Dynamical Casimir-Polder force in a one-dimensional cavity with quasimodes

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In this article, we consider the dynamic Casimir-Polder force between an atom and a conducting wall in a one-dimensional cavity. Using quasimode theory to describe the dissipation of the electromagnetic fields in the cavity, our investigation shows that the force oscillations are damped in a short time, and tend to a final, steady, negative value. We discuss in detail the effects on the force of the quasimode decay rate, the cavity size, and the atom-wall distance.

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I. INTRODUCTION

The Casimir force and the Casimir-Polder force arise from a quantized electromagnetic field. The Casimir effect, which is due to the presence of vacuum quantum fluctuations of the electromagnetic field between two parallel plates, was first predicted theoretically by Casimir in 1948 [1]. The vacuum quantum fluctuations of the electromagnetic field can lead to an interaction between neutral atoms (molecules) and is defined as the Casimir-Polder force [2]. Recently, the atom-wall Casimir-Polder force has been measured precisely both in the near and in the far zone in high-quality cavities [3–6]. The atom-atom Casimir-Polder force has also been investigated theoretically [7]. However, there are rare discussions in the cavity with some dissipation arising from the interaction with the environment.

There are many methods used to describe the dissipation in open systems. First, heat reservoir theory is a general method used to describe the decay of the electromagnetic field in the cavity. In this theory, the master equation and the quantum Langevin equation are usually applied to discuss the dynamics of the system [8–10]. On the other hand, when the dissipation of the cavity is considered, the modes of the electromagnetic fields are much different from those without dissipation, in that the normal modes cannot be rigorously defined. In this case, Fox and Li introduced a set of discrete quasimodes to describe leakage of the cavity [11].

In this sense, we introduce the Lamb-Scully theoretical model which is based on the Fox-Li quasimode theory to describe the cavity, which is dissipative [12,13]. As shown in Fig. 1, the cavity is embedded in a larger cavity, which is regarded as the environment, and a neutral atom is put into the cavity. We discuss the dynamic (time-dependent) Casimir-Polder force between the atom and conducting wall in such a system.

The organization of the article is as follows: In Sec. II, we establish our model as a two-level atom interacting with electromagnetic fields in a dissipative cavity. In Sec. III, we obtain the second-order energy shift and the force by iteratively solving the Heisenberg equation for atomic and field operators to lowest significant order. In Sec. IV, we find that the Casimir-Polder force initially executes damped oscillations and tends over longer times to a steady negative value. Effects of the decay rate of the quasimode, the size of the cavity, as well as the atom-wall distance on the force are also discussed in detail.

II. THEORETICAL MODEL AND HAMILTONIAN

The system in our consideration is schematically illustrated in Fig. 1. A two-level atom is put into a cavity (Region 1 shown in Fig. 1), which is the system we will focus on in the following. The cavity is located in a much larger cavity (Region 2 shown in Fig. 1). The larger cavity can be considered as the environment coupling to the system. The walls at $x = 0$ and $x = L$ are completely reflective, while the one at $x = l$ ($l \ll L$) is a semitransparent mirror with large dielectric constant. The dielectric constant nearby $x = l$ is

$$\varepsilon(x) = \varepsilon_0[1 + \eta\delta(x - l)], \tag{1}$$

where η is a parameter with dimensions of length that determines the transparency of the wall. For simplicity, the cavity is assumed to be one dimensional, and only a single polarization of the electromagnetic field is taken into account. The electromagnetic field in the entire cavity is determined by the Maxwell equation

$$\frac{\partial^2 E}{\partial x^2} - \mu_0\varepsilon(x)\frac{\partial^2 E}{\partial t^2} = 0. \tag{2}$$

In the above equations, ε_0 and μ_0 are the dielectric constant and magnetic susceptibility for the vacuum, respectively.

The boundary condition for a standing wave and the continuity condition at $x = l$ give the spatial part of the electromagnetic field mode function in the cavity [12]:

$$f_{nj}(x) = \frac{\Gamma\Lambda}{[(\omega_{nj} - \Omega_n)^2 + \Gamma^2]^{\frac{1}{2}}}\sin\left(\frac{\omega_{nj}x}{c}\right), \tag{3}$$

where

$$\Omega_n = \frac{c}{l}\left(n\pi + \frac{1}{\Lambda}\right);\ (n = 0,1,\ldots), \tag{4}$$

$$\omega_{nj} \in \left[\frac{c}{l}\left(n\pi - \frac{\pi}{2}\right), \frac{c}{l}\left(n\pi + \frac{\pi}{2}\right)\right], \tag{5}$$

where $\Lambda = (\eta/l)\tilde{x}$ with $\tilde{x}$ being the solution of $\tan x = l/\eta x$, and $\Gamma = c/\Lambda^2 l$ is the decay rate of the cavity quasimode.

In Eq. (4), the quantity Ω_n represents the nth Fox–Li quasimode. Owing to the dissipation of the system, the Fox–Li quasimode is different from the normal mode ($\omega = cn\pi/l$ for finite space), which is usually applied to describe the ideal

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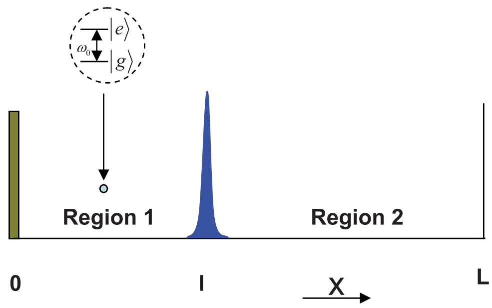

FIG. 1. (Color online) Schematic illustration of a setup in which a two-level atom is located inside a cavity bounded by a perfect mirror at $x = 0$ and a semitransparent mirror at $x = l$ (Region 1). The cavity is imbedded in a larger ideal cavity. The auxiliary cavity (Region 2) serves as the environment, which interacts with the cavity and is bounded by a perfect mirror at $x = L$.

cavity. We take the notation ω_{nj} to represent the jth mode in the nth quasimode.

We restrict our attention to a two–level atom coupling to the multimode electromagnetic field in the cavity. In analogy with [14,15], our model may be formulated with the following total Hamiltonian, which is given by $H_{\text{tot}} = H_0 + H_I$, where the Hamiltonian of the atom and the electromagnetic field H_0 is

$$H_0 = \hbar\omega_0 S_z + \sum_{n,j} \hbar\omega_{nj} a_{nj}^{\dagger} a_{nj}, \tag{6}$$

where $S_z = (|e\rangle\langle e| - |g\rangle\langle g|)/2$ is the pseudospin operator of the two–level atom with resonance frequency ω_0, and $|e\rangle$ ($|g\rangle$) is the excited (ground) state of the atom. $a_{nj}^{\dagger}$ and a_{nj} are creation and annihilation operators for the electromagnetic field with frequency ω_{nj} and $[a_{nj}, a_{n'j'}^{\dagger}] = \delta_{nn'}\delta_{jj'}$.

Under the electric dipole approximation, the Hamiltonian for the interaction between the atom and the field H_I can be written as [14]

$$H_I = -i \sum_{n,j} \sqrt{\frac{2\pi\hbar\omega_{nj}}{L}} [\boldsymbol{\mu} \cdot \mathbf{f}_{nj}(x)](S_+ + S_-)(a_{nj} - a_{nj}^{\dagger}), \tag{7}$$

where $S_{\pm}$ are also pseudospin operators of the two–level atom with $S_+ = |e\rangle\langle g|$ and $S_- = |g\rangle\langle e|$. The factor $\sqrt{2\pi\hbar\omega_{nj}/L}[\boldsymbol{\mu}\cdot\mathbf{f}_{nj}(x)]$ is the coupling strength between the atom and the electromagnetic field. Obviously, the model discussed here is the same as the Dicke model [16].

We only consider the one-dimensional case because the quasimode theory is put forward in [12]. We also assume that the direction of the dipole moment for the atom is perpendicular to the surface of the conducting wall. Thus, the coupling strength simplifies to $\sqrt{2\pi\hbar\omega_{nj}/L}[\mu f_{nj}(x)]$.

III. CALCULATION OF THE DYNAMIC CASIMIR-POLDER FORCE

In the section above, we analyze the electromagnetic field in the cavity and the Hamiltonian for the system. In the following, we calculate the second-order energy of the system by means of perturbation theory and discuss the Casimir-Polder force, which is related to the second-order energy shift.

The Casimir-Polder force is the negative of the derivative of the potential energy with respect to the atom-wall distance x:

$$F(x,t) = -\frac{\partial(\Delta E^{(2)})}{\partial x}. \tag{8}$$

In our system, the atom-wall potential energy for a ground-state atom can be obtained from the second-order energy shift $\Delta E^{(2)}$ of the bare ground state $|0, \downarrow\rangle$, which is the electromagnetic field in its vacuum state and the atom in its ground state.

According to perturbation theory, the second-order energy shift can be obtained from the average value of the interaction Hamiltonian on the dressed ground state of the system [14]:

$$\Delta E^{(2)} = \frac{{}_D\langle 0, \downarrow | H_I | 0, \downarrow\rangle_D}{2}, \tag{9}$$

where $|0, \downarrow\rangle_D$ is the first-order dressed ground state of the system,

$$|0, \downarrow\rangle_D = |0, \downarrow\rangle - \sum_{|\psi\rangle \neq |0,\downarrow\rangle} \frac{\langle\psi|H_I|0, \downarrow\rangle}{E_\psi - E_0} |\psi\rangle. \tag{10}$$

It is clear that the initial state $|0, \downarrow\rangle$ is not an eigenstate of the total Hamiltonian. Therefore, the state and hence the atom-field interaction energy will be time dependent.

In the following, we calculate the second-order energy shift. First of all, it is our task to obtain the time-dependent interaction Hamiltonian in the Heisenberg picture to second order. Thus, we need expressions for the Heisenberg operators which can be iteratively solved from the Heisenberg equation [14,17].

The zeroth and first orders of $a_{nj}(t)$ and $S_+(t)$ are as follows:

$$a_{nj}^{(0)}(t) = a_{nj}(0) e^{-i\omega_{nj} t}, \tag{11}$$

$$a_{nj}^{(1)}(t) = \sqrt{\frac{2\pi\omega_{nj}}{\hbar L}} e^{-i\omega_{nj}t} [\mu f_{nj}(x)][S_+(0) g(\omega_{nj} + \omega_0, t) + S_-(0) g(\omega_{nj} - \omega_0, t)], \tag{12}$$

$$S_+^{(0)}(t) = S_+(0) e^{i\omega_0 t}, \tag{13}$$

$$S_+^{(1)}(t) = -2S_Z(0) e^{i\omega_0 t} \sum_{nj} \sqrt{\frac{2\pi\omega_{nj}}{\hbar L}} [\mu f_{nj}(x)] \times [a_{nj}(0) g^*(\omega_{nj} + \omega_0, t) - a_{nj}^{\dagger}(0) g(\omega_{nj} - \omega_0, t)]. \tag{14}$$

Here, we introduce an auxiliary function $g(x,t) = (e^{ixt} - 1)/(ix)$. Thus, if we put the above expressions into the interaction Hamiltonian, we can easily evaluate the time-dependent Casimir-Polder atom-wall energy shift as

$$\Delta E^{(2)}(x,t) = \frac{\langle 0, \downarrow | H_I^{(2)}(t) | 0, \downarrow\rangle}{2} = -\frac{i\pi c}{L} \sum_{nj} \omega_{nj} [\mu f_{nj}(x)]^2 \big[g^*(\omega_0 + \omega_{nj}, t) e^{i(\omega_0 + \omega_{nj})t}$$

$$
\begin{aligned}
&- g(\omega_0 + \omega_{nj}, t)e^{-i(\omega_0+\omega_{nj})t}\big] \\
&= \frac{2\pi\mu^2}{L}\sum_{nj}\frac{\omega_{nj}(\Gamma\Lambda)^2\sin^2\left(\frac{\omega_{nj}}{c}x\right)[1-\cos[(\omega_0+\omega_{nj})t]]}{[(\omega_{nj}-\Omega_n)^2+\Gamma^2](\omega_0+\omega_{nj})}.
\end{aligned} \tag{15}
$$

The density of states ρ in ω_{nj} space is found to be $\rho = L/(c\pi)$, so we can transform the sum over j into $[L/(c\pi)]\int_{c/l(n\pi-\frac{\pi}{2})}^{c/l(n\pi+\frac{\pi}{2})} d\omega_{nj}$. By calculating the integral of ω_{nj} and the sum over n, we obtain the energy shift. Following Eq. (8), we obtain the Casimir-Polder force as

$$
\begin{aligned}
F(x,t) &= -\frac{\partial}{\partial x}\Delta E^{(2)}(x,t) \\
&= -\frac{2\omega_0\mu^2}{cx}\Delta_1 + \frac{\omega_0\mu^2}{c}\frac{\partial}{\partial x}(\Delta_2),
\end{aligned} \tag{16}
$$

where we define the notations as follows:

$$
\begin{aligned}
\Delta_1 = \sum_{i=1}^{3}\Bigg\{&h_i\left[\sin\left(\frac{c\pi\tau_i}{2l}\right)\sin\left(\frac{c\tau_i}{l\Lambda}+\frac{2x\omega_0}{c}\right)e^{-\Gamma|\tau_i|}\right] \\
&-\frac{2x\omega_0 g_i}{c}\int_{\tau_i}^{\infty}\frac{\cos\left(\frac{ct'}{l\Lambda}+\frac{2x\omega_0}{c}\right)\sin\left(\frac{c\pi t'}{2l}\right)e^{-\Gamma|t'|}}{t'}dt'\Bigg\}, \\
\Delta_2 = \sin\left(\frac{2x\omega_0}{c}\right)&\sum_{i=1}^{3} g_i e^{-\Gamma|\tau_i|}\Bigg\{\Gamma\frac{|\tau_i|}{\tau_i}\left[\frac{2l}{c\pi}\cos\left(\frac{c\pi\tau_i}{2l}\right)\right. \\
&\left.+\tau_i Si\left(\frac{c\pi\tau_i}{2l}\right)\right]+Si\left(\frac{c\pi\tau_i}{2l}\right)\Bigg\}.
\end{aligned}
$$

Herein, we define $h_1 = 1$, $h_2 = -1/(2+ct/x)$, $h_3 = -1/(2-ct/x)$, $g_1 = 2$, $g_2 = g_3 = -1$, and $\tau_i = 2x/c + t\delta_{2i} - t\delta_{3i}$.

IV. DISCUSSION ABOUT THE DYNAMIC CASIMIR-POLDER FORCE

In Fig. 2, we see that the force oscillates with decreasing amplitude until it reaches a steady value. This steady value is negative and is same as the result given in [14,18]. The steady value of the force coincides with the result in the static case, which indicates that the atom is fully dressed after a long time period ($t \gg 2x/c$). We consider different decay rates of the quasimode in order to investigate the influence of Γ. In Fig. 2, the decay rate represented by the red (dashed) curve is twenty five times larger than the one represented by the green (solid) curve. We see clearly that the red (dashed) curve has a steady value that is smaller than that of the green (solid) curve. It is understandable that the dissipation of the cavity weakens the interaction between the atom and the wall.

Actually, the size of the cavity can also affect the Casimir-Polder force mentioned above. As illustrated in Fig. 3, we assume that the atom-wall distance is $x = 2 \times 10^{-7}$ m and the decay rate is $\Gamma = 10^{12}$ Hz. The steady value of the force in a larger cavity is smaller than the force in a smaller cavity. This is same result as for the case without dissipation. On the other hand, we can see that, in the smaller cavity, the force oscillations are slower than in the larger cavity. We can explain this result as follows: The dissipation that arises during the propagation of the electromagnetic wave in the cavity weakens the atom-wall interaction. A larger cavity means a longer time for the electromagnetic wave propagation.

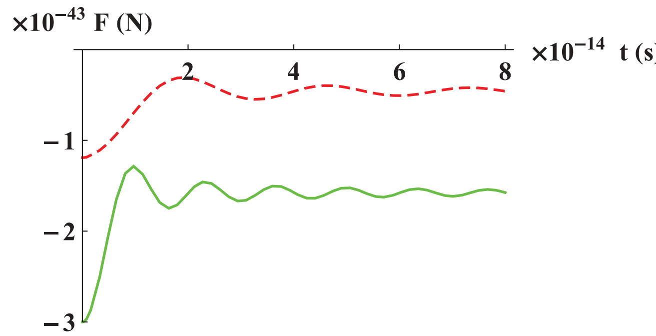

FIG. 3. (Color online) Time evolution of the dynamic Casimir-Polder forces F between an atom and the conducting wall in the cavity for different values of cavity size l. We have taken the atom-wall distance to be $x = 2 \times 10^{-7}$ m and decay rate $\Gamma = 10^{12}$ Hz. The red (dashed) line and green (solid) line represent the forces for $l = 2 \times 10^{-6}$ m and $l = 10^{-6}$ m, respectively.

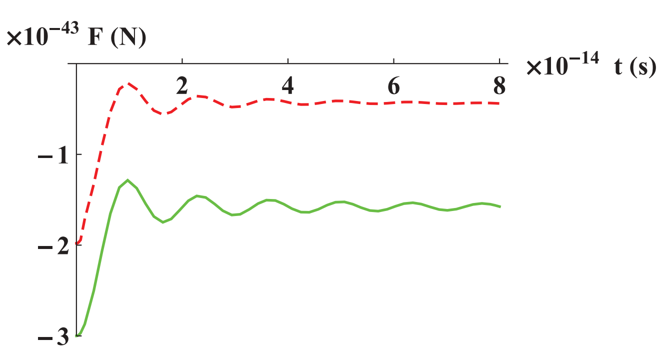

FIG. 2. (Color online) Time evolution of the dynamic Casimir-Polder forces F between an atom and conducting wall in the cavity for different values of Γ. Here, we have taken the atom-wall distance to be $x = 2 \times 10^{-7}$ m and the length of the cavity is $l = 10^{-6}$ m. The red (dashed) line and green (solid) line represent the forces for $\Gamma = 25 \times 10^{12}$ Hz and $\Gamma = 10^{12}$ Hz, respectively.

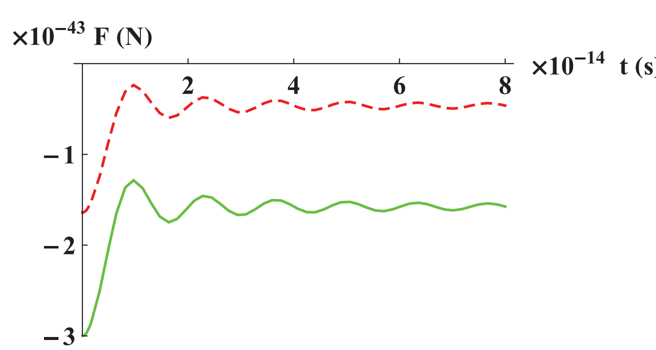

FIG. 4. (Color online) Time evolution of the dynamic Casimir-Polder forces F between an atom and the conducting wall in a cavity for different atom-wall distances x. We have taken the atom-wall distance to be $l = 10^{-6}$ m and the decay rate $\Gamma = 10^{12}$ Hz. The red (dashed) line and the green (solid) line represent the forces for $x = 2 \times 10^{-7}$ m and $x = 4 \times 10^{-7}$ m, respectively.

We also plot the time evolution of the Casimir-Polder force with the atom in two different positions within the cavity. The result can be clearly seen in Fig. 4. The shorter is the atom-wall distance, the larger is the steady value of the force. This is also in accord with the general conclusion about the behavior of the Casimir-Polder force with respect to the atom-wall distance.

V. CONCLUSIONS AND REMARKS

In this article, we consider the dynamic Casimir-Polder force between an atom and the conducting wall in a cavity with quasimodes. In evaluating $H_I^{(2)}(t)$, we calculate the first-order expressions of the Heisenberg operators, which are solved iteratively from the Heisenberg equations. We obtain the analytical expression of the time-dependent Casimir-Polder force in this system. We find that the force oscillates with an amplitude that attenuates over a short time. Finally, the force settles to a steady negative value. The force decreases with the size of the cavity and with the atom-wall distance. These results are similar to the behavior of the force in the ideal case. We also discuss the effects of the decay rate and the size of the cavity and the atom-wall distance on the force.

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