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Calibration of single-photon detectors using quantum statistics

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I show that calibration of the single-photon detector can be performed without knowledge of the signal parameters. Only partial information about the state statistics is sufficient for that. If one knows that the state is the squeezed one or the squeezed one mixed with the incoherent radiation, one can infer both the parameters of the state and the efficiency of the detector. For that one needs only to measure on/off statistics of detector clicks for the number of known absorbers placed before the detector. Thus, I suggest a scheme that performs a tomography of the signal and the measuring apparatus simultaneously.

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Nowadays single-photon detectors are necessary tools for quantum optics, informatics, and telecommunications. But to use them, they have to be calibrated. In any experiment where photon counting of a few-photon signal is to be performed, it is required to know, with precision, characteristics of the used detector. It is necessary to know the efficiency, the dark counts rate, the dead time, the twilight time, the afterpulsing rate, and other parameters needed to bridge a gap between the theoretical fiction of an "ideal detector" and a realistic photon-registering device. In practice, a calibration is most commonly achieved by using a standard source and/or precalibrated detector [1]. However, this procedure is quite involved, and can hardly be accomplished outside the metrology laboratory. A researcher really needs a simple and reliable procedure which would enable one to calibrate a detector with sufficient accuracy without leaving his or her own laboratory. Besides that, one cannot refrain oneself from questioning the optimality of the usual procedure when applied to a few-photon signal. Indeed, apart from knowledge of the signal field intensity required for calibration, the standard procedure uses virtually no information stored within the signal.

Attempts to use quantum statistical properties of the signal for a calibration have already three decades of history. In the 1980s it was suggested to calibrate using photons proclivity to be born in pairs during the spontaneous down-conversion process [2,3]. In this scheme one implements two detectors. A click on the first detector means that there is a certainty of having a photon on the way to the second detector. To infer an efficiency of the second detector it is sufficient to find a ratio of clicks on both detectors. Such a scheme makes it unnecessary either to compare the detector in question with the precalibrated detector, or to calibrate the source. Moreover, one does not need to know efficiencies of either detector to perform. Just the knowledge about photons arriving always in pairs makes the calibration possible. So, the procedure was termed as the "absolute" calibration. Nowadays, this procedure is actively researched, realized, and generalized (see, for example, [4–6]).

Obviously, a knowledge of the quantum state statistics can be exploited in a far more extensive way. Actually, the statistics itself can be a precise calibrating tool making it unnecessary to implement standard sources and detectors, or correlated and twin photon states. In this work I show that one can perform an accurate "absolute" calibration of the detector using a partial knowledge of the quantum statistics of a signal state. For example, information on a relation between just two elements of the signal state density matrix is sufficient to perform the calibration. It is interesting that the calibration performed in this manner is rather robust with respect to imperfections of this information, i.e., to contamination of the signal with noise.

In this work I consider a particular practically realizable scheme of the absolute detector calibration using a simple and easily generated signal state, namely the squeezed vacuum, possibly contaminated with the thermal noise. I emphasize that to perform the calibration one needs only a general knowledge about the quantum statistics. Initially, the degree of squeezing and the temperature of the incoherent component are supposed to be unknown. I demonstrate that it is possible to determine them simultaneously together with the detector efficiency, thus performing *simultaneous* tomography of the source and the measuring device. It can be accomplished by using already well-established procedure of the photon-number distribution reconstruction using on/off detectors [7–9]. For that one has to implement a set of absorbers with known transmittivities (notice, that they need not be either too high or too low).

To demonstrate a way of calibrating the detector using only a partial knowledge of the signal state statistics, let us start with the simple example of a mixture of the vacuum, one and two photons. It is described by the following density matrix:

$$\rho = (1-\rho_1-\rho_2)|0\rangle\langle 0| + \rho_1|1\rangle\langle 1| + \rho_2|2\rangle\langle 2|, \qquad (1)$$

where $|n\rangle$ denotes the Fock state with n photons. Let us assume that we know only about $\rho_1 = \rho_2$, but nothing else. So, we prepare a set of K absorbers with transmittivities T_k and measure frequencies, f_k, of "no clicks" on the detector for every one of them. Then, we reconstruct parameters ρ_n maximizing the following log-likelihood function:

$$\ln(L) = \sum_k f_k \ln[p(T_k)/P], \qquad (2)$$

where

$$p(T_k) = \sum_{n=0}^{\infty}(1-T_k\eta_d)^n \rho_n \qquad (3)$$

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is the estimated probability of no clicks for the assumed detector efficiency η_d, and

$$P = \sum_k p(T_k)$$

is the sum of estimated probabilities. Notice that η_d should be considered the total efficiency of the measurement setup, i.e., it includes also any losses arising on the way from the source to the detector. The maximization can be performed using a simple and fast iterative procedure extensively investigated in Refs. [9,10]. The results can be seen in Fig. 1. One can see that incorrect guessing of the detector efficiency distorts the photon-number distribution. Due to that fact, by estimating a difference between supposedly equal coefficients of the photon-number distribution, it is possible to hazard a fairly accurate guess of the detector efficiency. It is worth noting that both the number of absorbers and total number of measurements remain well within practically reasonable limits (K of about few tenths and the total number of measurements, $N \sim 10^6$–10^7) for an achieved estimation accuracy of about a few percent.

Of course, the state (1) is not a simple one to realize in practice. But one immediately recalls an easily producible state with the same property of the photon-number distribution, namely, the squeezed vacuum state. Indeed, for such a state, elements ρ_n, corresponding to odd n, are zeros. For the squeezed vacuum state, the probability, $p(T_k,S)$, of "no clicks" is given by the following formula [11]:

$$p(T_k,S) = \sqrt{\frac{2}{1+S+(1-T_k\eta_d)^2(1-S)}}, \qquad (4)$$

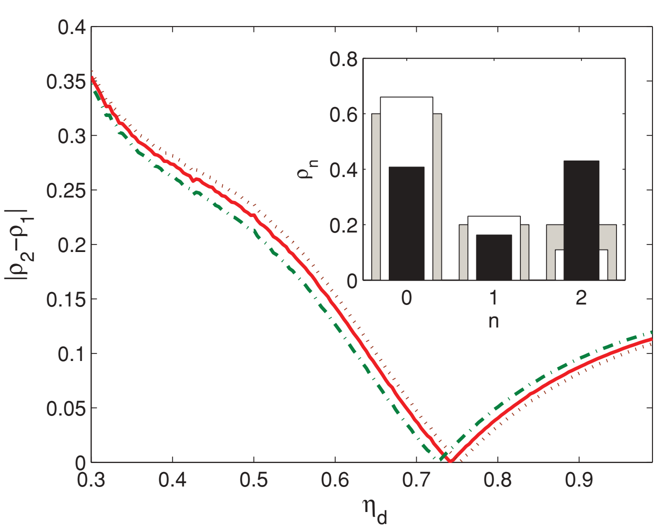

FIG. 1. (Color online) An example of estimated difference between one- and two-photon elements, $|\rho_2 - \rho_1|$, of the signal state (1) in dependence of the assumed detector efficiency, η_d, for three different realizations of the registered frequencies of "no clicks" on the detector (solid, dash-dotted, and dotted lines). The true efficiency of the detector is 0.75, $K = 40$ absorbers were taken with transmittivities equidistantly distributed within the interval $T_k \in [0.1, 0.9]$; 10^6 measurements were assumed for each absorber. In the inset it is shown how the estimated photon-number distribution is distorted by incorrect guessing of the detector efficiency. Grey bars correspond to the true distribution; black ones to the assumed efficiency $\eta_d = 0.9$; grey ones correspond to $\eta_d = 0.65$.

where S is the squeezing parameter. It is defined as

$$S = \cosh(r),$$

where $r = |\alpha|$, and the squeezed vacuum state is

$$|s\rangle = U(\alpha)|0\rangle, \quad U(\alpha) = \exp\{\alpha(a^\dagger)^2 - \alpha^* a^2\}, \qquad (5)$$

and a, $a^\dagger$ are photon annihilation and creation operators. It is easy to infer from Eq. (4) that by estimating the probability $p(T,S)$ and its derivative over the variable transmission, T, just for two values, say, T_1 and T_2, one obtains

$$\eta_d = \frac{F(T_1)T_2 - F(T_2)T_1}{F(T_1) - F(T_2)},$$
$$F(T) = \frac{4}{[p(T,S)]^3}\frac{dp(T,S)}{dT}. \qquad (6)$$

In the limit of an infinite number of measurements (no statistical errors) Eqs. (4) and (6) define uniquely both the detector efficiency and the squeezing parameter. But, of course, a realistic set of measurements is always finite. So, registered data are unavoidably fluctuating. Instead of the deterministic estimation given by Eqs. (6), it is more reasonable to implement a statistical estimation procedure (which supplies an estimation of possible errors, too). In Fig. 2 one can see a result of the log likelihood function (2) for the probability given by Eq. (4). For a total of 10^8 measurements and 100 different absorbers one gets about 2% uncertainty for estimating both the efficiency, η_d, and the squeezing parameter, S. An estimate for the uncertainty budget was obtained by repeating the measurement run 10^4 times. It is worth noting that this estimate is in a good agreement with the estimate obtained via the Fisher information matrix [12]. This Fisher matrix estimate gives for values of parameters used for Fig. 2 the following results: $S = 3.49 \pm 0.0196$, $\eta_d = 0.504 \pm 0.0061$.

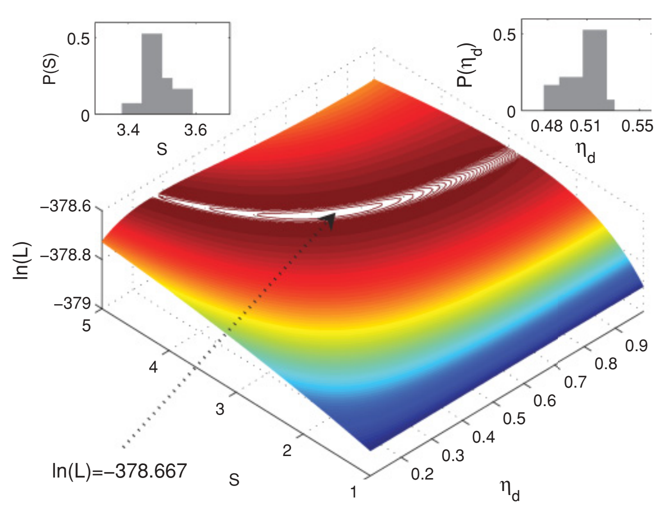

FIG. 2. (Color online) An example of log-likelihood function (2) for the probability distribution (4) The arrow points to the maximum. Left inset demonstrates a distribution of values of the squeezing parameter, S. Right inset shows a distribution of the estimated efficiency, η_d. Both distributions were obtained for 10^4 runs of the series of 10^6 measurements for each value of 100 absorber transmittivities equidistantly distributed within the interval [0.1,0.9]. True values of the parameters are $S = 3.5$, $\eta_d = 0.51$.

Now let us discuss the practicalness of the calibration scheme discussed above. First of all, let us consider a problem of dark counts. The presence of dark counts can be modelled as a superposition of the squeezed signal state with the incoherent light. Thus, the presence of dark counts (as well as a contamination of the generated state by thermal noise) can potentially be quite harmful for the presented method. However, there is a simple solution for the problem. It is possible to account for a possible incoherent contamination estimating simultaneously three parameters: the detector efficiency, the squeezing parameter, and the temperature of the incoherent component. Assuming the density matrix of the squeezed thermal signal as

$$\rho = \sqrt{2\sinh(\beta/2)}U(\alpha)\exp\left[-\beta(a^{\dagger}a + 0.5)\right]U^{\dagger}(\alpha), \quad (7)$$

one obtains for the probability of zero clicks the following formula [11]:

$$p(T,S,Q) = \frac{2}{\sqrt{q^{+} - (1-T\eta_d)q^{0} + (1-T\eta_d)^2 q^{-}}}, \quad (8)$$

where

$$q^0 = 2(Q^2-1), \quad q^{\pm} = 1 + Q^2 \pm 2QS, \quad Q = \coth(\beta/2).$$

An example of the log-likelihood function for Eq. (8) is given in Fig. 3. For a total number of measurements only four times higher than the one used for the two-parameter case depicted in Fig. 2, it is possible to achieve the same level of accuracy. For the realization depicted in Fig. 3 the estimated parameters are $S = 3.48 \pm 0.0067$, $Q = 1.22 \pm 0.0066$, $\eta_d = 0.509 \pm 0.00043$.

In view of the discussion about dark counts given above, one can conclude that the scheme is rather robust with respect to afterpulsing (the appearance of false clicking due to the detector being held off not long enough). Provided that afterpulsing can be considered just as excess dark counts, it can be accounted for by the procedure given above. Then, even if the afterpulsing is not thermal, for nonrandom pulses one can make the scheme impervious to it precisely due to the fact that, actually, we are measuring the absence of clicks. Namely, if there is no click, the next pulse goes as off usual. If there is a click, one can block a consequent signal pulse, thus avoiding the afterpulse effects.

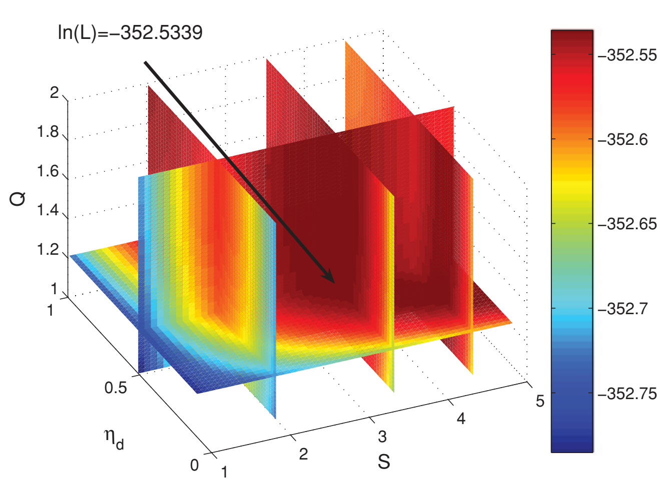

FIG. 3. (Color online) An example of log-likelihood function (2) for the probability distribution (8). The arrow points to the maximum. 4×10^6 measurements were assumed for each value of 100 absorber transmittivities equidistantly distributed within the interval [0.1,0.9]. True values of the parameters are $S = 3.5$, $\eta_d = 0.51$, $Q = 1.2$.

The dead time of the detector poses a more serious problem. Obviously, the dead time should be known at least with an accuracy up to an order of magnitude, to ensure that only a negligible portion of clicks is lost due to the dead time. It limits a practically achievable number of measurements. Taking a typical dead-time of modern single-photon detectors to be about 10 ns, one concludes that a few hours would be sufficient for performing the calibration up to units of a few percent accuracy. From the registered probability of "no clicks" it is possible also to derive a conclusion about the signal having a too large or too small number of photons. If one is getting registered frequencies too close to unity, or much lower than it, than, obviously, an accuracy of the estimation would be impaired, and a tuning of the source is necessary.

It is important to notice that one does not really need to calibrate with precision all the absorbers/beam splitters required to perform the calibration. To use just a few of them one can resort to the loop scheme already realized in experiment [8,14]. In the loop scheme the signal pulse is traveling through the same beam splitter several times, and the split signal is measured after each pass.

Finally, one should specify the kind of squeezed state source required for the calibration procedure described above. This procedure assumes the single-mode state of field. However, it can be easily seen that if the efficiency of the detector is constant within the spectral range of the signal, the field impinging on the detector can really be a modal superposition. Indeed, complex amplitudes of different modes are added. At the fixed position on the entrance of the detector the total field impinging on it can be always represented as the single collective mode, described, for example, by the following collective operator [13]:

$$a = \int d\omega f(\omega) A(\omega),$$

where $A(\omega)$ are operators for monochromatic modes satisfying the commutation relations $[A(\omega), A^{\dagger}(w)] = \delta(\omega - w)$, and the spectral function $f(\omega)$ satisfying

$$\int d\omega |f(\omega)|^2 = 1.$$

Provided that the detector has the same efficiency in the spectral range of the incident field, the detector sees exactly this collective mode. One can say here that the detector defines the measured mode both spatially and spectrally. Since we are interested in the calibration of the detector, this fake "single-modality" will do perfectly for the purpose. Of course, then the reconstruction procedure will estimate the parameters (squeezing, etc.) of the collective state defined by operators a, $a^{\dagger}$. So, the main requirement for the source is (apart from being a well-defined spacial mode) a sufficient spectral narrowness. Here it is useful to mention that by using subthreshold optical parametric generators, it is possible to produce squeezed vacuum fields with quite narrow spectra. Moreover, recently a generation into well-defined, practically monochromatic spatiotemporal modes was achieved [15]. To add, a recent experiment on squeezing has demonstrated that the measured degree of squeezing depends nonexponentially on the efficiency of the measurement scheme [16]. This fact can be considered as an indication of the possibility to determine the loss of the scheme by comparison of the theoretical prediction for the assumed loss with the actually measured squeezing.

To conclude, I have presented a simple and efficient way to calibrate singe-photon detectors. To perform the calibration one does not need to make any comparison with standard, precalibrated detectors or implement a calibrated source. For the method one needs a source generating the squeezed vacuum state. It is not necessary to know *a priori* an exact average number of photons of such a signal. It is not necessary to know *a priori* a dark count rate of the detector and guess a degree of contamination of the signal by an incoherent light. Provided that the statistics of the input signal is known (and that statistics can be quite general; in particular, it is possible to consider a superposition with the coherent component, too), one is able to estimate from the set of registered data both the parameters of the signal and the detector. One needs to pay for such a possibility with an increased number of measurements in comparison with the case when the problem is just to reconstruct the photon number distribution of the signal. However, an increase is not crucial (of about $10-10^2$ times for reaching the same accuracy, see Refs. [8,9]), and is well within borders of practical feasibility.

Finally, it is worth emphasizing once more that, actually, the presented calibration scheme performs *simultaneous* tomography of both the measurement setup and the signal state. Starting with some piece of initial information, the researcher is able to greatly improve his or her knowledge of the state and measurement process just by performing the measurement.

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