

Bloch oscillations of polaritons of an atomic ensemble in magnetic fields

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We study a washboard potential in a light-driven ensemble of three-level atoms induced by the externally applied magnetic fields under electromagnetically induced transparency (EIT). We find that a dark-state polariton, which is formed by an atomic ensemble dressed by the photon can display a typical quantum interference effect (i.e., the quasiparticle Bloch oscillation). The dark-state polariton is subjected to a washboard configuration potential with a spatially uniform force and a periodic potential provided by the linearly and periodically external magnetic fields, respectively. Due to the slower group velocity of the photons in the EIT medium, the period of Bloch oscillation can achieve a millisecond order of magnitude. Also, in our protocol, the Bloch oscillation of the dark-state polariton is feasibly modulated through the applied control and magnetic fields.

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I. INTRODUCTION

A particle in periodic potential will exhibit Bloch oscillations when subjected to an external force. Bloch oscillations were first presented by Bloch [1] and Zener [2] with respect to the oscillations of the electrons in crystal lattices induced by an applied electric field. Then, for a long time, there was a great amount of controversy about the existence of the oscillations until the oscillations of the electrons in semiconductor superlattices were experimentally observed [3–5]. Thereafter, a large number of theoretical studies and experimental observations about Bloch oscillations in other periodically physical systems emerged, such as cold atoms and Bose-Einstein condensates in optical lattices [6–11], photons in periodic optical systems, which include optical waveguide arrays [12–19], optical superlattices [20,21], and related periodic optical structures [22–30]. Bloch oscillations have many practical applications, which include the realization of Bloch oscillation transistors [31], the measurement of forces such as gravity [32] and Casimir-Polder force [33] at the micrometer scale, and control of light in coupled-resonator optical waveguides [34].

In this paper, we study the polariton Bloch oscillations in an electromagnetically induced transparency (EIT) atomic ensemble induced by the externally applied linear and periodic magnetic fields. It is well known that, in an atomic system, EIT [35–39] occurs due to the destructive interference between excitation pathways to the upper level, and an incoming pulse shows extremely slow group velocity [40–42] within the transparency windows. The incident beam will deflect in the EIT medium when applying an inhomogeneous external magnetic field, which has been studied both experimentally and theoretically [43–48]. In the experiment [43], the light beam deflects when passing through an atomic vapor with a gradient magnetic field as if it has acquired a magnetic moment. This phenomenon has been theoretically explained by the concept of the dark-state polariton, which is a bosoniclike collective excitation (a mixture of a signal light field and an atomic spin wave), which behaves as a matter particle with an effective magnetic moment and an effective transverse mass [43,47].

Inspired by these results, we think that the dark-state polariton will be subjected to a static force in a periodic potential if we add two inhomogeneous magnetic fields with transverse linear and periodic variations to the EIT

atomic ensemble. The uniform static force is induced by the linear magnetic field, and the periodic potential is caused by the periodic magnetic field. Thus, the motion of the dark-state polariton will behave as Bloch oscillations instead of deflections. By using the single-band and tight-binding methods to study the spatial motion of the dark-state polariton, we find that, due to slower group velocity of the photons in the EIT medium, the observed period of Bloch oscillations for the dark-state polariton can achieve a millisecond order of magnitude. In our protocol, the polariton Bloch oscillations are feasibly tunable via externally applied light and magnetic fields. Not only is this polariton Bloch oscillation of fundamental interest as a quantum coherence phenomenon, but also it can open up opportunities for photon manipulation such as coherent control of the state of the single photon.

This paper is organized as follows. In Sec. II, we describe our scheme. In Sec. III, we give the evolution equation for the dark-state polariton. In Sec. IV, we demonstrate Bloch oscillations of the dark-state polariton by analytically solving the equations of motion, and we describe the Bloch oscillations in some experimental data in detail.

II. SETUP

In this section, we consider an ensemble of N atoms of Λ type, which are identical and noninteracting. The atoms are confined in a gas cell of volume V as shown in Fig. 1(a). Each atom possesses three energy levels with a Λ configuration with internal states $|g\rangle$, $|s\rangle$, and $|e\rangle$ as shown in Fig. 1(b). The transition from the ground state $|g\rangle$ to the excited state $|e\rangle$ is driven by the probe field with the resonant frequency ν and the wave number k , while the transition from the metastable state $|s\rangle$ to the excited state $|e\rangle$ is coupled by an intense classical laser field with Rabi frequency Ω . The control field has the resonant frequency ν_c and the wave number k_c . After applying the magnetic field B_z along the z direction, the probe and control beams are detuned from the excited state $|e\rangle$ with the detunings δ_g and δ_s , respectively, which are induced by energy shifts of the corresponding states $|i\rangle$ ($i = g, s, e$) from their origins by magnitudes $-\mu_i B_z$ with μ_i as the magnetic moments corresponding to the states $|i\rangle$. Here,

$$\delta_g = (\mu_e - \mu_g)B_z, \quad \delta_s = (\mu_e - \mu_s)B_z. \quad (1)$$

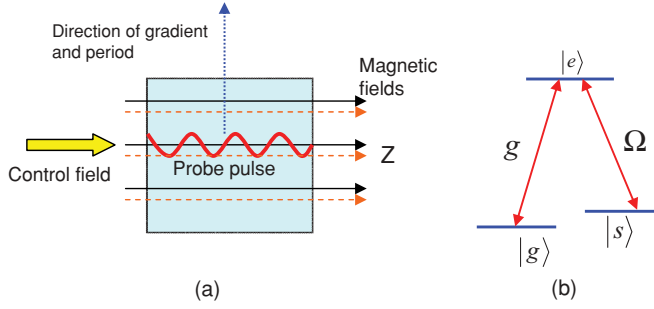


FIG. 1. (Color online) (a) Schematic of polariton Bloch oscillations in the atomic medium induced by the externally applied magnetic fields. The black solid arrows and the orange dashed arrows along the z direction represent the linear and the periodic magnetic fields, respectively. The blue dotted arrow is the varying direction of linear and periodic magnetic fields. (b) Configuration of atomic levels coupled to the probe and control fields.

Next, we introduce the slowly varying operators $\sigma_{\mu\nu}$ of the collective continuous atomic transitions. The collective continuous operators $\tilde{\sigma}_{\mu\nu}(\mathbf{r}) = \sum_{r_j \in N_r} \sigma_{\mu\nu}^j / N_r$ are the average over $N_r [= (2N/V)dr \gg 1]$ atoms in a small but macroscopic volume around position \mathbf{r} with $\sigma_{\mu\nu}^j = |\mu\rangle_j \langle \nu|$ as the internal state operators of the j th atom between states $|\mu\rangle$ and $|\nu\rangle$. The slowly varying operators $\sigma_{\mu\nu}$ for the atomic transitions are defined as $\tilde{\sigma}_{eg} = \sigma_{eg} \exp(-ikz)$ and $\tilde{\sigma}_{es} = \sigma_{es} \exp(-ik_c z)$, with k and k_c as the wave numbers for the central frequencies ν and ν_c of the probe and control fields, respectively.

With slowly varying amplitude, the probe field is

$$\tilde{E}^+(r, t) = \sqrt{\frac{v}{2\epsilon_0 V}} E(r, t) e^{i(kz - \nu t)}. \quad (2)$$

By considering the case that both probe and control fields propagate parallel to the z axis and by omitting the kinetic energy of the cold atoms in the rotating reference frame, the interaction Hamiltonian for the system reads [47]

$$H_I = -\frac{N}{V} \int d^3r [\delta_g \sigma_{gg} + \delta_s \sigma_{ss} + (gE\sigma_{eg} + \Omega\sigma_{es} + \text{H.c.})], \quad (3)$$

where the atomic-field coupling constant is $g = d_{eg} \sqrt{v/2\epsilon_0 V}$.

In this system, all atoms are initially prepared in the ground state $|g\rangle$. By considering the case that the intensity of the probe field is much weaker than that of the control field and that the number of the photons in the signal pulse is much less than the number of the atoms in the sample, we can treat the Heisenberg equations of atomic operators perturbatively with the perturbative parameter gE . For our study, the linear optical response (the first-order approximation) is enough to reflect the main physical characteristics of the spatial motion of the probe pulse with slow group velocity. From the Heisenberg equations of atomic operators [47], there are

$$gE = -\left[\frac{1}{\Omega} (\partial_t + i\delta_g + \gamma_1)(\partial_t + i\delta_g - i\delta_s + \gamma_2) + \Omega \right] \sigma_{gs}^{(1)}, \quad (4)$$

$$\sigma_{ge}^{(1)} = -\frac{i}{\Omega} (\partial_t + i\delta_g - i\delta_s + \gamma_2) \sigma_{gs}^{(1)}, \quad (5)$$

by the first-order approximation with the perturbative parameter gE . Here, we introduce the energy-level decay rates γ_1 and γ_2 phenomenologically, which correspond to the excited and metastable states, respectively.

III. EQUATIONS OF MOTION FOR DARK-STATE POLARITONS

In this section, we consider the spatial motion of the probe pulse in terms of the formation of the dark-state polariton, which is a bosoniclike collective excitation of a signal light field and an atomic spin wave [49–52]. By following the method of Ref. [47], the dark-state polariton is defined as

$$\Psi(r, t) = E \cos \theta - \sqrt{N} \sigma_{gs}^{(1)} \sin \theta, \quad (6)$$

where the mixing angle $\tan \theta = g\sqrt{N}/\Omega$.

To obtain the equation that describes the dark-state polariton, first, we write the paraxial wave equation of the slowly varying light field in classical optics as [47]

$$i \frac{\partial}{\partial t} E + ic \frac{\partial}{\partial z} E + \frac{c}{2k} \nabla_T^2 E = -g^* N \sigma_{ge}^{(1)}, \quad (7)$$

where c is the vacuum velocity of light and the transverse Laplacian is $\nabla_T^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

In adiabatic approximation, which follows the derivation in Ref. [47], the dynamics of the dark-state polariton field is described by the Schrödinger-like equation

$$i \frac{\partial}{\partial t} \Psi = \left[v_g P_z + \frac{1}{2m} (P_x^2 + P_y^2) + V'(r) \right] \Psi, \quad (8)$$

from the paraxial wave equation, Eq. (7), of the probe light field. Here, we set a sufficiently strong control field such that $\Omega^2 \gg \gamma_1 \gamma_2$ and a small magnetic field such that $|\delta_g| \ll \gamma_1$. In addition, $v_g = c \cos^2 \theta$ is the slow light velocity, $P_z = -i\partial_z$ is the momentum operator along the z direction, $m = k/v_g$ is the effective transverse mass, and $V'(r) = -\mu B_z(r)$ [with $\mu = (\mu_s - \mu_g) \sin^2 \theta$] is the effective potential induced by the steady atomic response in the external spatial-dependent field by setting $\gamma_2 = 0$.

Now, we assume that the magnetic field in the z direction has the transverse spatial distribution,

$$B_z(r) = B_{z1}(x) + B_{z2}(x), \quad (9)$$

with $B_{z1}(x) = -ax$ (a is a constant) as a linear magnetic field and $B_{z2}(x)$ as a periodic magnetic field of period d . Thus,

$$V'(r) = Fx + V(x), \quad (10)$$

with $F = a\mu$ and $V(x) = -\mu B_{z2}(x)$, which means that the dark-state polariton is subjected to a uniform force F along the x direction in a transverse periodic potential $V(x)$. A quantum particle that moves in a periodic potential is all delocalized in space, which is described by Bloch waves and which exhibits energy bands (the Bloch bands). When an additional uniform force is applied, the quasimomentum of the particle will be accelerated and will reach the Brillouin band edge, where the particle velocity reaches zero and changes sign due to the energy-band dispersion [20]. Thus, the particle will show Bloch oscillations.

In the following, we restrict our discussion to the two-dimensional system in the $x - z$ plane. Let us consider the evolution dynamics of a spatially well-localized wave packet, which is centered at $(x_0, z_0) = (0, 0)$ initially in a Gaussian form

$$\Psi(x, z, 0) = \phi(z, 0)\varphi(x, 0), \quad (11)$$

with

$$\phi(z, 0) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} \exp\left[-\frac{z^2}{4\sigma^2}\right], \quad (12)$$

and

$$\varphi(x, 0) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} \exp\left[-\frac{x^2}{4\sigma^2}\right]. \quad (13)$$

To describe the spatial motion of the polariton more clearly, we let

$$\Psi(x, z, t) = \phi(z, t)\varphi(x, t). \quad (14)$$

The Schrödinger-like equation, Eq. (8), of the dark-state polariton is decomposed into two parts,

$$i \frac{\partial}{\partial t} \phi(z, t) = v_g P_z \phi(z, t), \quad (15)$$

$$i \frac{\partial}{\partial t} \varphi(x, t) = \left\{ \frac{P_x^2}{2m} + [V(x) + Fx] \right\} \varphi(x, t), \quad (16)$$

which correspond to the spatial motion of the dark-state polariton along the z and x directions, respectively. Equation (15) indicates that the wave packet propagates along the z direction with group velocity v_g , which is solved by

$$\phi(z, t) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} \exp\left[-\frac{(z - v_g t)^2}{4\sigma^2}\right]. \quad (17)$$

Equation (16) describes the transverse motion of the dark-state polariton with the Hamiltonian,

$$H = \frac{P_x^2}{2m} + V(x) + Fx, \quad (18)$$

which means that the particle is subjected to a uniform force in a periodic potential.

IV. BLOCH OSCILLATIONS OF DARK-STATE POLARITONS

Now, we study Bloch oscillations of the dark-state polariton. Let us discuss, in detail, the dynamics of the transverse motion of the dark-state polariton in a one-dimensional periodic potential under the influence of a static force described by the previous Hamiltonian of Eq. (18). By neglecting the coupling between bands, we consider the tight-binding and single-band approximations, which only include the nearest-neighbor coupling. Here, we take $\hbar = 1$. The Hamiltonian of Eq. (18) is rewritten in the representation of Wannier states as [10, 11]

$$H = -\frac{\Delta}{4} \sum_{n=-\infty}^{n=\infty} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) + dF \sum_{n=-\infty}^{n=\infty} n |n\rangle\langle n|, \quad (19)$$

with the Wannier states

$$|n\rangle = \sqrt{\frac{d}{2\pi}} \int dk e^{-inkd} |k\rangle. \quad (20)$$

The Bloch states

$$|k\rangle = \sqrt{\frac{d}{2\pi}} \sum_{n=-\infty}^{n=\infty} e^{inkd} |n\rangle \quad (21)$$

are the eigenstates of the Hamiltonian,

$$H' = \frac{P_x^2}{2m} + V(x). \quad (22)$$

Here, Δ is the bandwidth of the ground energy band, and the quasimomentum k is confined to the Brillouin zone with $-b/2 \leq k \leq b/2$ ($b = 2\pi/d$). The last term in the Hamiltonian of Eq. (19) is induced by the term Fx , which is diagonal on the basis of Wannier states [9], that is,

$$Fx|n\rangle = dFn|n\rangle.$$

We obtain the eigenvalues and the eigenstates of the Hamiltonian of Eq. (19) in the representation of Bloch states. In the representation of Bloch states, the tight-binding Hamiltonian of Eq. (19) reads

$$H(k) = -\frac{\Delta}{2} \cos(kd) + iF \frac{d}{dk}. \quad (23)$$

This Hamiltonian has the Wannier-Stark energy spectrum,

$$E_m = mdF \quad (m = 0, \pm 1, \pm 2, \dots), \quad (24)$$

which possess the ladder configuration with equidistant level spacing $\Delta E = dF$. The eigenstates read

$$|\Psi_m(k)\rangle = \sqrt{\frac{d}{2\pi}} e^{-i[\gamma \sin(kd) + mdk]} \quad (m = 0, \pm 1, \pm 2, \dots), \quad (25)$$

which are

$$|\Psi_m\rangle = \sum_n J_{n-m}(\gamma) |n\rangle \quad (m = 0, \pm 1, \pm 2, \dots), \quad (26)$$

in Wannier state representation with $\gamma = \Delta/2dF$. These states are called Wannier-Stark states.

Now, we study the dynamic evolution of the wave packet of dark-state polaritons. In the Wannier representation, for $\sigma/d \gg 1$, the initially localized Gaussian wave packet of Eq. (13) reads

$$\varphi(x, 0) = \sum_{n'} f_{n'}(0) |n'\rangle, \quad (27)$$

with

$$f_{n'}(0) = \left(\frac{d^2}{2\pi\sigma^2} \right)^{1/4} \exp\left[-\frac{n'^2 d^2}{4\sigma^2}\right]. \quad (28)$$

By using the time-evolution operator,

$$U(t) = \sum_j |\Psi_j\rangle e^{-iE_j t} \langle \Psi_j|, \quad (29)$$

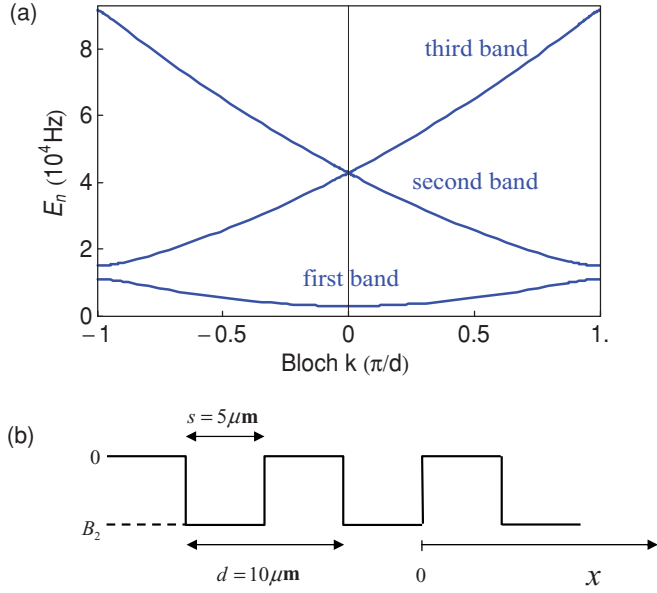


FIG. 2. (Color online) (a) The energy-band diagram. (b) The periodic square wave of the magnetic field $B_{z2}(x)$.

one obtains the propagator as [10,11]

$$\begin{aligned}
 U_{nn'}(t) &= \langle n|U(t)|n'\rangle \\
 &= J_{n-n'}\left(2\gamma\sin\left(\frac{\alpha}{2}\right)\right)e^{-in'\omega_B t} \\
 &\quad \times \exp\left[-i(n-n')\left(\frac{\omega_B t}{2}-\frac{\pi}{2}\right)\right], \quad (30)
 \end{aligned}$$

with $\alpha = dFt$ and $\omega_B = dF$.

Then, the transverse motion of the dark-state polariton with the initial state of Eq. (27) is described by the time-evolution equation

$$f_n(t) = \sum_{n'} U_{n,n'}(t)f_{n'}(0). \quad (31)$$

By following Eqs. (28) and (30), and by assuming that $\sigma/d \gg 1$, the summation over n' in Eq. (31) can approximately be replaced by integration. One obtains the explicit expression,

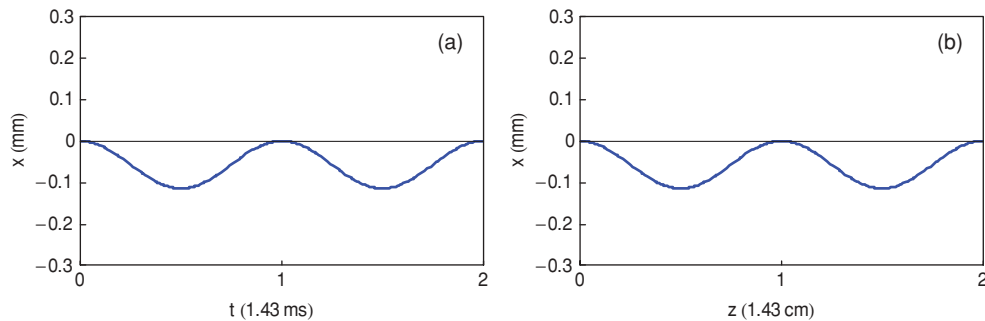


FIG. 3. (Color online) (a) Temporal Bloch oscillations of the dark-state polariton with period $T_B = 1.43$ ms. (b) Spatial Bloch oscillations of the dark-state polariton with period $z_B = 1.43$ cm.

approximately,

$$\begin{aligned}
 f_n(t) &\approx \left(\frac{d^2}{2\pi\sigma^2}\right)^{1/4} e^{-in\omega_B t + i\gamma\sin(\omega_B t)} \\
 &\quad \times \exp\left(-\frac{d^2}{4\sigma^2}\{n - \gamma[\cos(\omega_B t) - 1]\}^2\right). \quad (32)
 \end{aligned}$$

Thus, the transverse motion of the center of the wave packet $x(t) = L[\cos(\omega_B t) - 1]$ is time dependent with an amplitude $L = \Delta/(2F)$, which means that the center oscillates with the temporal period,

$$T_{\text{Bloch}} = \frac{2\pi}{Fd}, \quad (33)$$

by an amplitude L , which is inversely proportional to the uniform force F . Because the center of the wave packet propagates along the z direction with velocity v_g , the spatial Bloch oscillations reads

$$x(z) = L[\cos(\beta z) - 1], \quad (34)$$

with $\beta = dF/v_g$. The spatial period z_B of the Bloch oscillation is

$$z_B = \frac{2\pi}{\beta}. \quad (35)$$

Because the width of the ground energy band Δ varies with different transverse mass m , which can be modulated by changing the control field, we can control the amplitude to some extent by modulating the control field. As the uniform force F is proportional to the gradient a , the temporal period can also be tuned by changing the gradient of the external linear magnetic field.

Until now, we have theoretically predicted that there exist Bloch oscillations in the EIT medium induced by the external magnetic fields. Next, we will analyze, in detail, the Bloch oscillations by using some experimental data. Due to the frequency detunings induced by the external magnetic fields, we must limit the intensity of the magnetic fields to ensure that the probe field propagates within the transparency windows. Also, the conditions need to be satisfied (i.e., $\Omega^2 \gg \gamma_1\gamma_2$ and $|\delta_g| \ll \gamma_1$), which are the limitations for obtaining the propagating Eq. (8). We select the experimental data [43,47] as follows: the magnetic moment $\mu = 5.1 \times 10^{-24}$ JT⁻¹, the wavelength of the probe field $\lambda = 795$ nm, the transverse mass $m = 7.9 \times 10^5$ sm⁻², which correspond to the group

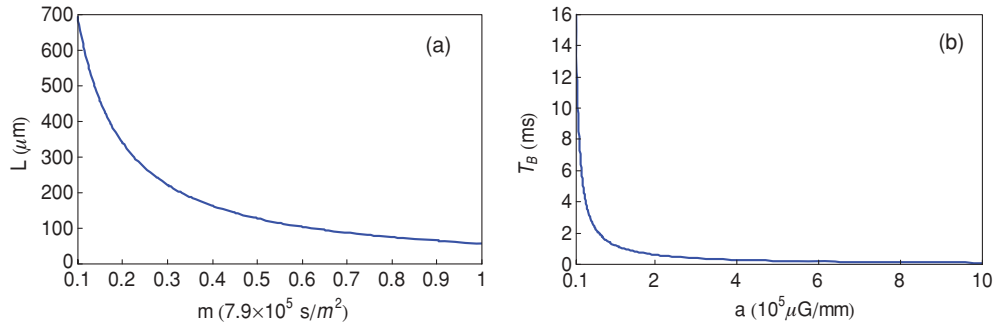


FIG. 4. (Color online) (a) The modulation of the amplitude of the Bloch oscillation by changing the control field by setting the gradient of the linear field $a = 9.1 \times 10^4 \mu\text{G mm}^{-1}$. (b) The modulation of the temporal period of the Bloch oscillation by changing the gradient of the linear magnetic field by setting period $d = 10 \mu\text{m}$ for the periodic magnetic field.

velocity $v_g = 10 \text{ ms}^{-1}$, and the gradient of the linear magnetic field $a = 9.1 \times 10^4 \mu\text{G mm}^{-1}$. We assume that $B_2(x)$ is the Kronig-Penney periodical function (see [e.g., Fig. 2(b)]) with period $d = 10 \mu\text{m}$ and amplitude $8.3 \times 10^3 \mu\text{G}$. First, we calculate the energy band of the Hamiltonian of Eq. (22) (see [e.g., Fig. 2(a)]). In this case, the width of the ground energy band is $\Delta = 0.8 \times 10^4 \text{ Hz}$, and the gap interval is about $0.4 \times 10^4 \text{ Hz}$ between the first energy band and the second energy band. The temporal period of the Bloch oscillation is $T_B = 1.43 \text{ ms}$, and the spatial period is $z_B = 1.43 \text{ cm}$ with the amplitude $57.1 \mu\text{m}$. The temporal and spatial Bloch oscillations are shown in Figs. 3(a) and 3(b), respectively. The position change of the wave-packet center in the x direction versus the time variation is shown in Fig. 3(a). Figure 3(b) shows the trajectories of the wave-packet center in the x - z plane, which indicates the wave packet of the dark-state polariton oscillates along the x direction with its propagation along the z direction by slow group velocity. Due to the slower group velocity, the millisecond order of magnitude of the Bloch

oscillations is observed in the atomic ensemble with a limited length of a few centimeters.

In our protocol, we can modulate the uniform force F and the transverse mass m by changing the gradient of the linear magnetic field and the intensity of the control field, respectively. Thus, for Bloch oscillations, we can obtain different amplitudes and time periods to some extent (see [e.g., Figs. 4(a) and 4(b)]). Figure 4(a) shows that, with an increase in the mass of the dark-state polariton (i.e., reduction of the group velocity), the amplitude of the Bloch oscillation decreases. Figure 4(b) shows that the temporal periods are inversely proportional to the gradient of the linear magnetic field.

We expect that these predicted phenomena can be observed based on substrates sensitive to the internal energy of metastable atoms [53]. Also, we may adopt the measurement method of Ref. [43], where the deflection angle of the signal pulse from the gas cell can be obtained by measuring the transverse position of the signal pulse with a charge-coupled device camera placed 2 m from the sample. These Bloch oscillations can be observed by selecting different lengths of samples, which allow for insight into the field evolution by observing the different outgoing angles of the pump pulse, which corresponds to different transverse positions measured by the camera. The outgoing angle is

$$\vartheta \approx \frac{v_x}{v_g} = -L\beta \sin(\beta z). \quad (36)$$

We demonstrate such a sinusoidal Bloch oscillation in Fig. 5, where the z axis means different lengths of sample from 1.43 to 2.86 cm, and the x axis is the transverse location of the signal pulse along the x direction measured by the camera 1 m away from the sample with the parameters $L = 57.1 \mu\text{m}$ and $\beta = 4.39 \text{ cm}^{-1}$.

V. CONCLUSION

To summarize, we have explored and have studied Bloch oscillations of the dark-state polariton in an EIT atomic ensemble with an effective moment and an effective transverse mass. The periodic potential and the uniform force are presented by the externally applied magnetic fields, which result in the Bloch oscillations of the dark-state polariton. The corresponding numerical results show that, just due to the slower group velocity of the pump light, there exist observable

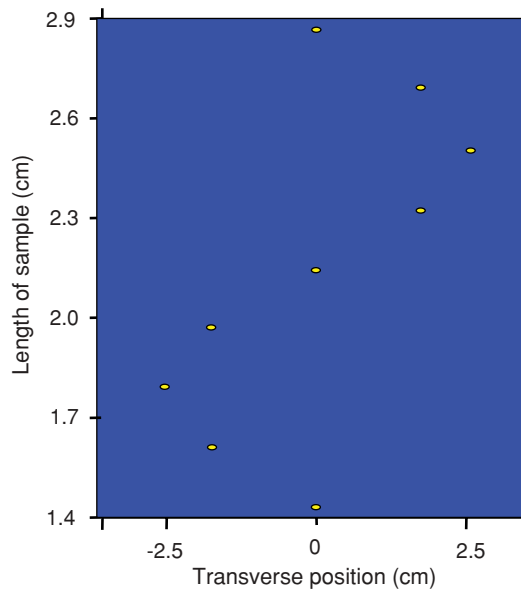


FIG. 5. (Color online) Demonstration of the transverse outgoing position of the wave packet of the signal beam along the x direction versus the different lengths of samples with $L = 57.1 \mu\text{m}$ and $\beta = 4.39 \text{ cm}^{-1}$.

polariton Bloch oscillations within the transparency window. When the group velocity of the pump light reaches 10 m/s, the period of the polariton Bloch oscillations is 1.43 ns. Also, the amplitude and the period are tuned by changing the control field and the gradient of the linear magnetic field, respectively.

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