one can tune these distances to obtain $P_C^1 \approx P_C^2$, which leads to the negligible total Casimir pressure on the metallic slab ($P_C^{T1} \approx 0$). However, for $d_2 \gg d_1$ or $d_2 \ll d_1$ one finds $|P_C^1| \gg |P_C^2|$ or $|P_C^1| \ll |P_C^2|$, so one can expect squeezing ($P_C^{T1} \approx P_C^1$) or relaxing ($P_C^{T1} \approx -P_C^2$) pressure on it, respectively.

Here we also want to analyze the influence of the outer medium on the total pressure P_C^{T1}. Figure 4 describes situations

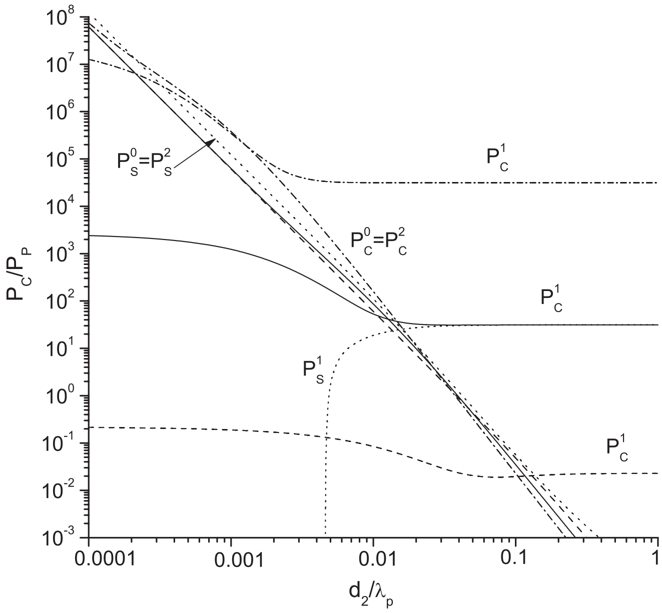

FIG. 2. Casimir pressures P_C^j, $j = (0,1,2)$, (in units of P_P) as a function of the vacuum layer thicknesses $d_0 = d_2$ (in units of λ_P), for different thicknesses of the metallic slab: $d_1 = 0.001\ \lambda_P$ (dash-dotted lines), $d_1 = 0.01\ \lambda_P$ (solid lines), and $d_1 = 0.1\ \lambda_P$ (dashed line). Dotted lines: SP contribution P_S^j to the Casimir pressures for $d_1 = 0.01\ \lambda_P$.

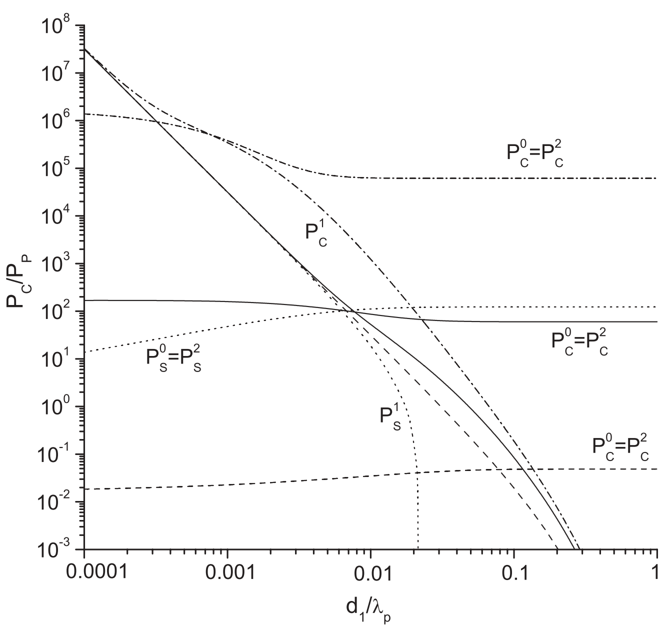

FIG. 3. Casimir pressures as a function of the thickness of the metallic slab d_1, for different vacuum layer thicknesses ($d_0 = d_2$): $d_2 = 0.001\ \lambda_P$ (dash-dotted lines), $d_2 = 0.01\ \lambda_P$ (solid lines), and $d_2 = 0.1\ \lambda_P$ (dashed line). Dotted lines: SP contributions to the total Casimir pressures for $d_2 = 0.01\ \lambda_P$.

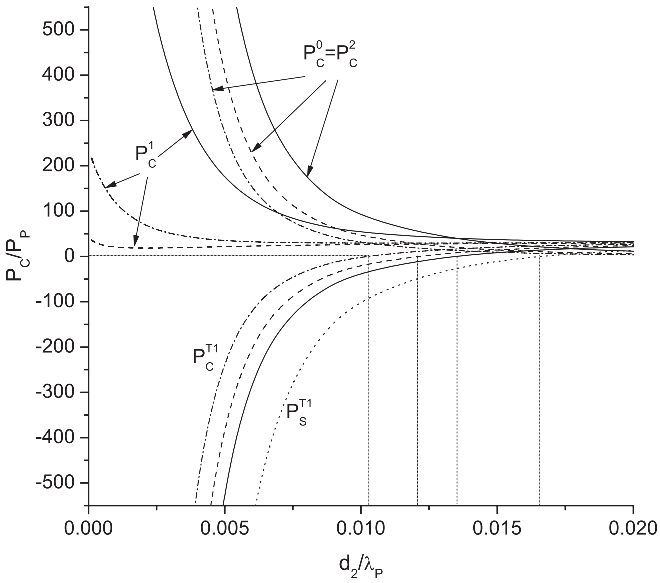

FIG. 4. Casimir pressures P_C^0, P_C^1, and P_C^2 and the total Casimir pressure $P_C^{T1} = P_C^1 - P_C^2$ on a thin Na slab ($d_1 = 0.01\ \lambda_P$) as a function of $d_0 = d_2$. The outer layers are taken to be ideal mirrors (solid lines), Al (dashed lines), and NaCl (dash-dotted lines). The dotted line represents the SP contribution $P_S^{T1} = P_S^1 - P_S^2$ to the total Casimir pressure in the ideal mirrors case.

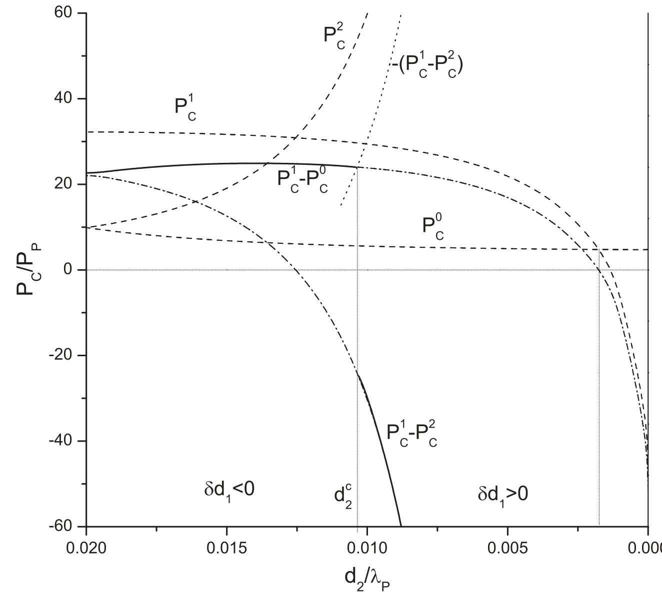

FIG. 5. Casimir pressures P_C^0, P_C^1, and P_C^2 (dashed lines) and the resulting Casimir pressures $P_C^1 - P_C^0$ and $P_C^1 - P_C^2$ (dashed-dotted lines) on a $d_1 = 0.01\ \lambda_P$ thick metallic slab as a function of the vacuum layer thickness d_2. The total Casimir pressure P_C^{T1} (solid line) has a discontinuity at $d_2^c = 0.0103\ \lambda_P$. The cavity walls are ideal mirrors, with the vacuum separations $(d_0 + d_2) = 0.04\ \lambda_P$.

with the thin Na slab ($\omega_P = 6.1$ eV) in between different outer slabs: ideal mirrors ($\omega_P \to \infty$), standard metals (Al, $\omega_P = 15.8$ eV), and polar dielectrics (NaCl, $\epsilon_\infty = 2.4$, $\omega_T = 0.022$ eV, $\omega_L = 0.034$ eV) [10,13]. Critical points d_2^c, denoted by vertical lines on Fig. 4, divide regions of squeezing behavior ($P_C^{T1} < 0$) from regions of relaxing behavior ($P_C^{T1} > 0$) of the total Casimir pressure P_C^{T1} on the Na slab. Note that (i) critical points are not particularly sensitive on properties of the outer medium, and (ii) when the space between the slab and the outer medium becomes very narrow ($d_2 < d_1$) there is always a *relaxing* pressure on the metallic slab, due to the strong attraction between the slab and the cavity walls.

Let us briefly comment on the contribution of SP modes to the Casimir pressure that is shown by dotted lines in Figs. 2, 3, and 4, for the case of ideal mirrors as the outer medium. As expected [10], pressure P_C^1 is dominantly due to SP modes for thin metallic slabs ($d_1 < d_2$), while pressures (P_C^0, P_C^2) are influenced by guided modes in all cases, since vacuum layers (d_0, d_2) are put in between two highly reflecting plates [9]. As a consequence, the SP contribution P_S^{T1} to the total Casimir pressure gives only a rough approximation of P_C^{T1} (Fig. 4).

Let us now consider the *asymmetric* situation by taking, for example, $d_0 > d_2$. In this case the "vacuum" Casimir pressure $P_C^0 - P_C^2 > 0$ would try to push the slab as a whole in one direction [14], but we have assumed that moving of the slab is prevented by external sources. It is illustrative to calculate the total Casimir pressure on the slab P_C^{T1} (15) while keeping the same $(d_0 + d_2)$ value. Figure 5 represents the Casimir pressures on the metallic slab that changes its position from the middle ($d_2 = d_0$) to the end ($d_2 = 0$) of the cavity. The parameters (p_0, p_2) follow from the requirement (10) and we found a critical distance d_2^c, so that $(p_0 = 1, p_2 = 0)$, that is, $P_C^{T1} = (P_C^1 - P_C^0)$ for $d_2 > d_2^c$, and $(p_0 = 0, p_2 = 1)$, that is, $P_C^{T1} = (P_C^1 - P_C^2)$ for $d_2 < d_2^c$. It gives a large *discontinuity* of P_C^{T1} at $d_2 = d_2^c$ so that we have a sudden change from squeezing ($\delta d_1 < 0$) to relaxing ($\delta d_1 > 0$) Casimir pressure on a metallic slab. The discontinuity $\Delta P_C = (P_C^2 - P_C^0)$ is obviously the result of different pressures on both sides of the slab. To point out the continuity of the energy change $\delta E_C/A = -P_C^{T1}\delta d_1$ (11) we have also plotted on Fig. 5 the curve $-(P_C^1 - P_C^2)$ that continuously keeps on the curve $(P_C^1 - P_C^0)$ at $d_2 = d_2^c$. Let us notice that if we have not required $\delta L = 0$, that is, if we have taken fixed (d_0,d_2) values, then $P_C^{T1} = P_C^1$ would have a *continuous* transition from $\delta d_1 < 0$ to $\delta d_1 > 0$ at the very end of the cavity ($d_2^c = 0.0013\ \lambda_P$).

In the case of thinner metallic slabs, the "sticking" pressure P_C^1 will be larger and this will lead to the bigger discontinuity ΔP_C, at the critical value d_2^c closer to the cavity wall (we expect $d_2^c \approx d_1$). For example, if we take the same cavity, but with $d_1 = 0.005\ \lambda_P$, we obtain $\Delta P_C = 420 P_P$ at $d_2^c = 0.0052\ \lambda_P$. However, the thickness $d_1 = 0.01\ \lambda_P$ is typically around 1 nm, so one would like to know what would be the result if all the involved thicknesses were an order of magnitude larger. That situation is shown in Fig. 6. Obviously, Figs. 5 and 6 are similar in shape and we have again a discontinuity of P_C^{T1} at $d_2^c = 0.107\ \lambda_P \approx d_1$, but in comparison with the previous case all relevant Casimir pressures are approximately 2×10^3 times weaker. This is a typical behavior of the Casimir pressure that becomes much lower when the electromagnetic field is confined in a larger space.

The simple "scaling" of a critical distance d_2^c with the corresponding length scale is a consequence of comparable Casimir pressures on a metallic slab and on a vacuum layer if their thicknesses correspond (see our discussion after Figs. 2 and 3). If we further enlarge relevant thicknesses so to approach

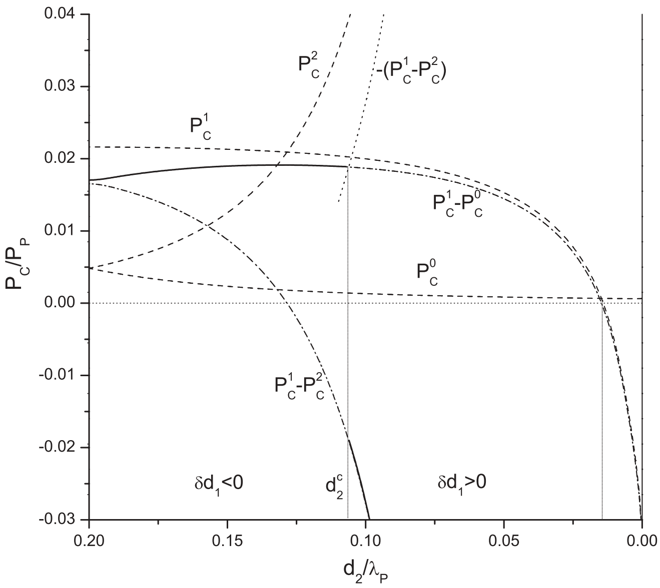

FIG. 6. Casimir pressures as in Fig. 5, but for ten times larger characteristic thicknesses [$d_1 = 0.1\ \lambda_P$, $(d_0 + d_2) = 0.4\ \lambda_P$].

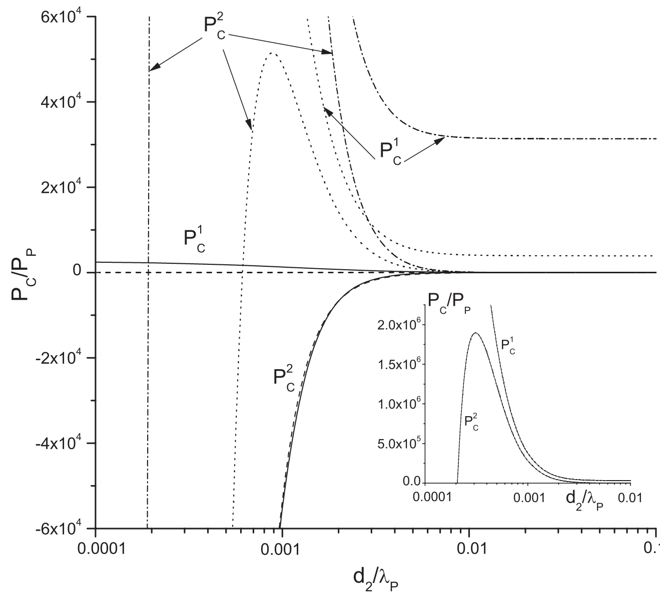

FIG. 7. Casimir pressures P_C^1 and $P_C^0 = P_C^2$ as a function of the thickness of the metallic slabs $d_0 = d_2$, for different vacuum layer thicknesses: $d_1 = 0.001\ \lambda_P$ (dash-dotted lines), $d_1 = 0.002\ \lambda_P$ (dotted lines), $d_1 = 0.01\ \lambda_P$ (solid lines), and $d_1 = 0.1\ \lambda_P$ (dashed lines). The cavity walls are ideal mirrors. Inset: P_C^1 and P_C^2 on larger scale, for $d_1 = 0.001\ \lambda_P$.

the micrometer range ($d_j/\lambda_p > 1$), we can still find critical distances d_2^c with the appropriate setup, but the corresponding Casimir pressures will be at least 10^3 weaker than those in Fig. 6. If one wants to measure the change of the thickness of a metallic layer due to the resulting Casimir pressure on it, the strong Casimir pressures (i.e., as thin as possible metallic layers) are favorable [14].

B. Vacuum layer between two metallic slabs

Two metallic slabs separated by a vacuum layer represent a well-known Casimir-like configuration. Again, we are faced with the three-layer geometry, but the vacuum layer ($j = 1$) is now in between two metallic plates ($j = 0,2$), all surrounded by the outer medium $N = 3$ (Fig. 1). If the plates are perfect mirrors then their thickness is of no importance and we find the standard Casimir pressure (16) that attracts the plates. However, for rather thin and nonperfect mirrors, one can expect significant deviation from the Casimir result [9].

When analyzing such a configuration one typically calculates the Casimir effect by assuming that metallic slabs interact with each other while keeping their thicknesses unchanged. We here analyze such a system, but we assume that only the total length of the system $L = (d_0 + d_1 + d_2)$ remains fixed. To ensure this, we put metallic slabs in an appropriate cavity with fixed walls. Depending on the thicknesses of the slabs, the dielectric properties of the walls (i.e., the outer medium $N = 3$) will influence the Casimir pressures (P_C^0, P_C^1, and P_C^2) and therefore the total Casimir pressure P_C^{T1} (15) that acts between the slabs.

For simplicity we first assume that the cavity walls are perfect mirrors and that the metallic slabs $j = (0,2)$ are equally thick and described by the same dielectric function (17). The results for the corresponding Casimir pressures are shown in Fig. 7. Obviously, the Casimir pressure P_C^1 on the vacuum layer is squeezing for all (d_1,d_2) values and it tries to attract the metallic slabs. However, the metallic slabs are surrounded by different media (ideal mirrors and vacuum). In this asymmetric situation, the Casimir pressures $P_C^0 = P_C^2$ on the slabs, for thicker vacuum layers ($d_1 \gtrsim 0.01\lambda_P$), are relaxing, and the total Casimir pressure $P_C^{T1} = P_C^1 - P_C^2$ is enlarged in comparison with P_C^1. For thinner vacuum layers, P_C^2 significantly depends on both thicknesses (d_1,d_2), and it can also change the sign [10], so, depending on (d_1,d_2), P_C^{T1} can be much larger or lower than P_C^1. However, for all (d_1,d_2) values, P_C^1 dominates over P_C^2; that is, the total Casimir pressure P_C^{T1} always attracts the metallic slabs.

To obtain a crossing point $P_C^1 = P_C^2$ and determine the corresponding critical distance d_2^c, we have to imply a smaller difference between the dielectric properties of media on both sides of the metallic plates. This situation is shown in Fig. 8 where we have taken the same configuration depicted in Fig. 7, but the outer medium that surrounds the metallic (Na) slabs is replaced with the polar dielectric (NaCl). The new setup significantly changes the corresponding Casimir pressures, particularly (P_C^0, P_C^2): like the "vacuum pressure" P_C^1, the pressures on the metallic slabs $P_C^0 = P_C^2$ are now always squeezing. As a result, we find critical distances d_2^c in all cases, which divides "thin slabs" ($d_2 < d_2^c$) with the relaxing total Casimir pressure ($|P_C^1| < |P_C^2|$) from "thick slabs" ($d_2 > d_2^c$) with the squeezing total Casimir pressure ($|P_C^1| > |P_C^2|$). The d_2^c value for $d_1 = 0.1\ \lambda_P$ cannot be resolved in Fig. 8, but we found $d_2^c = 0.084\,\lambda_P$, with $P_C^1 = P_C^2 = 0.027\ P_P$.

In order to analyze in more detail the influence of the outer medium on the appearance of critical values, we have analyzed the Casimir pressures on Na slabs with different outer media: Al ($\omega_P = 15.8$ eV), Na ($\omega_P = 6.1$ eV), Cs ($\omega_P = 3.3$ eV), and NaCl ($\omega_P = \sqrt{\omega_L^2 - \omega_T^2} = 0.026$ eV). The results are shown

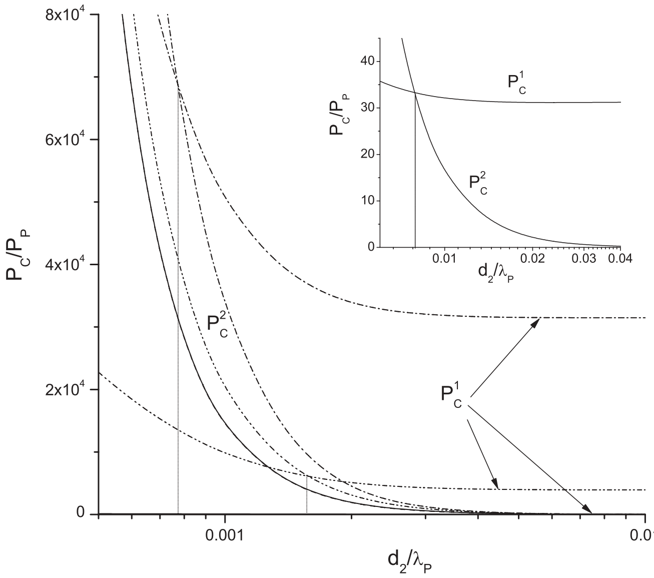

FIG. 8. The same parameters as in Fig. 7, but with Na slabs within the NaCl cavity walls. Inset: P_C^1 and P_C^2 on a smaller scale, for $d_1 = 0.01\ \lambda_P$.

in Fig. 9. The Na curves obviously do not depend on d_2, and $P_C^2 = 0$, since our system in that case consists of a thin vacuum layer in between two semi-infinite Na slabs. If Al is the outer medium, we find $\omega_P^{\rm Al} \gg \omega_P^{\rm Na}$, so the curves (P_C^1, P_C^2) are similar to the corresponding curves in Fig. 7 (with $\omega_P \to \infty$), and there is no crossing point $P_C^1 = P_C^2$. But if Cs or NaCl is the outer medium, their ω_P values are lower than $\omega_P^{\rm Na}$, and the situation resembles the situation depicted in Fig. 8, with clearly defined critical values d_2^c. Note that the critical values are not very sensitive to the properties of the outer medium

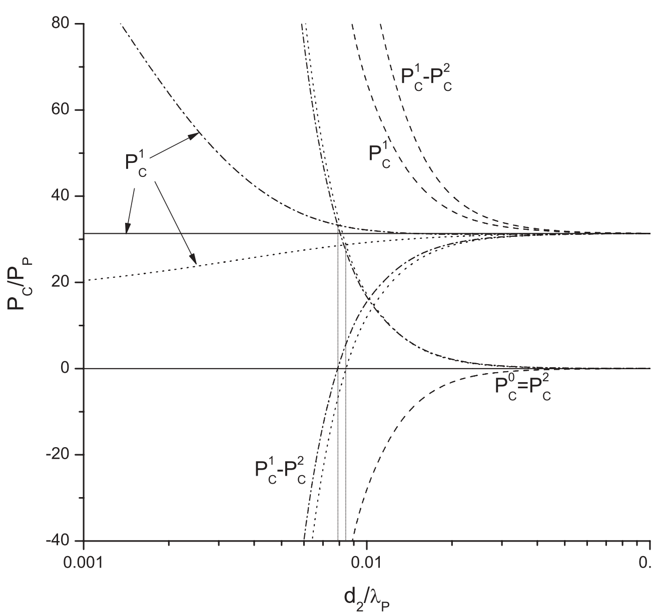

FIG. 9. Casimir pressures P_C^0, P_C^1, and P_C^2 and the total Casimir pressure $P_C^{T1} = P_C^1 - P_C^2$ acting on a thin ($d_1 = 0.01\ \lambda_P$) vacuum layer in between the Na slabs, as a function of the slab thicknesses $d_0 = d_2$. The outer layers are taken to be Al (dashed lines), Cs (dotted lines), NaCl (dash-dotted lines), and Na (solid lines).

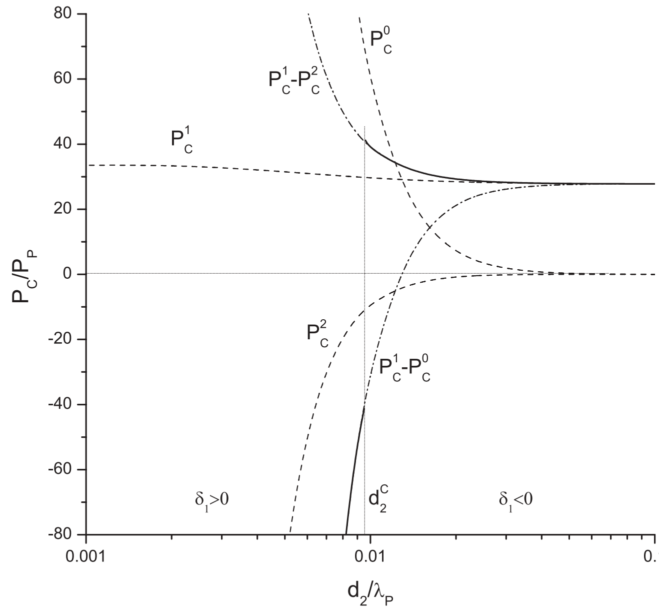

FIG. 10. Casimir pressures P_C^0, P_C^1, and P_C^2 (dashed lines) and the total Casimir pressure P_C^{T1} (solid lines) acting on the thin vacuum layer ($d_1 = 0.01\ \lambda_P$) in between the Al ($j = 0$) and Cs ($j = 2$) slabs, as a function of the slab thicknesses $d_0 = d_2$. The outer medium is Na.

and can be roughly approximated by $d_2^c \approx d_1$, just as in the complementary case discussed in the previous section.

If we want to obtain the discontinuity in the Casimir pressure, such as depicted in Fig. 5, we have to allow different pressures on each side of the vacuum layer. This can be achieved in various ways, but using the results of a previous discussion, we assume a configuration with two different metallic slabs (Al and Cs) of the same thicknesses and surrounded by the third metal (Na). This configuration satisfies $\omega_P^{\rm Al} > \omega_P^{\rm Na} > \omega_P^{\rm Cs}$, which should lead to significantly different pressures P_C^0 and P_C^2. The results are shown in Fig. 10.

Based on the previous considerations, all the curves can be easily understood. The pressure P_C^1 is mainly determined by the thin vacuum layer (d_1) and is squeezing at all $d_0 = d_2$ distances. The pressures (P_C^0, P_C^2) are negligible in comparison with P_C^1 at thick Al and Cs slabs ($d_2 \gg d_1$), while in the case of thin slabs ($d_2 \lesssim d_1$) the pressures P_C^0 and P_C^2 on the Al and Cs slabs become strongly squeezing and relaxing, respectively. As a consequence, in the case of thick (Al, Cs) slabs the resulting Casimir pressure $P_C^{T1} = P_C^1 - P_C^2 \approx P_C^1$ tries to attract them so that the squeezing of the vacuum layer is supported by the relaxation of the Cs layer ($\delta_1 = -\delta_2 < 0$). But below the critical value $d_2^c = 0.0096\,\lambda_P \approx d_1$, there is a sudden change in the resulting Casimir pressure; that is, at $d_2 < d_2^c$ the pressure $P_C^{T1} = P_C^1 - P_C^0 \approx -P_C^0$ tries to repel the Cs and Al slabs. In that case the pressure P_C^0 dominates over (P_C^1, P_C^2) so that the vacuum layer will be relaxed in order to compensate for the squeezing of the Al slab ($\delta_1 = -\delta_0 > 0$).

IV. CONCLUSION

In this article we have discussed the influence of the Casimir effect in the multilayer system. The motivation was to take into account some specific boundary conditions, that is, to analyze the Casimir pressure on layers in a cavity with a fixed length. In that case, for example, the squeezing of one plate obviously requires the relaxation of the others. All the results are compared with the standard approach, without such a constraint. Although the behavior of the Casimir energy or pressure is hard to predict in nontrivial systems, we have derived some general conclusions in two important configurations.

First, we have discussed the Casimir pressure on a d_1 thick metallic slab separated by vacuum layers (d_0,d_2) from the cavity walls. At large separations [$(d_0,d_2) \gg d_1$], the outer boundaries are not important and the resulting Casimir pressure on the slab is squeezing. For thinner vacuum layers [$(d_0,d_2) \lesssim d_1$], however, the total Casimir pressure on the slab is very sensitive to the imposed constraints, and it can be either squeezing or relaxing. For example, in the *symmetric* configuration ($\epsilon_0 = \epsilon_2, d_0 = d_2$) the Casimir pressure on the slab is always squeezing if we assume that only the slab thickness (and not the vacuum layers) can be changed [10]. However, if the distance between the cavity walls remains fixed so that the vacuum layers have to compensate for the change of the slab thickness, we have found a critical thickness $d_2 = d_2^c$ where the total Casimir pressure on the slab vanishes, and below which ($d_2 < d_2^c$) the resulting Casimir pressure on the slab becomes relaxing. The situation is more complicated in the *asymmetric* case, where (depending on configuration) one can obtain for both types of implemented boundary conditions either squeezing or relaxing Casimir pressure on the slab [10]. However, there is a significant difference between the d_2^c values at which it happens in those two cases, as well as the difference in the behavior of the total Casimir pressure at d_2^c: The transition from squeezing to relaxing pressure on the slab is continuous and occurs near a cavity wall, if only the slab thickness can be changed, and it is discontinuous and occurs at $d_2^c \approx d_1$, if the total length of the system remains fixed. We can conclude that only in the case of a free-standing slab is the result simple: the Casimir pressure is always squeezing. But if we put the slab in a cavity, the situation becomes rather complicated when the slab thickness and its separation from the cavity walls become comparable. As it turns out, answering the question: *What kind is the resulting Casimir pressure on a slab in a cavity?* is a very demanding task, and it strongly depends on the imposed constraints.

As a second example, we have analyzed a thin vacuum layer between two (different) metallic slabs adjacent to the cavity walls. In this Casimir-like system, we can again draw some general conclusions regarding the specific boundary conditions. Assuming that the thicknesses of the metallic slabs are fixed, the "vacuum" Casimir pressure that acts in between the slabs is always squeezing. The situation is more complicated if we assume that the cavity length is fixed. If the outer medium has higher reflectivity (i.e, higher ω_P value) than the metallic slab in the cavity, the Casimir pressure on this slab will be relaxing, which will enhance an attraction between the slabs. In the opposite case, that is, when the metallic slab has a higher ω_P value than the surrounding media, the Casimir pressure on this slab will be squeezing. In this case, the total Casimir pressure on the vacuum layer can be either squeezing or relaxing. We have found well-defined critical slab thicknesses that divide those two opposite behaviors, and, knowing the properties of the slabs and the outer medium, we can predict whether this transition will be continuous or discontinuous.

To detect squeezing or relaxing of a metallic layer due to the Casimir force one typically requires strong Casimir pressures on that layer [14], so we have normalized the Casimir pressure on P_P that does not contain the thickness of the slab rather than on the standard Casimir result P_C^c that becomes weak at larger thicknesses. We find $P_P/P_C^c = 3.8 \times 10^4 (d_j/\lambda_P)^4$, that is, $P_P/P_C^c \gg 1$ for $d_j > 0.1\,\lambda_P$. Strong Casimir pressures can be achieved at thin metallic layers, possibly in the nanometer range. At this range the simple macroscopic theory adopted in this article becomes questionable, but if the thin film remains well defined [10], it should give a main contribution to the Casimir pressure.

From the experimental point of view, one can possibly use a similar setup discussed in Ref. [15]. In that case one would have to fix the total length of the system, for example, by using the thick outer (Si) slabs, and measure the thicknesses of both the vacuum gap and the metallic (Cr) layers. Alternatively one can add a thin metallic layer in the vacuum gap and measure its thickness regarding the position in the gap. Note that the experiment [15] is realized with thicknesses in the micrometer range, while our results are typically shown on the nanometer length scale. But as explicitly shown in Fig. 6, the predictions on the behavior of the Casimir pressure remain the same if the relevant distance scale is enlarged. In particular, in both given examples, the critical thicknesses of the slabs at which the total Casimir pressure changes its sign are derived if all three involved layers have roughly the same thicknesses ($d_0 \approx d_1 \approx d_2$). And in both cases it holds regardless of the chosen materials of the slabs and regardless of how thick they are.

ACKNOWLEDGMENT

This work was supported by the Ministry of Science, Education and Sports of the Republic of Croatia.

[1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).

[2] C. Genet, A. Lambrecht, and S. Reynaud, Phys. Rev. A **62**, 012110 (2000); M. Bordag, U. Mohideen, and V. M. Mostepanenko, Phys. Rep. **353**, 1 (2001).

[3] P. W. Milonni, *The Quantum Vacuum: An Introduction to Quantum Electrodynamics* (San Diego, Academic Press, 1994).

[4] P. W. Milonni, R. J. Cook, and M. E. Goggin, Phys. Rev. A **38**, 1621 (1988); C. Genet, A. Lambrecht, and S. Reynaud, *ibid.* **67**, 043811 (2003).

[5] S. K. Lamoreaux, Rep. Prog. Phys. **68**, 201 (2005).

[6] G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Rev. Mod. Phys. **81**, 1827 (2009).

[7] A. Benassi and C. Calandra, J. Phys. A: Math. Theor. **40**, 13453 (2007); **41**, 175401 (2008).

[8] A. Lambrecht, I. Pirozhenko, L. Duraffourg, and Ph. Andreucci, Europhys. Lett. **77**, 44006 (2007).
[9] Z. Lenac and M. S. Tomaš, Phys. Rev. A **75**, 042101 (2007).
[10] Z. Lenac and M. S. Tomaš, Phys. Rev. A **78**, 023834 (2008); M. S. Tomaš and Z. Lenac, J. Phys. Conf. Ser. **161**, 012017 (2009).
[11] F. Zhou and L. Spruch, Phys. Rev. A **52**, 297 (1995).
[12] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 94 (1995) [Sov. Phys. JETP **2**, 73 (1956)].
[13] M. P. Marder, *Condensed Matter Physics* (Wiley & Sons, New York, 2000).
[14] Y. Imry, Phys. Rev. Lett. **95**, 080404 (2005).
[15] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002).