PHYSICAL REVIEW A **81**, 053808 (2010)

Perfect transmission and highly asymmetric light localization in photonic multilayers

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(Received 23 January 2010; revised manuscript received 11 March 2010; published 6 May 2010)

General principles for the existence of perfect transmission resonances in photonic multilayer structures are formulated in terms of light interference described by recurrent Airy formulas. Mirror symmetry in the multilayer is shown to be a sufficient but not necessary condition for perfect transmission resonances. Asymmetric structures displaying perfect transmission in accordance with the proposed principles are demonstrated. A hybrid Fabry-Pérot photonic-crystal structure of the type $(BA)^k(AB)^k(AABB)^m$ is proposed, combining perfect transmission and highly asymmetric electric field localization. Strength and asymmetry of localization can be controlled independently to be of use in tailoring nonreciprocal behavior of nonlinear all-optical diodes.

DOI: 10.1103/PhysRevA.81.053808 PACS number(s): 42.25.Bs, 78.67.Pt, 42.25.Hz, 42.65.Pc

I. INTRODUCTION

Probably the simplest case of inhomogeneous media, photonic multilayers are a good testing ground for structures with complex geometrical properties such as aperiodic long-range order (see, e.g., [1] and references therein). Indeed, the availability of simple, cheap, and reliable computational methods often makes it possible to relate geometrical and optical properties in an explicit manner. To name a few examples, scaling and self-similar features in optical spectra of quasiperiodic Fibonacci [2–4] and fractal Cantor [5–7] multilayers were recently found to result from geometrical self-similarities of the underlying structure. It is even possible to formulate general relations for spectral properties of structures with arbitrary layer arrangement [8].

One of the rather intriguing properties of aperiodic multilayers is the appearance of perfect transmission resonances (PTRs) in the optical spectra, that is, frequencies for which the multilayer has transmittance exactly equal to unity ($|T| = 1$). It is known that multilayers with mirror symmetry (e.g., Cantor) commonly exhibit PTRs, whereas those without it (e.g., Fibonacci) usually do not: transmission peaks in such multilayers, even if they look "perfect," really have $|T| < 1$ (see Fig. 1). Several accounts [9–12] report PTRs if a Fibonacci structure is symmetrized and show that perfect transmission is explicitly related to mirror symmetry [11]. However, more recent results show perfect transmission in asymmetric multilayers based on periodic [13], Fibonacci [14], and Thue-Morse [15] geometry. This suggests that mirror symmetry is sufficient but not necessary for PTRs.

Such PTRs in asymmetric structures are promising in designing nonreciprocal optical devices such as nonlinear all-optical diodes [16]. Indeed, an associated spatially asymmetric light localization at resonance (see [13,15]) induces a nonreciprocal nonlinear optical response, while perfect transmission ensures that reflection losses remain small. In this perspective, understanding the physical principles of PTR formation in multilayers is undoubtedly of importance. Most previous works, however, do not really arrive at such principles beyond attributing PTR existence to "hidden symmetries" in the structures. Instead they draw rather formal conclusions in terms of the widely employed transfer matrix method [13,14]. Such conclusions would benefit from an interpretation to reveal their physical meaning.

In this paper, the question of PTR presence in multilayer spectra is addressed from another, more physical than computational standpoint. Perfect transmission in any multilayer (however complex) is seen to be governed by the same principles of multiple-beam interference as in a simple Fabry-Pérot interferometer. Transmission and reflection spectra of any multilayer are recovered using recurrent Airy formulas, and conditions for any two structures to form PTRs when stacked together are derived explicitly. From these conditions, known results such as PTRs in mirror-symmetric multilayers naturally follow. Moreover, it becomes possible to engineer structures with PTRs on purpose. As an example, a structure comprising a Fabry-Pérot interferometer adjacent to a one-dimensional photonic crystal is proposed. This structure is shown to feature both perfect transmission and a highly asymmetric, strongly localized electric field profile. Localization strength and asymmetry can be controlled independently by structure design.

In Sec. II, the theoretical background on using Airy-like formulas for calculating the optical spectra of complex multilayers is given. Section III follows with application of these formulas to arrive at the principles encompassing all possible cases of PTRs in multilayers. Specific cases such as mirror-symmetric and Thue-Morse multilayers are considered, too. Section IV further employs these principles in proposing a design for a structure featuring PTRs as well as strongly localized and highly asymmetric electromagnetic field distribution. Finally, the paper is summarized in Sec. V.

II. RECURRENT AIRY FORMULAS

We begin by considering a single dielectric layer (labeled A), with refractive index $n = n_A$ and thickness d_A,

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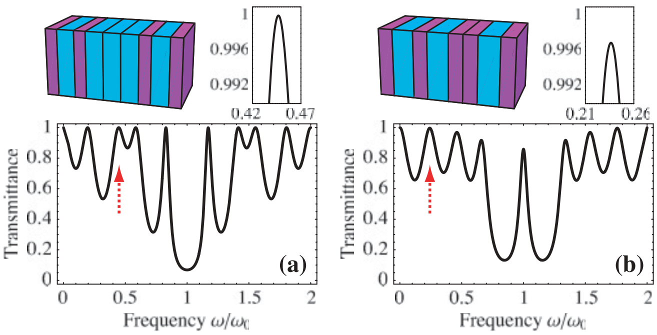

FIG. 1. (Color online) Example transmission spectra of (a) symmetric Cantor multilayer $BABAAABAB$ and (b) nonsymmetric Fibonacci multilayer $BABABBAB$. A and B correspond to single layers with $n_A = 1.55$, $d_A = 76$ nm, and $n_B = 2.3$, $d_B = 113$ nm, so that $n_A d_A = n_B d_B = \pi c/2\omega_0 = \lambda_0/4$ for $\lambda_0 = 700$ nm as in [15]. The insets show an enlarged view of the transmission peaks marked by arrows.

located in a homogeneous dielectric medium with $n = n_0$ [Fig. 2(a)]. Reflection and transmission coefficients of such a layer are given by well-known Airy formulas (see, e.g., [17,18])

$$R_A = r_{0A} + \frac{t_{0A} r_{A0} t_{A0} e^{2i\delta_A}}{1 - r_{A0}^2 e^{2i\delta_A}}, \quad T_A = \frac{t_{0A} t_{A0} e^{i\delta_A}}{1 - r_{A0}^2 e^{2i\delta_A}}, \tag{1}$$

where $\delta_A = (\omega/c) n_A d_A$ is the phase accumulated by the wave in the layer, and r_{ij} and t_{ij} are Fresnel reflection and transmission coefficients, respectively, of an interface between the two media labeled by i and j (the wave is incident on the interface from medium i to medium j). Note that R and T obtained by Eq. (1) are complex and contain information about the amplitude as well as the phase of the reflected and transmitted wave. The "usual" intensity-related reflectance and transmittance are given by $|R_A|^2$ and $|T_A|^2$, and it can be seen that $|R_A|^2 + |T_A|^2 = 1$, as is obvious from energy conservation.

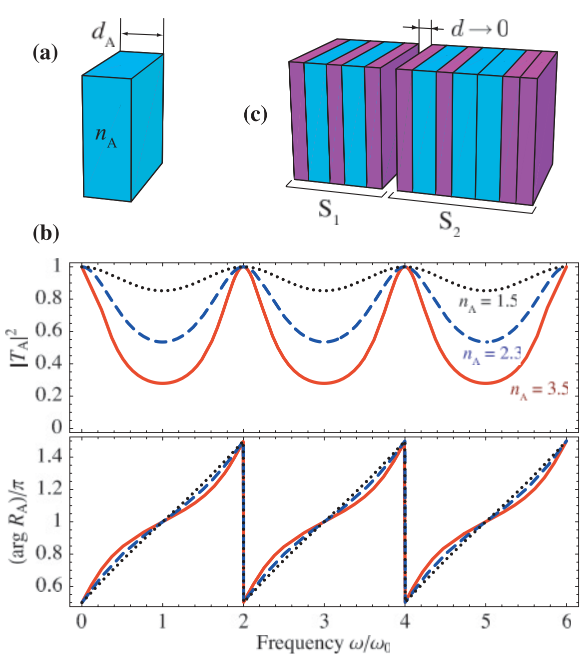

FIG. 2. (Color online) (a) A single layer and (b) its transmittance $|T_A|^2$ and phase shift of the reflected wave φ_A, given by Eq. (1), for different values of n_A [d_A is chosen in accordance with Eq. (2)]; (c) an example of a composite $S_1 S_2$ structure described by Eq. (3).

In such a simple system, the only frequency-dependent quantity is the phase δ_A. Since $r_{0A} = -r_{A0}$ and $t_{0A} t_{A0} = 1 - r_{A0}^2$, it follows that $T_A = 1$ whenever $\delta_A = m\pi$ for integer m. Physically, this corresponds to constructive interference of forward-propagating partial beams inside the layer, to occur when its optical thickness is an integer multiple of a half-wave. Hence, a single layer features equidistant PTRs like a Fabry-Pérot interferometer, albeit with poor-quality mirrors [see Fig. 2(b)]. The PTR frequencies are $2m\omega_0$ with ω_0 defined by a well-known quarter-wave (QW) condition

$$(n_B d_B =) n_A d_A = \pi c/(2\omega_0) = \lambda_0/4. \tag{2}$$

Similarly, let S_1 and S_2 be arbitrary multilayers (e.g., arbitrary combinations of A and B layers as in Fig. 1; but of course, what follows remains valid way beyond this example). Let the reflection and transmission coefficients R_S and T_S be known for $S = S_1, S_2$, and $\bar{S}_1$ (where a bar over S_1 denotes that S_1 is traversed in the reverse direction). Inserting an infinitely thin layer of the ambient medium between the structures [Fig. 2(c)], we can recover the reflection and transmission for the composite $S_1 S_2$ multilayer stack:

$$R_{S_1 S_2} = R_{S_1} + \frac{T_{S_1} R_{S_2} T_{\bar{S}_1}}{1 - R_{\bar{S}_1} R_{S_2}}, \quad T_{S_1 S_2} = \frac{T_{S_1} T_{S_2}}{1 - R_{\bar{S}_1} R_{S_2}}. \tag{3}$$

Note that Eqs. (3) follow from Eqs. (1) for $\delta = 0$, and that the energy conservation $|T_S|^2 + |R_S|^2 = 1$ holds. Also note that it is critical that both the amplitude *and* the phase of R_S and T_S are known. In a lossless, linear system one can make use of time reversal to relate the spectra of S_1 and $\bar{S}_1$ as $T_{\bar{S}} = T_S$, $R_{\bar{S}}/T_{\bar{S}} = -(R_S/T_S)^*$.

By first taking $S_{1,2}$ to be single layers with reflection and transmission spectra given by Eqs. (1) and then using Eqs. (3) and (1) in a recurrent fashion, we have a way to calculate transmission and reflection spectra for a multilayer of any degree of complexity. Because such recurrent calculation involves obtaining reflection and transmission coefficients for many intermediate structures, it is numerically less efficient than the transfer matrix method. However, the recurrent procedure is often adopted for the sake of analytical insight into the spectral properties of structures with internal symmetries (as was demonstrated, e.g., for fractal multilayers [6,7,17,18]).

III. CONDITIONS FOR PERFECT TRANSMISSION

Our goal is to formulate the existence conditions for a PTR in the transmission spectrum of an $S_1 S_2$ stack. From Eqs. (3), $|T_{S_1 S_2}(\omega)|$ can be obtained as

$$|T_{S_1 S_2}| = \frac{|T_{S_1}||T_{S_2}|}{\left|1 - |R_{\bar{S}_1}||R_{S_2}| e^{i(\varphi_{\bar{S}_1} + \varphi_{S_2})}\right|}, \tag{4}$$

where $\varphi_{\bar{S}_1}$ and φ_{S_2} are the phases of $R_{\bar{S}_1}$ and R_{S_2}, respectively. Since $|R|^2 + |T|^2 = 1$ in lossless structures, Eq. (4) can be rewritten in the form

$$\left|T_{S_1S_2}\right|^2 = \frac{(1-|R_1|^2)(1-|R_2|^2)}{|1-|R_1||R_2|e^{i\varphi}|^2}, \tag{5}$$

where we have denoted $|R_1| \equiv |R_{S_1}| = |R_{\bar{S}_1}|$, $|R_2| \equiv |R_{S_2}|$, and $\varphi \equiv \varphi_{\bar{S}_1} + \varphi_{S_2}$ for brevity. If the denominator in Eq. (5) is nonzero, the PTR condition $|T_{S_1S_2}| = 1$ is equivalent to

$$\begin{aligned}(1-|R_1|^2)&(1-|R_2|^2)\\ &= (1-|R_1||R_2|\cos\varphi)^2 + (|R_1||R_2|\sin\varphi)^2,\end{aligned}$$

which reduces to

$$|R_1|^2 + |R_2|^2 = 2|R_1||R_2|\cos\varphi.$$

Obviously, this equation always holds if $|R_1| = |R_2| = 0$, which becomes one possible case for PTR, and never holds if $|R_1| = 0, |R_2| \neq 0$ or vice versa. In all other cases, $|R_1||R_2| \neq 0$ so we obtain

$$\cos\varphi = \frac{|R_1|^2+|R_2|^2}{2|R_1||R_2|} = 1 + \frac{(|R_1|-|R_2|)^2}{2|R_1||R_2|} \geqslant 1. \tag{6}$$

If $|R_1| \neq |R_2|$, the right-hand side of Eq. (6) is strictly greater than unity, so no PTR can exist because the condition $\cos\varphi > 1$ cannot be met. If $|R_1| = |R_2|$, PTRs can and do occur whenever $\cos\varphi = 1$.

For completeness, note that the limiting case when the denominator in Eq. (5) equals zero results in

$$1 + (|R_1||R_2|)^2 - 2|R_1||R_2|\cos\varphi = 0.$$

If $|R_1||R_2| = 0$, this equation is false. Otherwise, it can be rewritten as

$$\cos\varphi = \frac{1+|R_1|^2|R_2|^2}{2|R_1||R_2|} = 1 + \frac{(1-|R_1||R_2|)^2}{2|R_1||R_2|} \geqslant 1$$

and can only be satisfied if $\cos\varphi = 1$ and $|R_1||R_2| = 1$. Since the reflectance can never exceed unity, the latter implies that $|R_1| = |R_2| = 1$ (i.e., the structure should consist of two *perfect* mirrors). Such an extreme case causes the right-hand sides of Eqs. (4) and (5) to be indeterminate. This indicates that the approach based on the interference of partial waves [Eqs. (3)] becomes invalid with perfect mirrors when there are no partial waves to interfere. However, this extreme can safely be ruled out by assuming that perfect transmission is impossible in structures involving perfect mirrors.

As a result, we have obtained two possibilities for PTR existence. The first is when $|R_1| = |R_2| = 0$, or, in the original notation of Eq. (4),

$$\left|T_{S_1}\right| = \left|T_{S_2}\right| = 1. \tag{7}$$

The second is when $|R_1| = |R_2|$ and $\cos\varphi = 1$; that is,

$$\left|T_{S_1}\right| = \left|T_{S_2}\right| \neq 1, \tag{8}$$

$$\varphi_{\bar{S}_1} + \varphi_{S_2} = 2m\pi. \tag{9}$$

The first condition given by Eq. (7) essentialy means that, whenever the individual structures S_1 and S_2 *both* have a PTR at *exactly* the same frequency, the composite stack S_1S_2 will always have a PTR at that frequency. In fact, this conclusion could have been drawn directly from Eq. (4). It is easily explained by the fact that if S_1 and S_2 are both perfectly transparent, no reflection at the S_1/S_2 interface can occur. Hence, the incident wave is fully transmitted and there is no possibility for the reflected wave to form. This is why, for example, all QW multilayers, where all layers conform to Eq. (2), have PTRs at $\omega = 2m\omega_0$ just as any one of the constituent layers.

The second condition [Eqs. (8) and (9)] is more interesting because it explains how PTRs are formed in the spectral regions of the composite structure where there were no PTRs for either S_1 or S_2. Indeed, $\exp[i(\varphi_{\bar{S}_1} + \varphi_{S_2})] = 1$ in the denominator in Eq. (4) renders it equal to the numerator and causes $|T_{S_1S_2}| = 1$ although $|T_{S_1}| = |T_{S_2}| \neq 1$. The PTR formation here can be explained by regarding the composite structure as a Fabry-Pérot interferometer with very complex mirrors. To begin with, the resonance occurs if the interference between partial waves is constructive, that is, if all the partial waves arising from multiple reflection are in phase, as given by Eq. (9). Then the resonance is perfect if the mirrors in the interferometer are balanced and have equal reflectivity [Eq. (8)].

Equations (8) and (9) let us easily see why a mirror-symmetric structure readily supports PTRs while most other structures do not. Mirror symmetry means $S_2 = \bar{S}_1$, so it is obvious that $|T_{S_1}| = |T_{S_2}|$ and $\varphi_{\bar{S}_1} = \varphi_{S_2}$ for all frequencies. The only remaining condition to be fulfilled is Eq. (9) (i.e., $\varphi_{\bar{S}_1} = m\pi$). Since the phase of the reflected wave varies monotonically between transmission resonances in any multilayer with rather few exceptions [19,20], there should be numerous points where it crosses $m\pi$ [e.g., for one layer it happens for $\omega = (2m-1)\omega_0$; see Fig. 2(b)]. These points necessarily result in PTRs, as can be seen in Fig. 3(a). It is seen that for any PTR $\cos 2\varphi_{\bar{S}_1} = 1$, except at $\omega = 0$ and $\omega = 2\omega_0$ where PTRs result from Eq. (7) rather than from Eqs. (8) and (9).

Another simple example would be $S_2 = S_1$ (i.e., when the same structure is repeated twice in the stack). Again we have $|T_{S_2}| = |T_{\bar{S}_1}| = |T_{S_1}|$ for all frequencies. However, Eq. (5) here assumes a different form, namely, $\varphi_{S_1} + \varphi_{\bar{S}_1} = 2m\pi$, which is more difficult to satisfy [compare Figs. 3(a) and 3(b)]. As a result, the double-stack structure S_1S_1 exhibits only half as many PTRs as does its mirror-symmetric counterpart $S_1\bar{S}_1$. Both mirror symmetry and stack doubling contribute to PTR formation in periodic structures (e.g., one-dimensional photonic crystals). Note that if S_1 is asymmetric, so is S_1S_1, and this case can be regarded as the simplest asymmetric multilayer featuring PTRs.

Equations (8) and (9) also encompass more exotic cases involving intrinsically asymmetric structures. Consider S_1 consisting of arbitrarily arranged A and B layers so that $n_Ad_A = n_Bd_B$ as in Eq. (2), and S_2 obtained from S_1 by substitution $A \leftrightarrow B$. The resulting structure is very asymmetric [see Fig. 3(c)], yet it can be shown to feature PTRs. This was observed by Nava *et al.* [14] for Fibonacci structures and further pointed out by Grigoriev and Biancalana [15], who named such structures "Thue-Morse conjugated" because one particular case of such structures, obtained by repeatedly applying inflation rules $A \to AB$, $B \to BA$, represents the well-known Thue-Morse sequence [21]. Figure 3(c) shows calculation results for S_1S_2 with the same S_1 as for the previous examples [Figs. 3(a) and 3(b)]. Similarly to these, there are numerous frequencies where $\cos(\varphi_{\bar{S}_1} + \varphi_{S_2}) = 1$ and Eq. (9)

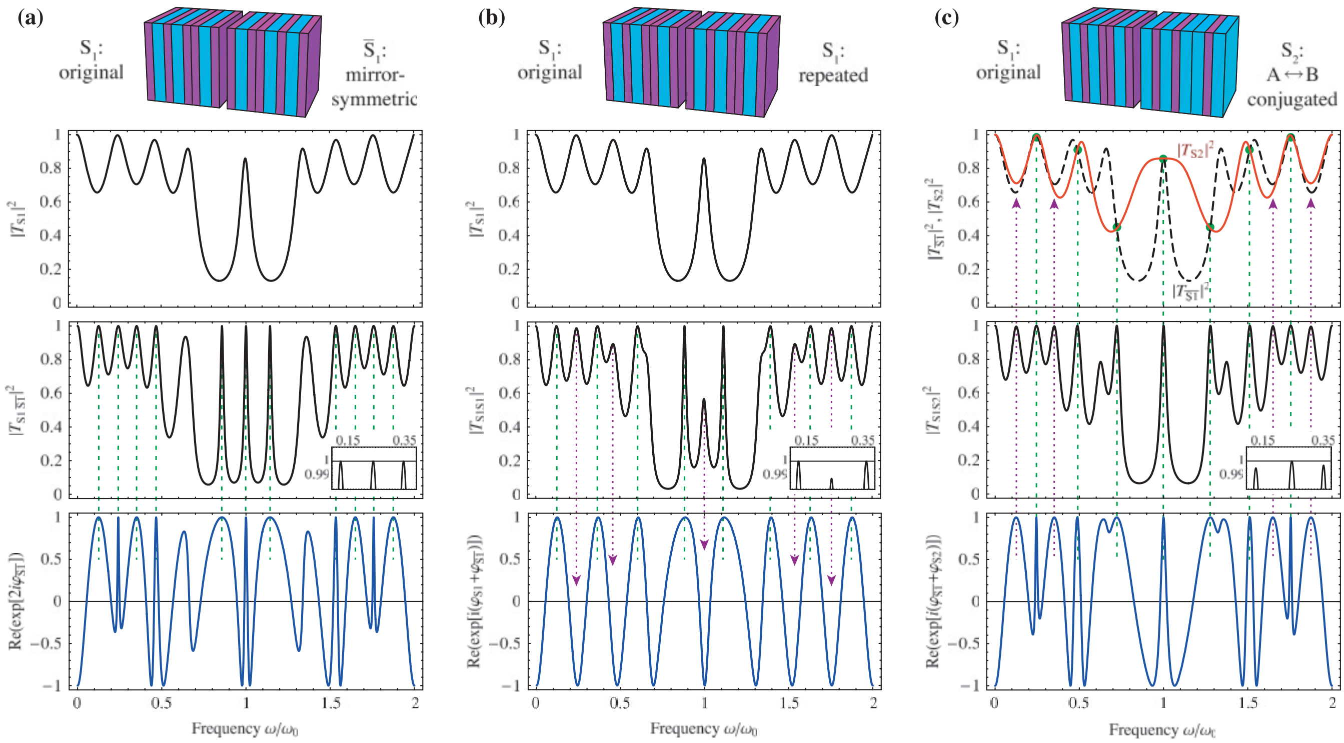

FIG. 3. (Color online) PTRs in composite structures S_1S_2: (a) mirror-symmetric ($S_2 = \bar{S}_1$), (b) double-stacked ($S_2 = S_1$), and (c) Thue-Morse conjugated. The transmission spectra of the constituent structures $S_{1,2}$, the transmission spectra of the whole structure, and the spectral dependence of the phase factor $\cos(\varphi_{\bar{S}_1} + \varphi_{S_2})$ as in Eq. (9) are shown in the second, third, and fourth rows, respectively. The dashed vertical lines show the location of PTRs when both Eqs. (8) and (9) hold. The dotted lines with arrows designate the peaks that fail to be PTRs due to violation of either Eq. (9) [in (b)] or Eq. (8) [in (c)]. The insets represent a blown-up view of some peaks to determine whether or not they are PTRs.

is satisfied, and each such frequency represents a transmission peak. However, only part of these peaks turn out to be PTRs (see insets in Fig. 3), namely the ones that simultaneously satisfy Eq. (8). The rigorous proof of how the fulfillment of these conditions results from the Thue-Morse symmetry can be given and is expected to appear in a forthcoming publication by Grigoriev *et al.*

IV. PERFECT TRANSMISSION IN HIGHLY ASYMMETRIC STRUCTURES

Equations (8) and (9) can be employed to engineer a structure of any predefined geometry with a PTR at the desired wavelength just by varying the refractive index and thickness of the layers involved. Indeed, modifying n_B/n_A in S_2 without violating Eq. (2) changes the value of transmittance and reflectance while keeping the phases relatively intact [see Fig. 2(b)]. This aids in fulfilling Eqs. (8) and (9) simultaneously and forms a PTR in the S_1S_2 structure. Subsequently varying ω_0 in Eq. (2) for both S_1 and S_2 causes all the spectra (both amplitude and phase) to scale uniformly, thus bringing the PTR to the chosen value of the wavelength. Similarly designed dual-interferometer structures of the type $(AB)^m(BA)^m(A'B')^m(B'A')^m$ were shown to possess PTRs [13].

Our objective for this paper is to arrive at a design for multilayers with highly asymmetric light localization at a PTR so as to facilitate the nonreciprocal operation in a nonlinear optical diode [15]. A straightforward way to achieve the desired asymmetry is to stack S_1 featuring a strongly localized mode with S_2 having an extended mode, and to match the frequencies of the corresponding resonances.

An obvious choice for S_1 with a maximally localized mode would be a periodic QW multilayer with a half-wave defect, or, in other words, a Fabry-Pérot interferometer surrounded by Bragg mirrors, so that $S_1 = (BA)^k(AB)^k$. If Eq. (2) holds, a sharp transmission resonance occurs exactly at ω_0 [see Fig. 4(b)]. On the contrary, the modes are known to be maximally extended at $\omega = 2\omega_0$ in any QW multilayer. By doubling the thickness of each layer, this frequency can be halved to exactly match the resonance for S_1. A double-periodic photonic-crystal structure of the type $(AABB)^m$ can thus be used as S_2. The resulting stack the has the geometry [Fig. 4(a)]

$$S_1S_2 = (BA)^k(AB)^k(AABB)^m. \tag{10}$$

This design has the obvious advantage that both resonances in question are PTRs (and they are exactly frequency matched), so there is no need to go as far as Eqs. (8) and (9) and the resulting PTR in S_1S_2 is ensured due to Eq. (7). The second advantage is that frequency matching always occurs at $\lambda = \lambda_0/4$, so it is easy to design the structure for any desired wavelength using any materials at hand. Figure 4(d) confirms the existence of a PTR, as does explicit numerical calculation of $|T_{S_1S_2}(\omega_0)|$, yielding 1 within limits of machine accuracy.

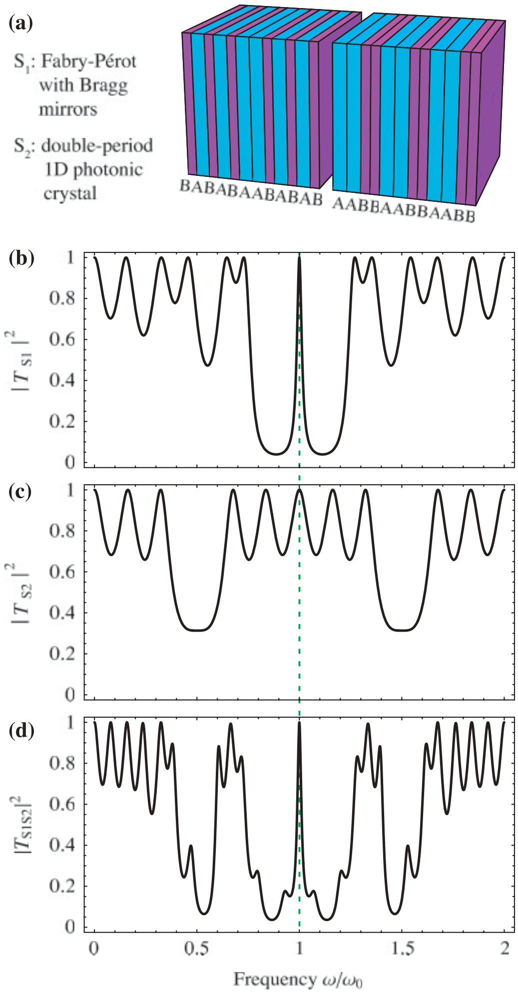

FIG. 4. (Color online) (a) The proposed design of a highly asymmetric multilayer featuring PTRs given by Eq. (10), along with transmission spectra for (b) S_1, (c) S_2, and (d) S_1S_2.

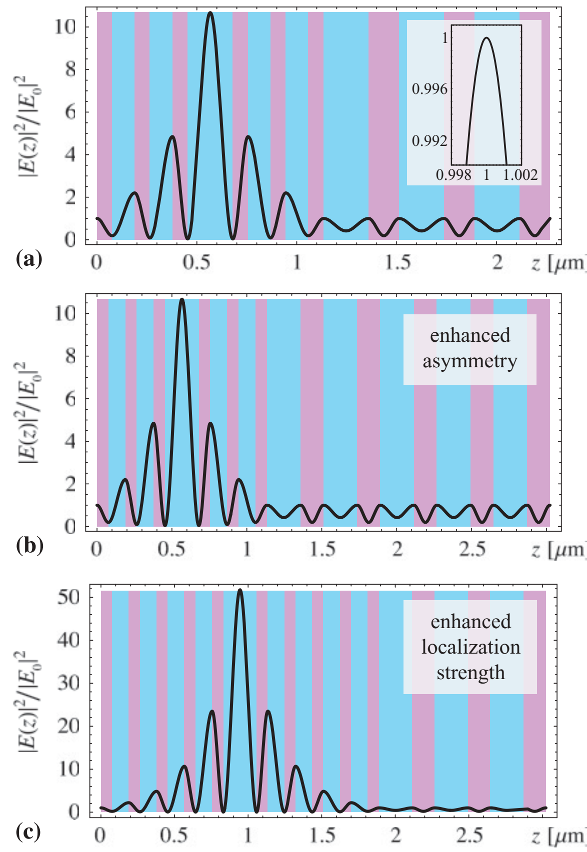

FIG. 5. (Color online) Electric field localization profile at the PTR frequency $\omega = \omega_0$ (cf. inset) for the proposed structure design [Eq. (10)] for (a) $k = m = 3$ as in Fig. 4; (b) $k = 3$, $m = 5$ (enhanced asymmetry); and (c) $k = 5$, $m = 3$ (enhanced localization strength).

The electric field intensity distribution at the resonant frequency is shown in Fig. 5. The localization is clearly asymmetric and mainly present in S_1. Using the same materials as in [15], comparable localization strength is observed for a structure about three times thinner and having 32 layers instead of 64 [see Fig. 5(c)]. The design of Eq. (10) also allows the localization strength (by varying k) and the asymmetry (by varying m) to be controlled independently and in a wide range for a relatively minor change in the number of layers [compare Figs. 5(a)–5(c)]. This is opposed to changing the number of generations in a Thue-Morse sequence, which would double or halve the number of layers at once. The possibility of building PTR-enabled structures with desired localization properties using relatively few layers is important from a practical point of view because losses would obviously be more detrimental to perfect transmission in thicker structures [14].

Note, finally, that the choice of geometry for S_2 is rather arbitrary, because any arrangement of AA and BB will produce the same extended-mode PTR at ω_0. This choice of geometry can be regarded as an additional design tool to influence the transmission spectrum around the PTR. For a periodic geometry $S_2 = (AABB)^m$ used in Eq. (10), the two band gaps around $(1 \pm 1/2)\omega_0$ brought about by S_2 [see Fig. 4(c)] can overlap with the gap around ω_0 for S_1 [Fig. 4(b)]. This would widen the region of predominantly low transmission surrounding the designed PTR, which can prove useful. For the materials adopted throughout this paper from [15], it is not yet the case, but the gap overlap can be achieved by increasing n_B/n_A (Fig. 6). It is seen that the PTR then becomes very isolated in the transmission spectrum.

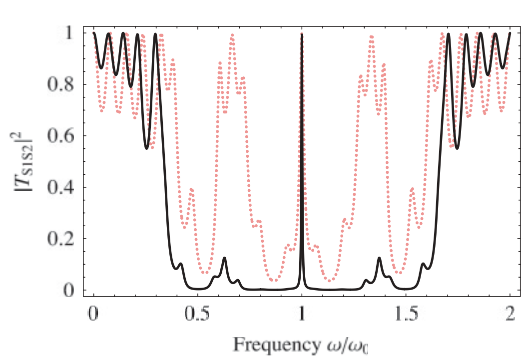

FIG. 6. (Color online) Transmission spectrum of the structure in Fig. 4(a) for $n_A = 1.55$, $n_B = 2.3$ as in [15] (dotted line) and for increased n_B/n_A by setting $n_A = 1$ (solid line).

V. CONCLUSIONS

Using the formalism of recurrent Airy formulas, the conditions for a mulilayer structure to exhibit perfect transmission resonances [Eqs. (7)–(9)] are formulated rigorously in such a way that possibilities for PTRs can be directly envisioned at the stage of structure design. Following the previous results [14], it was shown that mirror symmetry is a sufficient but not necessary condition for PTR existence. PTRs are shown to be possible in asymmetric structures, including Thue-Morse conjugated multilayers [15]. Based on frequency-matched PTRs in structure parts, the design for a combined Fabry-Pérot/double-period photonic-crystal multilayer was proposed [Fig. 4(a)]. This structure was shown to feature perfect transmission resonances with strongly localized and highly asymmetric spatial distribution of electric field intensity (Fig. 5). The strength and asymmetry of localization can be controlled independently by changing the design parameters, keeping the number of layers reasonably small. It is expected that multilayers of this kind would enhance nonreciprocal transmission if they contained nonlinear materials, improving the performance of optical diodes and similar devices.

ACKNOWLEDGMENTS

The author is very grateful to Victor Grigoriev for inspiring and fruitful discussions and helpful suggestions. Thanks go to Dmitry Chigrin and all organizers of the International Workshop on Theoretical and Computational Nanophotonics (TaCoNa-2009) for creating an excellent environment for a very lively exchange of ideas. This work was supported in part by the Deutsche Forschungsgemeinschaft (DFG Research Unit 557) and by the Natural Sciences and Engineering Research Council of Canada.

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