

Signatures of the superfluid-insulator phase transition in laser-driven dissipative nonlinear cavity arrays

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We analyze the nonequilibrium dynamics of a gas of interacting photons in an array of coupled dissipative nonlinear cavities when driven by a pulsed external coherent field. Using a mean-field approach, we show that the response of the system is strongly sensitive to the underlying (equilibrium) quantum phase transition from a Mott insulator to a superfluid state at commensurate filling. We find that the coherence of the cavity emission after a quantum quench can be used to determine the phase diagram of an optical many-body system even in the presence of dissipation.

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Recent theoretical advances in cavity quantum electrodynamics (QED) have indicated that arrays of coupled nonlinear cavities may be candidates for exploring quantum many-body phenomena of light [1]. Initial proposals for realizing a Mott phase of polaritons [2] have been scrutinized in great detail [3], and different schemes have been put forward to simulate a variety of correlated quantum states [4–7]. Given the importance of dissipation in state-of-the-art solid-state (single) cavity QED devices (see, for example, [8]), experiments in cavity QED arrays are expected to be performed under strongly nonequilibrium conditions, with an external source compensating for the loss of photons (see Fig. 1). These conditions, dictated by the experimental constraints, put forward QED arrays as natural candidates for exploring the rich world of nonequilibrium quantum many-body systems [10–12], which is the subject of recent interest in the context of ultracold atoms as well [13,14]. At the same time, new important questions arise related to whether it is possible to realize and detect, under realistic nonequilibrium conditions, the very rich phase diagram [1] predicted for QED arrays at equilibrium.

In this Rapid Communication, we propose an answer to this last question, thus providing the missing link between the initial ideas of cavity arrays as quantum simulators and future experimental realizations. Specifically, we show how to detect the superfluid-insulator phase transition for a dissipative nonlinear cavity array driven by an external pulsed laser. In analogy to what happens in a quantum quench [15,16], photons are excited in the cavities by a periodic train of short, coherent pulses and then evolve according to the complex many-body dynamics determined by the simultaneous action of hopping, strong interactions, and losses. The properties of the secondary emission are measured in the time lapse between subsequent pulses: even though the losses drive the system toward the vacuum state, distinct signatures of the quantum phase transition are found in the statistics of the transient emitted light. First- and second-order photon correlations

are shown to be powerful tools to detect the existence of a nonzero order parameter as they are very sensitive to the delocalization of photons through the array, although more complete measurement schemes exist [17].

The time evolution of the array in the presence of driving and losses is described by the master equation

$$\partial_t \rho(t) = -i[\hat{\mathcal{H}}, \rho] + \mathcal{L}[\rho], \quad (1)$$

for the density matrix ρ of the system. The first term on the right-hand side of Eq. (1) accounts for the unitary evolution of the system, while the second term accounts for the damping. As discussed in [1], an array of coupled cavities in the absence of losses can realize an effective Bose-Hubbard model whose Hamiltonian reads

$$\begin{aligned} \hat{\mathcal{H}} = & -J \sum_{\langle \ell \ell' \rangle} \hat{a}_\ell^\dagger \hat{a}_{\ell'} + \frac{U}{2} \sum_{\ell} \hat{n}_\ell (\hat{n}_\ell - 1) \\ & + \Delta \sum_{\ell} \hat{n}_\ell + \sum_{\ell} [\varepsilon(t) \hat{a}_\ell + \varepsilon(t)^* \hat{a}_\ell^\dagger], \end{aligned} \quad (2)$$

with \hat{a}_ℓ , \hat{a}_ℓ^\dagger , and \hat{n}_ℓ denoting, respectively, the annihilation, creation, and number operator of the electromagnetic mode of the ℓ th cavity. The first term of Eq. (2), quantified by the coupling constant J , describes the hopping of photons between neighboring cavities (the lattice has a coordination number z). The second term, quantified by U , represents the effective nonlinearity arising from the specific light-matter interaction mechanism. The achievable values of J and U depend on the specific implementation of the model [1]. The last two terms in Eq. (2) describe the coupling of the photons in the cavities with a uniform coherent pump in the rotating frame—here $\varepsilon(t)$ is the (slowly varying) envelope of the external driving field and Δ is the detuning of the laser frequency from the bare cavity resonance. The dominant source of dissipation in the system is the leakage of photons out of the cavities. In the Markov approximation, the decay process is described by a Liouvillian in the Lindblad

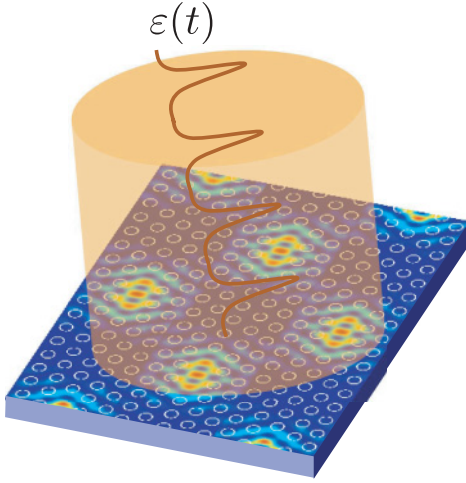


FIG. 1. (Color online) Schematic representation of a possible realization of the proposed system, employing a two-dimensional array of photonic crystal defect cavities [9]. Photons in the cavities have a finite lifetime, which requires losses to be compensated by some external pump, in our case a coherent pulsed laser field. Nonlinear processes in each cavity lead to an effective Bose-Hubbard repulsion between the cavity photons [1]. Neighboring cavities are tunnel coupled to each other by spatial overlap of the cavity mode profiles.

form $\mathcal{L}[\rho] = \kappa \sum_{\ell} (2\hat{a}_{\ell} \rho \hat{a}_{\ell}^{\dagger} - \hat{n}_{\ell} \rho - \rho \hat{n}_{\ell})$, where $(2\kappa)^{-1}$ is the photon lifetime.

Solving Eq. (1) is a formidable task, as it describes an open quantum many-body system out of equilibrium. We thus resort to a self-consistent cluster mean-field approach, which should give reliable results if the array is (at least) two-dimensional. The cluster mean-field (for its equilibrium version, see, for example, [18]) is based on approximating the Hamiltonian (2) with that of a cluster of cavities that interact with the rest of the lattice via a mean-field term, that is, $\hat{\mathcal{H}}_{\text{MF}} = \sum_{\ell, \ell' \in \Omega_c} \hat{\mathcal{H}}_{\ell, \ell'} - J \sum_{(\ell', \ell), \ell \in \Omega_b, \ell' \notin \Omega_c} (\hat{a}_{\ell}^{\dagger} \psi_{\ell'} + \text{H.c.})$. Here, $\hat{\mathcal{H}}_{\ell, \ell'}$ is the Hamiltonian (2) restricted to the sites within the cluster (Ω_c). The second term instead is the mean-field expression for the hopping from the cavities ℓ in the cluster boundary (Ω_b) to their nearest neighbors ℓ' outside the cluster. It is a function of the time-dependent mean-field order parameter $\psi_{\ell}(t)$, which, for the ℓ th cavity, is determined by the self-consistency condition $\psi_{\ell}(t) = \text{Tr}[\hat{a}_{\ell} \rho(t)]$. In the remainder of the Rapid Communication, we consider only the two cases in which the cluster consists of one or two sites. For the numerical integration of the master equation, we use a fourth-order Runge-Kutta algorithm and truncate the local Fock basis $\{|n\rangle_{\ell}\}_{n=0}^{n_{\text{max}}}$ for the ℓ th cavity, with an upper cutoff $n_{\text{max}} \simeq 20$.

In order to simplify the presentation, we first consider an idealized case in which the laser pulse sets, at $t = 0$, the cavity array in a Fock state $\rho_{\text{in}} = \bigotimes_{\ell} |1\rangle_{\ell} \langle 1|_{\ell}$ with one photon per cavity. We discuss the precise shape of the required pulses and the robustness of the response of the system to imperfections in the pulse shape further. We first focus on single-site mean field and analyze the coherence properties of the light emitted from the cavity array after each pulse. The time-dependence of the photon population $n(t)$ of each cavity is trivial, and

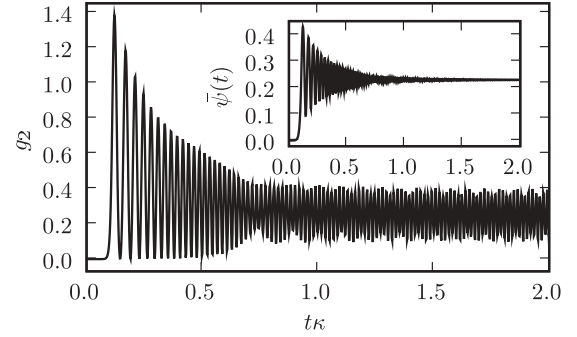


FIG. 2. Time evolution of the zero-time-delay second-order correlation function and of the rescaled order parameter (inset), after a pulse that projects the cavities into a Fock state with one photon per cavity. The rescaled order parameter $\bar{\psi}(t) = |\psi(t)|/\sqrt{n(t)}$ compensates for the decay of the photon number in the cavities and is related to the coherent fraction of the emission ($\kappa = 10^{-2} U$, $zJ = 3.0 U$, $\Delta = 0$).

the corresponding equation of motion can be integrated, yielding an exponential decay $n(t) = n(0)e^{-2\kappa t}$. Much more interesting are the properties of the order parameter $\psi_{\ell}(t)$ and the zero-time-delay second-order correlation function, $g_2 = \langle \hat{n}_{\ell}^2 - \hat{n}_{\ell} \rangle / \langle \hat{n}_{\ell} \rangle^2$, for $t > 0$, that is, after the laser pulse is switched off. If $J/U < (J/U)_c$ [$(J/U)_c$ is the value at which the transition from the superfluid to the Mott state occurs], any initial fluctuation in the order parameter does not grow in time, and the density matrix in photon-number representation is essentially diagonal throughout the relaxation process. In the opposite case, $J/U > (J/U)_c$, a superfluid instability can develop and lead to nonlinear oscillations of both $|\psi(t)|$ and g_2 . This kind of instability has been discussed, in the absence of dissipation, in [15]. Here, we show that the instability is present also in the presence of losses. It can be generated by means of pulsed laser and detected by measuring the properties of the light emitted from the cavities. In contrast, within our mean-field analysis, continuous shining of coherent light on the cavities would wash out the effects of the instability.

For the open system we are considering, a generic example of the instability induced by the pulsed pump is shown in Fig. 2, where we show the evolution of $g_2(t)$ and (normalized) $|\psi(t)|$ calculated for $t > 0$. The dynamics is characterized by three different regimes. At short times, a linear instability sets in and both quantities increase exponentially, notwithstanding the slow decay of the photon population. At intermediate times, the collective nonlinear dynamics of the array leads to oscillations. In the long time limit, these oscillations damp out with a time constant of the order of κ^{-1} [in this regime, when $n(t) \ll 1$, it can be shown that $|\psi(t)| \sim \sqrt{n(t)}$].

An experimentally measurable observable that yields a clear signature of the different regimes in the initial transient is the zero-time-delay second-order correlation function averaged over a certain interval of time. To calculate such time-averaged observables, we integrate the equation of motion, Eq. (1), up to the time $2.0\kappa^{-1}$ and determine the time-averaged values $\langle |\psi(t)| \rangle_t$ and $\langle g_2 \rangle_t$ in the time interval $1.0\kappa^{-1} < t < 2.0\kappa^{-1}$. Considering time-integrated quantities allows us to relax the experimental requirements on the time resolution of the

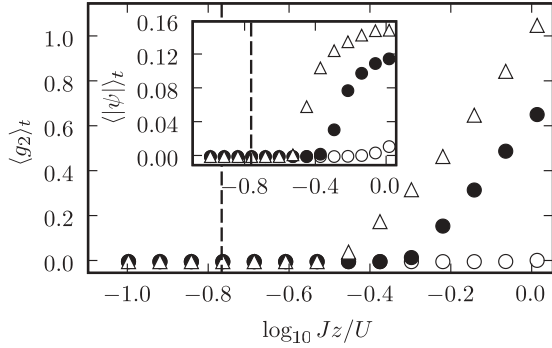


FIG. 3. Time average of $g_2(t)$ (main panel) and integral of $|\psi(t)|$ (inset), in the time interval $1.0\kappa^{-1} < t < 2.0\kappa^{-1}$ for different values of the damping: $\kappa = 2.0 \times 10^{-2}U$ (empty circles), $1.0 \times 10^{-2}U$ (filled circles), and $0.5 \times 10^{-2}U$ (empty triangles); $\Delta = 0$. The vertical dashed line marks the critical value of the Mott-to-superfluid transition for the equilibrium Bose-Hubbard model $[(zJ/U)_c \sim 0.17]$.

measurements, which can hardly exceed κ^{-1} in realistic state-of-the-art devices.

In Fig. 3, we show $\langle |\psi(t)| \rangle_t$ and $\langle g_2 \rangle_t$ as a function of the ratio J/U , for different values of the dissipation rate. Both quantities vanish if J/U is smaller than the critical value, meaning that the order parameter does not develop any instability, and consequently the light emitted from the cavities is strongly antibunched. In the opposite case, both $\langle |\psi(t)| \rangle_t$ and $\langle g_2 \rangle_t$ are different from zero. The time-averaged g_2 as a function of J/U monotonically rises from zero to almost unity, thus showing a crossover from antibunched to Poissonian statistics. Integrating the equation of motion for times larger than $2.0\kappa^{-1}$ leaves the features depicted in Fig. 3 qualitatively unaltered. We emphasize that the crossover of both $\langle |\psi(t)| \rangle_t$ and $\langle g_2 \rangle_t$ takes place in a very narrow range of J/U values around the transition point of the closed system. The sensitivity of the light statistics to the coupling between neighboring cavities is a signature of the many-body origin of this phenomenon. The second-order correlation is thus an excellent candidate to detect the different quantum phases in cavity arrays.

A linear stability analysis, valid for $tU, t\kappa \ll 1$, shows that the rate of the initial exponential buildup of the order parameter is reduced by a quantity κ at fixed J/U ; for large enough κ , the superfluid instability is then entirely suppressed. As a consequence, the critical ratio $(J/U)_c$ increases by $\simeq \kappa^2/(2U^2)$. Such trend is already visible for $U/\kappa \simeq 10^2$. We note that this suppression of the superfluid phase is not a consequence of the coupling to a bath [13]. It is important to note that $U/\kappa \simeq 10^2$ is experimentally achievable in a solid-state cavity with a line width of $\simeq 10 \mu\text{eV}$ (corresponding to $Q \simeq 10^5$ at optical or near-infrared frequencies) and interaction strength $\sim 1 \text{ meV}$, which are within reach in current solid-state cavity QED systems [19]. In the case of coupled photonic crystal cavities (see Fig. 1), the range of parameters that one can expect (assuming $U = 0.1 \text{ meV}$) is $0 < J/U < 50$ [20]. Alternatively, circuit QED also provides an ideal experimental realization for the present proposal [21].

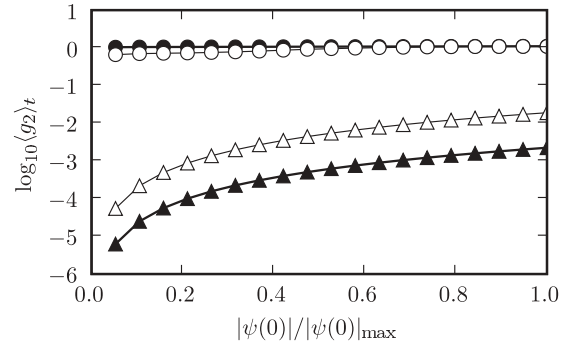


FIG. 4. Time average of $g_2(t)$, for $\kappa = 10^{-2}U$. The population ρ_0 of the vacuum in the initial state is $\rho_0 = 0.02$ for the filled symbols and $\rho_0 = 0.2$ for the empty symbols [$zJ/U = 0.1$ (triangles) and $zJ/U = 1.0$ (circles), below and above the critical value; $\Delta = 0$].

All the results presented here are robust even when some of the assumptions made so far are relaxed. In the vicinity of the critical point, the order parameter evolves on time scales which are much longer than the cavity parameters [15]. In contrast, the correlation function might show a more complex short-distance evolution, which could invalidate the analysis presented previously. In order to check this possibility, we carried out the same analysis in the two-site mean-field approximation, where both the dynamics of the order parameter and the short time-scale dynamics governed by J and U are present. All the results shown so far are fully confirmed. In addition, we checked that the uniform solution for the order parameter is stable against possible spatial inhomogeneities of the initial state.

We also checked that our findings do not depend on the exact form of the state after the pulsed excitation. We considered an initial state that is not necessarily pure but that entirely projects the system onto the subspace spanned only by the two Fock states $|0\rangle_\ell$ and $|1\rangle_\ell$. We considered depletion of the average filling up to 20% and scanned the whole range of initial coherences. For all the possible initial states, $\langle g_2 \rangle_t$ in the superfluid regime was always markedly different (at least three orders of magnitude) from the insulating one, as shown in Fig. 4. This insensitivity to the initial conditions in the superfluid regime is due to the nonlinearity of the time evolution for times shorter than the photon lifetime. In fact, in the insulating regime, the initial correlations are suppressed due to the photon blockade, while in the superfluid region, even the absence of any initial correlation is quickly compensated by the cooperative action of the photons. We remark that while $\langle g_2 \rangle_t$ predicts strongly antibunched cavity emission in the insulating regime for all considered variations in the initial states, the actual degree of antibunching does depend on the initial state coherence; at present, we do not understand this feature.

We now discuss pulse shapes that project the coupled cavity array into a desired initial state (such as ρ_{in} , discussed previously). A π pulse applied to an isolated cavity drives the vacuum state into the desired initial state, provided that the duration of the pulse is shorter than the photon lifetime κ^{-1} . To design a suitable pulse shape for the whole array, we resorted to a rapidly converging quantum optimal-control

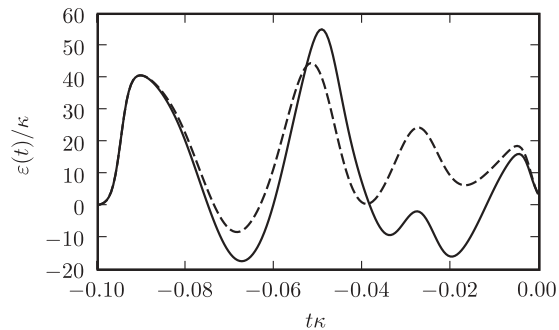


FIG. 5. Optimized pulse envelopes for the preparation of a Fock state with one photon per cavity, with a fidelity $\mathcal{F} \simeq 99\%$. The dashed line is the pulse $\varepsilon^{(0)}(t)$ for $J = 0$. The solid line is the pulse $\varepsilon^{(1)}(t)$, detuned by $\Delta = 43.25 \kappa$, with $zJ/U = 0.5$. The duration $T_p = 0.1/\kappa$ of the pulse is much shorter than the photon lifetime ($\kappa = 10^{-2} U$ in the plot).

algorithm [22]: first, we found an optimized envelope $\varepsilon^{(0)}(t)$ (see Fig. 5, dashed line) for a single cavity ($J = 0$). As dissipation does not play a substantial role during the application of the pulse in the time interval $-T_p < t < 0$, we use an algorithm of quantum optimal control for pure states, obtaining a fidelity $\mathcal{F} \simeq 99\%$ with respect to the desired state ρ_{in} . In the case

of an array of coupled cavities, $J \geq 0$ substantially modifies the response of the system to the pumping field, and it is necessary to devise a different shape $\varepsilon^{(1)}(t)$ of the pulse for each value of the hopping amplitude (see Fig. 5, solid line). It is nevertheless possible to prepare the initial state with high fidelity (99% in Fig. 5) in a substantial range of values up to $zJ \lesssim U$.

In conclusion, we have shown that the coherence properties of the secondary cavity emission, induced by a pulsed excitation, provide signatures of the collective many-body phase of the array of nonlinear cavities. Our analysis fully takes into account the intrinsically driven dissipative nature of the strongly correlated quantum many-body system and identifies how dissipation influences the underlying equilibrium quantum phase transition that this system exhibits in the absence of losses. Coupled cavities are promising candidates to simulate open quantum many-body systems, and a study of their nonequilibrium behavior will certainly unveil a variety of new phenomena.

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