Suppressing decoherence and improving entanglement by quantum-jump-based feedback control in two-level systems

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We study the quantum-jump-based feedback control on the entanglement shared between two qubits with one of them subject to decoherence while the other qubit is under the control. This situation is very relevant to a quantum system consisting of nuclear and electron spins in solid states. The possibility of prolonging the coherence time of the dissipative qubit is also explored. Numerical simulations show that the quantum-jump-based feedback control can improve the entanglement between the qubits and prolong the coherence time for the qubit subject directly to decoherence.

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I. INTRODUCTION

Superposition of states and entanglement make quantum information processing much different from its classical counterpart, but a quantum system would unavoidably interact with its environment, resulting in a degradation of coherence and entanglement. For example, spontaneous emission in atomic qubits [1] would spoil the coherence of quantum states and limit the entanglement time.

Recent experimental advances have enabled individual systems to be monitored and manipulated at the quantum level [2]. This makes the quantum feedback control realizable. Among the feedback controls, the homodyne-mediated feedback [3,4] and quantum-jump-based feedback controls have been proposed to generate steady-state entanglement in a cavity [5,6]. These two feedback schemes are Markovian; that is, a feedback information proportional to the quantum-jump detection is synchronously used. These control schemes can also be used to suppress decoherence [7–10].

Meanwhile, researchers are looking for good systems for experimental implementation of quantum information processing. Among the various candidates, solid-state quantum devices based on superconductors [11] and lateral quantum dots [12] are promising; however, the decoherence from intrinsic noise originating from two-level fluctuators is hard to engineer [13]. For this reason, the nuclear spins have attracted considerable attention [14] due to their long coherence times [15]. However, their weak interactions with others make the preparation, control, and detection on them difficult. Thanks to their intrinsic interactions with electron spins, electron spins can be used as ancillas to access single nuclear spins. This naturally leads to the following question: Can feedback strategy be used to suppress decoherence as well as to prepare and protect entanglement between the nuclear and electron spins by controlling the electron spin? In this article, using a generalized model, we study this problem by considering a nuclear spin (as a qubit) coupled to electron spin (as the other qubit) that is exposed to its environment. We show that a Markovian feedback based on quantum jumps can be used to suppress decoherence, produce entanglement, and protect it.

The article is organized as follows: In Sec. II, we describe our model and present the dynamics in the absence of feedback. In Sec. III, we introduce the quantum-jump-based feedback control and give the dynamical equation under the feedback control. The effect of feedback control on decoherence and entanglement is discussed in Secs. IV and V, respectively. Section VI concludes our results.

II. MODEL

Our system consists of a pair of two-level systems, called qubit 1 and qubit 2, where only qubit 2 interacts with its environment. We present a scheme employing quantum-jump-based feedback control on qubit 2 to affect the decoherence of qubit 1 and increase entanglement between the two qubits. The Hamiltonian of the system reads

$$H = \tfrac{1}{2}\hbar\omega_1\sigma_1^z + \tfrac{1}{2}\hbar\omega_2\sigma_2^z + \hbar g(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+). \tag{1}$$

The first two terms represent the free Hamiltonian of the two qubits, and the last term describes their interactions under the rotating-wave approximation. The terms ω_1 and ω_2 are the transition frequencies of the two qubits, respectively, g is the coupling strength of the two qubits, and σ_z is the Pauli matrix, that is, $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma^+ = |e\rangle\langle g|$, and $\sigma^- = |g\rangle\langle e|$.

The state of this quantum system can be described by the density operator ρ, which is obtained by tracing out the environment. The dynamics of open quantum systems can be described by quantum master equations. The most general form of master equation for the density operator is [16,17]

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}(\rho), \tag{2}$$

where H is the system Hamiltonian and $\mathcal{L}$ is a superoperator defined by $\mathcal{L}(\rho) = \Sigma_k\gamma_k(L_k\rho L_k^\dagger - \frac{1}{2}L_k^\dagger L_k\rho - \frac{1}{2}\rho L_k^\dagger L_k)$, in which different k's characterize different dissipative channels.

In our system, the first qubit is assumed to be isolated from the environment. The decoherence comes from the spontaneous emission of the qubit 2 (the second qubit). This situation is of relevance to a system consisting of nuclear and electron spins in the aforementioned solid-state devices. The dynamics of such a system takes

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \gamma\left(\sigma_2^-\rho\sigma_2^+ - \frac{1}{2}\sigma_2^+\sigma_2^-\rho - \frac{1}{2}\rho\sigma_2^+\sigma_2^-\right). \tag{3}$$

Here $\sigma_2^{\pm} = I_1 \otimes \sigma_2^{\pm}$. The second part of Eq. (3) describes the dissipation of our system with γ as the decay rate.

Though the first qubit is assumed to be isolated from the environment, it still loss coherence due to the coupling to the second qubit. The decoherence process can be showed by the decay of off-diagonal elements of the reduced density matrix for the first qubit. In order to investigate this decoherence, we calculate the evolution of system density operator ρ and then trace out the second qubit to get the reduced matrix

$$\rho_1 = \mathrm{Tr}_2(\rho) = \sum_{k=e,g} {}_2\langle k|\rho|k\rangle_2 = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}. \quad (4)$$

The diagonal elements are the populations in the excited and ground states of the first qubit, and the off-diagonal elements represent the coherence of the qubit 1.

III. QUANTUM-JUMP-BASED FEEDBACK CONTROL

Quantum feedback controls play an increasingly important role in quantum information processing. They are widely used to create and stabilize entanglement as well as combat decoherence [5,6,8,10]. In our model, the second qubit is used as an ancilla through which the feedback can affect the dynamics of the first qubit; that is, by employing a feedback control on the second qubit, we control the first qubit. The goal is to suppress the decoherence of the first qubit and enhance the entanglement between the two qubits by feedback control on the second qubit [6].

Our feedback-control strategy is based on quantum-jump detection. The master equation with feedback can be derived from the general measurement theory [4]. In our article, Eq. (3) is equivalent to

$$\rho(t+dt) = \sum_{\alpha=0,1} \Omega_\alpha(T)\rho(t)\Omega_\alpha^\dagger(T), \quad (5)$$

with

$$\Omega_1(dt) = \sqrt{\gamma dt}\sigma_2^-, \quad (6)$$

$$\Omega_0 = 1 - \left(\frac{i}{\hbar}H + \frac{1}{2}\gamma\sigma_2^+\sigma_2^-\right)dt. \quad (7)$$

When the measurement result is $\alpha = 1$, a detection occurs, which causes a finite evolution in the system via $\Omega_1(dt)$. This is called a quantum jump. Then the unnormalized density matrix becomes $\tilde{\rho}_{\alpha=1} = \sigma_2^-\rho(t)\sigma_2^+dt$. The feedback control is added by giving $\tilde{\rho}_{\alpha=1}$ a finite unitary evolution, and then $\tilde{\rho}_{\alpha=1}$ become $\tilde{\rho}_{\alpha=1} = F\sigma_2\rho(t)\sigma_2^+F^\dagger dt$. In the limit that the feedback acts immediately after a detection and in a very shot time (much smaller than the time scale of the system's evolution), the master equation is Markovian:

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \gamma\left(F\sigma_2^-\rho\sigma_2^+F^\dagger - \frac{1}{2}\sigma_2^+\sigma_2^-\rho - \frac{1}{2}\rho\sigma_2^+\sigma_2^-\right). \quad (8)$$

Here $F = e^{iH_f}$ and $H_f = -\frac{1}{\hbar}H_f' t_f$. We see that the operator H_f contains a relatively large operator H_f' multiplied by a very short time t_f (Markovian assumption), but the product represents a certain amount of evolution, so it is convenient to discuss H_f instead of H_f' and t_f. Here H_f is a 2×2 Hermitian operator which can be decomposed by Pauli matrices $H_f = A_x\sigma_x + A_y\sigma_y + A_z\sigma_z$ (A_x, A_y, and A_z are real numbers). So we have

$$F = I_1 \otimes e^{i\vec{A}\cdot\vec{\sigma}} = I_1 \otimes \left(\cos|\vec{A}| + i\frac{\sin|\vec{A}|}{|\vec{A}|}\vec{A}\cdot\vec{\sigma}\right). \quad (9)$$

Here $\vec{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$ and $\vec{A} = (A_x,A_y,A_z)$, representing the amplitude of σ_x,σ_y, and σ_z controls.

In order to understand the physical meaning of feedback operator F, we rewrite it as $F = I_1 \otimes e^{-i\frac{\omega}{2}\vec{n}\cdot\vec{\sigma}}$ where $\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and $\vec{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$. This feedback operator is equivalent to a time evolution with evolution operator $F = I_1 \otimes e^{iH_f}$, and it is clear that the operator F rotates the Bloch vector of the second qubit with the angle ω around the $\vec{n}$ axis. The relationship between the two forms of F are $A_x = -\frac{\omega}{2}\sin\theta\cos\phi, A_y = -\frac{\omega}{2}\sin\theta\sin\phi$, and $A_z = -\frac{\omega}{2}\cos\theta$, so an σ_x control ($A_y = 0, A_z = 0$) means rotating the Bloch vector with a certain amount of angle around the x axis of Bloch sphere; so do the A_y and A_z controls. Different $\vec{A}$'s represent different feedback evolution; that is, rotate the Bloch vector with a particular angle around a given direction in the Bloch sphere. For simplicity, we discuss the $\sigma_x,\sigma_y,\sigma_z$ controls one by one in the following.

This control mechanism has the advantage of being simple to apply in practice, since it does not need real-time state estimation as the Bayesian feedback control does [18]. The emission of the second qubit is measured by a photo detector, whose signal provides the information to design the control F. In this kind of monitoring, the absence of signal predominates the dynamics and the control is triggered only after a detection click (i.e., a quantum jump occurs).

IV. DECOHERENCE SUPPRESSION

Before investigating the influence of the feedback control, we first analyze the evolution of our system without the control. Assume that the two qubits are initially in the same pure superposition state, for example, $|\psi\rangle = \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1) \otimes \frac{1}{\sqrt{2}}(|e\rangle_2 + |g\rangle_2)$. The corresponding density matrix is

$$\rho_0 = |\psi\rangle\langle\psi| = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}. \quad (10)$$

We assign the Planck constant $\hbar$ to be 1, $\omega_1 = \omega_2 = \omega$ in Eq. (1), and $g/\omega = 1, \gamma/\omega = 0.5$. After numerical calculation, we get the evolution of the density matrix for the first qubit without control. Since $\rho_{eg} = \rho_{ge}^*, \rho_{ee} + \rho_{gg} = 1$, we discuss only coherence $|\rho_{eg}|$ and excited-state population ρ_{ee} for simplicity. The evolution of $|\rho_{eg}|$ and ρ_{ee} without the control is depicted in Figs. 1(a) and 1(b) (dashed lines).

In Fig. 1(a), a fast decay of $|\rho_{eg}|$ (dashed line) can be found. This demonstrates that the first qubit lost coherence due to the second qubit's spontaneous emission and their interaction. Meanwhile, the first qubit lost energy due to couplings with the second qubit [Fig. 1(b) (dashed line)]. The results also show that the populations in excited states decay away. This

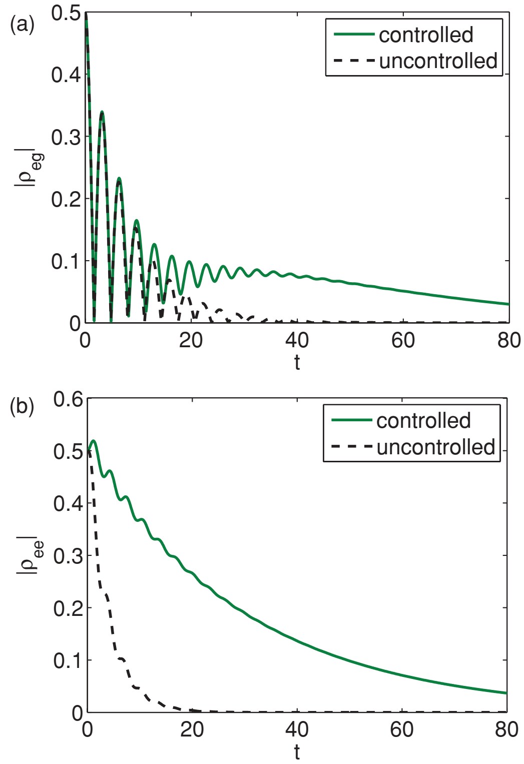

FIG. 1. (Color online) (a) Time evolution of $|\rho_{eg}|$ with and without control; t is in units of $\frac{1}{\omega}$. The different curves correspond to $A_x = 1.2$, $A_y = A_z = 0$ (solid line) and $A_x = A_y = A_z = 0 = 0$ (dashed line) for $g/\omega = 1, \gamma/\omega = 0.5$. The feedback-control strategy results in an improvement in decoherence time. (b) Excited-state population ρ_{ee} evolution with and without control for the same parameters as (a); the decay of excited-state population is slower in the controlled scheme.

is because the first qubit exchanges energy with the second qubit; see Eq. (1).

Now we add feedback control F to our system. The master equation then becomes Eq. (8). Our system is initially in the state ρ_0, and other parameters remain unchanged.

We first analyze the σ_x control by choosing feedback amplitude $A_x = 0 \sim \pi$, $A_y = A_z = 0$. Note that when $A_y = A_z = 0$, the feedback amplitude A_x influences the system's evolution with a period of π, which comes from the term $F\sigma_2^- \rho \sigma_2^+ F^\dagger$ in Eq. (9). It can be analytically proved that $e^{iA_x\sigma_x}\sigma^-\rho_2\sigma^+ e^{-iA_x\sigma_x} = e^{i(A_x+\pi)\sigma_x}\sigma^-\rho_2\sigma^+ e^{-i(A_x+\pi)\sigma_x}$ and $e^{iA_y\sigma_y}\sigma^-\rho_2\sigma^+ e^{iA_y\sigma_y} = e^{i(A_y+\pi)\sigma_y}\sigma^-\rho_2\sigma^+ e^{i(A_y+\pi)\sigma_y}$ under any A_x and A_y. Here ρ_2 is the reduced density matrix of the second qubit. The absolute value for the first qubit's off-diagonal density-matrix element evolves as shown in Fig. 2(a). The figure indicates that for an appropriate feedback amplitude, $A_x \approx 1.3$ and $A_x \approx 1.9$, the absolute value of off-diagonal element can be evidently enhanced compared with the uncontrolled case ($A_x = 0$). This means the decoherence is partially suppressed. The improvement of coherence caused by feedback is shown explicitly in Fig. 1(a). We plot $|\rho_{eg}|$, representing the coherence of the first qubit, as a function of time with $A_x = 1.2$, $A_y = A_z = 0$ (a selected controlled case). In comparison with the uncontrolled case, a stronger oscillation amplitude and longer dechoherence time appear. Meanwhile, the ρ_{ee} decays slowly compared to the uncontrolled case as shown in Fig. 2(b).

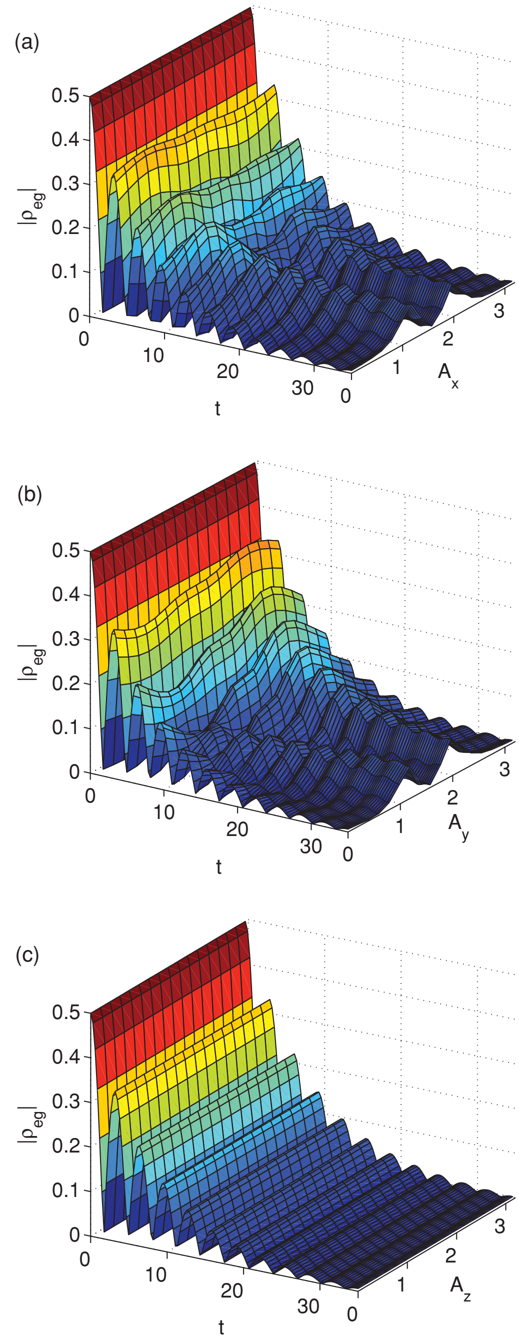

FIG. 2. (Color online) The evolution of absolute value of the first qubit's off-diagonal element with different control parameters, for $g/\omega = 1, \gamma/\omega = 0.5$, and t in the units of $\frac{1}{\omega}$. (a) The σ_x control for $A_x = 0 \sim \pi, A_y = A_z = 0$. (b) The σ_y control for $A_y = 0 \sim \pi, A_x = A_z = 0$. (c) The σ_z control for $A_z = 0 \sim \pi, A_x = A_y = 0$. When the feedback amplitude is chosen to be about 1.3 and 1.9 for both σ_x and σ_y controls, the oscillation of the off-diagonal element is remarkably enhanced. The σ_z control does not work in our model.

Similarly, the σ_y control is also able to slow down the decay of $|\rho_{eg}|$. We make $A_y = 0 \sim \pi, A_x = A_z = 0$. The numerical results of $|\rho_{eg}|$ are shown in Fig. 2(b). Unlike the σ_x and σ_y controls, the σ_z control ($A_z = 0 \sim \pi, A_x = A_y = 0$) has no

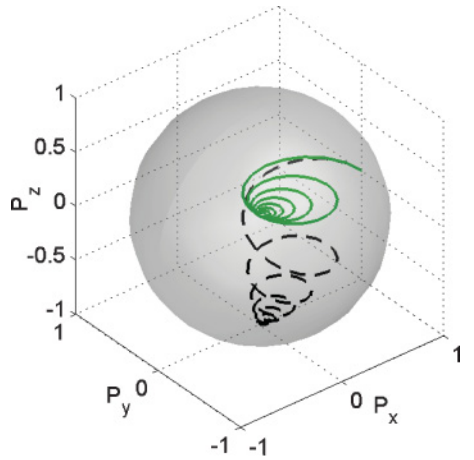

FIG. 3. (Color online) Polarization vector evolution in a Bloch sphere for feedback amplitudes $A_x = \frac{\pi}{2}$, $A_x = A_y = 0$ (solid line), and $A_y = A_x = A_y = 0$ (dashed line). The parameters are $g/\omega = 1$, $\gamma/\omega = 0.5$, and the initial state is $|\psi\rangle = \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1) \otimes \frac{1}{\sqrt{2}}(|e\rangle_2 + |g\rangle_2)$.

effect on the evolution of the system as shown in Fig. 2(c). This is because $e^{iA_z\sigma_z}\sigma_-\rho_2\sigma_+ e^{-iA_z\sigma_z} = \rho_2$ for any A_z. The physics behind this result is as follows: After emitting a photon, the controlled qubit must stay in the ground state with the Bloch vector pointing the bottom of the Bloch sphere, so the rotation around the z axis does not change the Bloch vector; that is, the state of the qubit remains unchanged.

The present results show that decoherence of the first qubit can be suppressed by controlling its partner. The decoherence source in our system is the spontaneous emission of the second qubit: Once the detector detects a photon (i.e., a quantum jump of the second qubit happens), the feedback beam instantaneously acts on the second qubit, and then the first qubit is impacted through the coupling of the second qubit. The feedback-control scheme can reduce the destructive effects of coherence and slow down the dissipation of energy. The control effect is relevant to the coupling strength g. When g is small, the first qubit is unlikely to be impacted by the second qubit, so it is hard to prepare, measure, and control the state of the first qubit. As the interaction gets stronger, the effect of feedback control becomes more evident.

For the case discussed in Fig. 1, the first qubit is dissipative. We found that when the control parameters are chosen as $A_x = \frac{\pi}{2}, A_y = A_z = 0$, or $A_y = \frac{\pi}{2}, A_x = A_y = 0$ with the two qubits initially being prepared in the same states, the decoherence dynamics turns to be the phase-damping type. The population in the ground state and excited state do not change, while the off-diagonal elements evolves in the same way as in the uncontrolled case. We show this in a Bloch sphere [19] in Fig. 3. Here the reduced density matrix of the first qubit can be written by $\rho_1 = \frac{1}{2}(I + \vec{P}\cdot\vec{\sigma})$. We can get the polarization vector components $P_x = \mathrm{Tr}(\sigma_x\rho_1)$, $P_y = \mathrm{Tr}(\sigma_y\rho_1)$, and $P_z = \mathrm{Tr}(\sigma_z\rho_1)$.

V. ENTANGLEMENT CONTROL

Quantum feedback control has been recently used to improve the creation of steady-state entanglement in open quantum systems. A highly entangled states of two qubits in a cavity can be produced with an appropriate selection of the feedback Hamiltonian and detection strategy [6,20]. We show that the quantum-jump-based feedback scheme can produce and improve entanglement in our model. We choose the concurrence [21] as a measure of entanglement. For a mixed state represented by the density matrix ρ, the "spin-flipped" density operator reads

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{11}$$

where the $*$ denotes complex conjugate of ρ in the bases of $\{|gg\rangle, |ge\rangle, |eg\rangle, |ee\rangle\}$ and σ_y is the usual Pauli matrix. The concurrence of the density matrix ρ is defined as

$$C(\rho) = \max(\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0), \tag{12}$$

where λ_i are eigenvalues of matrix $\rho\tilde{\rho}$ and sorted in decreasing order $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$. The range of concurrence is from 0 to 1, and $C = 1$ represents the maximum entanglement.

In the absence of the spontaneous emission (i.e., $\gamma = 0$), the system evolves without dissipation. We find that for the system initially in a separable state except for $|\psi\rangle = |e\rangle_1|e\rangle_2$ or $|\psi\rangle = |g\rangle_1|g\rangle_2$ (the eigenstates of system Hamiltonian H), an entangled state can be generated due to the interaction between the two qubits. The amount of entanglement depends on the initial states of the system and the coupling strength g, but when the spontaneous emission effect is taken into account, the performance of entanglement preparation get worse considerably.

Now we investigate if our feedback-control strategy can improve the entanglement preparation with the effect of spontaneous emission of the second qubit. The master equation with control is Eq. (8). The effect of feedback control lies in different choices for the feedback parameters A_x, A_y, and A_z, the coupling strength g, and different initial states. Here we present two typical results with two different states.

Our first choice is the initial state $|\psi\rangle = |g\rangle_1|e\rangle_2$ with σ_y control for $A_y = 0 \sim \pi$, $A_x = 0$, and $A_z = 0$. The concurrence evolution is plotted as a function of time and feedback amplitude A_y in Figs. 4(a) and 4(b) denotes the concurrence evolution with a selected feedback amplitude compared with the uncontrolled case. We see that entangled states can be generated with any feedback parameters, but they decrease with time because of the dissipative effect. When an appropriate feedback amplitude $A = 0.5\pi$ is chosen, the concurrence amplitude is remarkably enhanced, and the entanglement lasts for a long time. For the system initially in the state $|\psi\rangle = |e\rangle_1|e\rangle_2$ with σ_y control, the dynamics of the concurrence is shown in Fig. 5(a). Note that in this case if there is no spontaneous effect, this is a steady state of the system, and the density matrix elements do not change with time. Figure 5(a) demonstrates that the dissipation and feedback can produce entanglement. We show this explicitly in Fig. 5(b) by choosing feedback amplitude $A_y = 1.2$. We can see that for a proper feedback amplitude, after an entanglement death, a larger amount entanglement is regenerated.

These results show that the feedback-control strategy can be used to prepare and protect entanglement in our model. The

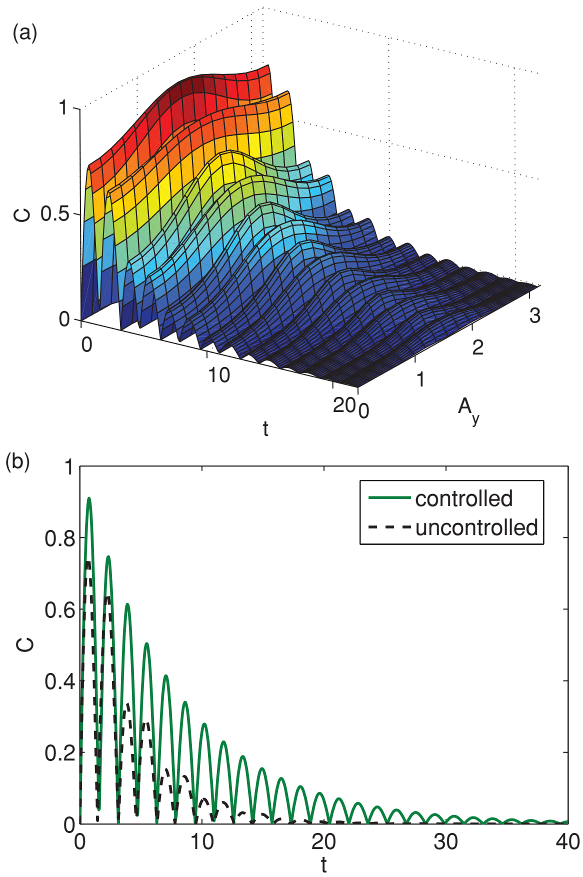

FIG. 4. (Color online) (a) Conccurence as a function of time and A_y. The system is initially in the state $|\psi\rangle = |g\rangle_1|e\rangle_2$, for the parameters $g/\omega = 1, \gamma/\omega = 0.5$. (b) A controlled evolution for $A_y = 0.5\pi$, $A_x = A_z = 0$ vs. the uncontrolled case. The entanglement is improved by choosing an appropriate feedback; t is in units of $\frac{1}{\omega}$ for (a) and (b).

effect of entanglement control strongly depends on the initial state. For a certain initial state, we found that the σ_x control and σ_y control have similars effect but that the σ_z control does not work.

Before closing this section, we note that although the model in Eq. (1) does not describe the hyperfine interaction (i.e., the interaction between nuclear and electron spins), we can simulate Hamiltonian Eq. (1) in nuclear-electron spin systems by recent technology [22–28]. On the other hand, by using the hyperfine interaction Hamiltonian, we can obtain results similar to that with the Hamiltonian Eq. (1), as follows. The Hamiltonian that describes hyperfine interactions is $H(t) = \omega_I(t)I_z + \omega_s(t)S_z + \omega_{IS}(t)2I_zS_z + \omega_{rf}^I(t)I_x + \omega_{rf}^s(t)S_x$. The Hamiltonian for the dipole-dipole coupling in the interaction frame of the rf irradiation averages over a rotor period to [29] $\tilde{H}_{IS} = (AZ^+BY^+) + (CZ^- + DY^-)$, where A,B,C, and D are parameters [28], $Z^\pm = I_zS_z \mp I_yS_y$, and $Y^\pm = I_yS_z \pm I_zS_y$. In the widely used double cross polarization, the system Hamiltonian may be described by $\bar{H}_{IS} = \kappa[\cos(\alpha)Z^- + \sin(\alpha)Y^-]$ [28]. By this Hamiltonian [instead of the Hamiltonian (1)], we numerically simulate the evolution of the nuclear spin and the entanglement between these spins; the results are presented in Figs. 6 and 7. The results show that in a nuclear-

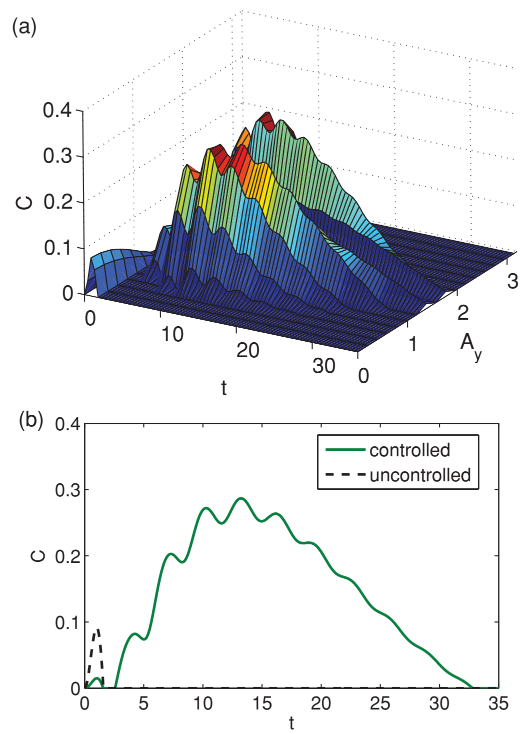

FIG. 5. (Color online) (a) Concurrence as a function of time and A_y. The system is initially in the state $|\psi\rangle = |e\rangle_1|e\rangle_2$, for the parameters $g/\omega = 1$, $\gamma/\omega = 0.5$. (b) The controlled concurrence evolution for $A_y = 1.2$, $A_x = A_z = 0$ vs. the uncontrolled case; t is in units of $\frac{1}{\omega}$ for (a) and (b).

electron spin system, the feedback-control scheme presented here is also available, and the results are similar to those with the Hamiltonian Eq. (1). The decoherence of electron spins in solids comes mainly from the inhomogeneity of the magnetic field. From the other point of view, this inhomogeneity induced decoherence can be described by atom-environment couplings [30]. Reference [31] shows that in the itinerant electrons model, after an appropriate transformation, the spin decoherence can be described by a boson-fermion Hamiltonian that would lead to the decoherence model chosen in this article.

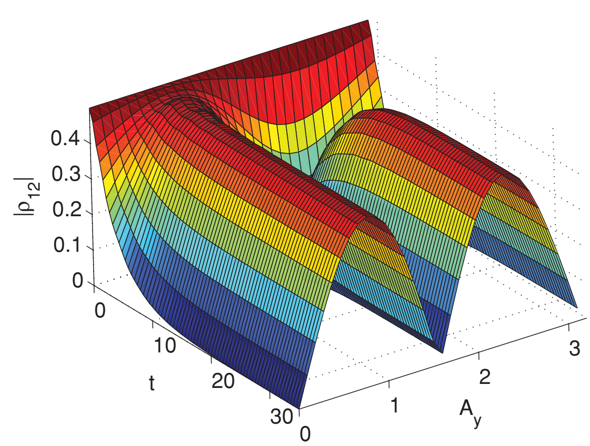

FIG. 6. (Color online) Off-diagonal element of the reduced density matrix for the nuclear spin vs. time with σ_y control. The parameters chosen are $g/\omega = 1, \gamma/\omega = 1, \kappa = 1$, and $\alpha = \frac{\pi}{4}$.

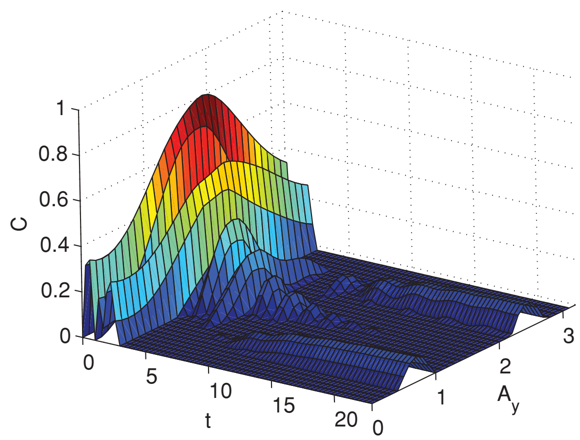

FIG. 7. (Color online) Concurrence as a function of time and A_y. The system is initially in the state $|\psi\rangle = |e\rangle_1|e\rangle_2$, for $g/\omega = 1, \gamma/\omega = 1, \kappa = 1$, and $\alpha = \frac{\pi}{4}$.

Therefore, the decoherence model presented here can be used to describe alternatively the decoherence of electrons in solids. On the other hand, the master equation can describe a wide range of decoherence in physical systems, so choosing such a model to study the feedback control is interesting on its own.

VI. CONCLUSION AND REMARKS

In this article, we studied the effect of quantum-jump-based feedback control on a system consisting of two qubits, where only one of them was subject to decoherence. By numerical simulation, we found that it is possible to suppress decoherence of the first qubit by a local control on the second qubit. We observed that the decoherence time of the first qubit is increased remarkably. The control scheme can also used to protect the entanglement between the two qubits. These features can be understood as the feedback control changing the dissipative dynamics of the system through the quantum-jump operators. We note that Hamiltonian Eq. (1) does not describe the hyperfine interaction. However, by the recent technology, we can simulate Hamiltonian Eq. (1) in nuclear-electron spin systems; in this sense, the scheme presented here is available for nuclear-electron spin systems. On the other hand, by using the hyperfine interaction Hamiltonian, our further simulations show that we can obtain results similar to that for Hamiltonian Eq. (1).

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[1] C. F. Roos, G. P. T. Lancaster, M. Riebe, H. Häffner, W. Hänsel, S. Gulde, C. Becher, J. Eschner, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. **92**, 220402 (2004).
[2] T. Puppe, I. Schuster, A. Grothe, A. Kubanek, K. Murr, P. W. H. Pinkse, and G. Rempe, Phys. Rev. Lett. **99**, 013002 (2007).
[3] H. M. Wiseman and G. J. Milburn, Phys. Rev. Lett. **70**, 548 (1993).
[4] H. M. Wiseman, Phys. Rev. A **49**, 2133 (1994).
[5] J. Wang, H. M. Wiseman, and G. J. Milburn, Phys. Rev. A **71**, 042309 (2005).
[6] A. R. R. Carvalho and J. J. Hope, Phys. Rev. A **76**, 010301 (2007).
[7] L. Viola and S. Lloyd, Phys. Rev. A **58**, 2733 (1998).
[8] G. Katz, M. A. Ratner, and R. Kosloff, Phys. Rev. Lett. **98**, 203006 (2007).
[9] N. Ganesan and T. J. Tarn, Phys. Rev. A **75**, 032323 (2007).
[10] J. Zhang, C. Li, R. Wu, T. Tarn, and X. Liu, J. Phys. A **38**, 6587 (2005).
[11] P. Bertet, I. Chiorescu, G. Burkard, K. Semba, C. J. P. M. Harmans, D. P. DiVincenzo, and J. E. Mooij, Phys. Rev. Lett. **95**, 257002 (2005).
[12] T. Hayashi, T. Fujisawa, H. D. Cheong, Y. H. Jeong, and Y. Hirayama, Phys. Rev. Lett. **91**, 226804 (2003).
[13] P. Rebentrost, I. Serban, T. Schulte-Herbrüggen, and F. K. Wilhelm, Phys. Rev. Lett. **102**, 090401 (2009).
[14] L. Vandersypen and I. Chuang, Rev. Mod. Phys. **76**, 1037 (2004).
[15] T. D. Ladd, D. Maryenko, Y. Yamamoto, E. Abe, and K. M. Itoh, Phys. Rev. B **71**, 014401 (2005).
[16] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, UK, 1997).
[17] C. W. Gardiner and P. Zoller, *Quantum Noises* (Springer-Verlag, Berlin, 1991).
[18] H. M. Wiseman, S. Mancini, and J. Wang, Phys. Rev. A **66**, 013807 (2002).
[19] C. Altafini, J. Math. Phys. **44**, 2357 (2003).
[20] A. R. R. Carvalho, A. J. S. Reid, and J. J. Hope, Phys. Rev. A **78**, 012334 (2008).
[21] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[22] P. Cappellaro, L. Jiang, J. S. Hodges, and M. D. Lukin, Phys. Rev. Lett. **102**, 210502 (2009).
[23] R. S. Said and J. Twamley, Phys. Rev. A **80**, 032303 (2009).
[24] P. Cappellaro, C. Ramanathan, and D. G. Cory, Phys. Rev. Lett. **99**, 250506 (2007).
[25] C. Ramanathan, S. Sinha, J. N. Baugh, T. F. Havel, and D. G. Cory, Phys. Rev. A **71**, 020303 (2005).
[26] C. H. Tseng, S. Somaroo, Y. Sharf, E. Knill, R. Laflamme, T. F. Havel, and D. G. Cory, Phys. Rev. A **61**, 012302 (1999).
[27] G. A. Avarez and Dieter Suter, Phys. Rev. Lett. **104**, 230403 (2010).
[28] N. Khaneja, C. Kehlet, S. J. Glaser, and N. C. Nielsen, J. Chem. Phys. **124**, 114503 (2006).
[29] M. Bjerring, J. T. Rasmussen, R. S. Krogshave, and N. C. Nielsen, J. Chem. Phys. **119**, 8916 (2003); N. Khaneija, C. Kehlet, S. J. Glaser, and N. C. Nielsen, *ibid.* **124**, 114503 (2006).
[30] M. Orszag, *Quantum Optics: Including Noise Reduction, Trapped Ions, Quantum Trajectories, and Decoherence* (Springer, Berlin, 2008).
[31] M. Frasca, Phys. Rev. B **68**, 193413 (2003).