Quantum tunneling time of a Bose-Einstein condensate traversing through a laser-induced potential barrier

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We theoretically study the effect of atomic nonlinearity on the tunneling time in the case of an atomic Bose-Einstein condensate (BEC) traversing the laser-induced potential barrier. The atomic nonlinearity is controlled to appear only in the region of the barrier by employing the Feshbach resonance technique to tune interatomic interaction in the tunneling process. Numerical simulation shows that the atomic nonlinear effect dramatically changes the tunneling behavior of the BEC matter wave packet and results in the violation of the Hartman effect and the occurrence of negative tunneling time.

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Quantum tunneling of a wave packet through a potential barrier is one of the fundamental topics in quantum physics [1–3]. The issue of tunneling time has attracted a lot of attention for decades since it was first put forward by Condon [4]. In the early 1960s, Hartman predicted that tunneling time becomes independent of barrier length for thick-enough barriers, ultimately resulting in unbounded tunneling velocities [5]. Such a phenomenon, termed the Hartman effect later on, seems to imply the superluminal velocities inside the barriers and leads to a wide interest in many different fields [6–9].

Mathematically, quantum tunneling is governed by the Schrödinger equation, which is a linear equation describing the quantum wave nature of a single particle. So far the Hartman effect or related topics studied in the literature are limited to the single-particle linear case. After the mid-1990s, there are significant advancements in the realization of atomic Bose-Einstein condensates (BECs), a macroscopic quantum-mechanical wave packet with nonlinear behavior due to interatomic interactions. Such a macroscopic coherent-matter wave packet of BEC opens a new window for studying the nonlinear quantum dynamics governed by the nonlinear Schrödinger equation or the mean field Gross-Pitaevskii (GP) equation [10]. No doubt, the tunneling of a BEC wave packet would exhibit different behaviors compared with the single-particle tunneling in linear quantum mechanics. In fact, there already exist in the literature many studies on BEC tunneling through different kinds of potentials [11–13].

In this article, we theoretically investigate how the interatomic interaction affects the tunneling time in the case of a coherent BEC wave packet traversing through a potential barrier created by a laser beam. We consider a BEC wave packet confined in a quasi-one-dimensional atomic waveguide to traverse a potential barrier created by a far blue-detuned laser beam, which is shown in Fig. 1. Suppose that the quasi-one-dimensional atomic waveguide is a transverse trapping harmonic potential, and the dynamics of atomic BEC can be described by the three-dimensional nonlinear Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[-\frac{\hbar^2\nabla^2}{2m} + V(x) + \frac{1}{2}m\omega_\perp r_\perp^2 + gN|\Psi|^2\right]\Psi, \tag{1}$$

where Ψ is the normalized macroscopic wave function of BECs, $r_\perp = (y,z)$, m the atomic mass, and $\omega_\perp$ the trapping frequency in the radial (transverse) direction. $g = 4\pi\hbar^2 a_s/m$ describes the interatomic interaction, with a_s being the atomic s-wave scattering length. N is the total number of atoms in the condensate, and $V(x)$ is the potential barrier created by the blue-detuned laser beam.

When transverse confinement is very strong, the transverse motion of BEC atoms may be considered to remain in the ground state. As a result, we can approximate the total wave function of the BEC as

$$\Psi(x,r,t) \approx \psi(x,t)\varphi_\perp(r)e^{-i\omega_\perp t}, \tag{2}$$

where $\varphi_\perp(r)$ is the transverse ground-state wave function of the BEC, which can approximately be replaced by a Gaussian function in the weak nonlinear limit

$$\varphi_\perp(r) = \frac{1}{\sqrt{\pi}a_\perp}\exp\left(-\frac{r^2}{2a_\perp^2}\right), \tag{3}$$

with $a_\perp = \sqrt{\hbar/m\omega_\perp}$ being the ground-state length of harmonic trapping potential. The longitudinal wave function $\psi(x,t)$ is governed by the effective one-dimensional nonlinear Schrödinger equation:

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\psi + V(x)\psi + g|\Psi|^2\psi. \tag{4}$$

In Eq. (4), we express x in units of $a_\perp$, t in units of $\omega_\perp^{-1}$, the dimensionless nonlinear interaction $g = 2a_s N/a_\perp$, and the dimensionless wave function $\psi = \psi/\sqrt{a_\perp}$. $V(x)$ is the potential barrier experienced by the BEC wave packet and written in the form

$$V(x) = V_0 f(x), \tag{5}$$

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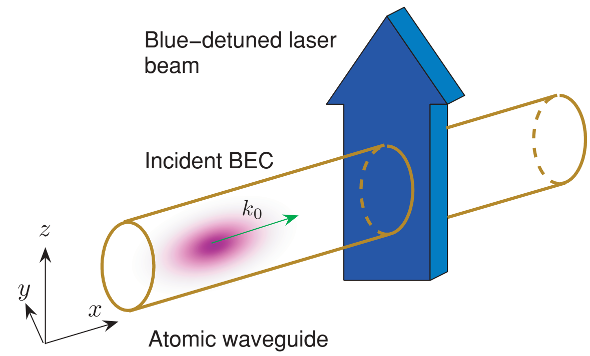

FIG. 1. (Color online) Schematic diagram of a BEC wave packet traversing a blue-detuning laser beam in an atomic waveguide.

where V_0 is the peak value of the potential normalized by $\hbar\omega_\perp$ and $f(x)$ is the barrier profile, which is controlled by the laser beam.

Now we have obtained a general dimensionless equation (4) to describe the BEC wave packet transmitting through a barrier. As pointed out in Ref. [14], Eq. (4) is invariant under the scaling transformation $\{x,t,\psi,V_0\} \rightarrow \{x\eta,t\eta^2,\psi/\eta,V_0/\eta^2\}$, where η is a dimensionless constant. To cover different experimental situations, it is convenient to introduce such scaling transformation in numerical simulation. In this article, we set $\omega_\perp \approx 2\pi \times 100$ Hz and $\eta = 10$, then for ^{87}Rb atoms, $a_\perp \approx 1\ \mu$m. In principle, Eq. (4) can describe a BEC wave packet transmitting through an arbitrary barrier. To simplify the problem, without loss of physical feature, in this article we just consider a rectangular barrier case. Such a rectangular barrier can be approximated by a super-Gaussian laser beam with a large-enough order [15–17]. Therefore, the barrier profile $f(x)$ has the form

$$f(x) = \begin{cases} 1, & -\frac{L}{2} < x < \frac{L}{2}, \\ 0, & x < -\frac{L}{2} \text{ or } x > \frac{L}{2}. \end{cases} \tag{6}$$

In the following, we simulate finite wave packets traversing through the rectangular barrier via the split operator method [18]. Assume the normalized initial wave packet is Gaussian,

$$\psi(x,0) = \frac{1}{\sqrt{\sqrt{\pi}\Delta x}} \exp\left[-\frac{(x-x_0)^2}{2\Delta x^2} + ik_0(x-x_0)\right], \tag{7}$$

where x_0, Δx, and k_0 are the initial center position, the initial half width, and the initial center momentum of the wave packet, respectively. From Eq. (4) we can find that the interatomic interaction will induce the self-phase-modulation (SPM) [19,20] of a matter wave packet. The SPM occurs in both the free region and the potential and causes confusion of the nonlinear effect on the tunneling process inside the barrier with the SPM process outside the barrier region. To avoid this confusion we shall eliminate the nonlinear interaction outside of the potential region, that is, make the s-wave scattering length a_s vanish outside of the potential region. In principle, this can be realized by employing the Feshbach resonance technique to tune the atomic scattering length via a spatially varying magnetic field [21]. For a spatially varying magnetic field, the scattering length $a_s(x) = a_{s0}\{1 - \Delta B/[B(x) - B_0]\}$, where a_{s0} is the background scattering length, ΔB the resonance width, B_0 the magnetic field of resonance, and $B(x)$ the magnetic field. In our work, we set the profile of the scattering length to be consistent with that of the barrier, namely, $g(x) = g_0 f(x)$. Considering the finite width of initial atomic BEC wave packet used in experiment, we set $\Delta x = 50$ in all of the following simulations, corresponding to a half width 500 μm for a BEC wave packet composed of ^{87}Rb atoms.

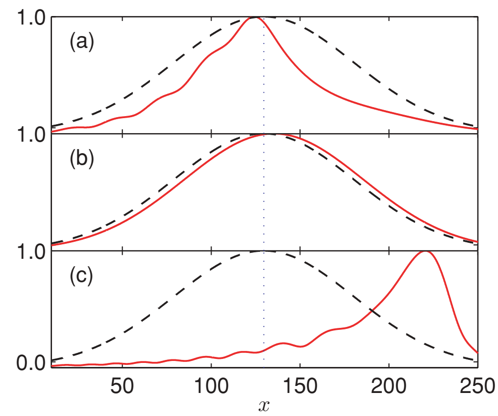

FIG. 2. (Color online) The normalized transmitted wave packets (solid curve) at time $t = 440$ with nonlinear interaction: (a) $g_0 = -16$, (b) $g_0 = 0$, and (c) $g_0 = 200$. Other parameters are $V_0 = 1$, $L = 12$, and $k_0 = 1.2$. The freely propagating reference wave packet is represented by the dashed curves.

With the preceding assumptions, we now investigate the tunneling of a BEC wave packet through the barrier with atomic nonlinearity. In Fig. 2 we present the transmitted wave packets with different nonlinear interaction at time $t = 440$, at which the transmitted wave packet just emerges from the barrier. Meanwhile, we also show the freely propagating wave packet as a reference, for which both the barrier and the nonlinear interaction are set to zero. From the top panel, one observes that the transmitted wave packet with negative nonlinearity slightly lags behind the reference one. The middle panel is the linear case, where the nonlinear interaction is neglected. In contrast to the negative nonlinear case, the transmitted wave packet is ahead of the reference one, due to the finite-width effect of the incident wave packet, as has been pointed out in previous works [22–24]. In the bottom panel, we find the transmitted wave packet with positive nonlinearity is far ahead of the reference, which is quite different from the other two cases. Figure 2 shows us that the nonlinear interaction indeed, as expected, affects the tunneling process of a BEC wave packet through the barrier. In what follows, we explain the physics behind the numerical results in detail.

We consider the transmitted spectra in the momentum space. Due to the SPM-induced spectral broadening, as well as the filtering of the barrier [3,25], the center-transmitted momenta exhibit great differences in the linear, positive, and negative nonlinear cases, as shown in Fig. 3. The filtering of the barrier takes effect on both the linear and the nonlinear cases, while the spectral nonlinear broadening only occurs in the nonlinear cases. The combination of the filtering effect and nonlinear broadening leads to a large transmitted central momentum in the positive nonlinear case. Therefore, with the same barrier parameters and incident wave number, the

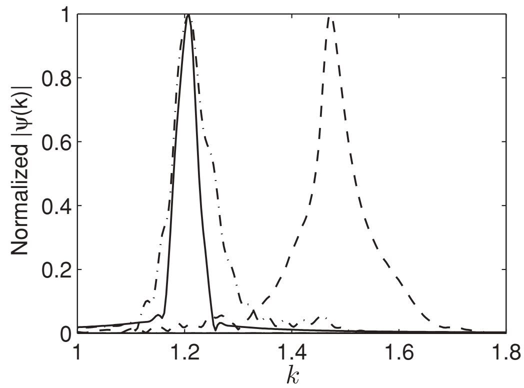

FIG. 3. The corresponding momentum spectra of transmitted wave packets in Fig. 2. The solid curve stands for the case without nonlinearity and the dashed and dash-dotted curves for the cases with positive and negative nonlinearity, respectively.

positive nonlinear case spends the least time in the barrier. However, we note that the transmitted central momenta of negative nonlinear and linear cases are less than that in the positive nonlinear case. This can also be well understood from the discussion in what follows. In fact, when the interatomic interaction exists, the atomic nonlinearity modifies the potential barrier and the atoms "see" an effective potential,

$$V_{\rm eff} = V_0 f(x) + g(x)|\psi(x,t)|^2. \qquad (8)$$

For a repulsive interaction, the nonlinear term has a positive sign. Consequently, the height of the linear barrier is raised, and vice versa for an attractive interaction. In terms of Hartman's calculation [5], the wave packet traverses faster through a higher barrier than a lower one with the same and wide-enough barrier width. Therefore, the transmitted wave packet with negative or positive nonlinearity spends more or less time than the linear one in the barrier.

Now we turn our attention to the quantitative calculation for the tunneling time of the BEC wave packet through a laser-induced barrier. For a finite wave packet transmitting through a barrier, we cannot use the usual stationary phase method to find the transmission time of the transmitted wave packet because the condition for the stationary phase method is not satisfied in this situation. However, the widely used time-of-flight method is applicable for measuring the transmission time of the transmitted wave packet [26]. The method is described as follows. Assume that the incident wave packet is placed at position $x(0) = -x_0$ $(x_0 > 0)$ at time $t = 0$, somewhere to the left of the barrier, and let it move to the right with initial momentum k_0. After transmitting through the barrier, the position of the transmitted wave packet is located in $x(t_T)$ at time $t = t_T$. Due to the effect of the barrier and nonlinear interaction, the transmitted wave packet is usually deformed. In this case, the appropriate way to describe the center of the transmitted wave packet is to define the expected position of the transmitted wave packet as $x(t_T) = \int_{x>0} x|\psi(x,t)|^2 dx / \int_{x>0} |\psi(x,t)|^2 dx$ [27]. In this way, one gets the tunneling time

$$\Delta t = t_T - \frac{x(0) - L/2}{k_0} - \frac{x(t_T) - L/2}{\bar{k}_0}, \qquad (9)$$

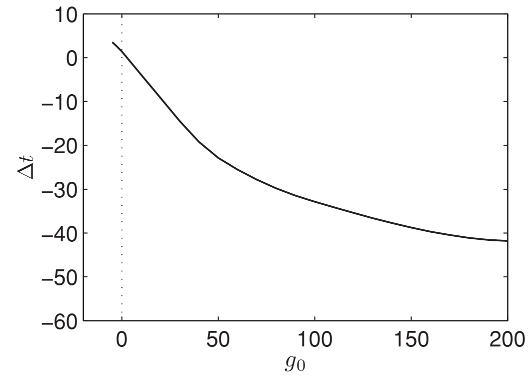

FIG. 4. Tunneling time as a function of nonlinear interaction strength. Other parameters are $k_0 = 0.6$, $V_0 = 1$, and $L = 6$.

where $\bar{k}_0$ is the center transmitted momentum, defined as

$$\bar{k}_0 = \int k|\psi_T(k)|^2 dk, \qquad (10)$$

with $\psi_T(k)$ the momentum distribution of the transmitted wave packet.

First, we study the dependence of tunneling time on the nonlinear interaction. Figure 4 plots the behavior of the tunneling time via nonlinear interaction strength. Obviously, the tunneling time decreases as the nonlinear interaction strength increases. This means that the stronger the repulsive interatomic interaction is, the faster the BEC wave packet traverses through the barrier. It is interesting to note that the tunneling time is negative for large positive nonlinearity. The negative tunneling time implies that the transmitted wave packet exits the barrier just before the incident wave packet arrives at the barrier. Such a phenomenon has been found and discussed in the literature for many situations [28,29]. Here the negative tunneling time is due to the repulsive interatomic interaction and is completely a nonlinear quantum mechanical phenomenon for an atomic Bose-Einstein condensate.

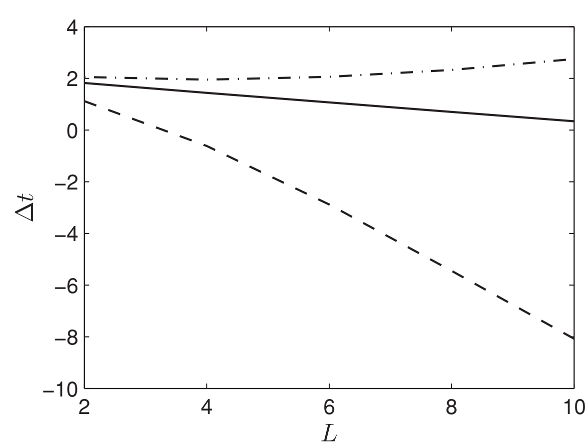

FIG. 5. Tunneling time as a function of barrier width with nonlinear strength $g_0 = 0$ (solid line), $g_0 = 5$ (dashed line), and $g_0 = -2$ (dash-dotted line). Other parameters are $k_0 = 0.6$ and $V_0 = 1$.

Now, we examine the dependence of tunneling time on the barrier width as shown in Fig. 5. We find that tunneling time in linear and positive nonlinear cases decreases, but it increases in the negative nonlinear case with increasing barrier width. These results imply that the Hartman effect is violated in both nonlinear and linear cases. For the linear case, such a violation is due to the finite width of the incident wave packet. This has been touched on in many theoretical and experimental works [22–24].

In summary, we have numerically studied the effect of nonlinear interatomic interaction on the quantum tunneling time of a BEC matter wave packet through a laser-induced barrier. Analysis shows that both the sign and the strength of nonlinear interaction significantly affect the tunneling time. As a result, the so-called the Hartman effect in linear quantum mechanics could be violated in nonlinear quantum mechanics with a macroscopic matter wave packet of a Bose-Einstein condensate.

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