Cavity quantum electro-optics

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(Received 9 March 2010; published 30 June 2010)

The quantum dynamics of the coupling between a cavity optical field and a resonator microwave field via the electro-optic effect is studied. This coupling has the same form as the optomechanical coupling via radiation pressure, so all previously considered optomechanical effects can in principle be observed in electro-optic systems as well. In particular, I point out the possibilities of laser cooling of the microwave mode, entanglement between the optical mode and the microwave mode via electro-optic parametric amplification, and back-action-evading optical measurements of a microwave quadrature.

DOI: 10.1103/PhysRevA.81.063837 PACS number(s): 42.50.Pq, 42.65.Ky, 42.65.Lm, 42.79.Hp

I. INTRODUCTION

Recent technological advances in optics [1,2], mechanics [2,3], and superconducting microwave circuits [4] have made observations of various quantum phenomena in microscopic systems possible. Such advances also suggest that quantum effects will play a major role in future communication, sensing, and computing technology. Each physical system has its own advantages and disadvantages for different applications, so it is desirable to transfer classical or quantum information to different systems for different purposes. At the quantum level, accurate information transfer requires coherent coupling among the systems. While optomechanical and electromechanical coupling have attracted a lot of attention recently [2,3], the coherent coupling between optical and microwave fields has been largely overlooked in the context of quantum information processing, even though electro-optic modulation is a well-known phenomenon in classical optics [5]. Previous studies on the quantum aspects of the electro-optic effect mainly treat the microwave as a classical signal for linear quantum optical processing [6], but treating both fields quantum-mechanically should give rise to novel physics.

In this paper, I consider both the optical field and the microwave field as quantum degrees of freedom and study the coupling between the fields via the electro-optic effect. Both fields are assumed to be modes in a cavity or a resonator, since it is likely that both of them need to be resonantly enhanced for quantum effects to be observable. In terms of prior work, Ilchenko *et al.* have previously outlined such a theory in their study of photonic microwave receivers [7], while Matsko *et al.* have studied the quantum back-action noise in electro-optic modulation [8]. As noted by Matsko *et al.*, the electro-optic coupling has the same form as the optomechanical coupling via radiation pressure, so all previously considered optomechanical effect can in principle be observed in electro-optic systems as well. In particular, I point out the possibilities of laser cooling of the microwave mode, entanglement between the optical mode and the microwave mode via electro-optic parametric amplification, and back-action-evading optical measurements of a microwave quadrature. All these effects require optical sideband pumping, which is not considered in Refs. [7,8] and is the main technical difference between this paper and the prior work on quantum electro-optics. If realized, the proposed effects should be useful for both fundamental science and information-processing applications in the quantum regime. For example, electro-optic cooling allows one to observe quantum optical effects at higher background temperatures or lower microwave or even radio frequencies, while coherent optical detection of microwave quadratures potentially allows one to leverage the high efficiency of optical detectors compared with conventional microwave detectors, the highest reported quantum efficiency of which is only 27% [9], for continuous low-noise microwave measurements.

II. FORMALISM

A full quantum treatment of the electro-optic effect can be done using standard quantum optics theory [5,10], but here I shall use a more phenomenological treatment to emphasize the similarity between electro-optics and optomechanics. Consider the cavity electro-optic system depicted in Fig. 1. The transverse electro-optic modulator, which consists of a second-order nonlinear-optical medium, such as lithium niobate or an electro-optic polymer, introduces a voltage-dependent phase shift to the intracavity optical field resonant at frequency ω_a. For simplicity, I first consider one optical mode. It is straightforward to show that the interaction Hamiltonian is

$$H_i = -\frac{\hbar}{\tau}\phi a^\dagger a, \tag{1}$$

where a and $a^\dagger$ are the optical annihilation and creation operators, respectively, which obey the commutation relation $[a,a^\dagger] = 1$, τ is the optical round-trip time, and ϕ is the single-round-trip phase shift. This Hamiltonian is exactly the same as the one for optomechanics, in which case ϕ is the phase shift introduced by the cavity mirror displacement [2]. The round-trip electro-optic phase shift, on the other hand, is given by [5]

$$\phi = \frac{\omega_a n^3 r l}{cd}V, \tag{2}$$

where n is the optical refractive index inside the electro-optic medium, r is the electro-optic coefficient in units of m/V, l is the length of the medium along the optical axis, d is the thickness, and V is the voltage across the medium. If

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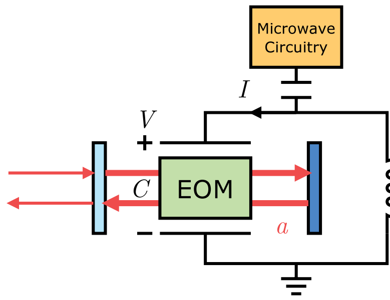

FIG. 1. (Color online) A basic cavity electro-optic system. EOM denotes an electro-optic modulator.

the modulator is modeled as a capacitor in a single-mode microwave resonator, one can quantize the voltage by defining

$$V = \left(\frac{\hbar\omega_b}{2C}\right)^{1/2}(b + b^\dagger), \tag{3}$$

where b and $b^\dagger$ are the microwave annihilation and creation operators, respectively, which obey $[b,b^\dagger] = 1$, ω_b is the microwave resonant frequency, and C is the capacitance of the microwave resonator. The full Hamiltonian becomes

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b - \hbar g(b + b^\dagger)a^\dagger a, \tag{4}$$

$$g \equiv \frac{\omega_a n^3 r l}{c\tau d}\left(\frac{\hbar\omega_b}{2C}\right)^{1/2}. \tag{5}$$

The fact that the Hamiltonian for electro-optics has the same form as that for optomechanics allows one to apply known results for the latter to the former. V then plays the role of the mechanical position and I, the current in the microwave resonator, plays the role of the mechanical momentum.

For the whispering-gallery-mode electro-optic modulator reported by Ilchenko *et al.* [7], the microwave frequency ω_b is close to the optical free spectral range $\Delta\omega$, so one should include multiple optical modes in the analysis [7,8]. Dobrindt and Kippenberg also performed a similar multimode analysis for optomechanics [11]. One can limit the analysis to three optical modes if $|\Delta\omega - \omega_b|$ is much larger than the optical linewidth γ_a, so that higher-order optical harmonics are not resonantly coupled, or if one introduces two or three nondegenerate optical modes via normal-mode splitting using two or three coupled optical cavities, so that higher-order modes are farther away in frequency [11,12]. To demonstrate cooling or parametric amplification alone, only two modes are needed. One can also detune the high-order modes by introducing an appropriate frequency dispersion to the cavity. This may be done by engineering the geometry of the cavity or using a photonic crystal structure.

Let a, a_1, and a_2 be the annihilation operators for the center optical mode, red-detuned mode, and blue-detuned mode, respectively. The Hamiltonian becomes [7,8]

$$\begin{aligned} H &= \hbar\omega_a a^\dagger a + \hbar(\omega_a - \Delta\omega)a_1^\dagger a_1 + \hbar(\omega_a + \Delta\omega)a_2^\dagger a_2 \\ &\quad + \hbar\omega_b b^\dagger b - \hbar g(b + b^\dagger)(a + a_1 + a_2)^\dagger(a + a_1 + a_2), \end{aligned} \tag{6}$$

assuming that the coupling coefficients for different optical modes are the same for simplicity and g is slightly modified to account for the partial overlap among the modes. In the following, I shall focus on the experimentally more relevant case of multiple optical modes.

III. COOLING AND PARAMETRIC AMPLIFICATION

To describe cooling and parametric amplification, it is more convenient to use the interaction picture. Let

$$b = \tilde{b}\exp(-i\omega_b t), \tag{7}$$

$$a = \tilde{a}\exp(-i\omega_a t), \tag{8}$$

$$a_1 = a_-\exp[-i(\omega_a - \Delta\omega)t], \tag{9}$$

$$a_2 = a_+\exp[-i(\omega_a + \Delta\omega)t]. \tag{10}$$

With the rotating-wave approximation, the Hamiltonian becomes

$$H_I \approx H_C + H_A, \tag{11}$$

$$H_C \equiv -\hbar g(\alpha_-\tilde{a}^\dagger\tilde{b} + \alpha_-^\dagger\tilde{a}\tilde{b}^\dagger), \tag{12}$$

$$H_A \equiv -\hbar g(\alpha_+\tilde{a}^\dagger\tilde{b}^\dagger + \alpha_+^\dagger\tilde{a}\tilde{b}), \tag{13}$$

$$\alpha_- \equiv a_-\exp[i(\Delta\omega - \omega_b)t] = a_1\exp[i(\omega_a - \omega_b)t], \tag{14}$$

$$\alpha_+ \equiv a_+\exp[-i(\Delta\omega - \omega_b)t] = a_2\exp[i(\omega_a + \omega_b)t]. \tag{15}$$

If one pumps the optical cavity at frequencies $\omega_a \pm \omega_b$ such that $\alpha_\pm$ can be approximated as classical complex numbers, H_C corresponds to linear coupling between $\tilde{a}$ and $\tilde{b}$ as in a beam splitter, while H_A corresponds to nondegenerate parametric amplification of the two modes. These two effects are schematically described in Fig. 2.

With red-detuned optical pumping at $\omega_a - \omega_b$, the microwave field is linearly coupled to an extra optical reservoir, so the microwave mode can be cooled. To account for coupling to traveling waves and thermal reservoirs, the easiest way is to use quantum Langevin equations [10,13]. Assuming $H_A \approx 0$, the equations of motion for $\tilde{a}$ and $\tilde{b}$ are

$$\frac{d\tilde{a}}{dt} = ig\alpha_-\tilde{b} - \frac{\gamma_a}{2}\tilde{a} + \sqrt{\gamma_a}A, \tag{16}$$

$$\frac{d\tilde{b}}{dt} = ig\alpha_-^*\tilde{a} - \frac{\gamma_b}{2}\tilde{b} + \sqrt{\gamma_b}B, \tag{17}$$

where γ_a and γ_b are the optical and microwave decay coefficients, respectively, and A and B are input quantum Langevin noise operators, which obey the commutation relations given by

$$[A(t),A^\dagger(t')] = \delta(t - t'), \quad [B(t),B^\dagger(t')] = \delta(t - t'). \tag{18}$$

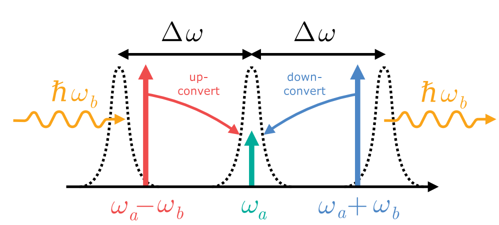

FIG. 2. (Color online) Red-detuned optical pumping cools the microwave mode by transferring energy from the microwave to the optical mode at ω_a via parametric up-conversion. Blue-detuned pumping causes nondegenerate parametric down-conversion to the optical mode at ω_a and the microwave mode.

Assuming thermal and white statistics for A and B such that

$$\langle A^\dagger(t')A(t)\rangle = N(\omega_a)\delta(t-t'), \tag{19}$$

$$\langle B^\dagger(t')B(t)\rangle = N(\omega_b)\delta(t-t'), \tag{20}$$

$$N(\omega) = \frac{1}{\exp(\hbar\omega/k_B T)-1}, \tag{21}$$

it can be shown that the microwave occupation number at steady state is

$$\langle \tilde{b}^\dagger \tilde{b}\rangle_{t\to\infty} = \frac{N(\omega_b)+GN(\omega_a)}{1+G}, \tag{22}$$

$$G \equiv \frac{G_0}{1+(\gamma_b/\gamma_a)(1+G_0)}, \quad G_0 \equiv \frac{4g^2|\alpha_-|^2}{\gamma_a\gamma_b}, \tag{23}$$

which implies cooling for $g \neq 0$, since $N(\omega_a) < N(\omega_b)$ and $\langle \tilde{b}^\dagger \tilde{b}\rangle_{t\to\infty} < N(\omega_b)$. This is the electro-optic analog of optomechanical sideband cooling [14]. Note that G can saturate with increasing pump power when $G_0 \sim \gamma_a/\gamma_b$, and the upper limit is

$$\lim_{G_0\to\infty} G = \frac{\gamma_a}{\gamma_b}, \tag{24}$$

in which case cooling is limited by the decay rate of the optical mode rather than the electro-optic coupling strength [15]. The linear coupling between $\tilde{a}$ and $\tilde{b}$ also allows classical or quantum information to be transferred coherently between the microwave mode and the optical mode and may be useful as a coherent microwave receiver [7].

With blue-detuned pumping at $\omega_a + \omega_b$, the parametric amplification process can create entangled photons or, equivalently, a two-mode squeezed state in the optical and microwave modes, if thermal fluctuations are negligible. The effect of thermal fluctuations on the entanglement can also be studied using quantum Langevin equations and is qualitatively the same as that on the analogous phenomenon of optomechanical entanglement [16]. One may also use parametric amplification beyond threshold to generate coherent microwaves, analogous to a phonon laser in opto-mechanics [12,17]. It can be shown that the oscillation threshold condition is $4g^2|\alpha_+|^2/(\gamma_a\gamma_b) \geqslant 1$, which requires a similar number of pump photons to that required for significant cooling.

IV. EFFECT OF PARASITIC DOWN-CONVERSION ON COOLING

As mentioned in Sec. II, if a whispering-gallery-mode optical cavity is used, parasitic coupling to higher-order modes can hamper the cooling or parametric amplification efficiency. To investigate the effect of this parasitic coupling on cooling, consider optical pumping at frequency $\omega_a + \delta$ and $\delta = \Delta\omega - \omega_b$, as shown in Fig. 3. The up-conversion process is then resonantly enhanced with respect to the down-conversion process. Assuming undepleted pumping, the steady-state center-mode field is

$$\tilde{a} \approx \alpha_0 \exp(-i\delta t), \quad \alpha_0 \equiv \frac{\sqrt{\gamma_a}\langle A\rangle}{-i\delta + \gamma_a/2}. \tag{25}$$

With the definition

$$a_- = \tilde{a}_- \exp(-2i\delta t), \tag{26}$$

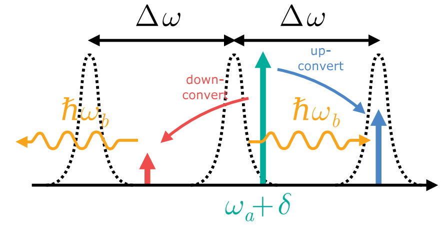

FIG. 3. (Color online) Electro-optic cooling via detuned pumping of the center optical mode. The parasitic down-conversion process can limit the lowest microwave occupation number one can achieve.

the coupled-mode equations become

$$\frac{d\tilde{a}_-}{dt} = 2i\delta\tilde{a}_- + ig\alpha_0\tilde{b}^\dagger - \frac{\gamma_a}{2}\tilde{a}_- + \sqrt{\gamma_a}A_-, \tag{27}$$

$$\frac{da_+}{dt} = ig\alpha_0\tilde{b} - \frac{\gamma_a}{2}a_+ + \sqrt{\gamma_a}A_+, \tag{28}$$

$$\frac{d\tilde{b}}{dt} = ig\alpha_0^* a_+ + ig\alpha_0\tilde{a}_-^\dagger - \frac{\gamma_b}{2}\tilde{b} + \sqrt{\gamma_b}B, \tag{29}$$

where A_- and A_+ are the input Langevin noise operators for the $\tilde{a}_-$ and a_+ modes, respectively. Assuming

$$N(\omega_a \pm \Delta\omega) \approx 0, \quad \gamma_b \ll \gamma_a, \quad 2g|\alpha_0| \ll \gamma_a \tag{30}$$

for simplicity, I obtain, after some straightforward but cumbersome algebra,

$$\langle \tilde{b}^\dagger \tilde{b}\rangle_{t\to\infty} \approx \frac{N(\omega_b)+\Gamma\mu}{1+\Gamma}, \tag{31}$$

$$\Gamma \equiv \frac{\Gamma_0}{1+\mu}, \quad \Gamma_0 \equiv \frac{4g^2|\alpha_0|^2}{\gamma_a\gamma_b}, \quad \mu \equiv \frac{\gamma_a^2}{16\delta^2}. \tag{32}$$

The lowest achievable microwave occupation number is thus

$$\lim_{\Gamma_0\to\infty} \langle \tilde{b}^\dagger \tilde{b}\rangle_{t\to\infty} = \mu \equiv \frac{\gamma_a^2}{16\delta^2}. \tag{33}$$

Interestingly, this expression is identical to that for the minimum mechanical occupation number in optomechanical cooling using one optical mode [14]. Since the pump photon number $|\alpha_0|^2$ also depends on δ, the optimal δ depends on the specific parameters of the system and can be calculated by minimizing $\langle \tilde{b}^\dagger \tilde{b}\rangle_{t\to\infty}$ with respect to δ. For example, when $N(\omega_b) \gg \Gamma\mu$, Γ is maximized and $\langle \tilde{b}^\dagger \tilde{b}\rangle_{t\to\infty}$ is minimized when $\mu = 0.5$.

V. OPTICAL QUANTUM NONDEMOLITION MEASUREMENTS OF A MICROWAVE QUADRATURE

To perform quantum nondemolition (QND) measurements of the microwave voltage, one can pump the optical cavity with laser light at ω_a in the single-optical-mode case and estimate the microwave voltage by measuring the phase of the output optical wave $A' = \sqrt{\gamma_a}\tilde{a} - A$. If the optical pump is strong enough, in addition to the intrinsic shot noise in the phase quadrature of A', one can also observe excess noise due to the back action of light on the microwave. The shot noise relative to the signal decreases with increasing optical power but the back-action noise increases with power, so there is a standard quantum limit (SQL) to voltage measurement sensitivity, much like the SQL of displacement measurement due to radiation pressure [8,18]. In both cases, the SQL can be overcome using squeezed light [19–21], variational measurement [20,22], or two mechanical or microwave resonators [23,24]. Although the SQL has not been observed in optomechanical or electro-optic systems, it can be simulated by adding classical excess noise to the optical pump [21,24], and the aforementioned noise cancellation techniques can be used to remove any classical back-action noise as well.

An interesting way of making back-action-evading measurements is to use double-sideband optical pumping [18,25]. Assuming equal-magnitude and undepleted double-sideband pumping, so that

$$\alpha_+ = |\alpha| \exp(i\theta_+), \quad \alpha_- = |\alpha| \exp(i\theta_-), \tag{34}$$

defining quadrature operators as

$$X_a \equiv \exp(-i\theta)\tilde{a} + \exp(i\theta)\tilde{a}^\dagger, \tag{35}$$

$$Y_a \equiv -i[\exp(-i\theta)\tilde{a} - \exp(i\theta)\tilde{a}^\dagger], \tag{36}$$

$$X_b \equiv \exp(-i\nu)\tilde{b} + \exp(i\nu)\tilde{b}^\dagger, \tag{37}$$

$$Y_b \equiv -i[\exp(-i\nu)\tilde{b} - \exp(i\nu)\tilde{b}^\dagger], \tag{38}$$

$$\xi \equiv \exp(-i\theta)A + \exp(i\theta)A^\dagger, \tag{39}$$

$$\eta \equiv -i[\exp(-i\theta)A - \exp(i\theta)A^\dagger], \tag{40}$$

$$\theta \equiv \frac{\theta_+ + \theta_-}{2}, \quad \nu \equiv \frac{\theta_+ - \theta_-}{2}, \tag{41}$$

and again making the rotating-wave approximation, the equations of motion become

$$\frac{dX_a}{dt} = -\frac{\gamma_a}{2}X_a + \sqrt{\gamma_a}\xi, \quad \frac{dX_b}{dt} = 0, \tag{42}$$

$$\frac{dY_a}{dt} = 2g|\alpha|X_b - \frac{\gamma_a}{2}Y_a + \sqrt{\gamma_a}\eta, \quad \frac{dY_b}{dt} = 2g|\alpha|X_a, \tag{43}$$

where coupling to traveling waves and reservoirs for the microwave mode is neglected for clarity. The measured microwave quadrature X_b is dynamically decoupled from the orthogonal quadrature Y_b, so the back action introduced to Y_b via X_a does not affect the estimation accuracy of X_b. X_b and Y_a can be changed independently by adjusting the phases of the two pump waves. This technique should also enable microwave squeezing by measurement and feedback [26]. The double-sideband-pumping back-action-evading measurement scheme has recently been demonstrated for the measurement of a mechanical quadrature with a microwave field acting as the meter [25].

VI. EXPERIMENTAL FEASIBILITY

For the cavity electro-optic modulator reported by Ilchenko *et al.* [7],

$$\omega_b \approx 2\pi \times 9 \text{ GHz}, \quad g \approx 2\pi \times 20 \text{ Hz},$$
$$\gamma_a \approx 2\pi \times 40 \text{ MHz}, \quad \gamma_b \approx 2\pi \times 90 \text{ MHz}. \tag{44}$$

If we assume a pump power P of 2 mW, $\lambda_0 \equiv 2\pi c/\omega_a =$ 1550 nm, and $\gamma_a^2/(16\delta^2) = 0.5$, then

$$|\alpha_-|^2 = \frac{\gamma_a P}{\hbar\omega_a(\delta^2 + \gamma_a^2/4)} \approx 1.7 \times 10^8, \quad G \approx 2 \times 10^{-5}. \tag{45}$$

G thus needs to be increased by a factor of 10^5 for cooling to be significant. Apart from increasing the pump power, one can also improve the g coefficient by reducing the size and capacitance of the microwave resonator. If one can make use of the maximum electro-optic coefficient $n^3r \sim 300$ pm/V in lithium niobate [5], make $d \sim 10$ μm instead of the 150 μm reported in Ref. [7], and assume $l/(c\tau) \sim 0.5$ and $C \sim 1$ pF, the g coefficient given by Eq. (5) can be as high as $2\pi \times 5$ kHz, which would make $G \sim 0.3$. This G is already close to the upper limit $\gamma_a/\gamma_b \sim 0.4$ according to Eq. (24), so one should reduce γ_b for more significant cooling. γ_b can be significantly reduced if the microwave resonator is unloaded or made of a better conductor. The quality factor of superconducting microwave resonators can be as high as 2×10^6–3×10^8 [4,27] and γ_b can then be reduced and G be enhanced by a factor of 10^4–10^6. Hence, cooling and the other quantum effects proposed in this paper should be experimentally observable in the near future, as the technology for electro-optic integration continues to improve.

VII. CONCLUSION

In conclusion, I have outlined several interesting quantum effects that can in principle be observed in electro-optic systems. Although these effects are challenging to demonstrate experimentally with current technology, they will enable additional classical and quantum information-processing capabilities that should be useful for both fundamental science and applied technology.

ACKNOWLEDGMENTS

Discussions with Tobias Kippenberg, Elanor Huntington, Matthew Woolley, Carlton Caves, and Jeffrey Shapiro are gratefully acknowledged. This work was supported in part by National Science Foundation Grants No. PHY-0903953 and No. 0653596 and Office of Naval Research Grant No. N00014-07-1-0304.

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