Three-body dwell time

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The lifetime of an unstable state or resonance formed as an intermediate state in two-body scattering is known to be related to the dwell time or the time spent within a given region of space by the two interacting particles. This concept is extended to the case of three-body systems and a relation connecting the three-body dwell time with the two-body dwell times of the substructures of the three-body system is derived for the case of separable wave functions. The Kapur-Peierls formalism is revisited to discover one of the first definitions of dwell time in the literature. An extension of the Kapur-Peierls formalism to the three-body case shows that the lifetime of a three-body resonance can indeed be given by the three-body dwell time.

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I. INTRODUCTION

Tunneling is one of the most exotic phenomenon in quantum mechanics and the study of tunneling times with its different controversial definitions is equally so. In an attempt to find out how long a particle needs to traverse a potential barrier, physicists gave rise to several definitions such as the dwell time, phase time, arrival time, Larmor time, traversal time, residence time, and more abstract complex times [1]. Of these, the dwell and phase times seem to be the most relevant for the study of unstable or resonant states occurring in tunneling as well as scattering problems. The dwell time (sometimes known as the sojourn or residence time) is defined as the average time spent by a particle in a given region of space. The introduction of the dwell-time concept is commonly attributed to Smith [2] in the literature. Smith introduced it in connection with quantum collisions and Büttiker [3] discussed it in the context of one-dimensional tunneling. However, it is interesting to note that long before, in 1938 [4], Kapur and Peierls had derived the formula for dwell time as we know it now. It was a byproduct of the formalism for cross sections with resonances in nuclear reactions and they did not explicitly mention it to be a quantum time. In the case of tunneling through a barrier, the average dwell time is the time spent by the particle in the barrier, irrespective of whether it got transmitted or reflected. However, there do exist definitions of reflection and transmission dwell times which are connected to the measured lifetimes of decaying objects [5].

The dwell-time concept finds applications in various branches of physics. In [6] for example, the author relates this concept to the qubit residence time measurements in the presence of Bose-Einstein condensates. In another recent work [7], it is shown that as a result of the Dresselhaus spin-orbit effect, the difference between the dwell times of spin-up and spin-down electrons can become greater as the semiconductor length increases. These studies could be useful in designing quantum spintronic devices. Some other applications include studies with semiclassical theories of quantum chaos [8] and the connection of dwell time with a quantum clock. Salecker and Wigner [9] proposed a microscopic clock to measure distances between space time events. Peres [10] extended the formalism to the measurement of an average time spent by particles in a given region of space. Leavens [11] established the connection between Peres's time spent in a given region of space and the standard definition of average dwell time. More recent discussions of the Peres clock and dwell times can be found in [12]. Finally, the dwell time is useful in characterizing resonances [13,14] as well as studying the time evolution of unstable states [15]. In the case of s-wave resonances, the dwell time is more useful than the phase time concept as it is free of the singularity present in the phase time near threshold. We shall discuss this point in Sec. II.

In the next section we shall first briefly introduce the concepts of dwell and phase times and the relations relevant in the present work. In Sec. III the formalism used by Kapur and Peierls (KP) is briefly presented and its connection with the dwell time of Smith and the closely connected definition of phase time and Wigner's phase time delay [16] is discussed. In Sec. IV, the three-body dwell time (τ^{3-b}) will be derived using two different approaches. The first one uses an extension of the K-P formalism to the three-body case. The second one starts with the standard definition of a dwell time involving a three-body wave function and current density. This derivation leads to a three-body dwell-time relation given in terms of the component two-body dwell times, exactly as obtained within the K-P approach. In Sec. V we summarize our findings.

II. DWELL AND PHASE TIME

The average dwell time for an arbitrary barrier $V(x)$ in one dimension (a framework which is also suitable for spherically symmetric problems) confined to an interval (x_1, x_2) is given by the number of particles in the region divided by the incident flux j:

$$\tau_D(E) = \frac{\int_{x_1}^{x_2} |\Psi(x)|^2 dx}{j}. \quad (1)$$

Here $\Psi(x)$ is the time-independent solution of the Schrödinger equation in the given region. The dwell and phase time are closely connected and for a particle of energy $E = \hbar^2 k^2/2\mu$ ($\hbar k$ is the momentum), incident on the barrier [17],

$$\tau_\phi(E) = \tau_D(E) - \hbar \frac{\mathrm{Im} R}{k} \frac{dk}{dE}, \quad (2)$$

where the phase time $\tau_\phi(E)$ is given in terms of a weighted sum of the energy derivative of the reflection and transmission phases. The phase time is essentially the time difference between the arrival and departure of a wave packet at the barrier. R is the reflection coefficient and the second term on the right-hand side arises due to the interference between the incident and reflected waves in front of the barrier. This term is important at low energies and becomes singular as $E \to 0$, thus making the phase time singular too. In the case of scattering in three dimensions, the above relation is replaced by a very similar one, namely [14],

$$\tilde{\tau}_D(E) = \tilde{\tau}_\phi(E) + \hbar\mu[t_R/\pi]\, dk/dE, \quad (3)$$

where t_R is the real part of the scattering transition matrix and μ the reduced mass of the two scattering particles. $\tilde{\tau}_D(E)$ and $\tilde{\tau}_\phi(E)$ are now the dwell and phase time "delays" given by $\tilde{\tau}_\phi(E) = \tau_\phi(E) - \tau^0(E)$ and $\tilde{\tau}_D(E) = \tau_D(E) - \tau^0(E)$, with $\tau^0(E)$ being the time spent in the same region of space without interaction (or in the absence of barrier). $\tilde{\tau}_\phi(E)$ is more commonly known as Wigner's time delay [16] and is given by the energy derivative of the scattering phase shift, $\tilde{\tau}_\phi(E) = 2\hbar\, d\delta/dE$.

The phase time delay has been found to be very useful in characterizing resonances in hadron scattering [18]. However, due to the singularity mentioned above, the phase time delay poses a serious problem in identifying the s-wave resonances occurring close to threshold. In these particular cases, the dwell-time delay emerges as the more useful concept [13–15] since it also has a physical significance of being connected to the density of states (DOS). A relation between the dwell time and the DOS for a system in three dimensions with arbitrary shape was derived in [19] and [20] discussed the same with the example of a symmetric barrier in one dimension. The dwell-time delay displays the correct threshold behavior expected for the density distribution of an unstable state formed as an intermediate state in two-body scattering [13,15]. We shall now see how the above definitions appear in the Kapur-Peierls formalism.

III. KAPUR-PEIERLS FORMALISM REVISITED

In an attempt to obtain a dispersion formula for nuclear reactions, KP considered first the scattering of one particle in a central field of force, assuming this field to be fully contained within a sphere of radius r_0. The partial wave ϕ with only $l = 0$ was taken into account. $\phi = r\Psi$ satisfies the radial equation [4],

$$(E - H)\phi = \frac{\hbar^2}{2m}\frac{d^2\phi}{dr^2} + [E - V(r)]\phi = 0. \quad (4)$$

For $r \geqslant r_0$, $V(r)$ vanishes and

$$\frac{d^2\phi}{dr^2} + k^2\phi = 0. \quad (5)$$

The solution of this equation is written as $\phi = (I/k)\sin(kr) + Se^{ikr}$, with I and S the amplitudes of the incident and scattered waves. At $r = r_0$,

$$I e^{-ikr_0} = \left(\frac{d\phi}{dr}\right)_{r_0} - ik\phi(r_0)$$
$$S = \cos(kr_0)\phi(r_0) - \frac{1}{k}\sin(kr_0)\left(\frac{d\phi}{dr}\right)_{r_0}. \quad (6)$$

At this point, KP impose a boundary condition and obtain a discrete set of complex energy eigenvalues. They consider a situation where no incident waves are present, which gives rise to the boundary condition,

$$\frac{d\phi}{dr} - ik\phi = 0 \quad (\text{at } r = r_0). \quad (7)$$

The boundary condition is obviously not compatible with (4) if E is real, but is rather satisfied by the solutions ϕ_n, such that

$$\frac{\hbar^2}{2m}\frac{d^2\phi_n}{dr^2} + [W_n - V(r)]\phi_n = 0, \quad (8)$$

where $V(r)$ is real but W_n are complex eigenvalues. Multiplying (8) by ϕ_n^*, subtracting the complex conjugate of this equation, and integrating gives

$$\frac{\hbar^2}{2mi}\left[\phi_n^*\frac{d\phi_n}{dr} - \phi_n\frac{d\phi_n^*}{dr}\right]\Bigg|_{r_0} = -2\,\mathrm{Im}(W_n)\int_0^{r_0}\phi_n^*\phi_n dr. \quad (9)$$

Identifying the left-hand side of the above equation with $\hbar j$, with j being the standard quantum mechanical definition of current density at r_0 and considering $W_n = E_n - i\Gamma_n/2$, a typical pole of an unstable state with width Γ_n,

$$\frac{\hbar}{\Gamma_n} = \frac{\int_0^{r_0}\phi_n^*\phi_n dr}{j(r_0)}. \quad (10)$$

The right-hand side is indeed similar to the definition of dwell time as in Eq. (1). KP did not identify the relation with a dwell-time definition as is now known in literature. Note, however, that the current density appearing in Eq. (10) is not the incident current density but rather the current density at r_0. In this sense, Eq. (10) could be compared with a transmission dwell time [5] rather than an average dwell time as in (1). The transmission dwell time involves the transmitted current density and was shown in [5] to be related to the lifetimes of unstable nuclei.

The boundary condition (7) has interesting implications. Indeed, in connection with the work of Smith [2], Wigner remarked [21] that the time delay in [2] should have been calculated using only the outgoing part of the scattered wave. We look at this possibility now. Using only the outgoing part of the scattered waves one would expect the second term on the right-hand side in Eq. (2) which arises due to the overlap of the incident and reflected waves to vanish and $\tau_D(E) = \tau_\phi(E)$. To demonstrate the above, we repeat the steps in Smith's [2] derivation of dwell-time delay with the asymptotic wave function given by ${}_\infty\Psi = (1/\sqrt{v})\,[e^{2i\delta}e^{ikx}]$ (v is the velocity) instead of the full incident plus scattered wave function, ${}_\infty\Psi = (1/\sqrt{v})\,[e^{-ikx} - e^{2i\delta}e^{ikx}]$. For a wave function Ψ which satisfies the Schrödinger equation, it is easy to see that

$$\Psi^*\Psi = -\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\left(\Psi^*\frac{\partial^2\Psi}{\partial x\partial E} - \frac{\partial\Psi}{\partial E}\frac{\partial\Psi^*}{\partial x}\right). \quad (11)$$

Since Ψ^* and $\partial\Psi/\partial E$ vanish at $x = 0$, integration from 0 to r_0 gives

$$\int_0^{r_0}\Psi^*\Psi = -\frac{\hbar^2}{2m}\left(\Psi^*\frac{\partial^2\Psi}{\partial x\partial E} - \frac{\partial\Psi}{\partial E}\frac{\partial\Psi^*}{\partial x}\right)_{r_0}. \quad (12)$$

At large r_0, we replace Ψ on the right-hand side of the above equation by ${}_\infty\Psi$ and obtain

$$\int_0^{r_0} \Psi^* \Psi dx - \frac{r_0}{v} = 2\hbar \frac{d\delta}{dE}. \quad (13)$$

With $r_0/v = \tau^0$, the time spent without interaction, the left-hand side of the above equation can be identified with a dwell-time delay and since $2\hbar d\delta/dE = \tilde{\tau}_\phi(E)$, we get $\tilde{\tau}_D(E) = \tilde{\tau}_\phi(E)$. We note in passing that the wave number k appearing in the above equations is real and is the same for all complex eigenvalues W_n. This is a result of the boundary condition (7). In contrast to this approach, Gamow [22] introduced standing waves in front of the barrier with the result that the asymptotic outgoing wave is a plane wave with a complex wave number k. Other approaches which deal with solutions of the Schrödinger equation for complex energies can be found in [23].

A. New definition of dwell time

Starting with the scattering amplitude as given in Eq. (6), namely, $S = \cos(kr_0)\phi(r_0) - \frac{1}{k}\sin(kr_0)(d\phi/dr)_{r_0}$ and noting the standard definition of $S = e^{2i\delta}$ (where δ is the scattering phase shift), we can write $e^{2i\delta} = \phi(r_0)e^{-ikr_0}$. Taking the energy derivative of this equation, it is easy to see that

$$2\hbar \frac{d\delta}{dE} + \frac{r_0}{v} = -i\hbar \frac{1}{\phi(r_0)} \frac{d\phi(r_0)}{dE}, \quad (14)$$

with $v = \hbar k/m$. $r_0/v = \tau^0$ is the time spent without interaction in the region of radius r_0. $2\hbar d\delta/dE$ is the phase time delay $[2\hbar d\delta/dE = \tilde{\tau}_\phi(E) = \tau_\phi(E) - \tau^0(E)]$ and, hence, the left-hand side of (14) is simply $\tau_\phi(E)$. If the boundary condition (7) is imposed, we have already seen that $\tau_\phi(E) = \tau_D(E)$ and from the equations above,

$$\tau_D(E) = -i\hbar \frac{d}{dE}[\ln \phi(r_0)], \quad (15)$$

which is a new definition of dwell time obtained within the K-P formalism.

IV. THREE-BODY DWELL TIME

Since the dwell and phase time concepts have been successfully used [5,14,18] to study resonances occurring in two-body elastic scattering, it appears timely to extend these ideas for the study of unstable systems which can be viewed upon as three-body systems. Such unstable states occur in different branches of physics. For example, in a recent study [24] of the s-wave resonances ^{9}Be and ^{9}B, it was shown that these unstable nuclei can be looked upon as genuine three-body resonances with ^{9}Be, for example, being composed of the substructure $\alpha + \alpha +$ n. Other examples could include hadronic systems of two mesons and a baryon, two-neutron halo nuclei, or even hypernuclei such as $^6\text{He}_\Lambda$ (with substructure $^4\text{He} + \Lambda + n$). Since this work is a first attempt to derive an expression for the dwell time of such three-body systems, we restrict ourselves to the case of s waves (i.e., we consider only the partial wave with $l = 0$). This allows us to develop the formalism in analogy to the dwell-time formalism in the one-dimensional case. We shall further assume that the wave function can be expressed in a separable form (which is often also the case in studies of three-body systems previously mentioned).

A. Lifetime of a three-body resonance

To describe the three-particle system, we start by writing the Hamiltonian as [25]

$$H = \frac{\mathbf{p}^2}{2\mu_1} + \frac{\mathbf{q}^2}{2\mu_2} + V^1(\boldsymbol{\rho}) + V^2\left(\mathbf{r} + \frac{m_2}{m_2+m_3}\boldsymbol{\rho}\right) + V^3\left(\mathbf{r} - \frac{m_3}{m_2+m_3}\boldsymbol{\rho}\right), \quad (16)$$

where $\mathbf{q}$ is the relative momentum of particles 2 and 3 and $\mathbf{p}$ that of particle 1 and the compound system made up of (2,3). These are conjugate momenta to the position vectors,

$$\mathbf{r} = \mathbf{r}_1 - \frac{m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_2+m_3}, \quad \boldsymbol{\rho} = \mathbf{r}_2 - \mathbf{r}_3. \quad (17)$$

In principle, there can be three such sets of coordinates $(\mathbf{r},\boldsymbol{\rho})$ depending on the choice of the subsystems. We now operate H in (16) on the wave function $\Psi(\mathbf{r},\boldsymbol{\rho})$ assuming a separable form for this wave function. As mentioned above, we shall restrict to spherical symmetry (s waves) and hence retain only the radial part of both the $\mathbf{r}$ and $\boldsymbol{\rho}$ coordinates in the above equation. Thus, writing $\Psi(\mathbf{r},\boldsymbol{\rho}) = F(\mathbf{r})G(\boldsymbol{\rho})$, with $F(r) = \chi(r)/r$, $G(\rho) = \Phi(\rho)/\rho$ and using

$$\mathbf{p}^2 = -\hbar^2 \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right),$$
$$\mathbf{q}^2 = -\hbar^2 \frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{d}{d\rho}\right),$$

one obtains in analogy to (4)

$$E\chi(r)\Phi(\rho) + \Phi(\rho)\frac{\hbar^2}{2\mu_1}\frac{d^2\chi(r)}{dr^2} + \chi(r)\frac{\hbar^2}{2\mu_2}\frac{d^2\Phi(\rho)}{d\rho^2} - \tilde{V}\chi(r)\Phi(\rho) = 0, \quad (18)$$

where $\tilde{V}$ is the sum of the three potentials and E the energy eigenvalue of the three-body system. From this point, we continue as in the K-P formalism where the potentials are real and the complex energy is $E = E_R - i\Gamma_R/2$. We now multiply the above equation by $\chi^*(r)\Phi^*(\rho)$, take the complex conjugate of the resulting equation and subtract it from the original one, and then integrate the resulting equation over the radial coordinates r and ρ to obtain

$$-iN_\chi N_\Phi \Gamma_R + N_\Phi \frac{\hbar^2}{2\mu_1}\left[\chi^*\frac{d\chi}{dr} - \chi\frac{d\chi^*}{dr}\right]\Bigg|_{r_\chi} + N_\chi \frac{\hbar^2}{2\mu_2}\left[\Phi^*\frac{d\Phi}{d\rho} - \Phi\frac{d\Phi^*}{d\rho}\right]\Bigg|_{\rho_\Phi} = 0, \quad (19)$$

where $N_\chi = \int_0^{r_\chi} |\chi|^2 dr$ and $N_\Phi = \int_0^{\rho_\Phi} |\Phi|^2 d\rho$. Dividing throughout by $N_\chi N_\Phi/(\hbar i)$ and identifying

$$j_\chi = \frac{\hbar}{2\mu_1 i}\left[\chi^* \frac{d\chi}{dr} - \chi \frac{d\chi^*}{dr}\right]\Bigg|_{r_\chi}$$
$$j_\Phi = \frac{\hbar}{2\mu_2 i}\left[\Phi^* \frac{d\Phi}{d\rho} - \Phi \frac{d\Phi^*}{d\rho}\right]\Bigg|_{\rho_\Phi}, \tag{20}$$

as the quantum mechanical current densities, we get

$$\frac{\Gamma_R}{\hbar} = \frac{j_\chi}{\int_0^{r_\chi} |\chi|^2 dr} + \frac{j_\Phi}{\int_0^{\rho_\Phi} |\Phi|^2 d\rho}. \tag{21}$$

Applying the definition (1) to the right-hand side, the lifetime of the three-body system $\tau^R = \Gamma_R/\hbar$ is thus given in terms of the two-body dwell times as

$$\frac{1}{\tau^R} = \frac{1}{\tau_D^\chi} + \frac{1}{\tau_D^\Phi}, \tag{22}$$

where τ_D^Φ is the time spent by particles 2 and 3 within a spherical region of radius ρ_Φ and τ_D^χ is the time spent by particle 1 and the composite system (2,3) within a sphere of radius r_χ. In the following we shall see that the τ^R derived above is indeed the definition of a three-body dwell time. At this point it is nice to note that such an inverse addition of dwell times was also found in [5] in connection with the reflection, transmission, and average dwell times for a particle tunneling a barrier.

B. Three-body current density and dwell time

Following the standard definition of the dwell time as in (1), one can define the three-body dwell time in terms of a quantum mechanical current density for a three-body system as

$$\tau^{3-b} = \frac{\int_0^{r_\chi}\int_0^{\rho_\Phi} |\Psi|^2 dr d\rho}{j_{3-b}}, \tag{23}$$

where j_{3-b} is the three-body current density and Ψ the three-body wave function which we write in terms of separable wave functions below. Though one would guess j_{3-b} to be a sum over the already defined j_χ and j_Φ, if one tries to derive a continuity equation for the three-body system, one finds that this is indeed not the case. In general, the nonrelativistic definition of a many-body current density is not trivial and has been dealt with using different approaches [26]. Here, we start with an approach (see the appendix) used for an N-body system with wave function $\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\ldots,\mathbf{r}_N)$ and define the current densities in terms of the coordinates $(\mathbf{r},\boldsymbol{\rho})$ instead of $(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$. For the separable wave function $\Psi(r,\rho,t) = \chi(r)\Phi(\rho)f(t)$ (with $l = 0$ only) which satisfies the Schrödinger equation $H\Psi = i\hbar\partial\Psi/\partial t$, we can write

$$\frac{\partial|\Psi|^2}{\partial t} = \frac{1}{i\hbar}(\Psi^* H\Psi - \Psi H\Psi^*)$$
$$= -\frac{\hbar}{2\mu_1 i}|\Phi|^2|f|^2\frac{d}{dr}\left(\chi^*\frac{d\chi}{dr} - \chi\frac{d\chi^*}{dr}\right)$$
$$- \frac{\hbar}{2\mu_2 i}|\chi|^2|f|^2\frac{d}{d\rho}\left(\Phi^*\frac{d\Phi}{d\rho} - \Phi\frac{d\Phi^*}{d\rho}\right) \tag{24}$$
$$= -\frac{\hbar}{\mu_1}|\Phi|^2|f|^2\nabla_r \mathrm{Im}\left(\chi^*\frac{d\chi}{dr}\right)$$
$$- \frac{\hbar}{\mu_2}|\chi|^2|f|^2\nabla_\rho \mathrm{Im}\left(\Phi^*\frac{d\Phi}{d\rho}\right),$$

implying

$$\frac{\partial|\Psi|^2}{\partial t} + \nabla_r J_r + \nabla_\rho J_\rho = 0, \tag{25}$$

where we denote $\partial/\partial r = \nabla_r$ and $\partial/\partial\rho = \nabla_\rho$. With $J_r = |\Phi|^2|f|^2 j_\chi$ and $J_\rho = |\chi|^2|f|^2 j_\Phi$, the above equation has the form $\partial|\Psi|^2/\partial t + \sum_{i=1,2}\nabla_i J_i = 0$ [with (1,2) corresponding to (r,ρ)] and is not a continuity equation of the standard form. Hence, we rather define new current densities, j_r and j_ρ (in analogy to those explained in the appendix), such that

$$j_r = \frac{\hbar}{\mu_1}|f|^2 N_\Phi \mathrm{Im}\left(\chi^*\frac{\partial\chi}{\partial r}\right) = |f|^2 N_\Phi j_\chi,$$
$$j_\rho = \frac{\hbar}{\mu_2}|f|^2 N_\chi \mathrm{Im}\left(\Phi^*\frac{\partial\Phi}{\partial\rho}\right) = |f|^2 N_\chi j_\Phi. \tag{26}$$

It can be easily checked that the above current densities satisfy the individual continuity equations, $\partial n_r/\partial t = -\nabla_r j_r$ and $\partial n_\rho/\partial t = -\nabla_\rho j_\rho$, with $n_r = N_\Phi|\chi|^2|f|^2$ and $n_\rho = N_\chi|\Phi|^2|f|^2$.

By replacing the current density $j_{3-b} = j_r + j_\rho$ in (23) along with the definitions (26) and using the separable form of the wave function as before, it is easy to verify that

$$\frac{1}{\tau^{3-b}} = \frac{j_\chi}{N_\chi} + \frac{j_\Phi}{N_\Phi}$$
$$= \frac{1}{\tau_D^\chi} + \frac{1}{\tau_D^\Phi}. \tag{27}$$

We have thus shown that the lifetime of the resonance τ^R derived previously is the same as τ^{3-b} which can be expressed in terms of the two-body dwell times, τ_D^χ and τ_D^Φ.

V. SUMMARY

There exist several concepts and definitions of quantum tunneling times in literature. However, among these, the dwell-time concept seems to be one of the most important concepts considering the variety of applications it finds in different branches of physics as mentioned in Sec. I. In the present work we make a first attempt to find a relation for the dwell time of a three-body system.

The findings of the present work can be summarized pointwise:

(1) The dwell-time relation derived by Kapur and Peierls is rediscovered and yet another new expression for the dwell time is obtained within their formalism. Within the approach used by Kapur and Peierls, the dwell time is shown to be equal to the phase time and, hence, also free of any singularities near threshold.

(2) Using a similar formalism as that of KP for a three-body resonance, the lifetime of such a resonance is shown to be related to the two-body dwell times of the substructures of the three-body system.

(3) Starting from the standard definition of dwell time taken along with a three-body current density it is shown that the

three-body dwell time is exactly equal to the lifetime found in point (2) and is thus related to the two-body dwell times of the substructures.

Though the approach of the present work relies on simplistic assumptions of spherically symmetric potentials and separable wave functions (which do find applications in certain physical examples), the results obtained are interesting and motivating enough to continue further investigations of the three-body dwell-time concept.

APPENDIX: PROBABILITY AND CURRENT DENSITIES IN MANY-BODY NONRELATIVISTIC QUANTUM MECHANICS

Given the wave function $\Psi(\mathbf{r},t)$ for a quantum system which satisfies the one-body Schrödinger equation $H\Psi = i\hbar\partial\Psi/\partial t$, with $H = -(\hbar^2/2m)\nabla^2 + V(\mathbf{r})$, one can derive the continuity equation,

$$\frac{\partial|\Psi|^2}{\partial t} = -\nabla\cdot\mathbf{J}, \tag{A1}$$

where $\mathbf{J}$ is the current density given by $\mathbf{J} = (\hbar/m)\mathrm{Im}(\Psi^*\nabla\Psi)$. Consider now a many-body system consisting of N particles and described by the wave function $\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4,\ldots,\mathbf{r}_N,t)$ which satisfies the Schrödinger equation $H\Psi = i\hbar\partial\Psi/\partial t$, however, with

$$H = -\frac{\hbar^2}{2}\sum_{i=1}^{N}\frac{1}{m_i}\nabla_i^2 + V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4,\ldots,\mathbf{r}_N). \tag{A2}$$

Starting in the standard way, with

$$\frac{\partial|\Psi|^2}{\partial t} = \frac{1}{i\hbar}(\Psi^* H\Psi - \Psi H\Psi^*), \tag{A3}$$

and using the Hamiltonian in (A2), one obtains

$$\frac{\partial|\Psi|^2}{\partial t} = -\hbar\sum_{i=1}^{N}\frac{1}{m_i}\nabla_i\cdot\mathrm{Im}(\Psi^*\nabla_i\Psi) = -\sum_{i=1}^{N}\nabla_i\cdot\mathbf{J}_i, \tag{A4}$$

where we have defined the current density for particle i as $\mathbf{J}_i(\mathbf{r}_i,t) = (\hbar/m_i)\mathrm{Im}(\Psi^*\nabla_i\Psi)$. Equation (A4) is obviously not of the same form as (A1) with $\mathbf{J} = \sum_i \mathbf{J}_i$. Hence, one defines [26] the number and current density rather for particle 1 as

$$\begin{aligned} n_1(\mathbf{r},t) &= \int d\mathbf{r}_2 d\mathbf{r}_3 \ldots d\mathbf{r}_N |\Psi|^2, \\ \mathbf{j}_1 &= \frac{\hbar}{m_1}\int d\mathbf{r}_2 d\mathbf{r}_3 \ldots d\mathbf{r}_N \mathrm{Im}(\Psi^*\nabla_1\Psi), \end{aligned} \tag{A5}$$

and so on for other particles, too. It can be easily checked [26] that particle 1 satisfies the continuity equation $\partial n_1/\partial t = \nabla_1\cdot\mathbf{j}_1$ and so does every particle in the system of N particles.

In analogy to the above procedure, we define the current densities for the three-body system. However, instead of the position coordinates $(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ of the three particles, we use the coordinates $\mathbf{r}$ and $\boldsymbol{\rho}$ described in the text. Thus, we define the two current densities:

$$\begin{aligned} j_r &= \frac{\hbar}{\mu_1}\int d\boldsymbol{\rho}\ \mathrm{Im}[\Psi^*(\mathbf{r},\boldsymbol{\rho},t)\nabla_r\Psi(\mathbf{r},\boldsymbol{\rho},t)], \\ j_\rho &= \frac{\hbar}{\mu_2}\int d\mathbf{r}\ \mathrm{Im}[\Psi^*(\mathbf{r},\boldsymbol{\rho},t)\nabla_\rho\Psi(\mathbf{r},\boldsymbol{\rho},t)], \end{aligned} \tag{A6}$$

which reduce to a simpler form in case of a separable wave function as used in the text.

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