Arbitrarily high super-resolving phase measurements at telecommunication wavelengths

Christian Kothe,[1,2] Gunnar Björk,[1] and Mohamed Bourennane[2]

[1]*School of Information and Communication Technology, Royal Institute of Technology (KTH), Electrum 229, SE-164 40 Kista, Sweden*

[2]*Department of Physics, Stockholm University, SE-109 61 Stockholm, Sweden*

(Received 22 July 2009; revised manuscript received 19 April 2010; published 30 June 2010)

We present two experiments that achieve phase super-resolution at telecommunication wavelengths. One of the experiments is realized in the space domain and the other is realized in the time domain. Both experiments show high visibility and are performed with standard lasers and single-photon detectors. The first experiment uses six-photon coincidences, whereas the latter experiment needs no coincidence measurements, is easy to perform, and achieves, in principle, arbitrarily high phase super-resolution. Here, we demonstrate a 30-fold increase of the resolution. We stress that neither entanglement nor joint detection is needed in these experiments, which demonstrates that neither is necessary to achieve phase super-resolution.

DOI: 10.1103/PhysRevA.81.063836 PACS number(s): 42.50.Dv, 03.67.−a, 42.50.St

I. INTRODUCTION

Interference plays a crucial role in many physical measurements, such as the detection of gravitational waves [1–3], metrology [4,5], interferometry and atomic spectroscopy [6–8], imaging [9], and lithography [10]. Improvements of these schemes can be achieved with the help of *phase super-resolution*, where n oscillations (fringes) appear in the interference pattern over a range, which usually would have given only one oscillation [11,12]. In a similar vein, *phase supersensitivity* decreases the phase uncertainty in such experiments so that the measurement sensitivity would surpass the classical limit (i.e., by beating the standard quantum limit [13,14]).

It was believed that entangled states were needed to achieve phase super-resolution [15]. One such state is a path-entangled number state, the so-called NOON state [16],

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|N\rangle|0\rangle + |0\rangle|N\rangle), \tag{1}$$

where N denotes the number of particles (most often photons). This state is a superposition of N particles in one path and no particles in the other path, and vice versa. The production of such states at satisfying rates already gets extremely difficult for small N. So far, only experiments with N up to 4 have been reported [11,17]. Recently, however, schemes were proposed and were shown, where phase super-resolution could be achieved by unentangled coherent light [13]. In that paper, an experiment was reported, where $n = N = 6$ oscillations occur over the period of one classical oscillation. For their experiment, Resch *et al.* used a six-photon coincidence, but received rather limited counting rates ($\leqslant$27 counts/10 s) and moderate visibility (50%–90%). Other experiments with unentangled states and phase super-resolution with $n > 4$ have been reported [18]. All those experiments used light at wavelengths around 800 nm.

In this paper, we report on two super-resolution experiments. One is in the *space domain* like all the experiments hitherto reported, but uses an innovative approach, which results in higher counting rates and very high visibility. The other experiment is performed in the *time domain* and shows very promising counting rates and visibility. Both experiments could, in principle, be scaled to high numbers of $n > 100$. Whereas the first experiment requires more components for higher numbers of n, the second experiment only requires a longer measurement time. Our space- and time-domain experiments were performed up to $n = 6$ and $n = 30$, respectively. Furthermore, these experiments are performed at telecommunication wavelengths, which allow efficient transmission of the photons over a long distance via an optical fiber and thereby allow more opportunities for applications of phase super-resolution.

II. PHASE SUPER-RESOLUTION—DEQUANTIFIED

We briefly introduce the theory of phase super-resolution. Consider the state given in Eq. (1). If a relative phase shift ϕ is imposed between the two modes the state transforms into

$$|\Psi(\phi)\rangle = \tfrac{1}{2}(|N\rangle|0\rangle + e^{iN\phi}|0\rangle|N\rangle), \tag{2}$$

since energy (difference) is the generator of (relative) phase. The phase $N\phi$ will, therefore, grow linearly with the number of particles N. If the two modes are treated as, for example, spatial modes and are combined via a 50:50 beam splitter (BS), then the detection probability $P \propto 1 \pm \cos(N\phi)$ in the two outputs of the BS when measuring N-fold coincidence. P exhibits phase super-resolution, since it oscillates $n = N$ times when ϕ varies from 0 to 2π. The same effect, however, can also be reached without entanglement by a so-called time-reversal measurement [13]. In the cited paper, the effect is explained by using the inherent time-reversal symmetry of quantum mechanics and measurement post-selected entanglement. A different view of the experiment is the following, based on the mathematical relation $\sin(2\phi) = 2\sin(\phi)\cos(\phi)$, or, more generally:

$$\frac{\sin(n\phi)}{2^{n-1}} = \sin(\phi)\sin\left(\phi + \frac{\pi}{n}\right)\sin\left(\phi + \frac{2\pi}{n}\right)\cdots \sin\left[\phi + \frac{(n-1)\pi}{n}\right]. \tag{3}$$

A phase super-resolving measurement can, hence, be implemented as a multiplication (e.g., coincidence detection) of n ordinary phase measurements, each shifted by $k\pi/n$, where $k = 0,1,\ldots,n-1$.

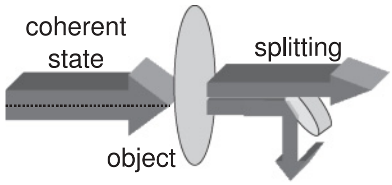

FIG. 1. A coherent state split spatially into two by a mirror.

A coherent state can be split into several modes, where each of these will be a coherent state [19]. Since the ensuing multimode state is separable, there are no quantum correlations between the states.

For example, a coherent state in an optical beam can be split in two halves by a mirror, as in Fig. 1. If the two half beams come from the same or from different identical sources makes no difference. It also does not matter if the beam is split after the interaction by a totally reflecting mirror inserted halfway into the beam or by a semitransparent BS, insofar as the measured object characteristic does not vary over the width of the beam. Hence, we already may as well split the beam before the object, as in Fig. 2. Also, it does not matter if the coherent-state mode is split spatially, temporally, or in frequency, insofar as the object does not vary over the corresponding space, time, or frequency range. In previous experiments, the state has been split spatially, by BSs, after the interaction. We find it simpler to split the state before the interaction, or in time, and this and the observation delineated in Eq. (3) are the basic ingredients of our experiments and the interpretation thereof.

III. EXPERIMENT—SPACE DOMAIN

In our experiment, to measure phase super-resolution in the space domain, we built the setup illustrated in Fig. 2. Except for the fact that we split the coherent state before the half-wave plate (HWP), the setup is essentially identical to that in Ref. [13]. The coherent state is generated by a $\lambda = 1550$-nm laser, which is pulsed at a rate of 2.5 MHz. Every pulse has a duration of around 500 ps, and the average power of the laser is 1 mW, which, directly after the laser, is attenuated by 50–55 dB (not shown in the figure) to avoid overexposure of our single-photon detectors (SPDs). A polarizing beam splitter (PBS) in front of the laser assures that the light is horizontally (H) polarized. Two 50:50 BSs divide the beam into three paths, which are superimposed on an HWP. The HWP can be rotated to any angle φ by a motor. One has to assure that the angle α between the beams is small so that the path lengths of the beams in the HWP (the object) are essentially the same. In our case, we use an angle of $\alpha = 2.9^\circ$. Since $\varphi = 45^\circ$ turns the polarization from H to vertical (V), one can see one oscillation by turning the HWP by 90° (which corresponds to a relative-phase shift of 360° in our figures) and detect, for example, only H-polarized photons. Two of the beams pass another HWP rotated by the angles 15° and 30°, respectively, to rotate the polarization further in such a way that they fulfill the relation in Eq. (3) after detection. Each beam then passes another PBS, carefully oriented so that the H-polarized light goes into one arm and the V-polarized light goes into the other arm. These six beams are then coupled into single-mode fibers, which lead to SPDs. The SPDs are gated and open only for 1 ns when a coherent-state pulse is coming, and the outputs of the SPDs are led to a six-channel coincidence counter, and subsequently, to a computer, where the coincidences are registered and are stored.

The results can be seen in Fig. 3. Every SPD detected one oscillation while turning the HWP by 90° (which corresponds to a phase shift of 360° in the figure), but from one SPD to the next, the peak of the oscillation was shifted by 60° Multiplication of the stored individual counts from all six SPDs in the computer gives the black dots in the figure, where the scale is in arbitrary units. In the case of the sixfold coincidences (gray dots), the scale to the left applies. One can clearly distinguish six peaks as expected, so a sixfold increase of the phase resolution is achieved. At every angle, we measured for 10 s and got counting rates of more than 1500 counts/s for the sixfold coincidences and about 10^6 counts/s for the individual SPDs. The visibility is high, between 98.6% and 96.7% in the case of the multiplication of the single detections and between 98.6% and 97.0% in the case of the coincidence detection. For calculating the visibility, we took the lowest of the two minima around a peak. The reason that two of the minima are higher than the other ones is due to imperfections in one of the PBSs,

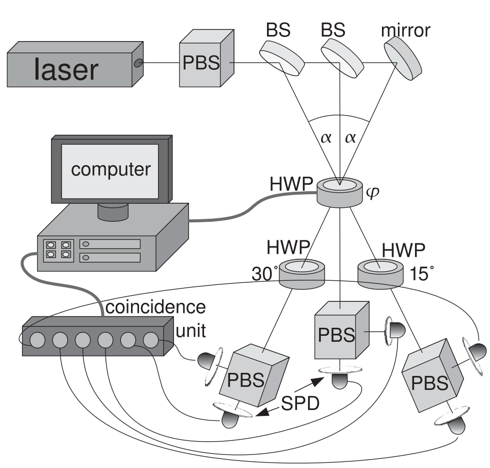

FIG. 2. Setup to measure sixfold phase super-resolution in the space domain. See the text for further details.

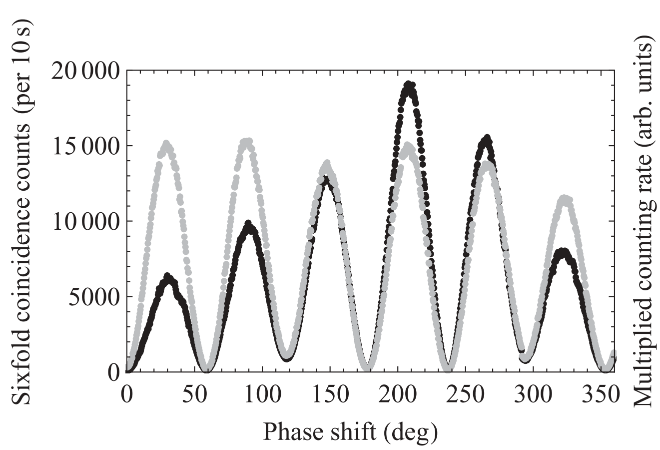

FIG. 3. Phase super-resolution in the space domain by measuring sixfold coincidence (gray dots) and by multiplying single-photon detections (black dots). The statistical errors are smaller than the dot size.

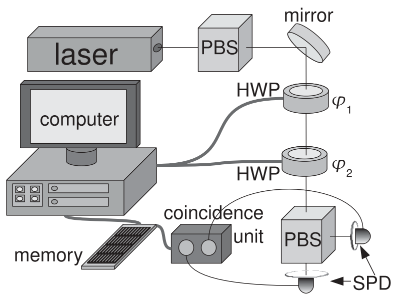

FIG. 4. Experimental setup to measure phase super-resolution in the time domain. See the text for further details.

which does not perfectly split the beam into H- and V-polarized components. To have better PBSs would probably result in a more uniform visibility $>98\%$. Another improvement would be to replace the first 50:50 BS with a 33% reflecting BS so that all the beams have the same intensity. This would neither change the visibility nor change the resolution, but would increase the coincidence rate. In principle, the setup can be extended to detect phase super-resolution with a factor $n > 6$ by adding more arms. The drawbacks are that this requires more components and that the coincidence counting rate gets exponentially lower so that one would have to increase the measurement time correspondingly.

IV. EXPERIMENT—TIME DOMAIN

To obtain phase super-resolution with $n > 6$, it, therefore, is more practical to perform the measurement in the time domain. To this end, we built the setup in Fig. 4. The parameters of the laser are the same as in the setup in the space domain, except at a slightly higher attenuation. Instead of having several physical arms in the setup, we used a computer to store the data sequentially to have, so to speak, n arms after each other in time. In this setup, we counted and stored the clicks in each SPD as a function of φ_1, turned the second HWP by an angle $\Delta\varphi_2 = 90°/n$, repeated the process n times, and numerically multiplied the results according to Eq. (3). For $n = 6$, we set the second HWP at the angles $\varphi_2 = 0°, 15°$, and $30°$ by using both SPDs, or at $\varphi_2 = 0°, 15°, 30°, \ldots, 75°$ when using only one SPD.

The results of our measurements can be seen in Fig. 5. We were rotating the first HWP by small increments over the range $0 \leqslant \varphi_1 \leqslant 90°$ to introduce a differential phase shift. Then, we change the angle of the second HWP after each scan of φ_1 by $\Delta\varphi_2 = 9°$ to get phase super-resolution by a factor of $n = 10$. At each combination of angles, we measured for 1 s. The upper panel of Fig. 5 shows the results when we used both SPDs followed by a coincidence counter, and the lower panel shows the results when we used only one SPD, which required twice as many settings of the angle φ_2. As one can see, the first method gives higher visibility 99.6% and takes only half the time, and, therefore, is more efficient. On the other hand, the second method needs neither the second SPD nor the coincidence detector in Fig. 4. This clearly shows that phase super-resolution can be achieved with neither entanglement nor joint detection.

Further results can be seen in Fig. 6, where we achieved phase super-resolution by a factor of $n = 30$. (Note that the x axis, in this case, only spans the range 0°–45°.) In this case too, every combination of angles φ_1, φ_2 was measured for 1 s. The reason that the visibility is degraded is most certainly due to the insufficient stability of our laser's intensity over long time periods. In our experiments, since higher values of n require smaller measurement increments of φ_1 to resolve one oscillation period, we had to measure for a long time to get a

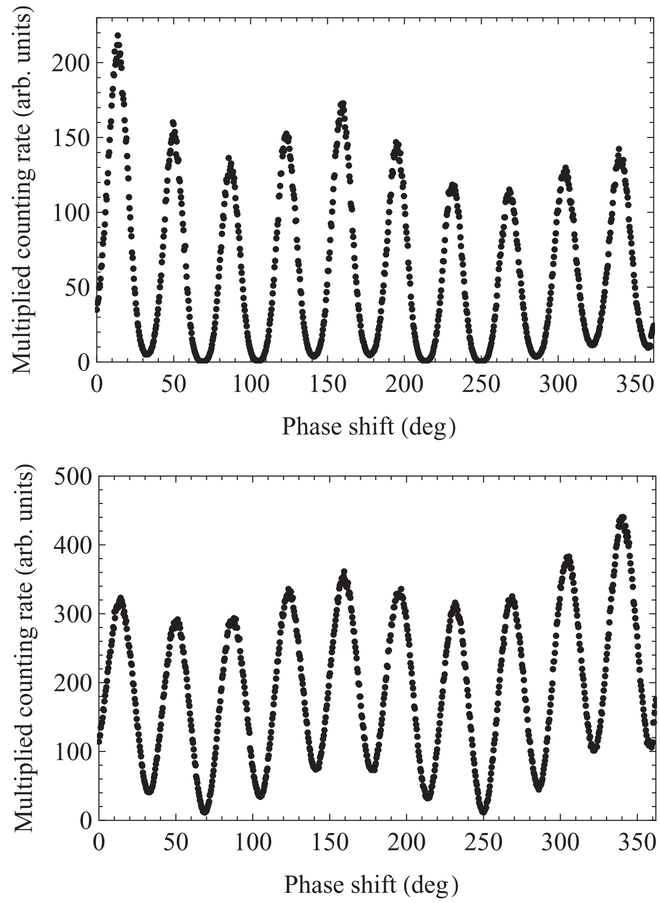

FIG. 5. Phase super-resolution with $n = 10$ in the time domain. The data were taken with the help of two SPDs and a coincidence unit (upper panel) and with the help of only one SPD (lower panel).

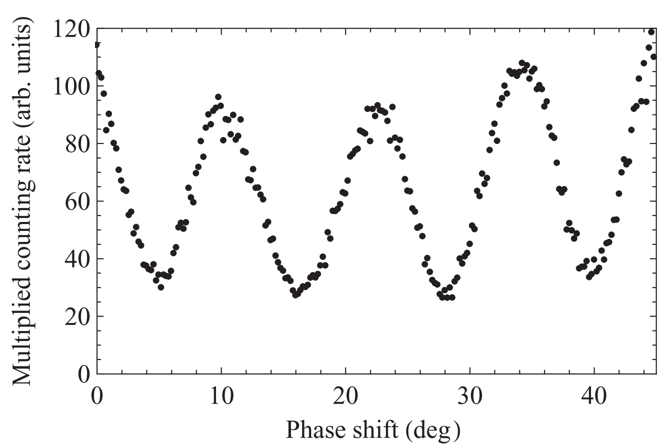

FIG. 6. Phase super-resolution in the time domain with a resolution enhancement by a factor of $n = 30$.

phase super-resolution with $n = 30$ over the whole range of φ_1 between 0° and 360°. (1800 settings of φ_1, 30 settings of φ_2, and a 1-s measurement time per setting plus time for rotation gives around 20 h.) Under practical conditions, however, this should not pose any substantial problem, since one usually is not interested in scanning the whole range between 0° and 360° with phase super-resolution, but rather in making some rough scans to start with, and then to limit oneself to a narrow range of angles φ_1 or phases. Furthermore, in most cases, measurement times of less than 1 s per setting should give sufficient visibility.

An advantage of the time-domain method is that one can get phase super-resolution with high factors n. A technical limitation of our setup is the step-size resolution $\Delta\varphi_1$ and $\Delta\varphi_2$ in turning the HWPs. Step sizes of $\Delta\varphi = 0.1°$ posed no problem in our setup, theoretically the rotator specification allows a resolution of around 1 min of arc. If m denotes the number of points in each fringe, this gives the possibility to achieve $n = 90/(m\Delta\varphi)$, for example, $m = 3$, and 1 min of arc as the step size of the rotator would allow $n = 1800$. A more fundamental problem is the measurement time. The object under investigation should not undergo any changes in phase during the measurement time. Additionally, the intensity of the laser should be stable during the whole measurement time because the SPDs have no way to tell whether a change in the counting rate is due to a phase change or due to an intensity variation of the laser. Since our laser was not sufficiently stable over long periods of time, we could not take full advantage of the small step sizes achievable by our rotators, but we only achieved $n = 30$.

V. INTERPRETATION

Phase super-resolution turns out to be quite a simple and mostly classical phenomenon. We have shown that neither entanglement nor joint detection was needed to achieve it (note that the coincidence unit and the second detector were not necessary in Fig. 4, as explained in Sec. IV). There is neither a need for time-reversal symmetry as introduced in Ref. [13] nor, for example, measurement-induced postselection entanglement. Basically, all one has to do is to shift sine functions as in Eq. (3) and to multiply them. This task can be done with quite ordinary standard optical components and multiplication, either by coincidence detection or by numerical multiplication of the acquired data in a computer.

In principle, these findings were known since Glauber's pioneering work on quantum optics [19]. The correlation functions $|g^{(n)}(x_1 \cdots x_{2n})|$ of any order n, where x denotes the time and space coordinates are symmetric in time and space. This, together with Glauber's finding that n-fold delayed coincidences, which are detected by ideal photon counters reduce to a product of the detection rates of the individual counter leads to the fact that our experiments in time and space domains give the same kind of physics as all the other hitherto reported experiments, yet with much better results. We have capitalized on this knowledge for the design and explanation of our setup, which outperforms the experiments shown so far, and we hope that it will clarify the requirements for achieving super-resolving phase measurements.

A. Phase supersensitivity

To get phase supersensitivity, one has to achieve an uncertainty in-phase estimation, which is smaller than the standard quantum limit allows. Whereas phase super-resolution is easily seen from the interference-fringe pattern, the resulting phase-estimation uncertainty is not that obvious. To determine the phase-estimation uncertainty close to a given phase in our experiments, we use the relation [14]

$$\Delta\phi_n = \Delta P_n \Big/ \left|\frac{\partial P_n}{\partial \phi}\right|, \tag{4}$$

where P_n denotes the probability of detecting n photons in coincidence. The variance of P_n can be written as $\Delta P_n^2 = P_n(1 - P_n)$, since any event either gives a coincidence detection or does not give a coincidence detection.

To calculate the standard quantum limit, one can look at a coherent state $|\alpha\rangle$. By sending this state through an interferometer, we can generate the output state $|\alpha\cos(\phi/2)\rangle$ in one of the arms [and $|\alpha\sin(\phi/2)\rangle$ at the other arm]. A measurement of the photon number at one of the interferometer outputs results in the probability $P_1 = 1 - \exp[-|\alpha|^2\cos^2(\phi/2)]$ of detecting at least one photon. When this expression is inserted into Eq. (4) one obtains the smallest phase-estimation uncertainty $\Delta\phi_{\mathrm{SQL}} = 1/\sqrt{\bar{N}}$, which is the standard quantum limit, where $\bar{N} = |\alpha|^2$ is the expectation value of the photon number in the coherent state.

A phase-resolution doubling, that is, $n = 2$, can be obtained by measuring the photon number in both output arms and either by multiplying the results or by recording the coincidences. The probability of coincidentally recording at least one photon in each output arm is given by

$$P_2 = \{1 - \exp[-|\alpha|^2\cos^2(\phi/2)]\}\{1 - \exp[-|\alpha|^2\sin^2(\phi/2)]\}. \tag{5}$$

By using Eq. (4), one can subsequently calculate the minimum phase-estimation uncertainty of this $n = 2$ interference pattern. The result is $\Delta\phi_2 = 1/\sqrt{\bar{N}}$. That is, the phase-estimation uncertainty is unchanged from the $n = 1$ case.

A coherent state $|\alpha\rangle$ can be split into m identical states $|\alpha/\sqrt{m}\rangle$ (up to an overall phase) by linear components (e.g., a series of BSs). Each of the states can give rise to interferometer outputs $|\alpha m^{-1/2}\cos(\phi/2 + \theta)\rangle$ and $|\alpha m^{-1/2}\sin(\phi/2 + \theta)\rangle$, where θ is an adjustable offset phase. By setting the offset phases in the different interferometers to $0, 2\pi/m, \ldots, 2(m-1)\pi/m$, we get the joint (coincidence) detection probability for all the $2m = n$ interferometer outputs,

$$P_n = \prod_{k=1}^{n/2}\left(1 - \exp\left\{-\frac{2|\alpha|^2\cos^2\left[\phi/2 + \frac{4(k-1)\pi}{n}\right]}{n}\right\}\right) \times \prod_{k=1}^{n/2}\left(1 - \exp\left\{-\frac{2|\alpha|^2\sin^2\left[\phi/2 + \frac{4(k-1)\pi}{n}\right]}{n}\right\}\right), \tag{6}$$

where n, quite obviously, is even. By defining the phase sensitivity $\mathcal{S}(n)$ as the ratio between the standard quantum limit and the phase error as

$$\mathcal{S}(n) = \frac{1}{\sqrt{\bar{N}}\Delta\phi_n},$$

TABLE I. Phase sensitivity (depending on n) maximized over all values of ϕ.

n	2	4	6	8
$\mathcal{S}(n)$	$\sqrt{1-e^{-\bar{N}}}$	$\frac{(1-e^{-\bar{N}/2})\sqrt{1-e^{-\bar{N}/4}}}{\sqrt{2}}$	$\frac{(1-e^{-\bar{N}/4})(1-e^{-\bar{N}/12})\sqrt{1-e^{-\bar{N}/3}}}{\sqrt{3}}$	$\frac{(1-e^{-\bar{N}/8})[1+e^{-\bar{N}/4}-\exp(\frac{-2+\sqrt{2}}{16}\bar{N})-\exp(\frac{-2-\sqrt{2}}{16}\bar{N})]\sqrt{1-e^{-\bar{N}/4}}}{2}$
$\lim_{\bar{N}\to\infty} \mathcal{S}(n)$	1	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{4}}$

a value of $\mathcal{S} > 1$ would indicate phase supersensitivity. We have calculated some values of $\mathcal{S}$ in Table I.

All the values in the table were optimized with respect to ϕ, and one finds that $\phi = 0, 2\pi/n, \ldots, 2(n-1)\pi/n$ gives the maximum sensitivity. For higher values of n, the analytical expressions for the sensitivity get rather messy, but from the calculated values in the table, we conjecture that $\mathcal{S}(n) = \sqrt{2/n}$ for even n in the limit $\bar{N} \to \infty$. For $\bar{N} \to 0$, the sensitivity goes to zero. As can be seen, $\mathcal{S} \leqslant 1$; and, therefore, the scheme we have investigated does not give phase supersensitivity for any n, rather, it gives the opposite. Our result supports the view that entanglement is required for phase supersensitivity [17].

VI. CONCLUSION

To summarize, we showed that super-resolving phase measurements with simultaneously high n (which denotes the relative increase in the number of fringes) and high visibility can be achieved. Although there are clear limitations to the increase of n in the space domain, it is less demanding to reach high values of n in the time domain. We showed the principle up to $n = 30$ and explained how one could reach higher values without requiring any extra components by just using a stabilized laser and high-quality optics. Furthermore, we performed the experiment at telecommunications wavelengths, which enabled possible remote applications, since the phase shifting (which includes the polarization analyzers), the detectors, the coincidence unit, and the control computer can be stationed at different remote locations.

ACKNOWLEDGMENTS

C.K. acknowledges support from the German National Academic Foundation. The work was supported by the Swedish Research Council (VR) and by the Knut and Alice Wallenberg Foundation (KAW). We thank E. Amselem, M. Rådmark, and J. Ahrens for discussions.

[1] B. C. Barish and R. Weiss, Phys. Today **52**(10), 44 (1999).
[2] G. Heinzel *et al.*, Class. Quantum Grav. **23**, 119 (2006).
[3] T. Uchiyama *et al.*, Class. Quantum Grav. **21**, 1161 (2004).
[4] S. Witte *et al.*, Science **307**, 400 (2005).
[5] N. P. Georgiades *et al.*, Opt. Lett. **21**, 1688 (1996).
[6] J. P. Dowling, Phys. Rev. A **57**, 4736 (1998).
[7] R. A. Campos, C. C. Gerry, and A. Benmoussa, Phys. Rev. A **68**, 023810 (2003).
[8] J. J. Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heinzen, Phys. Rev. A **54**, R4649 (1996).
[9] M. Pluta, Proc. SPIE **1846**, 10 (1994).
[10] A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. **85**, 2733 (2000).
[11] P. Walther *et al.*, Nature (London) **429**, 158 (2004).
[12] M. W. Mitchell *et al.*, Nature (London) **429**, 161 (2004).
[13] K. J. Resch, K. L. Pregnell, R. Prevedel, A. Gilchrist, G. J. Pryde, J. L. O'Brien, and A. G. White, Phys. Rev. Lett. **98**, 223601 (2007).
[14] R. Okamoto *et al.*, New J. Phys. **10**, 073033 (2008).
[15] V. Giovannetti *et al.*, Science **306**, 1330 (2004).
[16] N. Margolus and L. B. Levitin, Physica D **120**, 188 (1998); B. C. Sanders, Phys. Rev. A **40**, 2417 (1989); J. Söderholm, G. Björk, T. Tsegaye, and A. Trifonov, *ibid.* **59**, 1788 (1999); H. Lee, P. Kok, and J. P. Dowling, J. Mod. Opt. **49**, 2325 (2002).
[17] T. Nagata *et al.*, Science **316**, 726 (2007).
[18] B. L. Higgins *et al.*, Nature (London) **450**, 393 (2007).
[19] R. J. Glauber, Phys. Rev. **130**, 2529 (1963); **131**, 2766 (1963).