

# Quantum decoherence in a hybrid atom-optical system of a one-dimensional coupled-resonator waveguide and an atom

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Decoherence for a one-dimensional coupled-resonator waveguide with a two-level system inside one of the resonators, induced by their interaction with corresponding environments, is investigated. Each environment is modeled as a continuum of harmonic oscillators. By finding the eigenstates of the hybrid system, which is the dressed state of the hybrid system, we calculate the lifetime of one excitation, which characterizes the existence of quantum coherence in such hybrid systems and the basic quantum nature.

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## I. INTRODUCTION

Any discrete state coupled to continua of states is subject to decay. The intrinsic dynamics of such quantum systems is irreversible (i.e., once a system is initially in the discrete state, it never returns to the initial state spontaneously). Such a kind of phenomenon is also described by the so-called resonant tunneling process. There are many examples of such processes in different branches of physics. A typical example is an excited two-level system interacting with the modes of electromagnetic fields.

Spontaneous emission is normally regarded as a loss and a decoherence mechanism. Recent works show that spontaneous emission of the excited two-level system can be exploited to influence the coherent transport properties of a single photon in a one-dimensional (1D) waveguide due to the interference between the spontaneous emission from two-level systems and the propagating modes in the 1D continuum [1]. For a discrete system, the simplest model which possesses the resonant tunneling process is the so-called Anderson-Fano-Lee model [2–5], in which the continuum is formed by a linear chain of sites with the nearest-neighbor interaction and the rate of emission is modified due to the change of the density of state.

With the development of microfabrication technology, the platforms, such as defect resonators in photonic crystals [7–10] and coupled superconducting transmission line resonators [11–14], are promising candidates for realizing a waveguide (or an array) [6], under the tight-binding approximation. Since a 1D coupled-resonator waveguide (CRW) possesses a band-gap spectrum and can transmit a wave packet of light, a single-photon quantum switch [15,16], made of a controllable two-level or three-level system, has been studied using a discrete-coordinate approach. It is then found that the transmission of a single-photon in a 1D CRW can be switched on and off by modulating the energy-level spacing of the two-level system (TLS) with a high-frequency signal [17].

The system with TLSs inside a 1D CRW [15–18] is investigated under the condition of ideal resonators and ideal couplings of the resonator modes to the respective atomic

transitions. Such a system is closed, and the existence of superpositions prescribed by quantum mechanics is valid. All the results in Refs. [15,16] are obtained by seeking the stationary states of the system with a TLS inside 1D CRW. However, a realistic quantum system can rarely be isolated from its surrounding completely, rather it is usually coupled to the external environment (also called “heat bath”) with a large number of degrees of freedom. There are two main loss processes which are serious obstacles against the preservation of quantum coherence over a long period of time: spontaneous emission from the excited state to the ground state due to its interaction with the modes outside the resonator, and leaking out of photons of the resonator mode. Obviously, when the external environment is taken into account, the decoherence of every resonator and the TLS would result in the incoherent or dissipative propagation of the incident photon.

It is pointed out in Refs. [15–17] that the decoherence or dissipation can be divided into two categories: One influences the free propagation of the single photon, and another influences the scattering process which broadens the line width.

Therefore, in the present work, we shall investigate the influence of environment to the decoherence of a 1D coupled resonator with a two-level system inside. We mainly focus on two issues, one is the lifetime of each eigenstate characterizing the timescale of quantum coherence in a 1D CRW with a TLS inside, and another is the rate of decay through introducing the leakage rate in each resonator and the decay rate of the TLS, which influences the free propagation of the single photon. The reflection spectrum is also obtained and it is found that the dissipation lowers the peak of the resonance and broadens the line width. However, the total reflection can still be achieved when the leakage rate in each resonator is equal to the decay rate of the TLS.

This paper is organized as follows. In Sec. II, we introduce a microscopic model where both the resonators and the TLS are coupled to its own surrounding through an exchange interaction. In Sec. III, we briefly review the stationary states of a CRW with a TLS inside in Ref. [15]. In Sec. IV, we derive the characteristic time for a single-photon staying in the system with a TLS embedded in 1D CRW. In Sec. V, we investigate the impact of dissipation on scattering amplitude. Conclusions are made in Sec. VI.

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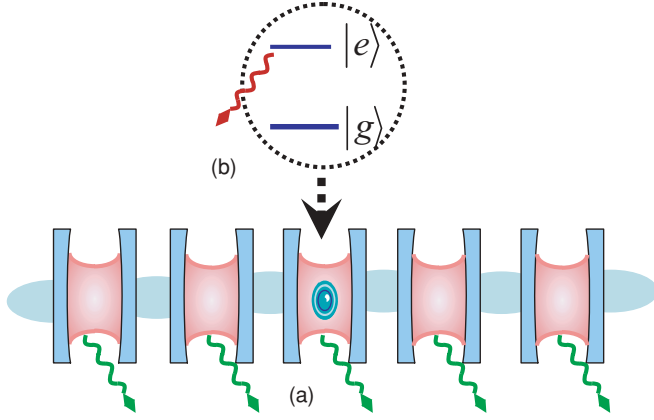


FIG. 1. (Color online) Schematic illustration of the model, where a two-level system (a) is inside a 1D coupled resonator waveguide (b). The two-level system and each of the resonators are coupled to the corresponding environment, which is modeled as an infinite set of harmonic oscillators.

## II. MODEL

Figure 1 shows a schematic diagram of what we consider in this paper. The total system includes the system S and the environment B. The environment B does not explicitly show in Fig. 1. The system S refers to a 1D CRW with a TLS inside one of the resonators as discussed in Ref. [15] and the environment B refers to all the subsystems interacting with each resonator in the 1D CRW and the TLS. In Fig. 1 the wavy line with an arrow indicates that a bath interacts with this subsystem.

Each resonator in the CRW is modeled as a harmonic oscillator mode with frequency  $\omega_c$ . Due to the overlap of the spatial profile of the resonator modes, photons can hop between neighboring resonators. Introducing the creation and annihilation operators  $a_j^\dagger$  and  $a_j$  for the  $j$ th resonator, the Hamiltonian for the CRW is given by

$$H_C = \sum_j \omega_c a_j^\dagger a_j - \xi \sum_j (a_j^\dagger a_{j+1} + \text{H.c.}), \quad (1)$$

where the intercavity coupling constant  $\xi$  is the same for all neighboring cavity-cavity interactions. A TLS with transition frequency  $\Omega$  is located in the 0th resonator. The atom-resonator interaction is described by the Jaynes-Cummings model,

$$H_I = \Omega |e\rangle\langle e| + J(|e\rangle\langle g| a_0 + \text{H.c.}), \quad (2)$$

under the dipole and rotating wave approximations. Here  $|e\rangle$  and  $|g\rangle$  are the excited state and ground state of the TLS, respectively. The dynamics of the system S is governed by the Hamiltonian,

$$H_S = H_C + H_I. \quad (3)$$

By employing the Fourier transformation,

$$a_j = \frac{1}{\sqrt{N}} \sum_k e^{ikj} a_k, \quad (4)$$

Hamiltonian  $H_S$  is transformed into a k-space representation,

$$H_S = \sum_k \Omega_k a_k^\dagger a_k + \Omega |e\rangle\langle e| + \frac{J}{\sqrt{N}} \sum_k (|e\rangle\langle g| a_k + \text{H.c.}), \quad (5)$$

where the lattice constant is assumed to be unity and  $\Omega_k = \omega_c - 2\xi \cos k$  is the well-known Bloch dispersion relation. Here, the Hamiltonian  $H_C$  given by a diagonal matrix describes the extended states of the continuum. The third term in Eq. (5) is responsible for the interaction between the TLS and the continuum. The Hamiltonian  $H_S$  describes that a quasiexcitation is created or annihilated in the  $k$ th mode, and the TLS transits from its excited state to the ground state or vice versa. Obviously, it is easy for the TLS to transit to its ground state, and very difficult to go back to its excited state. Therefore, the CRW with a TLS inside is a typical system of quantum dissipation phenomenon.

The environment B is modeled as a set of infinite numbers of harmonic oscillators. In this paper, we assume each resonator of the CRW is coupled to an individual bath of harmonic oscillators. The dynamic of the  $j$ th resonator and its corresponding environment is governed by the Hamiltonian,

$$H_{ER}^{[j]} = \sum_q \omega_{q[j]} b_{q[j]}^\dagger b_{q[j]} + \sum_q (g_{q[j]} a_j^\dagger b_{q[j]} + \text{H.c.}), \quad (6)$$

where  $b_{q[j]}$  and  $b_{q[j]}^\dagger$  are the annihilation and creation operators for the  $q$ th bath oscillator attached to the  $j$ th resonator of the CRW,  $\omega_{q[j]}$  is its frequency, and  $g_{q[j]}$  the coupling strength. The total Hamiltonian describing the interaction between the CRW and environments reads

$$H_{EC} = \sum_j H_{ER}^{[j]}. \quad (7)$$

We also introduce the exchange interaction between the TLS and its environment, which is described by the Hamiltonian,

$$H_{EA} = \sum_q \nu_q d_q^\dagger d_q + \sum_n \beta_n (d_n^\dagger |g\rangle\langle e| + |e\rangle\langle g| d_n), \quad (8)$$

where  $d_n$  and  $d_n^\dagger$  are the annihilation and creation operators corresponding to the  $q$ th mode with frequency  $\nu_q$ . The Hamiltonian of the total system S + B reads

$$H_{SB} = H_S + H_{EC} + H_{EA}. \quad (9)$$

Although there exists energy exchange between system S and environment B, the total number of excitations is preserved.

## III. STATIONARY STATES OF A CRW WITH A TLS INSIDE

In the one-excitation subspace, the stationary states of system S are either the localized states around the location of the TLS or a superposition of extended propagating Bloch waves incident reflected and transmitted by the TLS embedded in the CRW. Assume a photon is coming from the left of the TLS with energy:

$$\Omega_k = \omega_c - 2\xi \cos k. \quad (10)$$

The stationary state of the system is then

$$|\Omega_k\rangle = \sum_j u_k(j) a_j^\dagger |0g\rangle + u_{ek} |0e\rangle, \quad (11)$$

where  $|0\rangle$  is the vacuum state of the cavity field,  $u_{ek}$  is the probability amplitude with the TLS in the excited state and no photon, and  $u_k(j)$  is the probability amplitude for the TLS to move to the lower state, emitting a photon into a mode of the  $j$ th resonator. From the eigenvalue equation  $H_S |\Omega_k\rangle = \Omega_k |\Omega_k\rangle$ , together with the bosonic commutation relations, we derive a system of coupled equations among the amplitudes:

$$(V_k + JG_k \delta_{j0}) u_k(j) = \xi [u_k(j+1) + u_k(j-1)], \quad (12a)$$

$$u_{ek} = G_k u_k(0), \quad (12b)$$

with the Green function  $G_k = G_k(\Omega) = J/(\Omega_k - \Omega)$  and  $V_k = \omega_C - \Omega_k$ . Equation (11) has the solution in terms of incoming and outgoing waves with amplitudes:

$$u_k(j) = \begin{cases} e^{ikj} + r_k e^{-ikj}, & j < 0 \\ s_k e^{ikj}, & j > 0. \end{cases} \quad (13)$$

The reflection and transmission amplitudes yield

$$r_k = s_k - 1, \quad (14a)$$

$$s_k = \frac{2i\xi(\Omega_k - \Omega) \sin k}{2i\xi(\Omega_k - \Omega) \sin k - J^2}. \quad (14b)$$

Equation (12) implies that the transmission coefficient is sensitive to the transition frequency of the TLS, as the forward and backward propagating modes within the CRW are coupled via the TLS.

The periodicity of the CRW in  $r$  space gives rise to a continuum with Bloch waves grouped in energy bands, which is broken due to the coupling between the TLS and the CRW. Local modes are produced, and their corresponding eigenenergy is outside the energy bands. We denote the eigenvalue of the bound state as  $\Omega_\kappa$  (i.e., replace the index  $k$  with  $\kappa$ ) as does Eq. (11). Noticing that the probability of bound state at infinity in coordinate space is zero (i.e., bound state is spatially localized), we can assume that the bound states of even parity have the following amplitudes:

$$u_\kappa(j) = \begin{cases} C e^{(in\pi - \kappa)j}, & j > 0 \\ C e^{(in\pi + \kappa)j}, & j < 0, \end{cases} \quad (15)$$

where the normalization constant  $C$  is

$$C = \left[ \tanh k + \frac{J^2}{(\Omega_\kappa - \Omega)^2} \right]^{-1/2}. \quad (16)$$

The bound state energy lies either below or above the continuum with the magnitude,

$$\Omega_\kappa = \omega_C - 2\xi e^{in\pi} \cosh \kappa, \quad (17)$$

where the value of  $\kappa$  is determined by the following condition:

$$J^2 = 2\xi e^{in\pi} (\Omega - \Omega_\kappa) \sinh \kappa. \quad (18)$$

The bound states exist when Eq. (18) has solution.

#### IV. SINGLE-PARTICLE SPONTANEOUS EMISSION

Quantum states inevitably decay with time into a probabilistic mixture of classical states due to their interaction with the environment, which comprises much larger systems or ensemble of states. In this section we derive the relaxation time for single combined photonic-atomic excitation in system S by incorporating the effects of interaction with environment B. Notice that the number of excitations is conserved in the total system (S + B). Therefore, in one-excitation subspace, two situations will occur in system S: one is that the single excitation is in state  $|\Omega_k\rangle$  or  $|\Omega_\kappa\rangle$ ; and another is that no excitation is found in system S (i.e., the TLS is in the ground state and no photon is in the resonator). For the first case, there is no excitation in environment B, and we use the notation  $|\Omega_k 00\rangle$  to describe the state of the total system, where the first 0 in the ket indicates that the radiation modes coupled to the CRW are in the vacuum state, and the second 0 in the  $|\Omega_k 00\rangle$  denotes the vacuum state of the harmonic oscillators attached to the TLS. When system S is in the  $|0g\rangle$  state, environment B contains the single excitation. Here,  $d_n^\dagger |G\rangle$  ( $|G\rangle \equiv |0g00\rangle$ ) represents that the single particle has moved to the  $n$ th bath oscillator coupled to the TLS;  $b_{q[j]}^\dagger |G\rangle$  describes that the excitation in S is in the  $q$ th mode of environment B attached to the  $j$ th resonator of the CRW. Obviously, states  $\{|\Omega_k 00\rangle, d_q^\dagger |G\rangle, b_{q[j]}^\dagger |G\rangle\}$  provide a complete basis and thus we can expand the wave function of the S + B at arbitrary time  $t$  in terms of this basis as

$$|\Psi(t)\rangle = \sum_k U_k(t) |\Omega_k 00\rangle + \sum_n C_n(t) d_n^\dagger |G\rangle + \sum_{qj} B_{q[j]}(t) b_{q[j]}^\dagger |G\rangle, \quad (19)$$

where  $U_k$ ,  $B_{q[j]}$ , and  $C_n$  are the time-dependent probability amplitudes for finding the entire system in its corresponding states.

Inserting the wave function (19) into the Schrödinger equation with the governing Hamiltonian in Eq. (9), we find a system of coupled linear differential equations for the amplitudes:

$$i\dot{U}_k = \Omega_k U_k + \sum_{jq} g_{q[j]} u_k^*(j) B_{q[j]} + \sum_n u_{ek}^* \beta_n C_n, \quad (20a)$$

$$i\dot{B}_{q[j]} = \omega_{q[j]} B_{q[j]} + \sum_k g_{q[j]} u_k(j) U_k, \quad (20b)$$

$$i\dot{C}_n = v_n C_n + \sum_k \beta_n u_{ek} U_k, \quad (20c)$$

where the overdot indicates the derivative with respect to time. The evolution of  $U_k$  is coupled to  $B_{q[j]}$  and  $C_n$  via the coupling constant  $g_{q[j]} u_k^*(j)$  and  $u_{ek}^* \beta_n$ . To remove the high-frequency effect, we make the following substitution:

$$\begin{aligned} U_k(t) &= \phi_k(t) e^{-i\Omega_k t}, \\ B_{q[j]}(t) &= b'_{q[j]}(t) e^{-i\omega_{q[j]} t}, \\ C_n(t) &= c_n(t) e^{-iv_n t}. \end{aligned} \quad (21)$$

Then, in the interaction picture, Eq. (20) is rewritten as

$$\begin{aligned} i\dot{\phi}_k &= \sum_{jq} g_{q[j]} u_k^*(j) b'_{q[j]} e^{-i\Delta_{qk}^{[j]} t} + \sum_n u_{ek}^* \beta_n c_n e^{-i\delta_{nk} t}, \\ i b'_{q[j]} &= g_{q[j]} \sum_k u_k(j) \phi_k e^{i\Delta_{qk}^{[j]} t}, \\ i\dot{c}_n &= \sum_k \beta_n u_{ek} \phi_k e^{i\delta_{nk} t}, \end{aligned}$$

where  $\Delta_{qk}^{[j]} = \omega_{q[j]} - \Omega_k$ ,  $\delta_{nk} = \nu_n - \Omega_k$  are the detunings between system S and environment B. Formally integrating the equations for  $b_{q[j]}$  and  $c_n$  in the above equations yields

$$b'_{q[j]} = -i g_{q[j]} \sum_k u_k(j) \int \phi_k(\tau) e^{i\Delta_{qk}^{[j]} \tau} d\tau \quad (22a)$$

$$c_n = -i \sum_k \beta_n u_{ek} \int \phi_k(\tau) e^{i\delta_{nk} \tau} d\tau. \quad (22b)$$

Inserting Eq. (22) into the equation for  $\phi_k$ , we obtain the exact integro-differential equation,

$$\begin{aligned} \dot{\phi}_n &= - \sum_{kj} g_{q[j]}^2 u_n^*(j) u_k(j) e^{-i\Delta_{qn}^{[j]} t} \int_0^t \phi_k(\tau) e^{i\Delta_{qk}^{[j]} \tau} d\tau \\ &\quad - \sum_{kq} \beta_q^2 u_{en}^* u_{ek} e^{-i\delta_{qn} t} \int_0^t \phi_k(\tau) e^{i\delta_{qk} \tau} d\tau, \end{aligned} \quad (23)$$

where the coupling constants  $g_{q[j]}$  and  $\beta_q$  are assumed to be real.

Suppose that at time  $t = 0$  there is no interaction between system S and bath B, and the S + B is in the state  $|\Omega_k 00\rangle$ , corresponding to a single excitation in system S. Mathematically, solving Eq. (23) is to solve an initial value problem. It is well known that such an initial value problem can be solved by the Laplace transform. Denoting the Laplace transform of  $\phi_n(t)$  by  $\bar{\phi}_n(s)$  and taking into account the initial conditions  $\phi_k(0) = \delta_{nk}$ , we have

$$\bar{\phi}_n = \frac{1}{s + I(s)}, \quad (24)$$

where

$$I(s) = \sum_q \left[ \sum_j \frac{g_{q[j]}^2 |u_n(j)|^2}{s + i\Delta_{qn}^{[j]}} + \frac{\beta_q^2 |u_{en}|^2}{s + i\delta_{qn}} \right].$$

The roots of the denominator in Eq. (24) can be split into a sum of the singular and principal value parts. The principal part is merely included into redefinition of the energy. The singular part gives the decay rate due to the coupling to bath B. Under the Wigner-Weisskopf approximation [19], the system decay is dominantly exponential with rate:

$$\Gamma_n = \pi \sum_j |u_n(j)|^2 \Lambda_j(\Omega_n) + \pi |u_{en}|^2 \Lambda_A(\Omega_n). \quad (25)$$

This rate is proportional to the modulus square of the coupling between S and B.

The functions in Eq. (25),

$$\begin{aligned} \Lambda_j(\omega) &= \sum_q g_{q[j]}^2 \delta(\omega - \omega_{q[j]}) \\ \Lambda_A(\omega) &= \sum_q \beta_q^2 \delta(\omega - \nu_q), \end{aligned}$$

are called the reservoir response (memory) functions, which are the spectral densities of the states  $b_{q[j]}^\dagger|G\rangle$  and  $d_q^\dagger|G\rangle$  weighted with the coupling strengths  $g_{q[j]}^2$  and  $\beta_q^2$ , respectively. The memory function  $\Lambda(\omega)$  characterizes the spectral shape of the reservoir. Since the states in the reservoir are very dense (continuum), one can replace the sums over  $q$  by integrals, for instance,  $\sum_q \rightarrow 2V/(2\pi)^3 \int q^2 \sin\theta d\phi d\theta dq$ . Then the memory functions read

$$\begin{aligned} \Lambda_j(\omega) &\rightarrow g_{[j]}^2(\omega) \rho_{[j]}(\omega), \\ \Lambda_A(\omega) &\rightarrow \beta^2(\omega) \rho_A(\omega), \end{aligned}$$

where  $g_{[j]}^2(\omega)$  are the coupling constants to states  $b_{\omega_{[j]}}^\dagger|G\rangle$ , whose density of state is given by  $\rho_{[j]}(\omega)$ . Obviously, the decay rate is determined by two factors: (1) the form of  $\Lambda(\omega)$ ; (2) the width of the reservoir, which depicts the overlap of the eigenfrequencies of the system and the spectral shape of the reservoir. Usually, it is assumed that the reservoir is spectrally flat and the frequencies of the system of interest is deeply embedded in the continuum, therefore, the argument of the spectral density is replaced by the eigenfrequency of the system  $\Omega_n$  in Eq. (25). For the sake of simplicity, we further assume that the coupling constant  $g_{q[j]}$  is independent of the energy  $\omega_{q[j]}$  as well as the index  $j$  [i.e., let  $\Lambda_{[j]}(\omega) = g(\omega)$  and let  $\Lambda_A(\omega) = \beta_A(\omega)$ ]. The constant coupling gives us the explicit relationship between the decay rate and the wave number of system S:

$$\Gamma_n = \pi g^2 + \pi (\beta_A^2 - g^2) |u_{en}|^2, \quad (26)$$

where the normalization of the eigenstates  $|\Omega_n\rangle$  has been used in deriving the above equation. Obviously, when  $\beta_A = g$ , the decay rate is a constant for any eigenstates of system S. For a given wave number  $n$ , when  $\beta_A$  is smaller than  $g$ , the rate decreases, which opens up a possibility of prolonging the lifetime of the single excitation in system S. In Fig. 2, we plot the decay rate  $\Gamma_k$  as a function of the wave number  $k$ . It indicates that the widths of the eigenstates in Eq. (13) are different, which is a periodical function of the wave number  $k$ , and the detuning between the TLS and the resonator biases the symmetry of the line shape. By substituting the condition in Eq. (18) into the expression of the probability amplitude  $u_{ek}$ , the decay rate of a bound state has the form  $\Gamma_\kappa = \pi g^2 + \pi(\beta_A^2 - g^2)[1 - (J^2 + 2\xi^2 \sinh 2\kappa)^{-1}]$ . It indicates that the decay rate is an increasing function of  $\kappa$ , and when the resonator resonates with the TLS,  $\Gamma_\kappa = \pi(2g^2 + \beta_A^2)/3$ . Since the value of the imaginary wave number is determined for given parameters  $\{J, \xi, \omega_c - \Omega\}$ , the decay rate of the bound state is a given constant.

## V. DISSIPATION ON SCATTERING AMPLITUDE

To see the impact of dissipation of the system on the transmission and reflection coefficients, we begin with Hamiltonian (9) in the configuration space. Here, it is assumed that each bath attached to its resonator is identical and the coupling

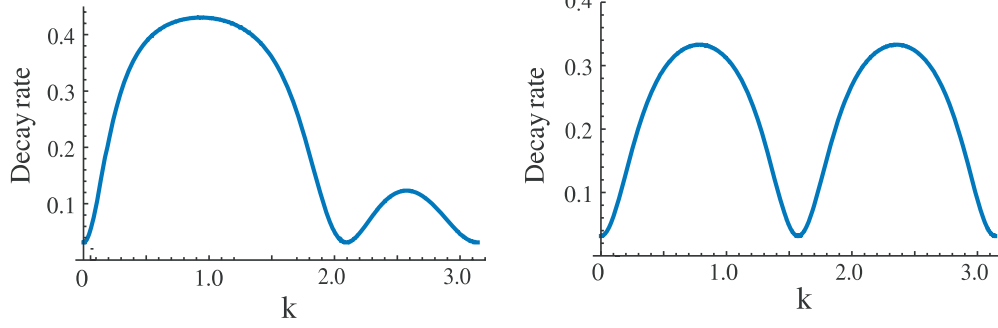


FIG. 2. (Color online) The decay rate via wave number  $k$  starting from one eigenstate of system S, with parameters  $g = 0.1$ ,  $\beta = 0.4$ ,  $J = 1.5$ , (a)  $\omega_C = 5$ ,  $\Omega = 6$ , and (b)  $\omega_C = \Omega = 5$ , which are in units of  $\xi$ .

strength between the resonator and its corresponding bath is independent of the location index  $j$ . The state at arbitrary time is a superposition of four parts: the photon at the  $j$ th cavity with atom in the ground state  $a_j^\dagger|G\rangle$ , no photon in all cavities with atom in the excited state  $\sigma_+|G\rangle$ , the photon in the  $q$ th mode of the bath attached to the  $j$ th resonator  $b_{q[j]}^\dagger|G\rangle$ , and that of the bath coupled to the TLS  $c_q^\dagger|G\rangle$ :

$$|\Psi(t)\rangle = \sum_j \Phi_{jk}(t)a_j^\dagger|G\rangle + \sum_{qj} A_{q[j]}(t)b_{q[j]}^\dagger|G\rangle + \Phi_{ek}(t)\sigma_+|G\rangle + \sum_q D_q(t)c_q^\dagger|G\rangle. \quad (27)$$

The Schrödinger equation results in a system of coupled linear differential equations for the amplitudes,

$$\begin{aligned} i\dot{\Phi}_{jk} &= \omega_C\Phi_{jk} - \xi(\Phi_{j+1k} + \Phi_{j-1k}) + J\Phi_{ek}\delta_{0j} + \sum_q g_q A_{q[j]}, \\ i\dot{\Phi}_{ek} &= \Omega\Phi_{ek} + J\Phi_{0k} + \sum_q \beta_q D_q \\ i\dot{A}_{q[j]} &= \omega_q A_{q[j]} + g_q \Phi_{jk}, \\ i\dot{D}_q &= \beta_q \Phi_{ek} + \nu_q D_q, \end{aligned} \quad (28)$$

where the dot denotes a derivative with respect to time. Applying the Fourier transform and expressing  $A_{q[j]}$ ,  $D_q$  with  $\Phi_{jk}$ ,  $\Phi_{ek}$ , respectively, the equations of motion in the frequency domain are obtained in the reduced dimensionality,

$$(E - \omega_C - \delta E)\psi_{jk} = -\xi(\psi_{j+1k} + \psi_{j-1k}) + \frac{J^2\psi_{0k}\delta_{0j}}{E - \Omega - \Delta E}, \quad (29)$$

where  $E$  is the eigenenergy of the single-photon wave in the whole system. The eigenfrequency of the  $j$ th resonator of the ideal CRW is renormalized into  $\omega_C + \delta E$ , and the transition energy of the TLS is also renormalized into  $\Omega + \Delta E$ :

$$\delta E = \sum_q \frac{g_q^2}{E - \omega_q}, \quad \Delta E = \sum_q \frac{\beta_q^2}{E - \nu_q}, \quad (30)$$

which is the influence of the baths on the state of each resonator and the TLS. The singular part of  $\delta E$  and  $\Delta E$  yields the dissipation factors  $\gamma_c = \pi g^2(\omega_C)\rho_c(\omega_C)$  and  $\gamma_A = \pi\beta(\Omega)\rho_A(\Omega)$ . The real part of  $\delta E$  and  $\Delta E$  contributes to the Lamb shift of the levels and change of  $\Omega$ , respectively, which is

merely included in the redefinition of the energy. Then Eq. (29) is reduced to the following set of equations:

$$(E - \omega_c + i\gamma_c)\psi_{jk} = -\xi(\psi_{j+1k} + \psi_{j-1k}) + \frac{J^2\psi_{0k}\delta_{0j}}{E - \Omega + i\gamma_A}. \quad (31)$$

We begin our analysis from the case with coupling strength  $J = 0$ . In this case, Eq. (29) has a complex dispersion curve that can be identified with the plane-wave solution  $\Phi_{jk} \propto e^{i(kj - Et)}$ , where  $E = E_r + iE_i$  is the complex energy and  $k$  is the wave momentum inside the CRW. The real and imaginary parts of energy  $E$  satisfy  $E_r = \Omega_k$ ,  $E_i = \gamma_c$ . When only one resonator is initially excited  $|\Psi(0)\rangle = |n\rangle$  (say  $n = 0$ ), where  $|n\rangle$  is the Wannier state localized at the site  $n$ , the field profile at time  $t$  is given by

$$|\Psi(t)\rangle = e^{-i\omega_c t - \gamma_c t} \sum_{kq} J_q(2\xi t) e^{iq\pi/2} e^{-ikq} |k\rangle, \quad (32)$$

where  $|k\rangle$  is the Bloch state. The amplitude at the site  $l$  reads

$$\Phi_{lk} = e^{-i\omega_c t - \gamma_c t} J_l(2\xi t) e^{il\pi/2}, \quad (33)$$

where  $J_l(x)$  is a Bessel function of the first kind of integer order  $l$ . Obviously, leakage rate in each resonator  $\gamma_c$  influences the free propagation of the single photon. The distance that the photon travels along the 1D resonator waveguide is depicted by the product of the group velocity and  $\gamma_c^{-1}$ .

We now consider the case with coupling strength  $J \neq 0$ . Due to the coupling between the atom and the 0th resonator, a complex  $\delta$ -like potential stands in the way that single photon travels. Consequently, the photon experiences scattering. We assume that the single photon has momentum  $k$  initially. Within the allowed distance of photon traveling along the 1D CRW, the transmission and reflection amplitudes,  $t$  and  $r$ , can be defined via the asymptotes of the wave function:

$$\psi_{jk} = \begin{cases} e^{ikj} + r_k e^{-ikj}, & j < 0 \\ s_k e^{ikj}, & j > 0. \end{cases} \quad (34)$$

By substituting the asymptotes of the wave function into Eq. (31) for  $j = \pm 1, 0$  sites, the reflection amplitude is obtained as

$$r_k = \frac{J^2}{2i\xi \sin k(\Omega_k - \Omega) - (\gamma_A - \gamma_c)2\xi \sin k - J^2}. \quad (35)$$

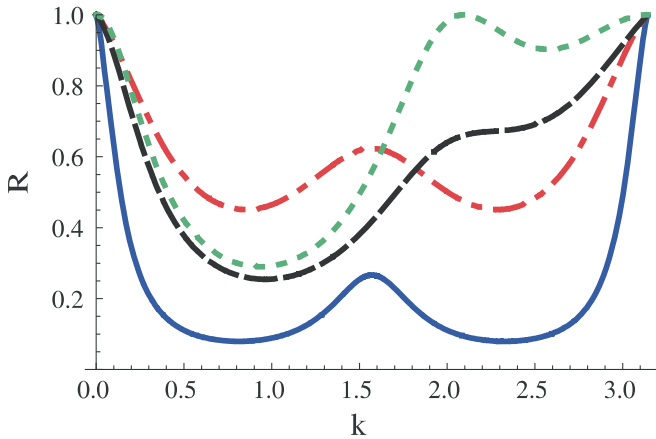


FIG. 3. (Color online) The reflection coefficient versus the wave number  $k$ . The parameters are in units of  $\xi$  and are set as follows:  $\gamma_c = 0.1$ ,  $\gamma_A = 0.4$ ,  $J = 0.8$ ,  $\omega_c = \Omega = 5$  for blue solid line;  $\gamma_c = 0.1$ ,  $\gamma_A = 0.4$ ,  $J = 1.5$ ,  $\omega_c = \Omega = 5$  for red dot-dashed line;  $\gamma_c = 0.1$ ,  $\gamma_A = 0.4$ ,  $J = 1.5$ ,  $\omega_c = 5$ ,  $\Omega = 6$  for black dashed line;  $\gamma_c = 0.1$ ,  $\gamma_A = 0.1$ ,  $J = 1.5$ ,  $\omega_c = 5$ ,  $\Omega = 6$  for green dotted line.

Equation (35) shows that as long as the rate of decay to each resonator  $\gamma_c$  is equal to the decay rate of the TLS  $\gamma_A$ , a resonant scattering occurs when the incident energy of the single photon is equal to  $\Omega$ , the transition energy of the TLS (i.e., when the single-photon incident from the left encounters the TLS, it is completely reflected back to the left), as shown via the green dotted line in Fig. 3. For unequal  $\gamma_c$  and  $\gamma_A$ , the decay rates lower the peak of the resonance, and the width of the line shape is broadened from  $J^2/(2\xi \sin k)$  for ideal system  $S$  to  $[(\gamma_A - \gamma_c)2\xi \sin k + J^2]/(2\xi \sin k)$ . In Fig. 3, we plot the reflection coefficient as a function of the wave number  $k$ . Comparing the blue solid line and the red dash-dot line with the black dashed line and the green dotted line, it can be found that the symmetry of the line shape is determined by the transition energy  $\Omega$  of the TLS. As the coupling strength

$J$  between the 0th resonator and the TLS goes to infinity, the reflection coefficient approaches to one.

## VI. CONCLUSION

In summary, we have studied the dissipative process of the CRW with a TLS inside, originating from the coupling to the environment. Our discussions are based on a simple decoherence scenario in which each resonator and the TLS individually interact with their own environments. Each environment is modeled as a continuum of harmonic oscillators and assumed to be on the vacuum state initially. Since the coupling with the environment is generally weak compared to the system of interest, the Wigner-Weisskopf approximation applies, and the lifetime of an excitation in system  $S$  is thus obtained, which represents the timescale of the transition from quantum to classical behavior. We further investigate the impact of the dissipation on the transport property of the single photon along the 1D CRW for identical baths attached to the CRW. The dissipation lowers the peak of the resonance and broadens the width of the line shape of the reflection spectrum except in the case that magnitude of the leakage rate is equal to the decay rate of the TLS. The leakage rate in each resonator not only influences the free propagation of the single photon, but also results in the inelastic scattering of the single photon together with the dissipation of the TLS.

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