Subwavelength spatial correlations in near-field speckle patterns

R. Carminati*
Institut Langevin, ESPCI ParisTech, CNRS, 10 rue Vauquelin, FR-75231 Paris Cedex 05, France
(Received 30 December 2009; revised manuscript received 21 February 2010; published 4 May 2010)

At subwavelength distance from the exit surface of a disordered medium, speckle patterns generated by multiple scattering of waves exhibit nonuniversal near-field correlations. A calculation of the field spatial correlation function shows that the correlation length is driven by the microscopic structure of the medium. The averaged speckle spot size can be smaller than the wavelength, even for nonresonant dielectric media.

DOI: 10.1103/PhysRevA.81.053804 PACS number(s): 42.25.Dd, 42.25.Kb, 78.35.+c, 42.30.Ms

I. INTRODUCTION

Wave scattering in disordered media leads to a complicated spatial distribution of intensity known as a speckle pattern [1,2]. Short-range intensity correlations characterize the averaged size of a speckle spot [3]. Multiple scattering generates long-range intensity correlations, which are responsible for enhanced mesoscopic fluctuations [4]. Although the far-field properties of speckle patterns have been widely studied [5–9], little is known about the near-field properties measured at subwavelength distance from the exit surface of a disordered medium. In this region, nonpropagating (evanescent) fields dominate and are expected to substantially influence the statistical properties.

The study of near-field speckle patterns is motivated by the existence of imaging techniques based on spatial field correlations of acoustic or seismic waves [10], and by the possibility of focusing waves through disordered media by time reversal in acoustics [11] and electromagnetism [12], or by wavefront control in the optical regime [13]. Since the spatial resolution is influenced by the speckle spot size, it is interesting to study the lower limit of the speckle spot size in the near field, which is *a priori* not given by the diffraction limit when evanescent fields contribute [14].

Near-field speckle properties have been studied experimentally in optics, using scanning near-field optical microscopy on volume-disordered dielectric samples [15–17] and on semicontinuous metal films [18]. In the case of volume-disordered samples, which we will address here, evidence of short-range correlations that deviate from the far-field behavior have been reported [16], as well as measurements of subwavelength correlation lengths. Spatial correlations in near-field speckle patterns measured on disordered photonic crystals have also been shown to provide information about the incomplete band gap of such structures [19]. In this paper, we study theoretically the field spatial correlation function in a near-field speckle pattern produced by wave transmission through a volume-disordered sample. We consider electromagnetic waves, since to date most of the relevant experiments have been carried out in this context [13,15–18]. We derive analytical formulas that describe the near-field regime and that coincide with known results in the infinite medium and far-field limits. We show that the correlation length depends on the observation distance and is connected to the microstructure of the sample in the extreme near field. This enables the existence of subwavelength speckle spots.

II. GENERAL EXPRESSION OF SPATIAL FIELD CORRELATION

We consider a disordered medium described by a real dielectric function (no absorption) of the form $\epsilon(\mathbf{r}) = 1 + \delta\epsilon(\mathbf{r})$, where $\delta\epsilon(\mathbf{r})$ is the fluctuating part with the following statistical properties:

$$\langle\delta\epsilon(\mathbf{r})\rangle = 0, \quad \langle\delta\epsilon(\mathbf{r})\,\delta\epsilon(\mathbf{r}')\rangle = \frac{U}{\pi^{3/2}\,\ell_\epsilon^3}\exp\left(-\frac{|\mathbf{r}-\mathbf{r}'|^2}{\ell_\epsilon^2}\right). \tag{1}$$

The brackets $\langle\cdots\rangle$ denote averaging over an ensemble of realizations of the random medium. The correlation length ℓ_ϵ reflects the microscopic structure of the medium [20]. The limit $\ell_\epsilon \to 0$ corresponds to the white-noise model $\langle\delta\epsilon(\mathbf{r})\,\delta\epsilon(\mathbf{r}')\rangle = U\,\delta(\mathbf{r}-\mathbf{r}')$ that has been used in early calculations of speckle correlations [3,5]. In this paper, we consider the mesoscopic regime $\ell_\epsilon \ll \lambda \ll \ell \ll L$, where λ is the wavelength, ℓ is the scattering mean free path, and L is the system size. The constant U is determined by calculating the imaginary part of the effective dielectric function $\boldsymbol{\epsilon}_{\text{eff}}(\mathbf{k}) = \mathbf{I} + k_0^{-2}\boldsymbol{\Sigma}(\mathbf{k})$, where $k_0 = \omega/c = 2\pi/\lambda$ is the wave number in vacuum (more generally in the averaged medium), with ω the wave frequency and c the speed of light, and $\mathbf{I}$ is the unit tensor. $\boldsymbol{\Sigma}(\mathbf{k})$ is the self-energy containing the sum of all multiply-connected scattering events [1,2]. On scales larger that the correlation length ℓ_ϵ, the effective dielectric function is isotropic and local, so that $\boldsymbol{\epsilon}_{\text{eff}}(\mathbf{k}) = \epsilon_{\text{eff}}\mathbf{I}$. A perturbative calculation in terms of the small parameter $(k_0\,\ell)^{-1}$ and for vector electromagnetic waves leads to $\text{Im}\,\epsilon_{\text{eff}} = (k_0\ell)^{-1}$ and $U = 6\pi/(k_0^4\,\ell)$.

Assuming that the sources of the incident field are located outside the medium, the electric field in the medium obeys the vector propagation equation $\nabla\times\nabla\times\mathbf{E}(\mathbf{r}) - \epsilon(\mathbf{r})\,k_0^2\,\mathbf{E}(\mathbf{r}) = 0$. The averaged field obeys $\nabla\times\nabla\times\langle\mathbf{E}(\mathbf{r})\rangle - \epsilon_{\text{eff}}\,k_0^2\,\langle\mathbf{E}(\mathbf{r})\rangle = 0$ as a consequence of Dyson's equation [1]. The fluctuating field $\delta\mathbf{E}(\mathbf{r}) = \mathbf{E}(\mathbf{r}) - \langle\mathbf{E}(\mathbf{r})\rangle$ satisfies

$$\nabla\times\nabla\times\delta\mathbf{E}(\mathbf{r}) - \epsilon_{\text{eff}}\,k_0^2\,\delta\mathbf{E}(\mathbf{r}) = i\,\mu_0\,\omega\,\mathbf{j}(\mathbf{r}), \tag{2}$$

where the source term is $\mathbf{j}(\mathbf{r}) = -i\omega\epsilon_0\,(1-\epsilon_{\text{eff}})\,\mathbf{E}(\mathbf{r}) - i\omega\epsilon_0\,\delta\epsilon(\mathbf{r})\,\mathbf{E}(\mathbf{r})$. Under the condition of weak scattering $\langle\delta\epsilon^2\rangle k_0^2\ell_\epsilon^2 \ll 1$, or equivalently $k_0^2\ell\ell_\epsilon \gg 1$, which we assume in the following, one has $|1-\epsilon_{\text{eff}}| \ll |\delta\epsilon(\mathbf{r})|$ [21], and the source term reduces to $\mathbf{j}(\mathbf{r}) = -i\omega\epsilon_0\,\delta\epsilon(\mathbf{r})\,\mathbf{E}(\mathbf{r})$.

*remi.carminati@espci.fr

The formal solution of Eq. (2) can be written as

$$\delta\mathbf{E}(\mathbf{r}) = k_0^2 \int_V \langle \mathbf{G}(\mathbf{r},\mathbf{r}')\rangle\, \delta\epsilon(\mathbf{r}')\, \mathbf{E}(\mathbf{r}')\, \mathrm{d}^3 r', \qquad (3)$$

where $\langle \mathbf{G}\rangle$ is the averaged (dyadic) Green's function, solution of the Dyson equation [1], obeying $\nabla\times\nabla\times\langle\mathbf{G}(\mathbf{r},\mathbf{r}')\rangle - \epsilon_{\text{eff}}\, k_0^2\, \langle\mathbf{G}(\mathbf{r},\mathbf{r}')\rangle = \delta(\mathbf{r}-\mathbf{r}')\,\mathbf{I}$ with appropriate boundary conditions that depend on the geometry and an outgoing (retarded) wave condition at infinity. The integral is extended to the volume V of the disordered medium.

The spatial correlation function between two components of the electric field reads

$$\langle \delta E_k(\mathbf{r})\, \delta E_l^*(\mathbf{r}')\rangle = k_0^4 \int_V \langle G_{km}(\mathbf{r},\mathbf{r}_1)\rangle\, \langle G_{ln}^*(\mathbf{r}',\mathbf{r}_1')\rangle \times \langle \delta\epsilon(\mathbf{r}_1) E_m(\mathbf{r}_1)\, \delta\epsilon(\mathbf{r}_1') E_n^*(\mathbf{r}_1')\rangle\, \mathrm{d}^3 r_1 \mathrm{d}^3 r_1', \qquad (4)$$

where repeated indices mean implicit summation. As a measure of the overall spatial correlation of the vector field, we define

$$\gamma_E(\mathbf{r},\mathbf{r}') \equiv \sum_k \langle \delta E_k(\mathbf{r})\, \delta E_k^*(\mathbf{r}')\rangle. \qquad (5)$$

This quantity will be referred to as field correlation function. In coherence theory, the normalized field correlation function is called the degree of spatial coherence, and its width defines the spatial coherence length. In a speckle pattern, we take this spatial coherence length as a measure of the averaged spot size [22].

For an explicit calculation of the field correlation function, we need to specify the correlator $\langle \delta\epsilon(\mathbf{r}_1) E_m(\mathbf{r}_1)\, \delta\epsilon(\mathbf{r}_1') E_n^*(\mathbf{r}_1')\rangle$ in Eq. (4). It can be expressed using the four-points irreducible vertex $\Gamma_{mn,pq}$ in the form

$$\langle \delta\epsilon(\mathbf{r}_1)\, E_m(\mathbf{r}_1)\, \delta\epsilon(\mathbf{r}_1')\, E_n^*(\mathbf{r}_1')\rangle = \int \Gamma_{mn,pq}(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_2,\mathbf{r}_2') \langle E_p(\mathbf{r}_2)\, E_q^*(\mathbf{r}_2')\rangle\, \mathrm{d}^3\mathbf{r}_2\, \mathrm{d}^3\mathbf{r}_2'. \qquad (6)$$

Inserting this relation into (4) gives the Bethe-Salpeter equation [1,2]. To lowest order in $(k_0\, \ell)^{-1}$, the ladder approximation leads to [1,2]

$$\Gamma_{mn,pq}(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_2,\mathbf{r}_2') \simeq \langle \delta\epsilon(\mathbf{r}_1)\delta\epsilon(\mathbf{r}_1')\rangle\, \delta(\mathbf{r}_1-\mathbf{r}_2)\, \delta(\mathbf{r}_1'-\mathbf{r}_2')\, \delta_{mp}\delta_{nq}. \qquad (7)$$

We end up with

$$\langle \delta E_k(\mathbf{r})\, \delta E_l^*(\mathbf{r}')\rangle = k_0^4 \int_V \langle G_{km}(\mathbf{r},\mathbf{r}_1)\rangle\, \langle G_{ln}^*(\mathbf{r}',\mathbf{r}_1')\rangle \times \langle \delta\epsilon(\mathbf{r}_1)\delta\epsilon(\mathbf{r}_1')\rangle \langle E_m(\mathbf{r}_1) E_n^*(\mathbf{r}_1')\rangle\, \mathrm{d}^3 r_1 \mathrm{d}^3 r_1'. \qquad (8)$$

Equations (5) and (8) are the basic equations for the calculation of the field correlation function. The construction of this model has required some approximations that have been specified. There is a striking similarity between this model and that used to describe thermal electromagnetic fluctuations [23]. The fluctuating thermal currents are replaced here by an effective source term $\mathbf{j}(\mathbf{r}) \propto \delta\epsilon(\mathbf{r})\mathbf{E}(\mathbf{r})$. Therefore the calculation of near-field speckle correlations is technically very close to the calculation of correlations of thermal near fields [24,25].

III. INFINITE MEDIUM

Let us first examine the field correlation function in the bulk of the random medium and neglect the effect of interfaces (infinite medium). The medium being statistically homogeneous and isotropic, one can write $\langle \delta\epsilon(\mathbf{r}_1)\delta\epsilon(\mathbf{r}_1')\rangle \langle E_m(\mathbf{r}_1)\, E_n^*(\mathbf{r}_1')\rangle = C_{mn}(|\mathbf{r}_1-\mathbf{r}_1'|)\delta_{mn}$. The factor δ_{mn} results from the assumption that the field inside the medium is unpolarized. Using this form in Eq. (8) leads to [26]

$$\langle \delta E_k(\mathbf{r})\delta E_l^*(\mathbf{r}')\rangle = k_0^3 \ell \int \operatorname{Im}\langle G_{kl}(\mathbf{r}-\mathbf{r}'-\mathbf{v})\rangle C_{mm}(|\mathbf{v}|)\mathrm{d}^3 v. \qquad (9)$$

In the infinite medium, one has

$$\langle \mathbf{G}(\mathbf{r}-\mathbf{r}')\rangle = \left[\mathbf{I} + \frac{1}{k_{\text{eff}}^2}\nabla\nabla\right] \frac{\exp(ik_{\text{eff}}|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|} \text{ for } \mathbf{r}\neq\mathbf{r}', \qquad (10)$$

where $k_{\text{eff}} = \sqrt{\epsilon_{\text{eff}}}\, k_0 \simeq k_0 + i/(2\ell)$. From this expression, we get $\sum_k \operatorname{Im}\langle G_{kk}(\rho)\rangle = (2\pi\rho)^{-1}\sin(k_0\rho)\exp[-\rho/(2\ell)]$, with $\rho = |\mathbf{r}-\mathbf{r}'|$. Therefore the first term in the integral in Eq. (9) has a width $\simeq \lambda/2$, while the second term $C_{mm}(|\mathbf{v}|)$, which includes the exponential term in Eq. (1), has a width $\ell_\epsilon \ll \lambda$. This allows us to simplify the integral, leading to

$$\gamma_E(\mathbf{r},\mathbf{r}') = \langle |\mathbf{E}|^2\rangle\, \operatorname{sinc}(k_0\,\rho)\, \exp\left(-\frac{\rho}{2\ell}\right). \qquad (11)$$

This expression, valid in an infinite medium, is identical to that obtained for scalar waves [3]. It is independent of the correlation length ℓ_ϵ of the medium, at least in the regime $\ell_\epsilon \ll \lambda \ll \ell$ considered in this work. Under the assumption of a statistically homogeneous and isotropic medium, general statements show that this correlation function is universal for electromagnetic vector waves [27]. It is found, e.g., for blackbody radiation in a weakly absorbing homogeneous medium [23].

IV. SPATIAL CORRELATION ABOVE AN INTERFACE

We now turn to the study of the field spatial correlation close to the exit surface of a random medium. We consider a thick medium, separated from a vacuum by a surface that is flat on average and that defines the plane $z = 0$ (see the geometry in the inset in Fig. 1). To proceed, we need to use the averaged Green's function corresponding to a flat interface separating a homogeneous medium with dielectric function ϵ_{eff} (half-space $z < 0$) from a vacuum (half-space $z > 0$). For a source at $\mathbf{r}'$ inside the medium and an observation point $\mathbf{r}$ in vacuum, this Green's function reads [28]

$$\langle \mathbf{G}(\mathbf{r},\mathbf{r}')\rangle = \frac{i}{8\pi^2}\int \frac{1}{\gamma_0}(\mathbf{s}t_s\mathbf{s} + \mathbf{p}_0 t_p \mathbf{p}_1)\exp[i\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}')] \times \exp(i\gamma_0 z - i\gamma_1 z')\, \mathrm{d}^2 K, \qquad (12)$$

where $\mathbf{R} = (x,y)$ and $\mathbf{K}$ is the component of the wave vector along the interface. The dyadic terms describe the polarization behavior at the interface, with $\mathbf{s} = \hat{\mathbf{K}}\times\hat{z}$,

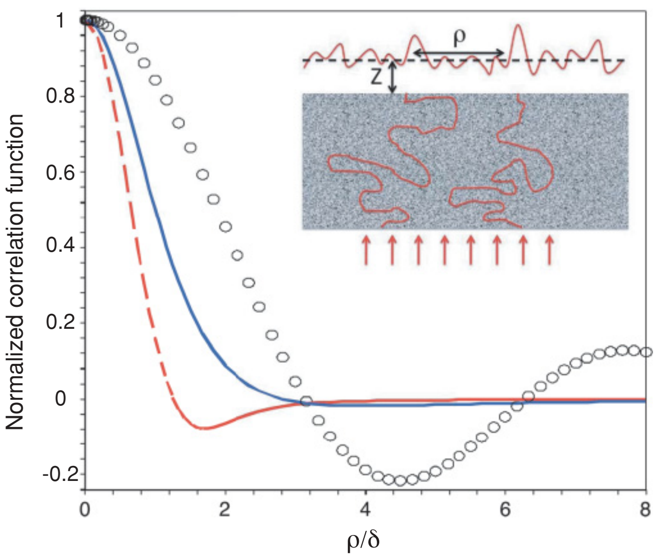

FIG. 1. (Color online) Normalized field spatial correlation function $\gamma_E(\mathbf{r},\mathbf{r}')/\gamma_E(\mathbf{r},\mathbf{r})$ in a plane at a distance z vs $\rho/\delta = |\mathbf{R}-\mathbf{R}'|/\delta$. δ is a reference length scale. Black markers: Far-field regime $z \gg \lambda$ ($\delta = \lambda/2\pi$). Blue solid line: near-field intermediate regime $\ell_\epsilon \ll z \ll \lambda$ ($\delta = z$). Red dashed line: extreme near-field regime $z \sim \ell_\epsilon \ll \lambda$ ($\delta = \ell_\epsilon$).

$\mathbf{p}_j = (K\hat{z} + \gamma_j\hat{\mathbf{K}})/k_j$, the symbol ^ denoting a unit vector and $K = |\mathbf{K}|$. The components of the wave vector along the z direction are $\gamma_j = (k_j^2 - K^2)^{1/2}$, with $k_1 = \sqrt{\epsilon_{\text{eff}}}\,k_0$, and the determination $\text{Re}(\gamma_j) > 0$ and $\text{Im}(\gamma_j) > 0$. $t_s(K)$ and $t_p(K)$ are the Fresnel transmission factors for s- and p-polarized waves [29]. For $z > 0$ (upper medium), the plane-wave expansion (12) possesses both propagating waves and evanescent waves. The latter correspond to high spatial frequencies $K > k_0$ and imaginary values of γ_0, and dominate in the near-field zone.

In the bulk of the medium, the field spatial correlation [Eq. (11)] varies on the scales of λ and ℓ, both being much larger than ℓ_ϵ. This means that in Eq. (8), one can make the approximation $\langle\delta\epsilon(\mathbf{r}_1)\delta\epsilon(\mathbf{r}_1')\rangle\langle E_m(\mathbf{r}_1)\,E_n^*(\mathbf{r}_1')\rangle \simeq \langle\delta\epsilon(\mathbf{r}_1)\delta\epsilon(\mathbf{r}_1')\rangle\langle|\mathbf{E}|^2\rangle\delta_{mn}/3$ in the integral. Using this approximation and inserting (12) into Eq. (8), we obtain after a little algebra

$$\gamma_E(\mathbf{r},\mathbf{r}') = \frac{\langle|\mathbf{E}|^2\rangle}{4\ell}\int_0^\infty f(K,z) \times \exp\left\{-[K^2 + (\text{Re}\gamma_1)^2]\ell_\epsilon^2/4\right\} J_0(K\rho)\text{d}K, \tag{13}$$

where $\rho = |\mathbf{R}-\mathbf{R}'|$ and $K = |\mathbf{K}|$. The function in the integrand is

$$f(K,z) = \frac{K}{2\text{Im}\,\gamma_1}\left[\frac{|t_s|^2}{|\gamma_1|^2} + \frac{(K^2+|\gamma_0|^2)(K^2+|\gamma_1|^2)}{|\sqrt{\epsilon_{\text{eff}}}|^2 k_0^4} \times \frac{|t_p|^2}{|\gamma_1|^2}\right]\exp(-2\text{Im}\gamma_0 z). \tag{14}$$

Equation (13) is valid for any observation distance z from the exit surface [30]. The function $f(K,z)$ describes waves transmission at the interface between the effective medium and the observation medium as well as the attenuation of the high-spatial-frequency components (evanescent waves) when z increases,

We first address the far-field regime with $z \gg \lambda$. This means that we consider a speckle pattern measured in real space in a plane at a given distance z from the exit surface, but at a distance large enough so that only propagating (homogeneous) plane waves contribute to the scattered field. In practice, a distance $z \sim \lambda$ is already in the far field [14,16]. Since $z \gg \lambda \gg \ell_\epsilon$, we consider the medium as uncorrelated (white-noise model). In this regime, the exponential filter $\exp(-2\text{Im}\,\gamma_0\,z)$ reduces the range of $f(K,z)$ in Eq. (13) to $0 \leqslant K \leqslant k_0$ (propagating waves). One obtains

$$\gamma_E(\mathbf{r},\mathbf{r}') = \frac{\langle|\mathbf{E}|^2\rangle}{2k_0}\int_0^{k_0}\frac{K}{\sqrt{k_0^2 - K^2}}[1 - R(K)]J_0(K\rho)\,\text{d}K, \tag{15}$$

where $R(K)$ is the intensity Fresnel reflection factor averaged over polarizations [29]. Since the effective medium satisfies $\epsilon_{\text{eff}} \simeq 1$, we can neglect the dependence of R on K. The integral simplifies to give

$$\gamma_E(\mathbf{r},\mathbf{r}') = \frac{(1-R)\langle|\mathbf{E}|^2\rangle}{2}\,\text{sinc}(k_0\rho) \quad \text{for} \quad z \gg \lambda. \tag{16}$$

Note that $\langle|\mathbf{E}|^2\rangle$ in this equation, and throughout the text, is the averaged intensity in the infinite medium. In the far field, the correlation function is independent on z. The far-field normalized correlation function is plotted in Fig. 1 (black markers) versus the normalized distance $2\pi\rho/\lambda$.

The $\text{sinc}(k_0\rho)$ behavior is similar to that obtained for blackbody radiation in the far field of a planar thermal source [24, 25]. The width of the correlation function is $\lambda/2$. Let us stress that the correlation function does not exhibit the $\exp(-\rho/2\ell)$ term obtained in an infinite medium. This exponential term only exists for the correlation function of the field *inside* the medium, the far-field correlation function observed outside being dependent on the wavelength only. The same observation holds for the spatial correlation function of blackbody radiation at a given wavelength, which exhibits a $\text{sinc}(2\pi\rho/\lambda)$ behavior in vacuum and a $\text{sinc}(2\pi\rho/\lambda)\exp(-\rho/2\delta)$ inside a medium at thermal equilibrium, δ being the skin depth in the material [23].

We now examine the near-field regime. When $z \ll \lambda$, inspecting the integrand in Eq. (13), we see that large wave vectors satisfying $k_0 \ll K \ll 1/z$ dominate the integral, since the exponential cutoff is $\exp(-2\text{Im}\,\gamma_0\,z) \simeq \exp(-2Kz)$ for large K and becomes effective only for $K \simeq 1/z$. The asymptotic behavior of the integral can be determined by expanding the integrand to leading order in the limit $K \gg k_0$, known as the quasistatic limit. This leads to

$$\gamma_E(\mathbf{r},\mathbf{r}') = \frac{2\langle|\mathbf{E}|^2\rangle}{k_0^4\ell|\epsilon_{\text{eff}}+1|^2}\int_0^\infty K^2\exp(-2Kz) \times \exp\left(-K^2\ell_\epsilon^2/4\right)J_0(K\rho)\,\text{d}K. \tag{17}$$

This expression includes as a particular case the limit $\ell_\epsilon = 0$ of the uncorrelated medium. This is relevant if $\ell_\epsilon \ll z \ll \lambda$, a regime that might be observed if ℓ_ϵ is very small compared to λ.

In this case, the integral in (17) can be calculated analytically [25]. We end up with

$$\gamma_E(\mathbf{r},\mathbf{r}') = \frac{\langle|\mathbf{E}|^2\rangle}{8k_0^4\ell\, z^3}\frac{1-\rho^2/(8z^2)}{[1+\rho^2/(4z^2)]^{5/2}} \quad \text{for} \quad \ell_\epsilon \ll z \ll \lambda, \tag{18}$$

where we have made the approximation $|\epsilon_{\text{eff}}+1|^2 \simeq 4$ in the denominator. The last term describes the lateral spatial dependence in a plane at a constant height z. The normalized correlation function in this regime is plotted in Fig. 1 (blue solid line), versus the normalized distance ρ/z. The width of the correlation function is on the order of z. Together with the $1/z^3$ behavior of the amplitude, these are features of the quasistatic regime in which the spatial structure of the field is that of an electrostatic field [14]. The same behavior has been obtained for near fields generated by thermal emission [25].

We now examine the behavior of the field spatial correlation function in the regime $z \lesssim \ell_\epsilon \ll \lambda$. Due to the exponential cutoff $\exp(-K^2\,\ell_\epsilon^2/4)$ in Eq. (17), we expect a transition to a regime that becomes independent of z when $z \sim \ell_\epsilon$, and driven by the length scale ℓ_ϵ. Indeed, when $z < \ell_\epsilon$, the exponential term $\exp(-2Kz)$ plays no role, and the integral in Eq. (17) gives a confluent hypergeometric function $M(a,b,x)$ [25]. We finally get

$$\gamma_E(\mathbf{r},\mathbf{r}') = \frac{\sqrt{\pi}\,\langle|\mathbf{E}|^2\rangle}{k_0^4\,\ell\,\ell_\epsilon^3}\, M\left(\frac{3}{2},\quad 1,\quad \frac{-\rho^2}{\ell_\epsilon^2}\right) \quad \text{for}\ z \lesssim \ell_\epsilon \ll \lambda. \tag{19}$$

The last term describes the spatial dependence and has a width ℓ_ϵ. It is plotted in Fig. 1 (dashed red curve), versus the normalized distance ρ/ℓ_ϵ. For a medium with a geometric correlation length ℓ_ϵ much smaller than λ, the correlation length of the near field is on the order of ℓ_ϵ when $z \lesssim \ell_\epsilon$. Moreover, the correlation function does not depend on z any more. The spatial-frequency cutoff due to ℓ_ϵ has removed the $1/z^3$ increase of the quasistatic contribution obtained in Eq. (18) with $\ell_\epsilon = 0$. Finally, let us remark that the dependence on ℓ_ϵ gives a nonuniversal character to the correlation function in the extreme near field, since it depends on a parameter that describes the microscopic structure of the sample. Also note that the exact shape of the correlation function in this regime depends on the form of the correlator $\langle\delta\epsilon(\mathbf{r})\,\delta\epsilon(\mathbf{r}')\rangle$. The result in Eq. (19) corresponds to a Gaussian correlated disorder as given in Eq. (1).

V. SUMMARY AND CONCLUSIONS

In summary, we have characterized short-range correlations in a near-field speckle pattern in the weak-scattering approximation. We have identified three regimes for the field correlation function $\gamma_E(\mathbf{r},\mathbf{r}')$ in a plane at a distance z from the averaged exit surface of the medium. For $z \gg \lambda$ (far field), $\gamma_E(\mathbf{r},\mathbf{r}')$ is given by Eq. (16) and the spatial correlation length $\Delta \simeq \lambda/2$. In the very near field, when $z \lesssim \ell_\epsilon \ll \lambda$, $\gamma_E(\mathbf{r},\mathbf{r}')$ is given by Eq. (19) and is independent of z. One has $\Delta \simeq \ell_\epsilon$, and the field correlation is driven by the microscopic structure of the disordered medium. The field correlation is nonuniversal in this regime (it strongly depends on the type of sample under study). The same behavior has been discussed previously in the case of speckle patterns produced by rough surfaces [31]. Nonuniversal intensity correlations have also been predicted in far-field speckle patterns produced by a point source located inside a scattering medium [32], the correlation function depending in this case on the local environment of the source, due to the near-field interaction between the source and the surrounding scatterers [8,33]. For $\ell_\epsilon \ll z \ll \lambda$, an intermediate near-field regime may be observed in which neither ℓ_ϵ nor λ play a role. $\gamma_E(\mathbf{r},\mathbf{r}')$ is given by Eq. (18) and $\Delta \simeq z$, the correlation being created by the quasistatic fields generated by the effective medium. These results suggest that subwavelength speckle spots can be created in the vicinity of nonresonant dielectric disordered media by quasistatic near fields. Moreover, they permit an identification of the length scales that define the speckle spot size.

ACKNOWLEDGMENTS

This work was supported by the EU Project *Nanomagma* NMP3-SL-2008-214107 and by the French ANR under project CAROL ANR-06-BLAN-0096.

[1] P. Sheng, *Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena* (Academic, San Diego, 1995).
[2] E. Akkermans and G. Montambaux, *Mesoscopic Physics of Electrons and Photons* (Cambridge University, Cambridge, UK, 2007).
[3] B. Shapiro, Phys. Rev. Lett. **57**, 2168 (1986).
[4] M. J. Stephen and G. Cwilich, Phys. Rev. Lett. **59**, 285 (1987); S. Feng, C. Kane, P. A. Lee, and A. D. Stone, *ibid.* **61**, 834 (1988).
[5] R. Pnini and B. Shapiro, Phys. Rev. B **39**, 6986 (1989).
[6] J. F. de Boer, M. P. van Albada, and A. Lagendijk, Phys. Rev. B **45**, 658 (1992).
[7] P. Sebbah, R. Pnini, and A. Z. Genack, Phys. Rev. E **62**, 7348 (2000); P. Sebbah, B. Hu, A. Z. Genack, R. Pnini, and B. Shapiro, Phys. Rev. Lett. **88**, 123901 (2002).
[8] S. E. Skipetrov, Phys. Rev. Lett. **93**, 233901 (2004).
[9] A. Garcí a-Martín, F. Scheffold, M. Nieto-Vesperinas, and J. J. Sáenz, Phys. Rev. Lett. **88**, 143901 (2002); G. Cwilich, L. S. Froufe-Pérez, and J. J. Sáenz, Phys. Rev. E **74**, 045603(R) (2006).
[10] R. L. Weaver and O. I. Lobkis, Phys. Rev. Lett. **87**, 134301 (2001); E. Larose *et al.*, Geophysics **71**, S11 (2006).
[11] A. Derode, A. Tourin, and M. Fink, Ultrasonics **40**, 275 (2002).
[12] G. Lerosey, J. de Rosny, A. Tourin, and M. Fink, Science **315**, 1120 (2007).
[13] I. M. Vellekoop and A. P. Mosk, Opt. Lett. **32**, 2309 (2007); S. M. Popoff, G. Lerosey, R. Carminati, M. Fink, A. C. Boccara, and S. Gigan, Phys. Rev. Lett. **104**, 100601 (2010).
[14] J.-J. Greffet and R. Carminati, Prog. Surf. Sci. **56**, 133 (1997).

[15] V. Emiliani, F. Intonti, M. Cazayous, D. S. Wiersma, M. Colocci, F. Aliev, and A. Lagendijk, Phys. Rev. Lett. **90**, 250801 (2003).

[16] A. Apostol and A. Dogariu, Phys. Rev. Lett. **91**, 093901 (2003); Opt. Lett. **29**, 235 (2004).

[17] A. Apostol and A. Dogariu, Phys. Rev. E **72**, 025602(R) (2005).

[18] S. Grésillon *et al.*, Phys. Rev. Lett. **82**, 4520 (1999); K. Seal *et al.*, *ibid.* **94**, 226101 (2005); J. Laverdant, S. Buil, B. Bérini, and X. Quélin, Phys. Rev. B **77**, 165406 (2008).

[19] V. M. Apalkov, M. E. Raikh, and B. Shapiro, Phys. Rev. Lett. **92**, 253902 (2004).

[20] S. E. Skipetrov and R. Maynard, Phys. Rev. B **62**, 886 (2000).

[21] L. Tsang and J. A. Kong, *Scattering of Electromagnetic Waves: Advanced Topics* (Wiley, New York, 2001), Chap. 4, p. 169.

[22] In practice, one usually measures intensity correlations. For short-range correlations, $\langle \delta I(\mathbf{r}) \delta I(\mathbf{r}') \rangle \propto \gamma_E(\mathbf{r},\mathbf{r}')^2$.

[23] S. M. Rytov, Yu. A. Kravtsov, and V. I. Tatarskii, *Principles of Statistical Radiophysics* (Springer-Verlag, Berlin, 1989), Vol. 3, Chap. 3.

[24] R. Carminati and J.-J. Greffet, Phys. Rev. Lett. **82**, 1660 (1999).

[25] C. Henkel, K. Joulain, R. Carminati, and J.-J. Greffet, Opt. Commun. **186**, 57 (2000); C. Henkel and K. Joulain, Appl. Phys. B **84**, 61 (2006).

[26] We have used the identity $k_0^2 \int \mathrm{Im}\, \epsilon_{\mathrm{eff}} \langle G_{km}(\mathbf{r}-\mathbf{r}'') \rangle \langle G_{lm}^*(\mathbf{r}'-\mathbf{r}'') \rangle \mathrm{d}^3 r'' = \mathrm{Im} \langle G_{kl}(\mathbf{r}-\mathbf{r}') \rangle$. See, e.g., H. T. Dung, L. Knöll, and D. G. Welsch, Phys. Rev. A **57**, 3931 (1998).

[27] T. Setälä, K. Blomstedt, M. Kaivola, and A. T. Friberg, Phys. Rev. E **67**, 026613 (2003).

[28] J. E. Sipe, J. Opt. Soc. Am. B **4**, 481 (1987).

[29] The amplitude transmission factors are $t_s = 2\gamma_1/(\gamma_0+\gamma_1)$ and $t_p = 2\sqrt{\epsilon_{\mathrm{eff}}}\gamma_1/(\epsilon_{\mathrm{eff}}\gamma_0+\gamma_1)$. The amplitude reflection factors are $r_s = (\gamma_0-\gamma_1)/(\gamma_0+\gamma_1)$ and $r_p = (\epsilon_{\mathrm{eff}}\gamma_0-\gamma_1)/(\epsilon_{\mathrm{eff}}\gamma_0+\gamma_1)$. The averaged intensity reflection factor is $R(K) = [|r_s(K)|^2 + |r_p(K)|^2]/2$.

[30] The use of a local effective dielectric function in this regime might be questioned, since ϵ_{eff} becomes a nonlocal function on scales on the order of ℓ_ϵ [1]. The term $\exp(-K^2\ell_\epsilon^2/4)$ in Eq. (13) prevents high spatial frequencies $K > 1/\ell_\epsilon$ from playing a role so the effect of nonlocality remains negligible.

[31] J.-J. Greffet and R. Carminati, Ultramicroscopy **61**, 43 (1995).

[32] B. Shapiro, Phys. Rev. Lett. **83**, 4733 (1999).

[33] B. A. van Tiggelen and S. E. Skipetrov, Phys. Rev. E **73**, 045601(R) (2006).