Electromagnetically induced transparency and Autler-Townes splitting: Two similar but distinct phenomena in two categories of three-level atomic systems

Tony Y. Abi-Salloum

Physics and Astronomy Department, Widener University, Chester, Pennsylvania 19013, USA (Received 2 December 2008; published 19 May 2010)

Electromagnetically induced transparency (EIT) and Autler-Townes (AT) splitting are two phenomena that could be featured in a variety of three-level atomic systems. The considered phenomena, EIT and AT, are similar "looking" in the sense that they are both characterized by a reduction in absorption of a weak field in the presence of a stronger field. In this paper, we explicitly set the threshold of separation between EIT and AT splitting in a unified study of four different three-level atomic systems. Two resonances are studied and compared in each case. A comparison of the magnitudes of the resonances reveals two coupling-field regimes and two categories of three-level system.

DOI: 10.1103/PhysRevA.81.053836 PACS number(s): 42.50.Gy

I. INTRODUCTION

Since the emergence of electromagnetically induced transparency (EIT) in 1990 [1], the phenomenon has been associated with destructive interference between two excitation pathways [2]. In order to briefly introduce EIT along with its corresponding assumptions and understanding, we review first the four three-level atomic systems Λ (Lambda) [Fig. 1(a)] [3], cascade-EIT [Fig. 1(b)] [4], cascade-AT (Autler-Townes) [Fig. 1(c)] [5], and V ("vee") [Fig. 1(d)] [6] that have always been assumed to feature EIT. The names cascade-EIT and cascade-AT are adopted in correspondence with our earlier work [7,8].

Each one of the four different three-level systems (Fig. 1) is engineered by the action of two fields, one weak field called the probe and one stronger field called the coupling, on two different atomic transitions which share a common level. If we monitor the absorption of the probe field, we find (Fig. 2) a reduction in absorption (a dip in the absorption line) at resonance ($\delta_p = 0$, the case when the frequency of the probe field matches the atomic transition frequency), where maximum absorption is expected in the absence of the coupling field. Note that Fig. 2 is a general plot for qualitative understanding only. For the sake of simplicity and clarity of the results, we consider the resonance coupling case (the frequency of the coupling field matches the corresponding atomic transition frequency) throughout this work.

The surprising reduction in absorption in EIT has always been understood in the literature as a result of destructive interference between two competing excitation pathways [2]. These two pathways have been explicitly studied and unambiguously presented in the Λ [Fig. 1(a)] [9] and cascade-EIT [Fig. 1(b)] [7] cases. We mentioned in ur earlier work [7] that, in the cascade-AT case, the second resonance (excitation pathway) is negligible compared to the first resonance in the low-saturation limit (the coupling Rabi frequency is much weaker than the corresponding atomic polarization decay rate, a point that will be covered in detail later in this paper). Some limitations in the scattering theory [7] constrict the study of the cascade-AT model to the low-saturation limit, leaving the picture slightly incomplete. In addition to the missing pieces in the cascade-AT configuration, no work to our knowledge clearly and unambiguously proves the existence of EIT in the V system. It is true that dips (reductions) in the absorption lines have been experimentally reported in all four three-level systems, but what needs to be clarified is whether the reported results are consequences of EIT or a similar phenomenon known as Autler-Townes splitting [10]. The latter phenomenon has a signature that looks very similar to that of EIT. Both phenomena display a reduction in absorption where a maximum is expected in the absence of the coupling field (Fig. 2). An explicit study of AT splitting was conducted by Cohen-Tannoudji and co-workers [11–13]. The studies were conducted in the secular limit (the coupling field much stronger than the polarization decay rates) and showed that the absorption line is made up of two Lorentzian-like lines that are located next to each other. Thus the dip in Fig. 2 can be interpreted as either a destructive interference between two competing pathways, which is known as EIT, or a gap between two resonances, which is known as AT splitting. It is the goal of this work to study in parallel all four three-level systems (Fig. 1) and explicitly clarify when the detected reduction in absorption is a result of EIT and when of AT splitting.

In the next section we use the scattering theory results [7] to split every one of the four probe absorption spectra (one spectrum for each three-level system) into two terms, which we call resonances. The comparison of the two resonances reveals two distinct coupling-field regimes and two categories of three-level atomic system. In Sec. III one sample of each category is explicitly explored. A study of the two considered configurations leads to conclusions about the existence or absence of EIT and AT splitting in each of the coupling-field regimes. Note that a homogeneously broadened medium and a resonant coupling field ($\delta_c = 0$) cases are considered in this work for the sake of simplicity and clarity of the results. A nonzero coupling detuning and an inhomogeneous Doppler broadening may add new physics to the systems but will not change the core of the physical phenomena that are studied in this paper, and correspondingly the presented results.

II. TWO COUPLING-FIELD REGIMES AND TWO CATEGORIES OF THREE-LEVEL SYSTEM

The absorption coefficient of the probe field is derived under the condition that the coupling field is much stronger than

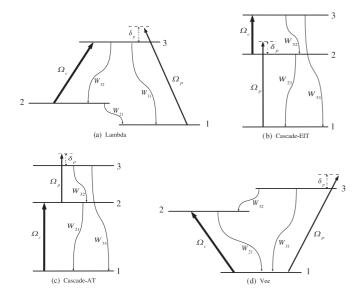


FIG. 1. Three-level atomic systems. W_{ij} is the population decay rate from level i to level j. Ω_p and Ω_c denote the Rabi frequencies of the probe and coupling fields, respectively. δ_p is the detuning of the probe field. The coupling field is at resonance.

the probe field, which is weak enough to be affected by the medium without changing its characteristics. Many techniques can be adopted to derive the explicit form of the absorption coefficient. One of the most favorable used in the literature is the semiclassical technique [14], where the atomic equations of motion are derived and then perturbatively solved in the steady state for different orders of the probe field. It is not the goal of this paper to cover any of the existing techniques. We consider in this work the spectra that have been derived in different publications. We modify the considered equations of the absorption coefficients, and accordingly the defined variables, to match our three-level systems. The absorption

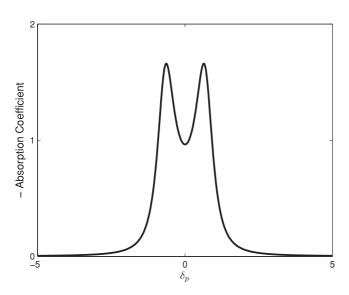


FIG. 2. Negative absorption coefficient of the probe field as a function of its detuning. The dip (reduction in absorption) could be a signature of EIT or AT coupling. This is a general plot added for qualitative reasons only.

spectra are then analyzed in light of the results of scattering theory [7]. We note that the absorption coefficient of the probe field is proportional to the imaginary part of the density-matrix element ρ_{ij} , where i-j is the atomic transition on which the probe field acts. In this paper, we define the needed variables in the following way: detuning of the probe field $\delta_p = \omega_p - \omega_{ij}$, detuning of the coupling field $\delta_c = \omega_c - \omega_{kl} = 0$ (resonance case), Rabi frequency of the probe field $\Omega_p = 2\mu_{ij}\mathcal{E}_p^0/\hbar$, Rabi frequency of the coupling field $\Omega_c = 2\mu_{kl}\mathcal{E}_c^0/\hbar$, and polarization decay rates $\gamma_{mn} = \sum_{t=1}^3 (W_{mt} + W_{nt})$, where i-j and k-l are respectively the transitions on which the probe and coupling fields act.

In the Λ case [Fig. 1(a)], the density-matrix element ρ_{13} [9],

$$\rho_{13} \propto \frac{\delta_p - i\gamma_{12}}{|\Omega_c|^2/4 - (\delta_p - i\gamma_{13})(\delta_p - i\gamma_{12})},\tag{1}$$

is derived as the sum (apart from a multiplication factor) of the following two resonances:

$$\mathscr{R}_{\rm I} \propto \frac{1}{(Z_{\rm I} - Z_{\rm II})} \frac{Z_{\rm I} + i\gamma_{12}}{\delta_p - Z_{\rm I}},$$
 (2a)

$$\mathscr{R}_{\mathrm{II}} \propto -\frac{1}{(Z_{\mathrm{I}} - Z_{\mathrm{II}})} \frac{Z_{\mathrm{II}} + i\gamma_{12}}{\delta_{p} - Z_{\mathrm{II}}},$$
 (2b)

where

$$Z_{\rm I} = \frac{1}{2} [-i\gamma_{23} + \sqrt{-(\gamma_{13} - \gamma_{12})^2 + |\Omega_c|^2}],$$
 (3a)

$$Z_{\rm II} = \frac{1}{2} [-i\gamma_{23} - \sqrt{-(\gamma_{13} - \gamma_{12})^2 + |\Omega_c|^2}].$$
 (3b)

In the cascade-EIT case [Fig. 1(b)], the density-matrix element of interest is ρ_{12} [7], given by

$$\rho_{12} \propto \frac{\delta_p - i\gamma_{13}}{|\Omega_c|^2/4 - (\delta_p - i\gamma_{12})(\delta_p - i\gamma_{13})}.$$
(4)

With the help of scattering theory [7], we split the absorption coefficient into the following two resonances:

$$\mathscr{R}_{\rm I} \propto \frac{1}{(Z_{\rm I} - Z_{\rm II})} \frac{Z_{\rm I} + i\gamma_{13}}{\delta_n - Z_{\rm I}},$$
 (5a)

$$\mathcal{R}_{\rm II} \propto -\frac{1}{(Z_{\rm I}-Z_{\rm II})} \frac{Z_{\rm II}+{\rm i}\gamma_{13}}{\delta_p-Z_{\rm II}},$$
 (5b)

where

$$Z_{\rm I} = \frac{1}{2} [-i\gamma_{23} + \sqrt{-(\gamma_{12} - \gamma_{13})^2 + |\Omega_c|^2}],$$
 (6a)

$$Z_{\rm II} = \frac{1}{2} [-i\gamma_{23} - \sqrt{-(\gamma_{12} - \gamma_{13})^2 + |\Omega_c|^2}].$$
 (6b)

In light of the work done in scattering theory and because all four absorption coefficients have similar algebraic forms, the absorption coefficients in the cascade-AT and V cases can also be written as sums of two resonances. In the cascade-AT [Fig. 1(c)] case, the density-matrix element ρ_{23} [7] is given by

$$\rho_{23} \propto \frac{\delta_p - i\gamma_{23}}{|\Omega_c|^2/4 - (\delta_p - i\gamma_{23})(\delta_p - i\gamma_{13})}$$
(7)

(we dropped the prefactor $\frac{|\Omega_c|^2/4}{\gamma_{l2}^2+2|\Omega_c|^2/4}),$ which splits into the two resonances

$$\mathscr{R}_{\rm I} \propto \frac{1}{(Z_{\rm I} - Z_{\rm II})} \frac{Z_{\rm I} + i\gamma_{23}}{\delta_p - Z_{\rm I}},$$
 (8a)

$$\mathscr{R}_{\mathrm{II}} \propto -\frac{1}{(Z_{\mathrm{I}} - Z_{\mathrm{II}})} \frac{Z_{\mathrm{II}} + \mathrm{i}\gamma_{23}}{\delta_{p} - Z_{\mathrm{II}}},$$
 (8b)

where

$$Z_{\rm I} = \frac{1}{2} \left[-i(\gamma_{23} + \gamma_{13}) + \sqrt{-\gamma_{12}^2 + |\Omega_c|^2} \right],$$
 (9a)

$$Z_{\rm II} = \frac{1}{2} \left[-i(\gamma_{23} + \gamma_{13}) - \sqrt{-\gamma_{12}^2 + |\Omega_c|^2} \right].$$
 (9b)

In a similar manner, the density-matrix element ρ_{13} [15] is given in the V case [Fig. 1(d)] by

$$\rho_{13} \propto \frac{(\delta_p - i\gamma_{13}) \frac{|\Omega_c|^2}{4\gamma_{12}^2} + (\delta_p - i\gamma_{23})}{|\Omega_c|^2/4 - (\delta_p - i\gamma_{23})(\delta_p - i\gamma_{13})}$$
(10)

(we dropped the prefactor $\frac{1}{1+|\Omega_c|^2/2\gamma_{12}^2}$), which can be rewritten as the sum of the two terms

$$\mathscr{R}_{\rm I} \propto \frac{1}{(Z_{\rm I} - Z_{\rm II})} \frac{Z_{\rm I} \left(1 + \frac{|\Omega_c|^2}{4\gamma_{12}^2}\right) + i\left(\gamma_{23} + \gamma_{13} \frac{|\Omega_c|^2}{4\gamma_{12}^2}\right)}{\delta_p - Z_{\rm I}},$$
 (11a)

$$\mathcal{R}_{\rm II} \propto -\frac{1}{(Z_{\rm I} - Z_{\rm II})} \frac{Z_{\rm II} \left(1 + \frac{|\Omega_c|^2}{4\gamma_{12}^2}\right) + i\left(\gamma_{23} + \gamma_{13} \frac{|\Omega_c|^2}{4\gamma_{12}^2}\right)}{\delta_p - Z_{\rm II}},$$
(11b)

where

$$Z_{\rm I} = \frac{1}{2} \left[-i(\gamma_{23} + \gamma_{13}) + \sqrt{-\gamma_{12}^2 + |\Omega_c|^2} \right],$$
 (12a)

$$Z_{\rm II} = \frac{1}{2} \left[-i(\gamma_{23} + \gamma_{13}) - \sqrt{-\gamma_{12}^2 + |\Omega_c|^2} \right].$$
 (12b)

We now know that all four absorption coefficients split into two resonances, and that in the two cascade-EIT [7] and Λ [9] cases the two resonances interfere in the weak-coupling-field regime (the coupling Rabi frequency less than the corresponding polarization decay rate). Based on the previously stated fact, we may assume that the same type of interference which exists in the cascade-EIT and A configurations must exist in the cascade-AT and V cases. We showed in our previous work [7] that, in the low-saturation limit (very weak-couplingfield regime), no interference can exist in the cascade-AT case. We plot in Fig. 3 the ratio of the maxima of the absolute values of the resonances, which reflects the relative strengths between the two resonances, versus what we call the "threshold factor," which we define as Ω_c divided by its threshold value Ω_t . This is the value of Ω_c that separates the weak- and strong-coupling-field regimes. The threshold varies from one system to another. A theoretical study (not included in this work) of the resonances shows that the thresholds are equal to the polarization decay rates under the square roots in the $Z_{\rm I}$ and $Z_{\rm II}$ complex numbers. The study of these complex numbers leads to the following thresholds in the four different systems:

$$\gamma_{13} - \gamma_{12}, \quad \Lambda, \tag{13a}$$

$$\gamma_{12} - \gamma_{13}$$
, cascade-EIT, (13b)

$$\gamma_{12}$$
, cascade-AT, (13c)

$$\gamma_{12}$$
, V. (13d)

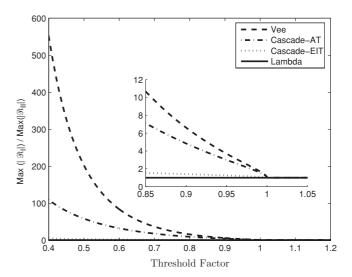


FIG. 3. Relative magnitudes of the two resonances [Eqs. (2) (Λ), (5) (cascade-EIT), (8) (cascade-AT), and (11) (V)] as functions of the threshold factor Ω_c/Ω_t [Eqs. (13a)–(13d)].

Note that the polarization decay rates have different values in the different three-level systems. We consistently use for the theoretical modeling in this paper the following scaled values of the population decay rates W_{ij} (spontaneous decay from level i to level j): cascade-EIT and cascade-AT ($W_{21}=1$, $W_{32}=0.2$, $W_{31}=0.001$); Λ ($W_{31}=1$, $W_{32}=0.9$, $W_{21}=0.001$); Λ ($W_{31}=1$, $W_{21}=0.9$, $W_{32}=0.001$). It is true that the study of the different three-level configurations as a function of a common coupling-field strength is not correct because of the different decay rates that affect the fields. For this reason, we unify the following study across the different configurations by plotting the ratio of the magnitudes of the resonances versus the threshold factor Ω_c/Ω_t .

Figure 3 shows clearly the existence of two regimes and two categories of three-level system. In the strong-coupling-field regime, the threshold factor is greater than 1 (Ω_c is greater than Ω_t), and the two resonances have exactly equal magnitudes. The magnitudes of the resonances vary as the strength of the coupling field varies in the strong-field regime, but the ratio of the magnitudes remains equal to 1. Once the couplingfield Rabi frequency becomes less than the threshold value (threshold factor less than 1), the four three-level systems show two completely different behaviors. In the Λ and cascade-EIT cases, the two resonances remain competitive, keeping a ratio that is very close to 1 (this characteristic is easily seen in the inset of Fig. 3). The surprising results are in the cascade-AT and V configurations, where one resonance dominates the other as the coupling field becomes weaker. Figure 3 shows clear evidence that two different coupling-field regimes must be distinguished and that two categories of three-level system must be considered. We know so far from this work that any possible interference effects that exist between the two resonances in the cascade-AT and V cases are going to be negligible in the low-saturation limit (very weak-coupling-field regime). The latter conclusion is in line with our study [7] of the cascade-AT system, where we state that only one resonance has to be retained in the low-saturation limit.

III. DISTINCTION BETWEEN ELECTROMAGNETICALLY INDUCED TRANSPARENCY AND AUTLER-TOWNES COUPLING

The study of the relative magnitudes of the resonances (Sec. II) revealed two categories of three-level system. We study in this section one system from each category for further understanding of the differences between these systems and the distinction between the EIT and AT phenomena. The two configurations that we are going to study in depth are the Λ [Fig. 1(a)] and V [Fig. 1(d)] regimes, which happen to be the most significant cases in their categories.

Figure 4 shows the "evolution" of the two resonances as functions of the threshold factor in the Λ case. Each subplot displays the two resonances separately and combined as the total absorption coefficient function of the detuning of the probe (a plot that is similar to Fig. 2). Figures 4(a)-4(d) show the evolution of the resonances and total absorption as we decrease the threshold factor. Figure 4(a) is in the strongcoupling-field regime. We can see that the total absorption is made up of two peaks, each of which is a resonance. The observed dip can be interpreted as a gap between the two resonances. This reduction in absorption is a characteristic of the AT effect, as explained by Cohen-Tannoudji and coworkers [12]. Figure 4(b) is also in the strong-coupling-field regime, but it is right above the threshold. We can see in this subplot that the dip is still a consequence of a gap between the two resonances even though the two resonances overlap. Once the threshold factor is less than 1 ($\Omega_c < \Omega_t$), we realize two crucial changes [Fig. 4(c)]. The two resonances totally overlap (peak on peak), and one resonance becomes totally negative.

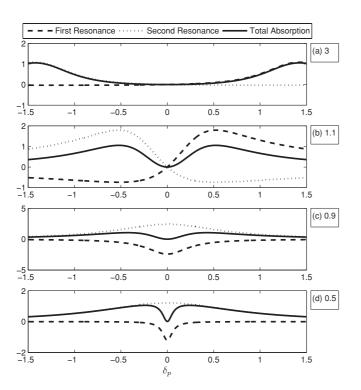


FIG. 4. Evolution of the resonances as functions of decreasing (a)–(d) threshold factor in the Λ case [Fig. 1(a)]. Each subplot includes the first resonance, second resonance, and total absorption of the probe field as a function of its detuning.

In this case, the weak-coupling-field regime, the dip is a result of a destructive interference between the two resonances. The detected reduction in absorption is an "imprint" of one resonance into the other. This destructive interference, which is a signature of EIT, becomes more apparent in Fig. 4(d), where the coupling field is weaker than it was in Fig. 4(c). This set of plots (Fig. 4) unambiguously shows that EIT and the AT effect are two similar-looking but distinct phenomena. Even though the two phenomena are consequences of two resonances, the detected reduction in absorption is a result of two different pieces of physics. This study of the Λ system clarifies the existence of AT coupling and EIT in the strongand weak-coupling-field regimes, respectively, in the Λ and cascade-EIT configurations.

In the same way that we previously studied the Λ system, we now study the V system. Figure 5 is a set of subplots similar (same threshold factors) to the ones presented in Fig. 4. The AT feature, a gap between two neighboring resonances, is the same in the V [Fig. (5(a))] as it is in the Λ case in the strong-couplingfield regime. Even though the two resonances overlap, with the peaks next to each other, the dip disappears in Fig. 5(b), where the coupling Rabi frequency is right above the threshold value. This disappearance of the dip has to do with the fact that the resonances become shallower as the coupling Rabi frequency decreases. In order to study the EIT phenomenon in the V system, we consider the weak-coupling-field regime [Figs. 5(c) and 5(d)]. Consistently with the Λ case, the two resonances overlap peak on peak. The difference here is that one of the resonances becomes very flat and starts vanishing as the coupling field becomes weaker (as expected from a study of Fig. 3). These dramatic changes in one of the resonances

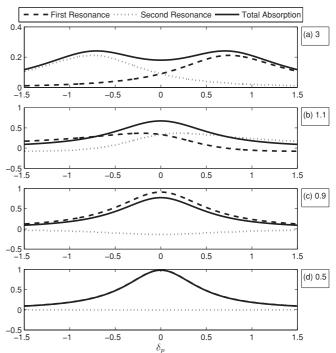


FIG. 5. Evolution of the resonances as functions of the decreasing (a)–(d) threshold factor in the V case [Fig. 1(d)]. Each subplot includes the first resonance, second resonance, and total absorption of the probe field as a function of its detuning.

prevent any possibility of seeing a dip in the absorption line. We can easily notice in Fig. 5(d) that one resonance is completely negligible while the other overlaps perfectly with the total absorption line. In other words, the absorption spectrum in the low-saturation limit (very weak coupling field) is a result of only one resonance. This conclusion is in line with our previous studies [7] presented in the cascade-AT case. The evolution of the resonances in the V case verifies the absence of EIT in the V [Fig. 1(d)] and cascade-AT [Fig. 1(c)] configurations.

We introduced in the previous section the variable threshold factor, which does not simply unify the study across all four different schemes (Fig. 1), but also leads to a separation between two different coupling-field strength regimes. It is in this section that we realize that the strong-coupling-field regime $(\Omega_c/\Omega_t > 1)$ is the AT regime, where transparency is a result of a gap between two absorption lines (a split in the absorption line). Correspondingly, the weak-couplingfield regime $(\Omega_c/\Omega_t < 1)$ happens to be the regime where destructive interference leads to a reduction in absorption (a dip "drilled" in the absorption line), which we know as EIT. As a result of this, we concluded in the previous paragraph that no EIT (transparency due to destructive interference) can be claimed in the V and cascade-AT schemes. This conclusion contradicts several previously published works [5], [16], [17], [18], including some quite recent ones [6], [19], [20], all of which contain claims of EIT or a transparency caused by destructive interference in one of the two schemes, V [Fig. 1(d)] or cascade-AT [Fig. 1(c)].

IV. CONCLUSION

We explicitly studied in this paper the four three-level atomic systems (Fig. 1) that feature EIT and/or AT coupling. The different studies were conducted under the limit of coupling resonance only ($\delta_c=0$). We considered a homogeneously broadened medium and a resonant coupling field ($\delta_c=0$) for the sake of simplicity and clarity of the results. The conclusions drawn are based on the study of two resonances for each system that add up to the exact total absorption coefficient. The four schemes were compared in light of the variation of the threshold factor (Ω_c/Ω_t) which unifies the studies across the different systems. The value 1 ($\Omega_c=\Omega_t$) of the threshold factor is clearly a point of separation between the strong-

 $(\Omega_c > \Omega_t)$ and weak- $(\Omega_c < \Omega_t)$ coupling-field regimes, as shown in Fig. 3. This fact verifies the values of the thresholds [Eqs. (13)] and the unification of the studies conducted.

In the strong-coupling-field regime (threshold factor greater than 1) an exact equality between the absolute magnitudes of the resonances is common across the four systems (Fig. 3). The peaks of the resonances are located next to each other with overlapping wings [Figs. 4(a) and 5(a)]. The observed reduction in absorption is a consequence of a gap between the resonances, which is a characteristic of AT splitting. In summary, the AT splitting (two resonances with a gap in between) is commonly observed in the strong-coupling-field regime ($\Omega_c > \Omega_t$) in the four different three-level systems (Fig. 1).

In the weak-coupling-field regime (threshold factor less than 1), the four three-level systems split into two categories (Fig. 3). The Λ and cascade-EIT configurations continue to feature two highly competitive resonances, unlike the casade-AT and V systems, where one resonance dominates the other as the threshold factor decreases. This characteristic, which sets the two categories apart, is most apparent in Figs. 4(d) and 5(d). A dip in the absorption line can be seen [Figs. 4(c) and 4(d)] in the Λ and cascade-EIT systems, which belong to the same category of three-level system. This dip is a result of destructive interference between the two competing pathways, which is a characteristic of EIT. This latter dip is absent in the second category of three-level system, which includes the cascade-AT and V configurations [Figs. 5(c) and 5(d)]. As the threshold factor decreases one resonance dominates the other in these systems. In these latter cases, cascade-AT and V, and in the low-saturation limit [very weak coupling field, such as the case in Fig. 5(d)], the absorption spectrum becomes equal to the contribution of only one resonance in the presence of the other negligible resonance. In summary, an EIT type of dip, which is a result of destructive interference, can be observed only in the Λ [Fig. 1(b)] and cascade-EIT [Fig. 1(a)] systems, but never in the cascade-AT [Fig. 1(c)] and V [Fig. 1(d)] systems.

ACKNOWLEDGMENTS

We thank Dr. Frank Narducci for inspiring discussions. We also sincerely thank Professor Mark Havey for his careful reading of the manuscript and for his constructive feedback.

S. E. Harris, J. E. Field, and A. Imamoglu, Phys. Rev. Lett. 64, 1107 (1990).

^[2] S. Harris, Phys. Today **50**(9), 36 (1997).

^[3] Y.-q. Li and M. Xiao, Phys. Rev. A 51, 4959 (1995).

^[4] J. J. Clarke, W. A. van Wijngaarden, and H. Chen, Phys. Rev. A 64, 023818 (2001).

^[5] Y. Zhao, C. Wu, B.-S. Ham, M. K. Kim, and E. Awad, Phys. Rev. Lett. 79, 641 (1997).

^[6] S. Vdovic, T. Ban, D. Aumiler, and G. Pichler, Opt. Commun. 272, 407 (2007).

^[7] T. Y. Abi-Salloum, J. Mod. Opt. (in press), doi:10.1080/09500341003658147.

^[8] T. Abi-Salloum, J. Davis, C. Lehman, E. Elliott, and F. A. Narducci, J. Mod. Opt. 54, 2459 (2007).

^[9] B. Lounis and C. Cohen-Tannoudji, J. Phys. II 2, 579 (1992).

^[10] S. H. Autler and C. H. Townes, Phys. Rev. 100, 703 (1955).

^[11] C. Cohen-Tannoudji and S. Reynaud, J. Phys. B 10, 2311 (1977).

^[12] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, Atom-Photon Interactions: Basic Processes and Applications (Wiley Interscience, New York, 1992).

^[13] C. Cohen-Tannoudji, *Amazing Light* (Springer, Berlin, 1996), Chap. 11, pp. 109–123.

^[14] M. Sargent III, M. O. Scully, and W. E. Lamb Jr., *Laser Physics* (Westview Press, Boulder, CO, 1974).

- [15] A. Lazoudis, Ph.D. dissertation, Temple University, 2005.
- [16] D. J. Fulton, S. Shepherd, R. R. Moseley, B. D. Sinclair, and M. H. Dunn, Phys. Rev. A 52, 2302 (1995).
- [17] J. R. Boon, E. Zekou, D. J. Fulton, and M. H. Dunn, Phys. Rev. A **57**, 1323 (1998).
- [18] J. Zhao, L. Wang, L. Xiao, Y. Zhao, W. Yin, and S. Jia, Opt. Commun. 206, 341 (2002).
- [19] L. Li, W. Tang, and H. Guo, Phys. Rev. A 76, 053837 (2007).
- [20] P. Light, F. Benabid, G. Pearce, F. Couny, and D. Bird, Appl. Phys. Lett. **94**, 141103 (2009).