Spectral shifts and switches in random fields upon interaction with negative-phase materials

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Spectral shifts in stochastic beam-like fields on interaction with layers of positive- and negative-phase materials are examined on the basis of the ABCD-matrix approach and generalized Huygens-Fresnel principle. It is found that boundaries between such materials may cause spectral switches. Effect of absorption of negative-phase materials on the beam spectrum is discussed. Our results may find applications in connection with spectrum-selection optical interconnects, spectrally encoded information transfer, image formation in systems involving negative-phase materials, and geometrically tunable metamaterials.

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I. INTRODUCTION

Correlation-induced spectral changes in stochastic light fields propagating in free space have been discovered by Wolf [1] (see also [2]) and remained a subject of acute interest for a number of years. In particular, it was found that shifts toward both higher and lower frequencies are possible, which are called blue shifts and red shifts, respectively. Similar spectral changes were later found in more complex situations as well, for instance, on propagation of light scattered from static random media [3,4]. It was also demonstrated that spectral changes can be used for solution of the inverse problems of scattering [5].

Quite recently, so-called spectral switches were predicted and confirmed experimentally on passing of a random light beam through an aperture [6]. A spectral switch is a phenomenon in which the alternation of spectral shifts might occur: the light spectrum is first blue shifted and then red shifted. Spectral switches were also found to be pertinent to light propagation in image-forming optical systems, such as combinations of thin lenses and mirrors [7]. The phenomenon of such switches is explained by the fact that in a system involving free-space propagation and passage through a lens, the competing mechanisms of focusing and diffraction affect the transverse coherence properties of the beam at different planes, which, in their turn, influence the beam's spectral composition.

In this paper, we tackle the possibility of having spectral switches interacting with stochastic light beams in optical systems composed of several alternating layers of positive- (PPMs) and negative-phase materials (NPMs) [8,9]. NPMs, unlike PPMs, are still considered as anomalous media, in which the dot product of a Poynting vector $\mathbf{s}$ and the wave vector $\mathbf{k}$ is negative, that is, $\mathbf{s}\cdot\mathbf{k} < 0$, and the energy flows in a direction opposite to the direction of wave front's evolution [9].

Recently, general aspects of propagation of random fields in PPMs and NPMs were treated by the authors [10]. It was found, in particular, that the major statistical properties, such as spectral density, states of coherence, and polarization of a beam may exhibit an anomalous evolution on interaction with layered PPM/NPM media. For example, the degree of coherence of a beamlike field may decrease after passing through the NPM/free space interface, unlike on propagation in free space, where it usually grows with distance from the source [11].

In this paper, we explore the subject of correlation-induced spectral changes and switches in stochastic beams on passing through several alternating layers of PPMs and NPMs. We confine ourselves to scalar theory of beamlike stochastic fields and demonstrate the results using Gaussian Schell-model beams, being very broad class of such optical fields. Since the materials that exhibit negative-phase phenomenon can be practically synthesized so far only in the regime of quite significant absorption, we also include the relating mechanism in our study.

We find that on propagation in a layer of either a PPM or a NPM for a sufficiently long distance, the blue shift of the beam spectrum is generated. However, if the beam impinges on the PPM/NPM interface, the red shift is generated but is gradually mitigated by propagation in the NPM layer, until it eventually becomes blue again. The same is valid for the NPM/PPM interface. We show that it is possible for a given beam and layers to find a propagation distance at which the spectrum, after being red-shifted by the interface and blue-shifted by propagation, comes back to its original state. If the absorption is present in the layers, a stronger red shift is generated by the interface and the propagation-induced blue shift becomes insufficient to compensate for it. Under these circumstances, the spectrum can no longer be reconstructed.

The paper is organized as follows: in Sec. II, we provide the theory of propagation of spectral density in materials with arbitrary refractive and absorptive properties and describe the layered optical system that we use for illustrating spectral shifts and switches; in Sec. III, we give a numerical example dealing with a Gaussian Schell-model beam with initial narrow Gaussian spectral composition; and in Sec. IV, we summarize our findings and outline possible applications.

II. THEORETICAL ANALYSIS

We begin by a brief introduction of the media in which spectral switches in optical fields are demonstrated. Suppose that the layers of the PPMs and the NPMs are confined in planes transverse to the direction z, coinciding with axis of

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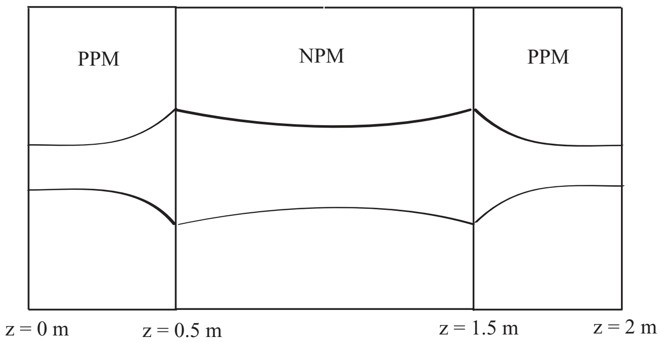

FIG. 1. Illustrating the layered medium arrangement.

beam propagation. Although our theoretical analysis is valid for any combination of layers, we are interested here in a particular setup which was previously used by Pendry [12] in his well-known paper on perfect imaging (see Fig. 1). For our purposes, such a system contains enough layers, all having proper width proportions, to illustrate various spectrum-related phenomena.

The $ABCD$ matrices for propagation in a medium at distance l and for passage through a boundary with refractive indexes n_1 and n_2 are well known [13]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_a = \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}, \tag{1}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_b = \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix}. \tag{2}$$

Then a system in Fig. 1 in which the PPM of length l_1, NPM of length l_2, and PPM of length l_3 are combined can be described as a product of corresponding matrices which, after simplification, reduces to the form

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & l_1 + l_3 + l_2 \frac{n_p}{n_n} \\ 0 & 1 \end{bmatrix}. \tag{3}$$

Let us now assume that fluctuations in the beam generated in the source plane $z = 0$ are statistically stationary in the wide sense and hence may be described with the help of the cross-spectral density function [11]

$$W^{(0)}(x_1,y_1,x_2,y_2,z=0;\omega) = \langle E^{(0)*}(x_1,y_1,z=0;\omega) \times E^{(0)}(x_2,y_2,z=0;\omega)\rangle, \tag{4}$$

where *denotes complex conjugate and $\langle\rangle$ stand for the average over the ensemble of monochromatic realizations [11]. Then, according to the generalized Huygens-Fresnel principle, the cross-spectral density function at distance z from the source plane can be found from the integral [14]

$$\begin{aligned} W(x_1,y_1,x_2,y_2,z;\omega) &= \frac{|K|^2}{(2\pi B)^2}\exp(2K_i z)\iiiint \exp\left\{-\frac{iA}{2B}\left[K\left(x_1'^2+y_1'^2\right)\right.\right. \\ &\quad \left.\left. -K^*\left(x_2'^2+y_2'^2\right)\right]\right\}\exp\left\{\frac{i}{B}\left[K(x_1x_1'+y_1y_1')\right.\right. \\ &\quad \left.\left. -K^*(x_2x_2'+y_2y_2')\right]\right\}\exp\left\{-\frac{iD}{2B}\left[K\left(x_1^2+y_1^2\right)\right.\right. \\ &\quad \left.\left. -K^*\left(x_2^2+y_2^2\right)\right]\right\}W^{(0)}(x_1',y_1',x_2',y_2',z=0;\omega) \\ &\quad \times dx_1'dy_1'dx_2'dy_2'. \end{aligned} \tag{5}$$

Here $K = K_r + iK_i$ is the wave number in the medium such that $K_r = kn_r$, k being the wave number in free space, n_r being the real index of refraction, and K_i being the parameter characterizing gain ($K_i > 0$) or absorbtion ($K_i < 0$). In what follows, we will be interested in evaluation of the normalized spectral density of the beam at distance $z \geqslant 0$ from the source plane and at any transverse location (x,y) given by the expression [11]

$$S_N(x,y,z;\omega) = \frac{W(x,y,x,y,z;\omega)}{\int_0^\infty W(x,y,x,y,z;\omega)d\omega}. \tag{6}$$

By substituting from Eq. (3) into Eq. (5) and using the result in Eq. (6), one can trace the evolution of the spectral density in the system of interest. Further, the shifted central frequency of the beam can be found from the expression

$$\omega_1 = \frac{\int_0^\infty \omega W(x,y,x,y,z;\omega)d\omega}{\int_0^\infty W(x,y,x,y,z;\omega)d\omega}. \tag{7}$$

The difference $\omega_1 - \omega_0$ is called the spectral shift, which is called blue shift if this value is positive and red shift if it is negative.

III. NUMERICAL EXAMPLES

In order to illustrate various possibilities for spectral changes and shifts numerically, we employ, as the beam model, the isotropic Gaussian Schell-model beams [11]. The cross-spectral density matrix of the beam in such a source plane $z = 0$ has the form

$$\begin{aligned} W^{(0)}(x_1',y_1',x_2',y_2',0;\omega) &= I_0(\omega)\exp\left(-\frac{x_1'^2+y_1'^2+x_2'^2+y_2'^2}{4\sigma^2}\right) \\ &\quad \times \exp\left(-\frac{(x_1'-x_2')^2+(y_1'-y_2')^2}{2\delta^2}\right), \end{aligned} \tag{8}$$

where the values of the parameters must obey the beam conditions [11]. We assume that the initial spectral composition is a single Gaussian spectral line, that is,

$$I_0(\omega) = \exp\left(-\frac{(\omega-\omega_0)^2}{2\bar{\omega}^2}\right), \tag{9}$$

being centered at frequency ω_0 and having rms width $\bar{\omega}^2$. By substituting from Eqs. (8) and (9) into Eq. (5) and evaluating the integrals, we find that (see also Ref. [14])

$$W(x_1,y_1,x_2,y_2,z;\omega) = \frac{I_0(\omega)|K|^2\delta^4 e^{2K_i z}}{B^2(4|g|^2-1)}\exp\left[-\frac{iD}{2B}\left[K\left(x_1^2+y_1^2\right)\right.\right.$$

$$-K^*\left(x_2^2+y_2^2\right)\Big]\Big]\exp\left[-\frac{\delta^2K^2g^*\left(x_1^2+y_1^2\right)}{B^2(4|g|^2-1)}\right]$$
$$\times\exp\left[\frac{\delta^2|K|^2(x_1x_2+y_1y_2)}{B^2(4|g|^2-1)}\right]\exp\left[-\frac{\delta^2K^{*2}g\left(x_2^2+y_2^2\right)}{B^2(4|g|^2-1)}\right], \tag{10}$$

where

$$g=\frac{1}{2}+\frac{\delta^2}{4\sigma^2}+i\frac{AK\delta^2}{2B}. \tag{11}$$

Finally, Eq. (10), if used in Eq. (6), provides the result for the spectral density at any position within the beam which passes through the layered medium of Fig. 1. We now evaluate this result numerically and demonstrate it via a number of curves. The following values of the parameters are chosen (otherwise different values are indicated in figure captions): $n_p=1$, $n_n=n_r+in_i=-1$, $\sigma=3\times10^{-3}$ m, $\delta=10^{-5}$ m, $\omega_0=10^{15}$ rad/s, and $\bar{\omega}=0.1\omega_0$.

To illustrate the typical switches in the spectrum, we show in Fig. 2 the spectra of the propagating beam [see Eq. (6)], on optical axis, at five transverse planes: $z=0$, 0.5, 1, 1.5, and 2 m from the source. One can clearly see from this figure that the alternation of blue shift and red shift of the original spectrum occurs after the beam passes through the interface between the PPM and the NPM layers.

Figures 3 and 4 show the behavior of the central frequency ω_1 [see Eq. (7)] of the propagating Gaussian Schell-model beam through the layered medium with different values of rms beam width σ (Fig. 3) and of the rms coherence width δ (Fig. 4). From these figures, we see that different values of source parameters significantly influence the resulting values of the central frequency when the beams passes through the medium. In particular, lower values of the rms beam width σ and the rms coherence width δ imply faster spectral changes with propagation distance. Also, for the layered medium

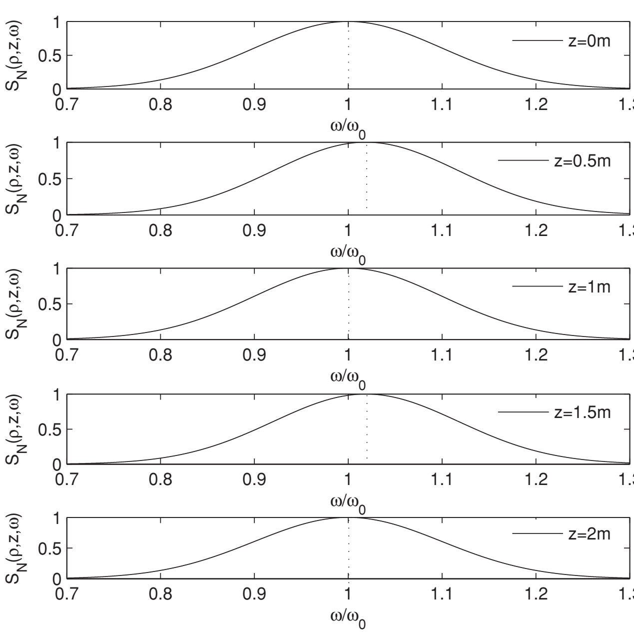

FIG. 2. Spectral changes in Gaussian Schell-model beams on propagation in layers of PPM and NPM.

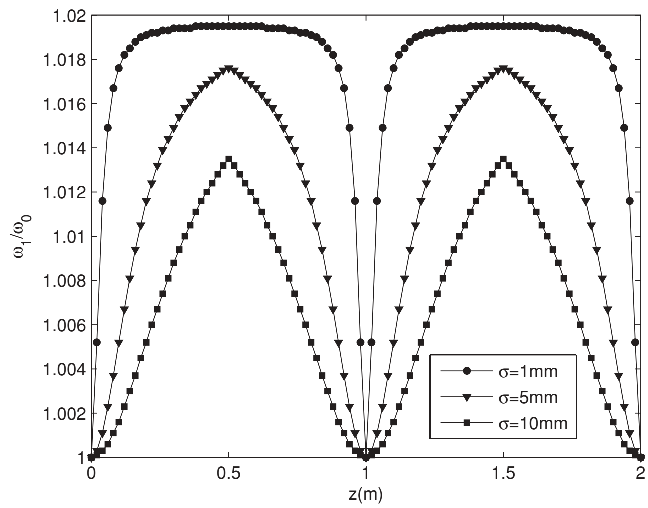

FIG. 3. Spectral changes in Gaussian Schell-model beams on propagation in layers of PPM and NPM for several values of rms beam width.

of interest, it is possible to retrieve the original value of the cental frequency of the propagating beams at certain distances ($z=1$ m, $z=2$ m). As was shown by the authors previously [10], other statistical properties of such beams (for instance, the degrees of coherence and polarization) may also be reconstructed in such optical arrangement.

Due to the fact that in practice the materials that induce negative refraction of light are highly absorptive, it is necessary to investigate how absorption might affect the spectral shifts and switches. As we demonstrate by all of the following numerical examples, the absorption in the NPM material induces stronger red shifts compared to a lossless NPM material with the same n_r. In particular, in Fig. 5 we show the behavior of the central frequency ω_1 of a typical Gaussian Schell-model beam propagating through the layered medium with absorptive NPM layer for several values of n_i. In this case, in the interval $z\in[0.5, 1\,\text{m}]$ the beam's spectrum

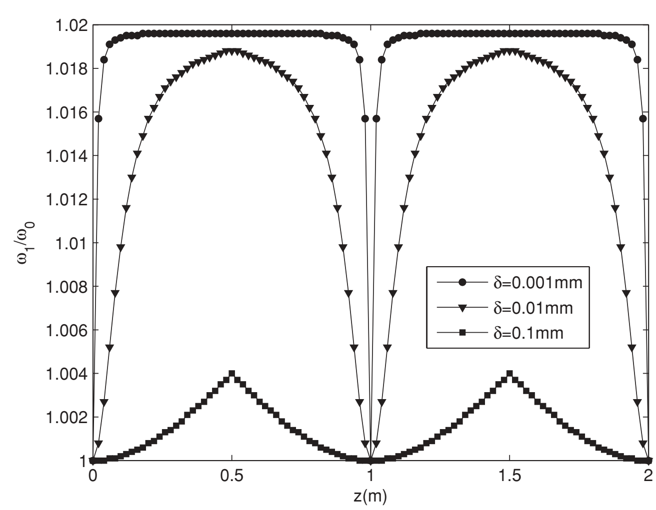

FIG. 4. Spectral changes in Gaussian Schell-model beams on propagation in layers of PPM and NPM for several values of rms coherence width.

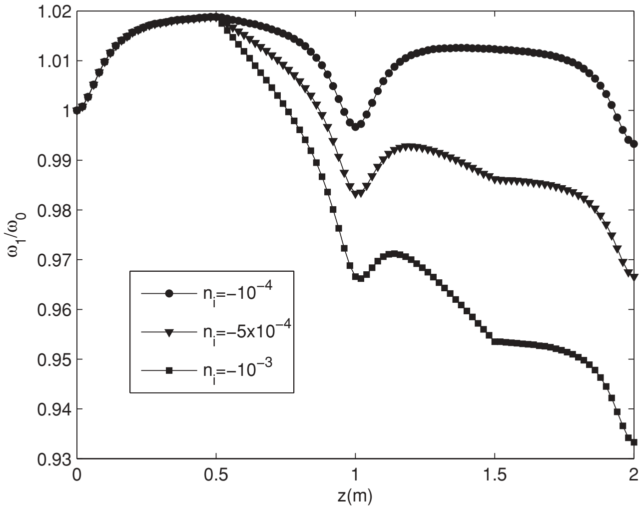

FIG. 5. Influence of absorption in layers of PPM and NPM on spectral changes in Gaussian Schell-model beams.

exhibits red shifts faster for greater values of n_i due to two mechanisms acting at the same time: focusing induced by the PPM-NPM interface and absorption of the NPM. After reaching a minimum turning point, the central frequency of the beam recuperates and reaches its second maximum point somewhere in the interval $z \in [1, 1.5\,\mathrm{m}]$. The value of the second maximum is lower than that of the first, since diffraction and absorption now compete. Then in the interval $z \in [1.5, 2\,\mathrm{m}]$, the central frequency continues to decrease but with a slower rate since only focusing caused by the NPM-PPM interface matters.

In Fig. 6, we show the on-axis spectra at the five transverse planes of our setup, $z = 0, 0.5, 1, 1.5$, and 2 m from the source, for the case when $n_i = -10^{-6}$. We can see that in the presence of absorption only single spectral switch (from blue shift to red shift) is present; the other one is suppressed because of absorption. This result is in striking difference with that shown in Fig. 2 where the beam passes through the lossless NPM layer and its spectral direction switches twice and has the ability to reconstruct.

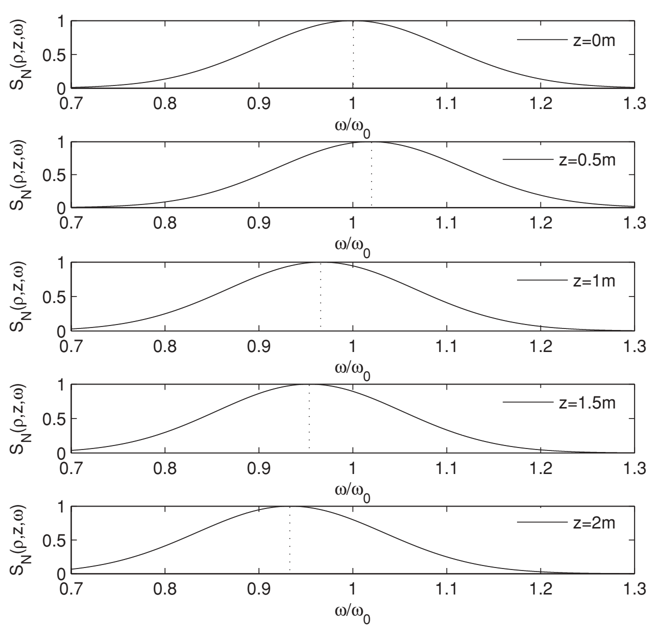

FIG. 6. Spectral changes of Gaussian Schell-model beams on propagation in layers of PPM and NPM with absorption.

IV. CONCLUDING REMARKS

In summary, we have studied the effect of the NPMs on the spectral changes in stochastic beamlike fields. For our study, we employed the perfect lens arrangement with two layers of PPM and one layer of NPM. Such configuration provides a very convenient mechanism for analysis of beam propagation in PPM and NPM and its interaction with both interfaces, that is, PPM/NPM and NPM/PPM. We have found that on propagation in either PPM or NPM a blue shift is always introduced, but the boundary between the PPM and NPM always introduces a spectral switch; that is, at that plane, the spectrum changes the direction of the spectral shift from blue to red. The turning points of the spectral changes closely depend on the parameters of the source. In the situation when the NPM medium is lossless, it is possible to recover the original spectral composition of the beam.

We have also investigated the effect of absorption in the NPM layer on the spectral changes and switches, finding that absorption can suppress and eliminate some of the switches. This phenomenon is due to the fact that increasing absorption in the NPM material helps in producing the strong blue shift and that the red shift due to the interface cannot compensate for it.

There are several potential applications of our findings. First of all, the very possibility of spectral recovery entails useful practical implementation. In a typical optical system, light propagates through free space between optical elements. Even though free-space propagation always induces blue shift, it appears possible to turn the spectrum back to its original value by putting a NPM between the two optical elements. By adjusting the position or the width of the NPM, we may retrieve the original spectrum, at least in the case when absorption is not significant. For example, with NPM with absorption $n_i = 10^{-4}$, we may retrieve the original spectral composition at around $z = 1.9$ m (see Fig. 5). More generally, the fact that the spectrum of light changes in NPM in a predictable fashion and can be controlled, to a large extent, via the choice of the source parameters and widths of PPM/NPM layers may be very helpful for development of spectrum-selection optical interconnects [15], spectrally encoded information transfer [16], and image formation [17]. Moreover, an entirely new class of tunable metamaterials [18] may be introduced based on our study, in which tuning is performed entirely via the geometry of the layers. For instance, the order and the thickness of the layers with fixed refractive and absorptive properties can be chosen for fine-tuning of the spectral density, hence layers of different homogeneous metamaterials can be viewed as an effective heterogeneous metamaterial.

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