Formation and propagation of ultraslow three-wave-vector optical solitons in a cold seven-level triple-Λ atomic system under Raman excitation

Liu-Gang Si,[1,*] Wen-Xing Yang,[2] Xin-You Lü,[3] Xiangying Hao,[1] and Xiaoxue Yang[1]
[1]*Wuhan National Laboratory for Optoelectronics and School of Physics, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China*
[2]*Department of Physics, Southeast University, Nanjing 210096, People's Republic of China*
[3]*School of Physics, Ludong University, Yantai 264025, People's Republic of China*
(Received 2 January 2010; published 27 July 2010)

In this article, a theoretical scheme is proposed to investigate the formation and propagation of three-wave coupled vector optical solitons with ultraslow group velocities in a lifetime-broadened seven-state triple-Λ atomic system under Raman excitation. We show that in the presence of a weak applied magnetic field that removes the degeneracy of the corresponding sublevels of the atomic medium, three continuous-wave control fields with circularly left or right polarized fields induce a quantum interference effect which can largely suppress the absorption of the three low-intensity pulsed fields, that is, the circularly σ^- (right), the linearly π, and the circularly σ^+ (left) polarized fields converted from one weak linear-polarized probe field. By means of the standard method of multiple scales, we solve the equations of motion of atomic response and probe-control electromagnetic fields and derive three-coupled nonlinear Schrödinger equations that govern the nonlinear evolution of the envelopes of the probe fields in this scheme. We then demonstrate that because of the nonlinear coupling to one another, the three probe fields can evolve into three-wave temporal, group velocity, and amplitude-matched optical solitons under appropriate conditions, which are produced from the delicate balance of the dispersion effects and the self- and cross-phase modulation effects. This scheme may thus pave the way to generate ultraslow vector optical solitons composed of three field components in a highly resonant atomic medium and result in a substantial impact on this field of nonlinear optics.

DOI: 10.1103/PhysRevA.82.013836 PACS number(s): 42.65.Tg, 05.45.Yv, 42.50.Gy, 42.81.Dp

I. INTRODUCTION

Vector solitons are solitons involving two and generally $N(N \geqslant 3)$ components [1–9], which can be generated as a consequence of a delicate balance between dispersion, self-, and cross-modulation (SPM and XPM) in all light components. As a particular class of solitons, vector solitons formed in each component of the optical field can remain almost stable and propagate over relatively long distances without spreading. Thus far, most vector optical solitons have been produced in passive media, such as optical fibers [3–5], in which there is no distinctive energy level structure. Therefore high-powered lasers and far-off resonance excitation schemes are generally employed in order to avoid unmanageable optical field attenuation and distortion. As a consequence, vector optical solitons produced this way require substantially long propagation distances and generally travel with group velocities very close to the speed of light in free space.

In the past few years, the propagation of optical fields in highly resonant media has been extensively studied. These works have opened up several significant features of wave propagation in such a highly resonant medium such as the remarkable reduction of the propagation group velocity, the dramatic modification of the dispersion properties, and the large enhancement of the Kerr nonlinearities of the optical medium [10–14]. Based on these techniques, many nonlinear optical phenomena, including XPM [15–18], optical bistability [19–22], quantum entanglement [23–30], and four-wave mixing (FWM) [31–35] in highly resonant optical media, have been realized. Recently, following the report of ultraslow optical solitons in a highly resonant atomic medium [36,37], vector solitons consisting of two components of the optical field have been demonstrated that can be also generated with ultraslow group velocity in atomic systems [38,39]. These vector solitons are produced from the nonlinear interaction between two waves with the same frequency but belong to two different polarizations, resulting in a proper balance between dispersion, SPM, and XPM in two light components, and can be described by two-coupled nonlinear Schrödinger (NLS) equations. A natural question, then, may be asked: In an atomic system, if three low-intensity pulsed electromagnetic fields with the same frequency but different polarizations interact with one another, can the dispersion, SPM, and XPM effects achieve a perfect balance under certain conditions and lead to three fields evolving into distortion-free three-wave temporal vector optical solitons? Subsequently, we will present a systematic study to address this question.

In this article, a theoretical scheme of a seven-level triple-Λ atomic system under Raman excitation with large single- and two-photon detunings is proposed to generate three-wave temporal vector optical solitons with ultraslow group velocities. For this purpose, one weak linear-polarized probe field is converted into a circularly σ^- (right), a linearly π, and a circularly σ^+ (left) polarized fields which drive, respectively, three transitions in the seven-level triple-Λ atomic system subject to an applied magnetic field (see Fig. 1). By using three strong continuous wave (cw) control fields circularly left or right polarized to couple the other three transitions, we show that the absorption of three low-intensity polarized pulsed probe fields (σ^-, π, and σ^+ polarized fields) can be largely suppressed, while simultaneously, their linear as

*siliugang@gmail.com

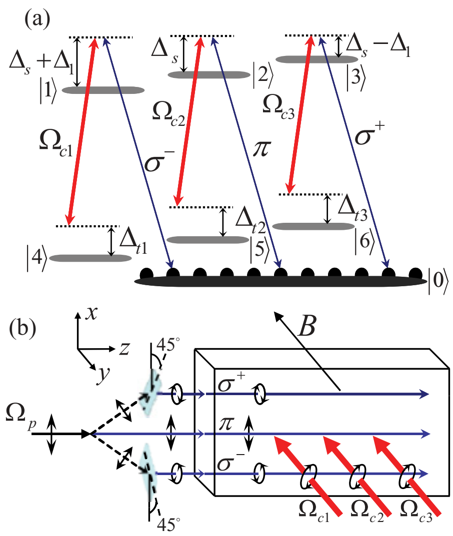

FIG. 1. (Color online) (a) Schematic of a cold seven-level triple-Λ atomic system. A circularly σ^- (right), a linearly π, and a circularly σ^+ (left) polarized field converted from one weak linear-polarized probe field drive the transitions $|0\rangle \leftrightarrow |1\rangle$, $|0\rangle \leftrightarrow |2\rangle$, and $|0\rangle \leftrightarrow |3\rangle$, respectively, while the other transitions $|1\rangle \leftrightarrow |4\rangle$, $|2\rangle \leftrightarrow |5\rangle$, and $|3\rangle \leftrightarrow |6\rangle$ are induced, respectively, by three strong circularly left polarized cw control fields with Rabi frequencies $\Omega_{c1,c2,c3}$. Variable Δ_s represents single-photon detuning, and $\Delta_{t1,t2,t3}$ are three separate two-photon detunings. (b) Possible arrangement of experimental apparatus. The probe field Ω_p is converted into the circularly σ^-, the linearly π, and the circularly σ^+ polarized fields which propagate in the $+z$ direction and are sent into the atomic medium to form ultraslow three-wave coupled temporal vector optical solitons. The circularly right- and left-polarized fields are obtained after the probe field Ω_p passing through the $\pm 45^\circ$-oriented $\lambda/4$ plates [40]. Three control fields $\Omega_{c1,c2,c3}$ are all circularly left polarized in order to couple to the corresponding transitions and propagate in the $-y$ direction parallel to the applied magnetic field B.

well as nonlinear dispersion is dramatically enhanced. Then a perfect balance between dispersion, SPM, and XPM is achieved under appropriate conditions, which leads to the Maxwell's equations for describing the propagation of three weak polarized pulsed probe fields evolving into three-coupled NLS equations that have solutions describing various types of three-wave temporal vector optical solitons. Besides, we have noticed other reports on the formation of scalar optical solitons [41–43], optical soliton pairs [44–46], and two-wave vector optical solitons [47] in highly resonant media with different configurations of levels. Also, there are a number of theoretical works on three-wave coupled vector solitons as the solutions of three-coupled NLS equations in biophysics, optical fibers, multicomponent Bose-Einstein condensates, Kerr-like photorefractive media, and so on [48–54]. However, to the best of our knowledge, the investigation of the formation of ultraslow three-wave coupled vector optical solitons in such an atomic system has rarely been touched up to now.

The rest of our article is organized as follows. Section II mainly focuses on describing the theoretical model proposed here. We derive the corresponding Hamiltonian and the differential equations governing the dynamics of the atomic system and the propagation of the three low-intensity pulsed fields. The linearized dynamics of this model is briefly discussed in this section. In Sec. III, we solve the dynamics equations of this system by means of the standard method of multiple scales and derive three-coupled NLS equations governing the time evolution of the circularly σ^-, the linearly π, and the circularly σ^+ polarized fields converted from the probe field. Then three-wave coupled temporal vector optical solitons like bright-bright-bright, bright-bright-dark, bright-dark-bright, bright-dark-dark, and dark-dark-dark vector solitons with ultraslow group velocities have been proved to exist stably in this atomic system. We end the article with conclusions and a discussion in Sec. IV.

II. MODEL AND LINEARITY RESULTS

The system under investigation is lifetime-broadened seven-level atoms driven in a triple-Λ configuration of energy levels labeled as $|j\rangle (j = 0-6)$, as shown in Fig. 1(a). Here three upper atomic sublevels $|1\rangle$, $|2\rangle$, and $|3\rangle$ as well as the sublevels $|4\rangle$, $|5\rangle$, and $|6\rangle$ are Zeeman splits because of the applied magnetic field B. The frequency difference between adjacent sublevels is given by $\Delta_1 = \mu_B g_1 B/\hbar$ for levels $|1\rangle$, $|2\rangle$, $|3\rangle$ and by $\Delta_2 = \mu_B g_2 B/\hbar$ for levels $|4\rangle$, $|5\rangle$, $|6\rangle$, where μ_B is the Bohr magneton and $g_{1(2)}$ is the gyromagnetic factor of the corresponding state with $g_2 \approx 3g_1$ [55]. One weak linear-polarized probe field Ω_p with optical frequency ω_p and wave vector $\vec{k}_p$ is converted into a circularly σ^-, a linearly π, and a circularly σ^+ polarized field [see Fig. 1(b)] to drive the transitions $|0\rangle \leftrightarrow |1\rangle$, $|0\rangle \leftrightarrow |2\rangle$, and $|0\rangle \leftrightarrow |3\rangle$, respectively. A strong circularly left or right polarized control field with optical frequency ω_{c1} (ω_{c2}, ω_{c3}) and wave vector $\vec{k}_{c1}$ ($\vec{k}_{c2}$, $\vec{k}_{c3}$) couples the atomic transition $|1\rangle \leftrightarrow |4\rangle$ ($|2\rangle \leftrightarrow |5\rangle$, $|3\rangle \leftrightarrow |6\rangle$) with Rabi frequency Ω_{c1} (Ω_{c2}, Ω_{c3}). Obviously, the seven-level system consists of three Λ configurations under Raman excitation, all of them sharing the ground-state level $|0\rangle$ [36]. In order to generate three-wave coupled temporal vector optical solitons experimentally, one may choose the hyperfine-split levels for the D lines of cold Na atoms trapped in a magneto-optical trap at sufficiently low temperature as an experimental candidate for the proposed system. Thus the designated states are chosen as follows: $3\,^2S_{1/2}$, $F = 1$, $M_F = 0$ as $|0\rangle$; $3\,^2P_{1/2}$, $F = 2$, $M_F = -1$ as $|1\rangle$; $3\,^2P_{1/2}$, $F = 2$, $M_F = 0$ as $|2\rangle$; $3\,^2P_{1/2}$, $F = 2$, $M_F = 1$ as $|3\rangle$; $3\,^2S_{1/2}$, $F = 2$, $M_F = -2$ as $|4\rangle$; $3\,^2S_{1/2}$, $F = 2$, $M_F = -1$ as $|5\rangle$; and $3\,^2S_{1/2}$, $F = 2$, $M_F = 0$ as $|6\rangle$ [55]. Then the control fields Ω_{c1}, Ω_{c2}, and Ω_{c3} should be circularly σ^+ (left) polarized. Figure 1(b) is shown as the corresponding possible arrangement of experimental apparatus.

Suppose the electric fields of the three low-intensity polarized pulsed probe fields (σ^-, π, and σ^+ polarized fields) have the forms $\vec{E}_{p1,p2,p3} = \vec{e}_{\sigma^-,\pi,\sigma^+} E_{\sigma^-,\pi,\sigma^+} \exp(-i\omega_p t + i\vec{k}_p \cdot \vec{r}) + \text{c.c.}$ and three strong circularly σ^+ polarized cw control fields can be written as $\vec{E}_{cj} = \vec{e}_{cj} E_{cj} \exp(-i\omega_{cj} t + i\vec{k}_{cj} \cdot \vec{r}) + \text{c.c.} (j = 1,2,3)$, with c.c. being the complex conjugate and $\vec{e}_\nu$ being the corresponding unit vectors of these fields. If the free Hamiltonian of the system is defined as $\hat{H}_0/\hbar = \omega_p |1\rangle\langle 1| + \omega_p |2\rangle\langle 2| + \omega_p |3\rangle\langle 3| + (\omega_p - \omega_{c1})|4\rangle\langle 4| + (\omega_p - \omega_{c2})|5\rangle\langle 5| + (\omega_p - \omega_{c3})|6\rangle\langle 6|$, then under electric-dipole and rotating-wave approximations, the interaction Hamiltonian of the system in the interaction picture can be obtained as follows:

$$\begin{aligned}\frac{\hat{H}^{int}}{\hbar} =& -(\Delta_s+\Delta_1)|1\rangle\langle 1| - \Delta_s|2\rangle\langle 2| - (\Delta_s-\Delta_1)|3\rangle\langle 3| \\ & - \Delta_{t1}|4\rangle\langle 4| - \Delta_{t2}|5\rangle\langle 5| - \Delta_{t3}|6\rangle\langle 6| \\ & - (\Omega_{p1}e^{i\vec{k}_p\cdot\vec{r}}|1\rangle\langle 0| + \Omega_{p2}e^{i\vec{k}_p\cdot\vec{r}}|2\rangle\langle 0| \\ & + \Omega_{p3}e^{i\vec{k}_p\cdot\vec{r}}|3\rangle\langle 0| + \Omega_{c1}e^{i\vec{k}_{c1}\cdot\vec{r}}|1\rangle\langle 4| \\ & + \Omega_{c2}e^{i\vec{k}_{c2}\cdot\vec{r}}|2\rangle\langle 5| + \Omega_{c3}e^{i\vec{k}_{c3}\cdot\vec{r}}|3\rangle\langle 6| + \text{H.c.}),\end{aligned} \quad (1)$$

where the symbol H.c. stands for Hermitian conjugate. In this expression, $\Delta_s = \omega_p - \omega_{20}$ is defined as single-photon detuning and $\Delta_{t1} = \omega_p - \omega_{c1} - \omega_{40}$, $\Delta_{t2} = \omega_p - \omega_{c2} - \omega_{50}$, and $\Delta_{t3} = \omega_p - \omega_{c3} - \omega_{60}$ represent three separate two-photon detunings with $\omega_{j0} = (\epsilon_j - \epsilon_0)/\hbar$ being the resonant transition frequency, with $(\epsilon_0)\,\epsilon_j$ being the energy of the atomic state $(|0\rangle)$ $|j\rangle (j = 1-6)$. The frequency difference $\Delta_2 = \mu_B g_2 B/\hbar$ for levels $|4\rangle$, $|5\rangle$, and $|6\rangle$ is incorporated in the resonant transition frequencies ω_{40}, ω_{50}, and ω_{60} with $\omega_{60} = \omega_{50} + \Delta_2 = \omega_{40} + 2\Delta_2$. Besides, $2\Omega_{p1,p2,p3,c1,c2,c3} = (\vec{\mu}_{10,20,30,14,25,36}\cdot \vec{e}_{\sigma^-,\pi,\sigma^+,c1,c2,c3})E_{\sigma^-,\pi,\sigma^+,c1,c2,c3}/\hbar$ denote the Rabi frequencies for the respective transitions, with $\vec{\mu}_{lm}$ being the dipole moment for the transition between levels $|l\rangle$ and $|m\rangle$.

Defining the state of the atomic system as $|\Psi\rangle = A_0(t)|0\rangle + A_1(t)e^{i\vec{k}_p\cdot\vec{r}}|1\rangle + A_2(t)e^{i\vec{k}_p\cdot\vec{r}}|2\rangle + A_3(t)e^{i\vec{k}_p\cdot\vec{r}}|3\rangle + A_4(t)e^{i\vec{k}_p\cdot\vec{r}-i\vec{k}_{c1}\cdot\vec{r}}|4\rangle + A_5(t)e^{i\vec{k}_p\cdot\vec{r}-i\vec{k}_{c2}\cdot\vec{r}}|5\rangle + A_6(t)e^{i\vec{k}_p\cdot\vec{r}-i\vec{k}_{c3}\cdot\vec{r}}|6\rangle$, we then obtain from the Schrödinger equation $i\hbar\partial|\Psi\rangle/\partial t = \hat{H}^{int}|\Psi\rangle$ in the interaction picture the evolution equations for the probability amplitudes $A_j(t)$, as follows:

$$\frac{\partial A_1}{\partial t} = i(\Delta_s+\Delta_1+i\gamma_1)A_1 + i\Omega_{c1}A_4 + i\Omega_{p1}A_0, \quad (2a)$$

$$\frac{\partial A_2}{\partial t} = i(\Delta_s+i\gamma_2)A_2 + i\Omega_{c2}A_5 + i\Omega_{p2}A_0, \quad (2b)$$

$$\frac{\partial A_3}{\partial t} = i(\Delta_s-\Delta_1+i\gamma_3)A_3 + i\Omega_{c3}A_6 + i\Omega_{p3}A_0, \quad (2c)$$

$$\frac{\partial A_4}{\partial t} = i(\Delta_{t1}+i\gamma_4)A_4 + i\Omega_{c1}{}^*A_1, \quad (2d)$$

$$\frac{\partial A_5}{\partial t} = i(\Delta_{t2}+i\gamma_5)A_5 + i\Omega_{c2}{}^*A_2, \quad (2e)$$

$$\frac{\partial A_6}{\partial t} = i(\Delta_{t3}+i\gamma_6)A_6 + i\Omega_{c3}{}^*A_3, \quad (2f)$$

where we have introduced the decay rate of the state $|k\rangle$ $2\gamma_k (k = 1-6)$ phenomenologically and A_0 can be determined by the relation $\sum_{j=0}^{6}|A_j|^2 = 1$, while three time-dependent low-intensity polarized pulsed probe fields $\Omega_{p1,p2,p3}$ (σ^-, π, and σ^+ polarized fields) satisfy the equations

$$\frac{\partial\Omega_{p1}}{\partial z} + \frac{1}{c}\frac{\partial\Omega_{p1}}{\partial t} = i\kappa_{10}A_1A_0{}^*, \quad (3a)$$

$$\frac{\partial\Omega_{p2}}{\partial z} + \frac{1}{c}\frac{\partial\Omega_{p2}}{\partial t} = i\kappa_{20}A_2A_0{}^*, \quad (3b)$$

$$\frac{\partial\Omega_{p3}}{\partial z} + \frac{1}{c}\frac{\partial\Omega_{p3}}{\partial t} = i\kappa_{30}A_3A_0{}^*, \quad (3c)$$

which can be gotten from Maxwell's equations under the slowly varying envelope approximation. In Eqs. (3a)–(3c), $\kappa_{10} = N\omega_p|\vec{\mu}_{10}\cdot\vec{e}_{\sigma^-}|^2/(2\hbar\epsilon_0 c)$, $\kappa_{20} = N\omega_p|\vec{\mu}_{20}\cdot\vec{e}_{\pi}|^2/(2\hbar\epsilon_0 c)$, and $\kappa_{30} = N\omega_p|\vec{\mu}_{30}\cdot\vec{e}_{\sigma^+}|^2/(2\hbar\epsilon_0 c)$, with N being the concentration and ϵ_0 being the vacuum dielectric constant.

Before seeking the three-wave coupled vector optical solitons in a seven-level triple-Λ atomic system, we begin to study the weak probe absorption and dispersion behavior of the system by solving Eqs. (2) and (3) in the linear regime. This may be useful for understanding the robust balance between the dispersion effect and nonlinearity resulting in the formation of three-wave solitons in this system, which will be shown in the next section, where a weak nonlinear theory is developed to obtain three-coupled NLS equations. Now under the initial conditions, $A_j(t=0) = \delta_{j0}$; that is, all atoms are initially pumped into the ground state $|0\rangle$ before the three weak polarized probe fields enter the medium at $t = 0$; we outline the solutions of Eqs. (2) and (3) in the low-density approximation, where the intensities of the σ^-, π, and σ^+ polarized pulsed probe fields are much weaker than those of the control fields. In the low-density approximation, the ground state $|0\rangle$ is not depleted. Thus we can make the nondepleted ground state approximation and take $A_0(t) \simeq 1$, which is always adapted in describing the phenomenon of EIT and EIT-related multiwave mixing and soliton phenomena [10–14,33–37,56–59]. We use the Fourier transform technique with respect to the time t by taking the Fourier transform of Eqs. (2) and (3) and then obtaining three branches of the linear dispersion relation for the σ^-, π, and σ^+ polarized pulsed probe fields:

$$K_j(\omega) = \frac{\omega}{c} + \kappa_{j0}\frac{\omega + d_{j+3}}{D_j(\omega)} = \sum_{k=0}^{2}\frac{1}{k!}K_{jk}\omega^k + \mathcal{O}(\omega^3), \quad (4)$$

where $D_j(\omega) = |\Omega_{cj}|^2 - (\omega + d_j)(\omega + d_{j+3}) (j = 1,2,3)$, with $d_1 = \Delta_s + \Delta_1 + i\gamma_1$, $d_2 = \Delta_s + i\gamma_2$, $d_3 = \Delta_s - \Delta_1 + i\gamma_3$, $d_4 = \Delta_{t1} + i\gamma_4$, $d_5 = \Delta_{t2} + i\gamma_5$, and $d_6 = \Delta_{t3} + i\gamma_6$. In Eq. (4), we have expanded the linear dispersion relations $K_j(\omega)$ into a rapid conversion power series around the center frequency ω_p with $K_{jk} = [\partial^k K_j(\omega)/\partial\omega^k]|_{\omega=0} (k = 0,1,2)$. The coefficients K_{jk} in Eq. (4) have rather clear physical interpretation. The imaginary parts of K_{j0} give the absorption coefficient $\alpha_j = 2\mathrm{Im}(K_{j0})$, while the corresponding real parts describe the phase shift $\phi_j = \mathrm{Re}(K_{j0})$ per unit length of the σ^-, π, and σ^+ polarized pulsed probe fields; the group velocities of the three polarized pulsed probe fields are given by $K_{j1} = 1/V_{gj} = 1/c + \kappa_{j0}(|\Omega_{cj}|^2 + d_{j+3}{}^2)/D_j{}^2$, with $D_j = |\Omega_{cj}|^2 - d_j d_{j+3}$, and $K_{j2} = 2\kappa_{j0}(2|\Omega_{cj}|^2 d_{j+3} + d_{j+3}{}^3 + |\Omega_{cj}|^2 d_j)/D_j{}^3$ represent the group-velocity dispersions which contribute to the corresponding probe pulse spreading and additional field intensity loss.

To illustrate the linear properties of this model, we plot in Fig. 2 the absorption and dispersion spectra of the σ^- (dashed line), π (solid line), and σ^+ (dash-dotted line) polarized probe fields. For simplicity, we have set $\Delta_s \simeq \Delta_{t1} \simeq \Delta_{t2} \simeq \Delta_{t3} = 0$ and $\gamma_4 \simeq \gamma_5 \simeq \gamma_6 = \gamma$. The other parameters are all in units of γ and chosen as $\Delta_1 = 4.0\times 10^3$, $\gamma_1 \simeq \gamma_2 \simeq \gamma_3 = 6.0\times 10^3$, and $\kappa_{10} \simeq \kappa_{20} \simeq \kappa_{30} = 1.0\times 10^5$. As shown in Fig. 2(a), in the presence of three control fields with intensities $\Omega_{c1} \simeq \Omega_{c2} \simeq \Omega_{c3} = 1.0\times 10^4$, the absorptions of three polarized pulsed probe fields are almost completely suppressed, accompanied by the steep and approximately linear slope of the dispersions

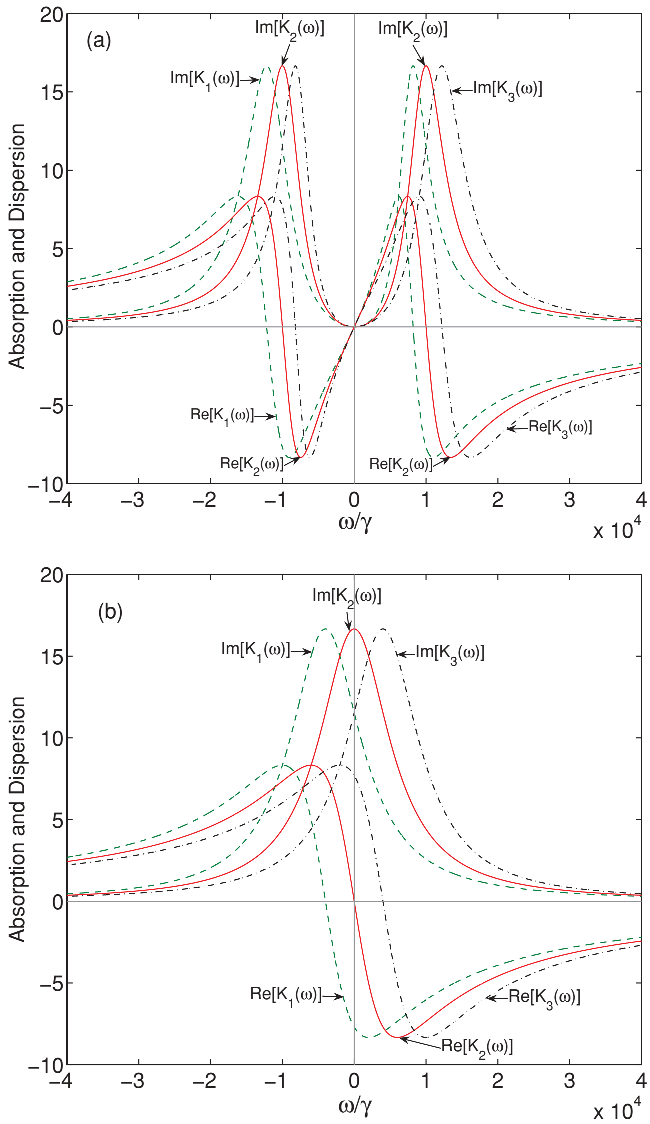

FIG. 2. (Color online) Absorption and dispersion spectra, in arbitrary units, for the σ^- (dashed line), π (solid line), and σ^+ (dash-dotted line) polarized pulsed probe fields in the presence of an applied magnetic field ($\Delta = 4.0 \times 10^3\gamma$) and three control fields with intensities (a) $\Omega_{c1} \simeq \Omega_{c2} \simeq \Omega_{c3} = 1.0 \times 10^4\gamma$ and (b) $\Omega_{c1} \simeq \Omega_{c2} \simeq \Omega_{c3} = 0$. The other parameters are explained in the text.

in the vicinity of $\omega = 0$, where the known effect of EIT is attained in the system. For comparison, we turn off the three control fields ($\Omega_{cj} = 0, j = 1,2,3$) and plot the absorption and dispersion spectra of the three polarized pulsed probe fields in Fig. 2(b), and we find that the absorption peaks appeared near $\omega = 0$ within the corresponding absorption lines where the EIT effect is held back. The differences between two figures show that it is exactly the three strong control fields that induce a quantum destructive interference effect which makes the three pulsed polarized probe fields propagate transparently with almost no absorption in this atomic system. Besides, we have noticed that because of the nonvanishing group-velocity dispersions K_{j2}, three polarized pulsed probe fields may suffer pulse spreading and intensity attenuation as the distance z increase. Therefore, to cancel the detrimental dispersion effects for the sake of producing three-wave coupled temporal vector optical solitons in this seven-level triple-Λ atomic system, it is necessary to find an effective remedy to balance the rapid increase in pulse width in the time domain. This is the main objective of the next section: to find and take the SPM and XPM effects as the effective remedy to balance the pulse spreading where three-coupled NLS equations describing the envelope evolution of the σ^-, π, and σ^+ polarized probe fields are derived by means of the standard method of multiple scales.

III. MULTIPLE SCALE METHOD APPLIED AND THREE-COUPLED NLS EQUATIONS

A detailed analysis of the dispersion properties of this seven-level triple-Λ atomic system shows that not only the quantities K_{j2} but also K_{j1} obtained from Eq. (4) contribute to the group velocities and pulse spreading of the corresponding fields. It is noteworthy that K_{j2} introduces a z dependence to the group velocities and the pulse width in the time domain. These dispersion effects are harmful for optical information processing but are always observed in the propagation of ultraslow waves in highly resonant media. Therefore it is necessary for the formation of the vector optical solitons to use nonlinear effects (SPM, XPM) to balance dispersion effects. In the remainder of this section, we will demonstrate that a delicate balance between SPM, XPM, and group-velocity dispersion effects of the σ^-, π, and σ^+ polarized pulsed probe fields can be achieved and lead to the generation and propagation of ultraslow three-wave coupled temporal vector optical solitons in our system.

In order to demonstrate how the σ^-, π, and σ^+ polarized pulsed probe fields evolve into three coupled stable shape-preserving wave forms, we employ the standard method of multiple scales [4] to analyze the interaction of the three weak polarized pulsed probe fields and the triple-Λ atomic system. Thus, owing to the weak nonlinear effect, we should consider corrections to the nondepleted ground state approximation, allowing a small population depletion of the ground state, which is characterized by a small parameter λ. Then we can introduce the perturbation expansion, as follows:

$$\Omega_{pj} = \lambda\Omega_{pj}^{(1)} + \lambda^2\Omega_{pj}^{(2)} + \lambda^3\Omega_{pj}^{(3)} + \cdots, \; j = 1,2,3, \tag{5a}$$

$$A_k = A_k^{(0)} + \lambda A_k^{(1)} + \lambda^2 A_k^{(2)} + \cdots, \; k = 0-6, \tag{5b}$$

with $A_k^{(0)} = \delta_{k0}$ and $A_0^{(1)} = 0$. Following the standard procedure of multiple scales perturbation analysis, we write z and t in multiple scale forms $z_0 = z, z_1 = \lambda z, z_2 = \lambda^2 z$ and $t_0 = t, t_1 = \lambda t$, which can be temporarily treated as independent variables. Thus, instead of determining the quantities $\Omega_{pj}^{(m)}(m = 1,2,3,\cdots)$ and $A_k^{(n)}(n = 0,1,2,\cdots)$ in Eqs. (5) as the functions of $(z;t)$, we determine $\Omega_{pj}^{(m)}(m = 1,2,3,\cdots)$ and $A_k^{(n)}(n = 0,1,2,\cdots)$ as the functions of $(z_0,z_1,z_2;t_0,t_1)$. Using a chain rule, we have

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z_0} + \lambda\frac{\partial}{\partial z_1} + \lambda^2\frac{\partial}{\partial z_2}, \tag{6a}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \lambda\frac{\partial}{\partial t_1}. \tag{6b}$$

Substituting Eqs. (5) and (6) into Eqs. (2) and (3), we obtain ($j = 1,2,3$)

$$\left(i\frac{\partial}{\partial t_0} + d_j\right) A_1^{(l)} + \Omega_{cj} A_{j+3}^{(l)} + \Omega_{pj}^{(l)} = P_j^{(l)}, \quad (7a)$$

$$\left(i\frac{\partial}{\partial t_0} + d_{j+3}\right) A_{j+3}^{(l)} + \Omega_{cj}{}^{*} A_j^{(l)} = P_{j+3}^{(l)}, \quad (7b)$$

$$i\left(\frac{\partial}{\partial z_0} + \frac{1}{c}\frac{\partial}{\partial t_0}\right) \Omega_{pj}^{(l)} + \kappa_{j0} A_j^{(l)} = Q_j^{(l)}, \quad (7c)$$

where the explicit expressions of $P_j^{(l)}$, $P_{j+3}^{(l)}$, and $Q_j^{(l)}$ can be systematically and analytically obtained and will be given subsequently. In addition, the relation $\sum_{j=0}^{6}|A_j|^2 = 1$ gives the condition $A_0^{(2)} + [A_0^{(2)}]^* = -(A_1^{(1)}[A_1^{(1)}]^* + A_2^{(1)}[A_2^{(1)}]^* + A_3^{(1)}[A_3^{(1)}]^* + A_4^{(1)}[A_4^{(1)}]^* + A_5^{(1)}[A_5^{(1)}]^* + A_6^{(1)}[A_6^{(1)}]^*)$ and higher orders of $A_0^{(l)} (l \geqslant 3)$ are not needed and thus neglected.

The analytic solutions of Eqs. (7a)–(7c) can be obtained straightforwardly and denoted symbolically as the following forms:

$$A_j^{(l)} = \frac{1}{\kappa_{j0}}\left[Q_j^{(l)} - i\left(\frac{\partial}{\partial z_0} + \frac{1}{c}\frac{\partial}{\partial t_0}\right)\Omega_{pj}^{(l)}\right], \quad (8a)$$

$$A_{j+3}^{(l)} = \frac{1}{\Omega_{cj}}\left[P_j^{(l)} - \left(i\frac{\partial}{\partial t_0} + d_j\right) A_j^{(l)} - \Omega_{pj}^{(l)}\right], \quad (8b)$$

$$\hat{\mathcal{L}}_j \Omega_{pj}^{(l)} = R_j^{(l)}, \quad (8c)$$

with $\hat{\mathcal{L}}_j = i[|\Omega_{cj}|^2 - (i\partial/\partial t_0 + d_j)(i\partial/\partial t_0 + d_{j+3})](\partial/\partial z_0 + c^{-1}\partial/\partial t_0) + \kappa_{j0}(i\partial/\partial t_0 + d_{j+3})$ and $R_j^{(l)} = \kappa_{j0}[(i\partial/\partial t_0 + d_{j+3})P_j^{(l)} - \Omega_{cj} P_{j+1}^{(l)}] + [|\Omega_{cj}|^2 - (i\partial/\partial t_0 + d_j)(i\partial/\partial t_0 + d_{j+3})]Q_j^{(l)}$. We can solve Eqs. (8a)–(8c) order by order, as shown subsequently.

At the leading order ($l = 1$), we have $P_j^{(l)} = P_{j+3}^{(l)} = Q_j^{(l)} = 0$. It is easy to find the first-order approximation solutions in the linear regime ($j = 1,2,3$):

$$\Omega_{pj}^{(1)} = \Omega_j e^{iz_0 K_j(\omega) - i\omega t_0}, \quad (9)$$

with $K_j(\omega)$ being the linear dispersion relations (4) and the envelope function Ω_j being yet to be determined by variables $(z_1,z_2;t_1)$.

At the second order ($l = 2$), the explicit expressions of $P_j^{(2)}$, $P_{j+3}^{(2)}$, and $Q_j^{(2)}$ can be obtained using the solutions at first order and denoted as $P_j^{(2)} = -i\partial A_j^{(1)}/\partial t_1$, $P_{j+3}^{(2)} = -i\partial A_{j+3}^{(1)}/\partial t_1$ and $Q_j^{(2)} = -i(\partial/\partial z_1 + c^{-1}\partial/\partial t_1)\Omega_{pj}^{(1)}$. Then we obtain the solvability conditions for obtaining a divergence-free solution for $\Omega_{pj}^{(2)}$ by eliminating secular terms:

$$\frac{\partial \Omega_j}{\partial z_1} + \frac{1}{V_{gj}}\frac{\partial \Omega_j}{\partial t_1} = 0, \quad (10)$$

with $V_{gj} = 1/K_{j1}$ being the group velocities of the wave packets Ω_j.

To form the three-wave coupled temporal vector optical solitons in this atomic system, it is necessary to find the effective nonlinear effect to balance the second-order dispersion effect that causes pulse spreading. To this end, we go to the asymptotic expansion to the third order ($l = 3$) with the preceding results. In this order, we have $P_j^{(3)} = -i\partial A_j^{(2)}/\partial t_1 - \Omega_{pj}^{(1)} A_0^{(2)}$, $P_{j+3}^{(3)} = -i\partial A_{j+3}^{(2)}/\partial t_1$, and $Q_j^{(3)} = -i(\partial/\partial z_1 + c^{-1}\partial/\partial t_1)\Omega_{pj}^{(2)} - i\partial\Omega_{pj}^{(1)}/\partial z_2 - \kappa_{j0} A_j^{(1)}[A_0^{(2)}]^*$. Then the solvability conditions of eliminating secular terms in this order yield the coupled differential equations governing the spatial-temporal evolution of Ω_j ($j = 1,2,3$) :

$$i\frac{\partial \Omega_j}{\partial z_2} - \frac{K_{j2}}{2}\frac{\partial^2 \Omega_j}{\partial t_1{}^2} = \kappa_{j0}\frac{d_{j+3}}{D_j}\left(\sum_{k=1}^{3} e^{-\tilde{\alpha}_k z_2}\frac{|\Omega_{ck}|^2 + |d_{k+3}|^2}{|D_k|^2}|\Omega_k|^2\right)\Omega_j, \quad (11)$$

where K_{12}, K_{22}, and K_{32} characterize the group-velocity dispersion of the σ^-, π, and σ^+ polarized pulsed probe fields, respectively, and $\tilde{\alpha}_{1,2,3} = \lambda^2\alpha_{1,2,3}$, with $\alpha_{1,2,3} = 2\mathrm{Im}(K_{10,20,30})$.

After combining Eqs. (10) and (11) and returning to the original variables, we obtain the three-coupled NLS equations aroused from SPM and XPM [3] ($j = 1,2,3$):

$$\frac{\partial F_j}{\partial z} + \delta_j\frac{\partial F_j}{\partial \tau} + \frac{iK_{j2}}{2}\frac{\partial^2 F_j}{\partial \tau^2} = -i\left(\sum_{k=1}^{3} e^{-\alpha_k z} G_{jk}|F_k|^2\right)F_j, \quad (12)$$

where $F_j = \lambda\Omega_j$, $\tau = t - z/V_{g2}$, $\delta_j = 1/V_{gj} - 1/V_{g2}$, and

$$G_{1j} = \frac{\kappa_{10} d_4 (|\Omega_{cj}|^2 + \Delta_{tj}{}^2 + \gamma_{j+3}{}^2)}{D_1 |D_j|^2}, \quad (13a)$$

$$G_{2j} = \frac{\kappa_{20} d_5 (|\Omega_{cj}|^2 + \Delta_{tj}{}^2 + \gamma_{j+3}{}^2)}{D_2 |D_j|^2}, \quad (13b)$$

$$G_{3j} = \frac{\kappa_{30} d_6 (|\Omega_{cj}|^2 + \Delta_{tj}{}^2 + \gamma_{j+3}{}^2)}{D_3 |D_j|^2}. \quad (13c)$$

We note that the coefficients G_{jj} characterize the SPM, and $G_{jk} (j \neq k, j,k = 1,2,3)$ characterize the XPM of the σ^-, π, and σ^+ polarized pulsed probe fields, respectively.

Through inspection of Eqs. (13a)–(13c), we find that nonlinear evolution Eqs. (12) generally have complex coefficients and hence do not allow soliton solutions. However, because of the contribution of three strong control fields, the imaginary parts of these complex coefficients may be much smaller than their corresponding real parts, while simultaneously, the absorption of the probe fields can be almost completely suppressed under appropriate conditions, which results in $G_{jk} = G_{jkr} + iG_{jki} \simeq G_{jkr}$, $K_{j2} = K_{j2r} + iK_{j2i} \simeq K_{j2r}$, and $\exp(-\alpha_j L) \simeq 1$, with $|G_{jkr}| \gg |G_{jki}|$, $|K_{jkr}| \gg |K_{jki}|$, and L being the length of the atomic medium. These important properties lead Eqs. (12) to be solved by the F-expansion method [53], which has proven to be a very useful method for building periodic and soliton solutions of some nonlinear partial differential equations, and hence three-wave coupled temporal vector soliton solutions are possible that can propagate for an extended distance without significant deformation in this system. For this purpose, we adopt the dimensionless coordinates and variables $s = z/(2L_d)$, $\sigma = \tau/\tau_0$, and $q_j = F_j/U_0$ by defining the characteristic dispersion length $L_d = \tau_0^2/|K_{22r}|$ and the characteristic nonlinear length $L_n = 1/(|G_{22r}|U_0^2)$, with τ_0 and U_0 being the characteristic pulse length and the typical Rabi frequency of the probe fields, respectively. In this situation, Eqs. (12) can be reduced to the dimensionless three-coupled NLS equations as

$$i\frac{\partial q_j}{\partial s}+i\beta_{\delta_j}\frac{\partial q_j}{\partial\sigma}-\beta_j\frac{\partial^2 q_j}{\partial\sigma^2}-2\left(\sum_{k=1}^{3}\beta_{jk}|q_k|^2\right)q_j=0, \tag{14}$$

where we have set $L_d=L_n$ to achieve the proper balance between dispersion, SPM, and XPM, which leads to the generation of three-wave coupled temporal vector optical solitons that traverse this triple-Λ atomic system with ultraslow matched group velocities. Here we also define $\beta_{\delta_j}=2\text{sgn}(\delta_j)L_d/L_{\delta_j}$, $\beta_j=K_{j2r}/|K_{22r}|(j=1,2,3)$, and $\beta_{jk}=G_{jkr}/|G_{22r}|(j,k=1,2,3)$, with $L_{\delta_j}=\tau_0/|\delta_j|$ characterizing the walk-off length which results from the group-velocity mismatch (GVM), that is, the difference between the group velocities of the involved waves in a dispersive medium. If one ignores the XPM nonlinear effects among interacting waves, the walk-off length will lead three pulsed fields to separate from one another after propagating a distance. It is noteworthy that the existence of GVM terms β_{δ_j} in Eqs. (14) is the major obstacle to the generation of ultraslow three-wave coupled temporal vector optical solitons in this system. Fortunately, solitons can shift their frequencies (wavelengths) that slow down the faster-moving pulse while simultaneously making the slower-moving one speed up to three polarized pulsed fields continuing to overlap indefinitely, which is the main mechanism for generating vector optical solitons in nonlinear system under a GVM. Therefore, in order to obtain soliton solutions from Eqs. (14), we suppose the solutions of this equations have the forms of $q_j(s,\sigma)=\mathcal{Q}_j(s,\sigma)\exp[-i\beta_{\delta_j}(s\beta_{\delta_j}-2\sigma)/(4\beta_j)]$ and then obtain the ordinary differential equation for $\mathcal{Q}_j(s,\sigma)$:

$$i\frac{\partial\mathcal{Q}_j}{\partial s}-\beta_j\frac{\partial^2\mathcal{Q}_j}{\partial\sigma^2}=2(\beta_{j1}|\mathcal{Q}_1|^2+\beta_{j2}|\mathcal{Q}_2|^2+\beta_{j3}|\mathcal{Q}_3|^2)\mathcal{Q}_j, \tag{15}$$

which may admit solutions describing the bright-bright-bright, bright-bright-dark, bright-dark-bright, bright-dark-dark, and dark-dark-dark vector solitons [53], depending on the choice of parameter values in this triple-Λ atomic system.

Following Ref. [53], we attempt to obtain bright-bright-bright, bright-bright-dark, bright-dark-bright, bright-dark-dark, and dark-dark-dark vector soliton solutions of Eqs. (15) when disregarding the small imaginary parts of the coefficients. The bright-bright-bright vector soliton solution is given by

$$\mathcal{Q}_1(s,\sigma)=\mathcal{C}_1\text{sech}(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{11}\sigma+i\mathcal{F}_{12}s), \tag{16a}$$

$$\mathcal{Q}_2(s,\sigma)=\mathcal{C}_2\text{sech}(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{21}\sigma+i\mathcal{F}_{22}s), \tag{16b}$$

$$\mathcal{Q}_3(s,\sigma)=\mathcal{C}_3\text{sech}(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{31}\sigma+i\mathcal{F}_{32}s), \tag{16c}$$

which indicates that the σ^-, π, and σ^+ polarized pulsed probe fields all evolve into bright solitons. Here we have defined $\mathcal{B}=2\mathcal{A}\mathcal{F}_{j1}\beta_j(j=1,2,3)$, $\mathcal{F}_{12}=\beta_1(\mathcal{F}_{11}^2-\mathcal{A}^2)$, $\mathcal{F}_{22}=\beta_2(\mathcal{F}_{21}^2-\mathcal{A}^2)$, $\mathcal{F}_{32}=\beta_3(\mathcal{F}_{31}^2-\mathcal{A}^2)$, $\mathcal{C}_1^2=\mathcal{A}^2[\beta_2(\beta_{13}\beta_{32}-\beta_{12}\beta_{33})+\beta_3(\beta_{13}\beta_{22}-\beta_{12}\beta_{23})]/[\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$, $\mathcal{C}_2^2=\mathcal{A}^2(\beta_1\beta_{23}\beta_{31}+\beta_3\beta_{11}\beta_{23})/[\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$, and $\mathcal{C}_3^2=\mathcal{A}^2[\beta_1\beta_{22}\beta_{31}-\beta_2(\beta_{12}\beta_{31}-\beta_{11}\beta_{32})+\beta_3\beta_{11}\beta_{22}]/[\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$.

The bright-bright-dark vector soliton solution can be deduced from Eqs. (15), in which two bright solitons appear in the σ^- and π polarized pulsed probe fields and the dark one appears in the σ^+ polarized pulsed probe fields:

$$\mathcal{Q}_1(s,\sigma)=\mathcal{C}_1\text{sech}(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{11}\sigma+i\mathcal{F}_{12}s), \tag{17a}$$

$$\mathcal{Q}_2(s,\sigma)=\mathcal{C}_2\text{sech}(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{21}\sigma+i\mathcal{F}_{22}s), \tag{17b}$$

$$\mathcal{Q}_3(s,\sigma)=\mathcal{C}_3\tanh(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{31}\sigma+i\mathcal{F}_{32}s), \tag{17c}$$

where $\mathcal{B}=2\mathcal{A}\mathcal{F}_{j1}\beta_j(j=1,2,3)$, $\mathcal{F}_{12}=\beta_1(\mathcal{F}_{11}^2-\mathcal{A}^2)-2\mathcal{C}_3^2\beta_{13}$, $\mathcal{F}_{22}=\beta_2(\mathcal{F}_{21}^2-\mathcal{A}^2)-2\mathcal{C}_3^2\beta_{23}$, $\mathcal{F}_{32}=\beta_3\mathcal{F}_{31}^2-2\mathcal{C}_3^2\beta_{33}$, $\mathcal{C}_1^2=\mathcal{A}^2[\beta_2(\beta_{13}\beta_{32}-\beta_{12}\beta_{33})-\beta_3(\beta_{13}\beta_{22}-\beta_{12}\beta_{23})]/[\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$, $\mathcal{C}_2^2=\mathcal{A}^2(\beta_1\beta_{23}\beta_{31}-\beta_3\beta_{11}\beta_{23})/[\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$, and $\mathcal{C}_3^2=\mathcal{A}^2[\beta_1\beta_{22}\beta_{31}-\beta_2(\beta_{12}\beta_{31}-\beta_{11}\beta_{32})-\beta_3\beta_{11}\beta_{22}]/[-\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$.

The bright-dark-bright vector soliton solution of Eqs. (15) reads as

$$\mathcal{Q}_1(s,\sigma)=\mathcal{C}_1\text{sech}(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{11}\sigma+i\mathcal{F}_{12}s), \tag{18a}$$

$$\mathcal{Q}_2(s,\sigma)=\mathcal{C}_2\tanh(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{21}\sigma+i\mathcal{F}_{22}s), \tag{18b}$$

$$\mathcal{Q}_3(s,\sigma)=\mathcal{C}_3\text{sech}(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{31}\sigma+i\mathcal{F}_{32}s), \tag{18c}$$

which shows that the σ^- and σ^+ polarized pulsed probe fields evolve into bright solitons and the π polarized pulsed probe field evolves into a dark one. The corresponding parameters are given by $\mathcal{B}=2\mathcal{A}\mathcal{F}_{j1}\beta_j(j=1,2,3)$, $\mathcal{F}_{12}=\beta_1(\mathcal{F}_{11}^2-\mathcal{A}^2)-2\mathcal{C}_2^2\beta_{12}$, $\mathcal{F}_{22}=\beta_2\mathcal{F}_{21}^2-2\mathcal{C}_2^2\beta_{22}$, $\mathcal{F}_{32}=\beta_3(\mathcal{F}_{31}^2-\mathcal{A}^2)-2\mathcal{C}_2^2\beta_{32}$, $\mathcal{C}_1^2=\mathcal{A}^2[\beta_2(\beta_{13}\beta_{32}-\beta_{12}\beta_{33})-\beta_3(\beta_{13}\beta_{22}-\beta_{12}\beta_{23})]/[\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$, $\mathcal{C}_2^2=\mathcal{A}^2(\beta_1\beta_{23}\beta_{31}-\beta_3\beta_{11}\beta_{23})/[-\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$, and $\mathcal{C}_3^2=\mathcal{A}^2[\beta_1\beta_{22}\beta_{31}-\beta_2(\beta_{12}\beta_{31}-\beta_{11}\beta_{32})-\beta_3\beta_{11}\beta_{22}]/[\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$.

One can also obtain the bright-dark-dark vector soliton solution from Eqs. (15):

$$\mathcal{Q}_1(s,\sigma)=\mathcal{C}_1\text{sech}(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{11}\sigma+i\mathcal{F}_{12}s), \tag{19a}$$

$$\mathcal{Q}_2(s,\sigma)=\mathcal{C}_2\tanh(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{21}\sigma+i\mathcal{F}_{22}s), \tag{19b}$$

$$\mathcal{Q}_3(s,\sigma)=\mathcal{C}_3\tanh(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{31}\sigma+i\mathcal{F}_{32}s), \tag{19c}$$

which describe the σ^- polarized pulsed probe field that evolves into a bright soliton and the π and σ^+ polarized pulsed probe field that evolve into dark solitons in the triple-Λ atomic system. Here we have $\mathcal{B}=2\mathcal{A}\mathcal{F}_{j1}\beta_j(j=1,2,3)$, $\mathcal{F}_{12}=\beta_1(\mathcal{F}_{11}^2-\mathcal{A}^2)-2\mathcal{C}_2^2\beta_{12}-2\mathcal{C}_3^2\beta_{13}$, $\mathcal{F}_{22}=\beta_2\mathcal{F}_{21}^2-2\mathcal{C}_2^2\beta_{22}-2\mathcal{C}_3^2\beta_{23}$, $\mathcal{F}_{32}=\beta_3\mathcal{F}_{31}^2-2\mathcal{C}_2^2\beta_{32}-2\mathcal{C}_3^2\beta_{33}$, $\mathcal{C}_1^2=\mathcal{A}^2[\beta_2(\beta_{13}\beta_{32}-\beta_{12}\beta_{33})-\beta_3(\beta_{13}\beta_{22}-\beta_{12}\beta_{23})]/[\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$, $\mathcal{C}_2^2=\mathcal{A}^2(\beta_1\beta_{23}\beta_{31}-\beta_3\beta_{11}\beta_{23})/[-\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$, and $\mathcal{C}_3^2=\mathcal{A}^2[\beta_1\beta_{22}\beta_{31}-\beta_2(\beta_{12}\beta_{31}-\beta_{11}\beta_{32})-\beta_3\beta_{11}\beta_{22}]/[-\beta_{31}(\beta_{12}\beta_{23}-\beta_{22}\beta_{13})]$.

A dark-dark-dark vector soliton solution is given by

$$\mathcal{Q}_1(s,\sigma)=\mathcal{C}_1\tanh(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{11}\sigma+i\mathcal{F}_{12}s), \tag{20a}$$

$$\mathcal{Q}_2(s,\sigma)=\mathcal{C}_2\tanh(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{21}\sigma+i\mathcal{F}_{22}s), \tag{20b}$$

$$\mathcal{Q}_3(s,\sigma)=\mathcal{C}_3\tanh(\mathcal{A}\sigma+\mathcal{B}s)\exp(i\mathcal{F}_{31}\sigma+i\mathcal{F}_{32}s). \tag{20c}$$

In this situation, the σ^-,π, and σ^+ polarized pulsed probe fields all evolve into dark solitons. Here we define

$\mathcal{B} = 2\mathcal{A}\mathcal{F}_{j1}\beta_j (j = 1,2,3)$, $\mathcal{F}_{12} = \beta_1(\mathcal{F}_{11}^2 + 2\mathcal{A}^2)$, $\mathcal{F}_{22} = \beta_2(\mathcal{F}_{21}^2 + 2\mathcal{A}^2)$, $\mathcal{F}_{32} = \beta_3(\mathcal{F}_{31}^2 + 2\mathcal{A}^2)$, $\mathcal{C}_1^2 = \mathcal{A}^2[-\beta_2(\beta_{13}\beta_{32} - \beta_{12}\beta_{33}) + \beta_3(\beta_{13}\beta_{22} - \beta_{12}\beta_{23})]/[\beta_{31}(\beta_{12}\beta_{23} - \beta_{22}\beta_{13})]$, $\mathcal{C}_2^2 = \mathcal{A}^2(-\beta_1\beta_{23}\beta_{31} + \beta_3\beta_{11}\beta_{23})/[-\beta_{31}(\beta_{12}\beta_{23} - \beta_{22}\beta_{13})]$, and $\mathcal{C}_3^2 = \mathcal{A}^2[-\beta_1\beta_{22}\beta_{31} + \beta_2(\beta_{12}\beta_{31} - \beta_{11}\beta_{32}) + \beta_3\beta_{11}\beta_{22}]/[\beta_{31}(\beta_{12}\beta_{23} - \beta_{22}\beta_{13})]$.

It is worthwhile to point out that these three-wave coupled temporal vector optical solitons described by Eqs. (16), (17), (18), (19), and (20) are generated from the delicate balance of the dispersion, SPM, and XPM effects and are allowed in this seven-level triple-Λ atomic system with matched ultraslow group velocity. In the preceding equations, the coefficients $\mathcal{A}$ and $\mathcal{B}$ are two free parameters ($\mathcal{A} \neq 0$). And to obtain these equations, we have made use of the relation $G_{jj}G_{kk} = G_{jk}G_{kj} (j,k = 1,2,3; j \neq k)$, which can be seen from the definition expressions of the SPM and XPM coefficients shown in Eqs. (13a)–(13c).

To generate the ultraslow three-wave coupled temporal vector optical solitons, we now consider a practical example for a realistic atomic medium driven in a triple-Λ configuration of energy levels. We consider a cold Na atomic vapor with the decay rates $2\gamma_1 \simeq 2\gamma_2 \simeq 2\gamma_3 \simeq 1.2 \times 10^8 \mathrm{s}^{-1}$ and $2\gamma_4 \simeq 2\gamma_5 \simeq 2\gamma_6 \simeq 2.0 \times 10^4 \mathrm{s}^{-1}$. We take $\kappa_{10} \simeq \kappa_{20} \simeq \kappa_{30} \simeq 1.0 \times 10^9 \mathrm{cm}^{-1}\mathrm{s}^{-1}$, $2\Omega_{c1} \simeq 2\Omega_{c2} \simeq 2\Omega_{c3} = 2.0 \times 10^8 \mathrm{s}^{-1}$, $\Delta_{t1} \simeq \Delta_{t2} \simeq \Delta_{t3} = 2.0 \times 10^6 \mathrm{s}^{-1}$, $\Delta_s = -1.0 \times 10^9 \mathrm{s}^{-1}$, and $\Delta_1 = 2.0 \times 10^6 \mathrm{s}^{-1}$. Then, with the preceding parameters, we obtain $K_{10} \simeq K_{20} \simeq K_{30} \simeq 0.167 + 0.003i \,\mathrm{cm}^{-1}$, $K_{11} \simeq (6.953 + 0.127i) \times 10^{-8} \mathrm{cm}^{-1}\mathrm{s}$, $K_{21} \simeq (6.948 + 0.127i) \times 10^{-8} \mathrm{cm}^{-1}\mathrm{s}$, $K_{31} \simeq (6.944 + 0.127i) \times 10^{-8} \mathrm{cm}^{-1}\mathrm{s}$, $K_{12} \simeq (-1.153 + 0.038i) \times 10^{-14} \mathrm{cm}^{-1}\mathrm{s}^2$, $K_{22} \simeq (-1.154 + 0.038i) \times 10^{-14} \mathrm{cm}^{-1}\mathrm{s}^2$, $K_{12} \simeq (-1.155 + 0.038i) \times 10^{-14} \mathrm{cm}^{-1}\mathrm{s}^2$, $G_{11} \simeq (1.159 + 0.016i) \times 10^{-17} \mathrm{cm}^{-1}\mathrm{s}^2$, $G_{22} \simeq (1.157 + 0.016i) \times 10^{-17} \mathrm{cm}^{-1}\mathrm{s}^2$, $G_{33} \simeq (1.156 + 0.016i) \times 10^{-17} \mathrm{cm}^{-1}\mathrm{s}^2$, $G_{12} \simeq G_{21} \simeq (1.158 + 0.016i) \times 10^{-17} \mathrm{cm}^{-1}\mathrm{s}^2$, $G_{13} \simeq (1.157 + 0.016i) \times 10^{-17} \mathrm{cm}^{-1}\mathrm{s}^2$, $G_{31} \simeq (1.158 + 0.016i) \times 10^{-17} \mathrm{cm}^{-1}\mathrm{s}^2$, and $G_{23} \simeq G_{32} \simeq (1.157 + 0.016i) \times 10^{-17} \mathrm{cm}^{-1}\mathrm{s}^2$. Notice that the imaginary parts of these quantities are indeed much smaller than their relevant real parts, and thus we can ignore the small imaginary parts to obtain the dimensionless three-coupled NLS Eqs. (14). With these quantities, we have $L_d \simeq 0.555$ cm, $L_{\delta_1} \simeq 1727.8$ cm, and $L_{\delta_2} \simeq 1729.5$ cm, with the negligible absorption $\alpha_1 \simeq \alpha_2 \simeq \alpha_3 \simeq 0.0047 \,\mathrm{cm}^{-1}$, $\tau_0 = 8.0 \times 10^{-8}$ s, and $U_0 \simeq 3.95 \times 10^8 \,\mathrm{s}^{-1}$. The group velocities of the σ^-, π, and σ^+ polarized pulsed probe fields are given by $V_{g1}/c \simeq 4.794 \times 10^{-4}$, $V_{g2}/c \approx 4.797 \times 10^{-4}$, and $V_{g3}/c \approx 4.800 \times 10^{-4}$, respectively, which means that the σ^-, π, and σ^+ polarized pulsed probe fields of the vector optical solitons propagate with nearly matched, ultraslow propagating velocities compared with c.

IV. DISCUSSION AND CONCLUSION

Before concluding, it is worthwhile to point out some important differences between the theoretical scheme proposed in this article and the system proposed in Ref. [54]. First, the present system is a triple-Λ atomic system in which three control fields operate in a stimulated Raman emission mode and a powerful and sophisticated perturbative method of multiple scales [4] is applied to solve equations of motion of the system. Second, although the nonlinear equations derived in this article and easier work are of similar form, they appear in two different physical situations. In fact, this work deals with the nonlinear coupling between three pulsed waves with the same frequency but belonging to different polarizations, and therefore, in substance, this differs from the nonlinear interaction between three waves with different frequencies. Last but not the least, this work gives systematically all types of possible three-wave vector solitons which may be generated via our system because of the wide parameter regimes of the triple-Λ atomic system, while the previous model [54] only gives the type of bright-bright-bright vector solitons.

In conclusion, we have analyzed the nonlinear dynamics of a circularly σ^-, a linearly π, and a circularly σ^+ polarized field converted from one weak linear-polarized probe field in a cold seven-level triple-Λ atomic system under Raman excitation. In the presence of three coherent driving control fields, the linear as well as nonlinear dispersion are dramatically enhanced, while simultaneously, the absorptions of the σ^-, π, and σ^+ polarized probe fields are suppressed in the medium. By means of the multiple scales technique, we derive three-coupled NLS equations which admit three-wave vector optical soliton solutions describing bright-bright-bright, bright-bright-dark, bright-dark-bright, bright-dark-dark, and dark-dark-dark vector solitons. We have shown that the three-wave vector optical solitons are produced from the perfect balance of the group-velocity dispersion, SPM, and XPM effects and can propagate through this atomic system with nearly matched, ultraslow propagating velocities compared with the speed of light in a vacuum.

ACKNOWLEDGMENTS

We would like to acknowledge Y. Wu for his enlightening suggestions and encouragement. The work is supported in part by the NSF of China (Grant Nos. 60478029, 90503010, 10575040, 10874050, 10975054, 10634060, and 10704017), the National Fundamental Research Program of China (Grant No. 2005CB724508), and the Research Fund for the Doctoral Program of Higher Education of China (Grant No. 200804870051). The authors also acknowledge the referees for their valuable comments and suggestions, which significantly improved the article.

[1] F. T. Hioe, Phys. Rev. Lett. **82**, 1152 (1999).
[2] Q. H. Park and H. J. Shin, Phys. Rev. E **61**, 3093 (2000).
[3] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, New York, 2001).
[4] A. Hasegawa and M. Matsumoto, *Optical Solitons in Fibers* (Springer, Berlin, 2003).
[5] K. W. Chow and D. W. C. Lai, Phys. Rev. E **68**, 017601 (2003).
[6] D. V. Skryabin, A. V. Yulin, and A. I. Maimistov, Phys. Rev. Lett. **96**, 163904 (2006).
[7] R. Grinfeld and B. A. Malomed, Phys. Rev. A **77**, 035801 (2008).
[8] T. Kanna and M. Lakshmanan, Phys. Rev. Lett. **86**, 5043 (2001).

[9] C. Montes, A. Picozzi, C. Durniak, and M. Taki, Eur. Phys. J. Special Topics **173**, 167 (2009).
[10] H. Schmidt and A. Imamoğlu, Opt. Lett. **21**, 1936 (1996); M. D. Lukin and A. Imamoğlu, Phys. Rev. Lett. **84**, 1419 (2000).
[11] S. E. Harris, Phys. Today **50**, 36 (1997); S. E. Harris and L. V. Hau, Phys. Rev. Lett. **82**, 4611 (1999).
[12] E. Paspalakis and P. L. Knight, Phys. Rev. A **66**, 015802 (2002).
[13] Y. Wu, L. Wen, and Y. Zhu, Opt. Lett. **28**, 631 (2003); Y. Wu and X. Yang, Phys. Rev. A **71**, 053806 (2005).
[14] M. Fleischhauer, A. Imamoğlu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
[15] H. Kang and Y. Zhu, Phys. Rev. Lett. **91**, 093601 (2003).
[16] D. Petrosyan and Y. P. Malakyan, Phys. Rev. A **70**, 023822 (2004).
[17] Z. B. Wang, K. P. Marzlin, and B. C. Sanders, Phys. Rev. Lett. **97**, 063901 (2006).
[18] X. Yang, S. Li, C. Zhang, and H. Wang, J. Opt. Soc. Am. B **26**, 1423 (2009).
[19] A. Joshi and M. Xiao, Phys. Rev. Lett. **91**, 143904 (2003).
[20] A. Joshi, A. Brown, H. Wang, and M. Xiao, Phys. Rev. A **67**, 041801(R) (2003).
[21] A. Joshi, W. Yang, and M. Xiao, Phys. Rev. A **70**, 041802(R) (2004).
[22] J. H. Li, X. Y. Lü, J. M. Luo, and Q. J. Huang, Phys. Rev. A **74**, 035801 (2006).
[23] Y. Wu and L. Deng, Opt. Lett. **29**, 1144 (2004); Y. Wu, M. G. Payne, E. W. Hagley, and L. Deng, Phys. Rev. A **69**, 063803 (2004).
[24] X. Yang and Y. Wu, J. Opt. B **7**, 54 (2005).
[25] H. Jing, X.-J. Liu, M.-L. Ge, and M.-S. Zhan, Phys. Rev. A **71**, 062336 (2005).
[26] X. M. Hu and J. H. Zou, Phys. Rev. A **78**, 045801 (2008).
[27] A. M. Marino, R. C. Pooser, V. Boyer, and P. D. Lett, Nature (London) **457**, 859 (2009).
[28] G. X. Li, S. S. Ke, and Z. Ficek, Phys. Rev. A **79**, 033827 (2009).
[29] H. T. Tan, H. X. Xia, and G. X. Li, Phys. Rev. A **79**, 063805 (2009).
[30] X.-Y. Lü, P. Huang, W.-X. Yang, and X. Yang, Phys. Rev. A **80**, 032305 (2009).
[31] Y. Li and M. Xiao, Opt. Lett. **21**, 1064 (1996).
[32] B. Lu, W. H. Burkett, and M. Xiao, Opt. Lett. **23**, 804 (1998).
[33] Y. Wu, X. Yang, C. P. Sun, X. J. Zhou, and Y. Q. Wang, Phys. Rev. A **61**, 043604 (2000).
[34] Y. Wu, J. Saldana, and Y. Zhu, Phys. Rev. A **67**, 013811 (2003); Y. Wu and X. Yang, *ibid.* **70**, 053818 (2004); Opt. Lett. **30**, 311 (2005).
[35] H. J. Li and G. Huang, Phys. Rev. A **76**, 043809 (2007).
[36] Y. Wu and L. Deng, Opt. Lett. **29**, 2064 (2004).
[37] Y. Wu and L. Deng, Phys. Rev. Lett. **93**, 143904 (2004).
[38] C. Hang and G. Huang, Phys. Rev. A **77**, 033830 (2008).
[39] L.-G. Si, J.-B. Liu, X.-Y. Lü, and X. Yang, J. Phys. B **41**, 215504 (2008); L.-G. Si, W.-X. Yang, J.-B. Liu, J. Li, and X. Yang, Opt. Express **17**, 7771 (2009).
[40] D. Petrosyan, J. Opt. B **7**, S141 (2005).
[41] X. J. Liu, H. Jing, and M. L. Ge, Phys. Rev. A **70**, 055802 (2004).
[42] G. Huang, L. Deng, and M. G. Payne, Phys. Rev. E **72**, 016617 (2005); C. Hang, G. Huang, and L. Deng, *ibid.* **73**, 036607 (2006).
[43] L.-G. Si, X.-Y. Lü, J.-H. Li, X. Hao, and M. Wang, J. Phys. B **42**, 225405 (2009); L.-G. Si, X.-Y. Lü, X. Hao, and J.-H. Li, *ibid.* **43**, 065403 (2010).
[44] Y. Wu, Phys. Rev. A **71**, 053820 (2005).
[45] L. Deng, M. G. Payne, G. Huang, and E. W. Hagley, Phys. Rev. E **72**, 055601(R) (2005).
[46] G. Huang, K. Jiang, M. G. Payne, and L. Deng, Phys. Rev. E **73**, 056606 (2006).
[47] L. G. Si, W. X. Yang, and X. Yang, J. Opt. Soc. Am. B **26**, 478 (2009); L. G. Si, W. X. Yang, X. Y. Lü, J. Li, and X. Yang, Eur. Phys. J. D **55**, 161 (2009).
[48] A. C. Scott, Phys. Scr. **29**, 279 (1984).
[49] N. Akhmediev and A. Ankiewicz, *Solitons: Nonlinear Pulses and Beams* (Chapman and Hall, London, 1997).
[50] N. Akhmediev, W. Krolikowski, and A. W. Snyder, Phys. Rev. Lett. **81**, 4632 (1998); A. Ankiewicz, W. Krolikowski, and N. N. Akhmediev, Phys. Rev. E **59**, 6079 (1999).
[51] M. J. Ablowitz, G. Biondini, and L. A. Ostrovsky, Chaos **10**, 471 (2000); A. Hasegawa, *ibid.* **10**, 475 (2000).
[52] Y. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic, San Diego, 2003).
[53] E. Yomba and G. R. Sell, J. Math. Phys. **50**, 053518 (2008).
[54] W.-X. Yang, A.-X. Chen, L.-G. Si, K. Jiang, X. Yang, and R.-K. Lee, Phys. Rev. A **81**, 023814 (2010).
[55] [http://steck.us/alkalidata].
[56] H. Kang, G. Hernandez, J. Zhang, and Y. Zhu, Phys. Rev. A **73**, 011802(R) (2006).
[57] Y. Zhang, A. W. Brown, and M. Xiao, Phys. Rev. A **74**, 053813 (2006).
[58] X.-J. Liu, X. Liu, L. C. Kwek, and C. H. Oh, Phys. Rev. Lett. **98**, 026602 (2007).
[59] Y. Wu and X. Yang, Phys. Rev. B **76**, 054425 (2007).