Svetlichny's inequality and genuine tripartite nonlocality in three-qubit pure states

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The violation of the Svetlichny's inequality (SI) [Phys. Rev. D **35**, 3066 (1987)] is sufficient but not necessary for genuine tripartite nonlocal correlations. Here we quantify the relationship between tripartite entanglement and the maximum expectation value of the Svetlichny operator (which is bounded from above by the inequality) for the two inequivalent subclasses of pure three-qubit states: the Greenberger-Horne-Zeilinger (GHZ) class and the *W* class. We show that the maximum for the GHZ-class states reduces to Mermin's inequality [Phys. Rev. Lett. **65**, 1838 (1990)] modulo a constant factor, and although it is a function of the three tangle and the residual concurrence, large numbers of states do not violate the inequality. We further show that by design SI is more suitable as a measure of genuine tripartite nonlocality between the three qubits in the *W*-class states, and the maximum is a certain function of the bipartite entanglement (the concurrence) of the three reduced states, and only when their sum attains a certain threshold value do they violate the inequality.

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I. INTRODUCTION

The correlations between outcomes of measurements on two spatially separated subsystems are said to be nonlocal, if they cannot be reproduced by any local-hidden variable (LHV) model [1]. To quantify bipartite nonlocality (NL) in any theory, one defines appropriate bipartite Bell-type inequalities (BTI's), which give an upper bound on correlations consistent with any LHV model [1]. Therefore, the magnitude of violation of such an inequality by the correlations is said to be a measure of its bipartite NL. In this article we study *irreducible* (genuine) NL of the tripartite quantum states. The motivation to do so is that it is precisely the NL which gives an advantage to quantum theory (QT) over classical theories for certain information processing (QIP) tasks [2]. However, the advantage, in a sense, scales with the number of subsystems which are genuinely nonlocally correlated, that is, genuine n-partite NL is more powerful than genuine $(n-1)$-partite NL. More importantly, entanglement does not necessarily imply NL [3]. Thus, one of the central aims for the field of quantum information science is to quantify the genuine NL of multipartite entangled states.

II. DISCUSSION

All two-qubit pure entangled states violate the simplest bipartite BTI, known as the CHSH inequality [4], and the violation increases as the entanglement of the state increases [5]; but the maximum that the expectation value of the CHSH operator can attain for a quantum state is $2\sqrt{2}$, which is known as the Tiserlson's bound [6]. More importantly, one can define a generalized nonlocal theory which is more nonlocal than QT, (i.e., such a nonlocal theory, for instance, can achieve the value of 4 for the expectation of CHSH operator [7]). Quantifying genuine tripartite NL of quantum states is not straightforward for the following reasons. Mermin's tripartite BTI is based on an absolute LHV model [8], (i.e., it is derived on the assumption that if the correlations can be simulated by an ensemble where all the three subsystems are locally correlated to each other, then the correlations are not genuinely tripartite nonlocal). Nonetheless, it is only a necessary condition, since some pure bi-separable entangled states also violate the inequality [3]. Svetlichny formulated a stronger kind of tripartite inequality: If the correlations cannot be simulated by a *hybrid nonlocal-local* ensemble [9], only then are the correlations genuinely tripartite nonlocal, where a hybrid ensemble consists of two of the subsystems being arbitrarily nonlocally correlated, but locally correlated to the third, and one takes an ensemble average over all such possible combinations. Thus, by construction, the violation of Svetlichny's inequality (SI) is a confirmation of genuine tripartite NL.

Note, the assumption of arbitrary NL between two of the subsystems in the hybrid ensemble makes the violation of SI only a sufficient condition for genuine tripartite NL, since a hybrid ensemble can be more bipartite nonlocal than QT. This simply means that a genuinely tripartite nonlocal entangled state may not violate SI, (i.e., there may exist a hybrid ensemble which can simulate the quantum correlations described by the state [3]). Nevertheless, at present, the violation of SI is the *only* criterion known for the confirmation of genuine tripartite NL of a quantum state. In this article we use SI to quantify genuine tripartite nonlocality of the following two subclasses of three-qubit pure states in terms of their genuine tripartite entanglement [11]: the Greenberger-Horne-Zeilinger (GHZ)-class states,

$$|\psi_{gs}\rangle = \cos\theta|000\rangle + \sin\theta|11\rangle\{\cos\theta_3|0\rangle + \sin\theta_3|1\rangle\}, \qquad (1)$$

and the W-class states,

$$|\psi_w\rangle = \alpha|001\rangle + \beta|010\rangle + \gamma|100\rangle, \qquad (2)$$

where α, β, and γ are real. The results presented in this paper can be generalized to n-qubit pure states via the

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generalized SI [3]. Our results may be of particular value to the experimental community interested in genuine multiqubit NL for the implementation of certain QIP tasks [10], because we provide explicit measurement settings which maximize the violation of SI.

A. Monogamy

The entanglement of the three-qubit pure states in the two classes are inequivalent [12]. The difference can be quantified by a measure of genuine tripartite entanglement called the three tangle [13]:

$$\tau(\psi) = \mathcal{C}^2_{1(23)} - \mathcal{C}^2_{12} - \mathcal{C}^2_{13}, \tag{3}$$

which is invariant under all permutations of subsystem indices; and where the concurrence $\mathcal{C}^2_{1(23)}$ is bipartite entanglement between qubit 1 and qubits 2 and 3 jointly, and $\mathcal{C}^2_{12}$ is the concurrence of the reduced state ρ_{12} [13]. $\tau \geqslant 0$ characterizes the generalized GHZ state, whereas $\tau = 0$ for all the W-class states. Since τ is an entanglement monotone, hence the inequivalence [12]. The difference arises in the way QT allows the distribution of bipartite entanglement among the qubits (i.e., the concurrences are constrained by the monogamy inequality [13]):

$$\mathcal{C}^2_{1(23)} \geqslant \mathcal{C}^2_{12} + \mathcal{C}^2_{13}, \tag{4}$$

which is saturated by W-class states, while the difference $\mathcal{C}^2_{1(23)} - \mathcal{C}^2_{12} - \mathcal{C}^2_{13}$ is maximized by the GHZ-class states. This implies that the W-class states are determined by the concurrences of the three reduced states (modulo local unitaries); and in contrast, the GHZ-class states are fixed by the tangle, which decreases by the states residual concurrences [14].

B. Svetlichny's inequality

Let the measurements by observers be spin projections onto unit vectors: $A = \vec{\sigma}_1 \cdot \vec{a}$ or $A' = \vec{\sigma}_1 \cdot \vec{a}'$ on qubit 1, $B = \vec{\sigma}_2 \cdot \vec{b}$ or $B' = \vec{\sigma}_2 \cdot \vec{b}'$ on qubit 2, and $C = \vec{\sigma}_3 \cdot \vec{c}$ or $C' = \vec{\sigma}_3 \cdot \vec{c}'$ on the third qubit. If a theory is consistent with the hybrid nonlocal-local realism, then the quantum prediction for any three-qubit state $|\Psi\rangle$ is bounded by Svetlichny's inequality [9]:

$$|\langle\Psi|S|\Psi\rangle| \equiv S(\Psi) \leqslant 4, \tag{5}$$

where the Svetlichny's operator S is defined as

$$S = A(DC + D'C') + A'(D'C - DC') = M + M', \tag{6}$$

where $D = B + B'$ and $D' = B - B'$, and $\langle M\rangle \leqslant 2$ and $\langle M'\rangle \leqslant 2$ are Mermin's inequalities [15].

Note that S can be further simplified by defining $\vec{b} + \vec{b}' = 2\vec{d}\cos t$ and $\vec{b} - \vec{b}' = 2\vec{d}'\sin t$, which implies

$$\vec{d}\cdot\vec{d}' = \cos\theta_d\cos\theta_{d'} + \sin\theta_d\sin\theta_{d'}\cos(\phi_d - \phi_{d'}) = 0. \tag{7}$$

Now by setting $D = \vec{d}\cdot\vec{\sigma}_2$ and $D' = \vec{d}'\cdot\vec{\sigma}_2$, gives

$$\begin{aligned} S(\Psi) = 2|&\cos t\langle ADC\rangle + \sin t\langle AD'C'\rangle \\ &- \cos t\langle A'DC'\rangle + \sin t\langle A'D'C\rangle| \end{aligned} \tag{8}$$

$$\begin{aligned} S(\Psi) \leqslant 2|&\{\langle ADC\rangle^2 + \langle AD'C'\rangle^2\}^{\frac{1}{2}} \\ &+ \{\langle A'DC'\rangle^2 + \langle A'D'C\rangle^2\}^{\frac{1}{2}}|, \end{aligned} \tag{9}$$

where we have used the fact that

$$x\cos\theta + y\sin\theta \leqslant (x^2 + y^2)^{\frac{1}{2}}, \tag{10}$$

the equality results when $\tan\theta = y/x$. The following,

$$x\sin^2\theta + y\cos^2\theta \leqslant \begin{cases} y, & x \leqslant y \\ x, & x \geqslant y, \end{cases} \tag{11}$$

will be useful later; the first inequality is realized when $\theta = 0$, and $\theta = \pi/2$ gives the second. In the next two sections, we obtain the maximum value of the expectation value Svetlichny's operator, $S_{\max}(\psi)$, with respect to the GHZ-class states $|\psi_{gs}\rangle$ (1) and the W-class states $|\psi_w\rangle$ (2).

C. The GHZ-class states

Let $P = (1 - 2\sin^2\theta\sin^2\theta_3)$, $Q = (\sin^2\theta\sin 2\theta_3)$, $\cos\phi_{adc} = \cos(\phi_a + \phi_d + \phi_c)$, and $\cos\phi_{ad} = \cos(\phi_a + \phi_d)$, then the first term $\langle ADC\rangle$ in (9) with respect to $|\psi_{gs}\rangle$ can be expressed as

$$\begin{aligned} &\cos\theta_a\cos\theta_d\{P\cos\theta_c + Q\cos\phi_c\sin\theta_c\} + \{\sin 2\theta\sin\theta_a \\ &\quad\times\sin\theta_d\{\cos\theta_3\cos\phi_{ad}\cos\theta_c + \sin\theta_3\cos\phi_{adc}\sin\theta_c\}\}, \end{aligned} \tag{12}$$

which when maximized with respect to $(\phi_d - \phi_{d'})$ by using (7) and considering $\theta_{d'}$, ϕ_d, and $(\phi_d - \phi_{d'})$ to be independent variables, one obtains $(\phi_d - \phi_{d'}) = 0$ and $\theta_d = \frac{\pi}{2}$. The iterative maximization of the Mermin operator (6) using inequalities (10) and (11) is summarized below:

$$M = 2\{\langle ADC\rangle^2 + \langle AD'C'\rangle^2\}^{\frac{1}{2}} \tag{13}$$

$$\begin{aligned} &\leqslant 2\{\sin^2\theta_a\sin^2 2\theta\{(\cos\theta_3\cos\phi_{ad}\cos\theta_c + \sin\theta_3\cos\phi_{adc}\sin\theta_c)^2 + (\cos\theta_3\sin\phi_{ad}\cos\theta_{c'} + \sin\theta_3\sin\phi_{adc'}\sin\theta_{c'})^2\} \\ &\quad + \cos^2\theta_a(P\cos\theta_{c'} + Q\cos\phi_{c'}\sin\theta_{c'})^2\}^{\frac{1}{2}} \end{aligned} \tag{14}$$

$$\leqslant \begin{cases} 2\sin 2\theta\{(\cos\theta_3\cos\phi_{ad}\cos\theta_c + \sin\theta_3\cos\phi_{adc}\sin\theta_c)^2 + (\cos\theta_3\sin\phi_{ad}\cos\theta_{c'} + \sin\theta_3\sin\phi_{adc'}\sin\theta_{c'})^2\}^{\frac{1}{2}} \\ 2(P\cos\theta_{c'} + Q\cos\phi_{c'}\sin\theta_{c'}) \end{cases} \tag{15}$$

$$\leqslant \begin{cases} 2\sin 2\theta\{(\cos^2\theta_3\cos^2\phi_{ad} + \sin^2\theta_3\cos^2\phi_{adc}) + (\cos^2\theta_3\sin^2\phi_{ad} + \sin^2\theta_3\sin^2\phi_{adc'})\}^{\frac{1}{2}} \\ 2(P^2 + Q^2\cos^2\phi_{c'})^{\frac{1}{2}} \end{cases} \tag{16}$$

$$\leqslant \begin{cases} 2\sin 2\theta\sqrt{1 + \sin^2\theta_3} \\ 2\sqrt{P^2 + Q^2} = 2(1 - \sin^2 2\theta\sin^2\theta_3)^{\frac{1}{2}}, \end{cases} \tag{17}$$

Maximization is over $\theta_{d'}$ in (14), θ_a in (15), and θ_c and $\theta_{c'}$ in (16). Equations (18) and (19) are a particular instance of the constraints that have to be satisfied for the top and bottom inequalities in (17), respectively,

$$\begin{cases} \theta_{d'} = \frac{\pi}{2}; \quad \theta_a = \frac{\pi}{2}; \quad \theta_c = \theta_3; \quad \theta_{c'} = \frac{\pi}{2} \\ \phi_{ad} = 0; \quad \phi_{adc} = 0; \quad \phi_{adc'} = \frac{\pi}{2} \end{cases} \tag{18}$$

$$\theta_{d'} = 0; \quad \theta_a = 0; \quad \theta_c = \frac{\pi}{2}; \quad \phi_{c'} = 0. \tag{19}$$

By symmetry in (9), M' is obtained by taking $A \leftrightarrow A'$ and $C \leftrightarrow C'$, and satisfying similar constraints to (18) and (19). More importantly, both sets of constraints can be *matched*; this implies that as far as the GHZ-class states is concerned, SI reduces to Mermins' inequality, modulo the constant value of 2, which ensures that the violation of SI is sufficient to detect genuine tripartite nonlocality.

Equation (17) implies that $|\psi_{gs}\rangle$ is

$$S_{\max}(\psi_{gs}) = \begin{cases} 4\sqrt{1-\tau}, & 3\tau + C_{12}^2 \leqslant 1 \\ 4\sqrt{C_{12}^2 + 2\tau}, & 3\tau + C_{12}^2 \geqslant 1, \end{cases} \tag{20}$$

where, as discussed earlier, the entanglement of $|\psi_{gs}\rangle$ is fixed by its tangle:

$$\tau(\psi_{gs}) = \sin^2 2\theta \sin^2 \theta_3, \tag{21}$$

and the residual concurrence of $\mathrm{tr}_3(|\psi_{\rm gs}\rangle\psi_{\rm gs}|) = \rho_{12}$:

$$C_{12}^2(\psi_{gs}) = \sin^2 2\theta \cos^2 \theta_3, \tag{22}$$

and, $C_{23} = C_{31} = 0$.

For instance, the first equality in (20) can be achieved by setting the measurement unit vectors as $\vec{a} = \hat{x}, \vec{a}' = \hat{y}, \vec{b} = \hat{x}\cos t - \hat{y}\sin t, \vec{b}' = \hat{x}\cos t + \hat{y}\sin t, \vec{c} = \hat{z}\cos\theta_3 + \hat{x}\sin\theta_3$, and $\vec{c}' = \hat{y}$; and the set $\vec{a} = \hat{z}, \vec{a}' = \hat{z}, \vec{b} = \hat{x}\cos t + \hat{z}\sin t, \vec{b}' = \hat{x}\cos t - \hat{z}\sin t, \vec{c} = \hat{x}$, and $\vec{c}' = \hat{x}$, where $\tan t = \sin\theta_3$, attains the second in (20). The behavior of $S_{\max}(\psi_{gs})$ as a function of tripartite entanglement of $|\psi_{gs}\rangle$ is surprising (see Fig. 1). When the state is tri-separable, $\tau = C_{12} = 0$, or bi-separable, $\tau = 0, 0 < C_{12} \leqslant 1$, then as expected $S_{\max}(\psi_{gs}) = 4$. In the region where the entanglement of $|\psi_{gs}\rangle$ satisfy $3\tau + C_{12}^2 \leqslant 1$, as the entanglement increases $S_{\max}(\psi_{gs})$ monotonically decreases below the value of 4 (this was also noted in Ref. [11]). The converse happens in the regime where $3\tau + C_{12}^2 \geqslant 1$ and the value of $S_{\max}(\psi_{gs})$ starts monotonically increasing as the entanglement increases; however, only when $C_{12}^2 + 2\tau \geqslant 1$ do the states violate SI. Note in the latter region one expects the residual bipartite entanglement C_{12}^2 to decrease the maximum value, instead of increasing it.

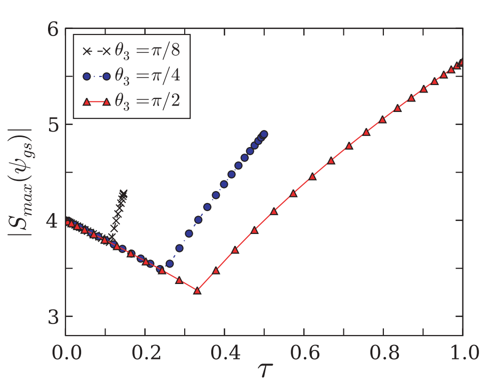

FIG. 1. (Color online) Maximum of the Svetlichny operator for varying tangle τ, for three values of $\theta_3 = \{\pi/8, \pi/4, \pi/2\}$.

D. The W-class states

For the W-class states it is convenient to obtain $S_{\max}(\psi_w)$ by simply adding all the eight terms involved in the Svetlichny's operator S, as all the terms in S contribute differently. This, unlike the GHZ class, makes SI significantly different from Mermin's inequality for the W-class states. Let $\cos\phi_{dc} = \cos(\phi_d - \phi_c)$, and likewise for similarly defined terms, then the term $\langle ABC\rangle$ in (6) with respect to $|\psi_w\rangle$ can be expressed as

$$\cos\theta_b(-\cos\theta_a\cos\theta_c + C_{31}\sin\theta_a\sin\theta_c\cos\phi_{ac}) + \sin\theta_b(C_{12}\cos\theta_a\sin\theta_c\cos\phi_{bc} + C_{23}\sin\theta_a\cos\theta_c\cos\phi_{ab}), \tag{23}$$

where $C_{12} = 2\alpha\beta, C_{23} = 2\beta\gamma, C_{31} = 2\gamma\alpha$ are the concurrences of the three reduced states of $|\psi_w\rangle$. Due to the inherent symmetry in (23), $S_{\max}(\psi_w)$ is achieved when all $\phi_i = 0$. Now adding all the terms (6), one obtains for the expectation of Mermin operator:

$$\begin{aligned}\langle M\rangle = \tfrac{1}{4}[&(-1 - C_{31} - C_{12} - C_{23})\{\cos(\theta_a + \theta_b + \theta_{c'}) \\ &+ \cos(\theta_{a'} + \theta_b + \theta_c) + \cos(\theta_a + \theta_{b'} + \theta_c) \\ &- \cos(\theta_{a'} + \theta_{b'} + \theta_{c'})\} + (-1 + C_{31} + C_{12} - C_{23}) \\ &\times\{\cos(\theta_a + \theta_b - \theta_{c'}) + \cos(\theta_{a'} + \theta_b - \theta_c) \\ &+ \cos(\theta_a + \theta_{b'} - \theta_c) - \cos(\theta_{a'} + \theta_{b'} - \theta_{c'})\} \\ &+ (-1 - C_{31} + C_{12} + C_{23})\{\cos(\theta_a - \theta_b + \theta_{c'}) \\ &+ \cos(\theta_{a'} - \theta_b + \theta_c) + \cos(\theta_a - \theta_{b'} + \theta_c) \\ &- \cos(\theta_{a'} - \theta_{b'} + \theta_{c'})\} + (-1 + C_{31} - C_{12} + C_{23}) \\ &\times\{\cos(\theta_a - \theta_b - \theta_{c'}) + \cos(\theta_{a'} - \theta_b - \theta_c) \\ &+ \cos(\theta_a - \theta_{b'} - \theta_c) - \cos(\theta_{a'} - \theta_{b'} - \theta_{c'})\}].\end{aligned}$$

In the same fashion, one can find the expression for $\langle M'\rangle$. The dependence on θ_i's can be suitably expressed by defining $\overline{\theta}_g = (\theta_g + \theta_{g'})/2, \widetilde{\theta}_g = (\theta_{g'} - \theta_g)/2, g\varepsilon\{a,b,c\}$. Allowing $\Sigma = (\overline{\theta}_a + \overline{\theta}_b + \overline{\theta}_c)$, and $\Sigma_g = \Sigma - 2\overline{\theta}_g$ one obtains,

$$\begin{aligned} S(\psi_w) = \tfrac{1}{2}\{&(-1 - C_{31} - C_{12} - C_{23})\sin(\overline{\theta}_a + \overline{\theta}_b + \overline{\theta}_c) \\ &\times\{G - 2\sin(\widetilde{\theta}_a - \widetilde{\theta}_b - \widetilde{\theta}_c)\} + (-1 + C_{31} + C_{12} \\ &- C_{23})\sin(\overline{\theta}_a + \overline{\theta}_b - \overline{\theta}_c)\{G - 2\sin(\widetilde{\theta}_a - \widetilde{\theta}_b + \widetilde{\theta}_c)\} \\ &+ (-1 - C_{31} + C_{12} + C_{23})\sin(\overline{\theta}_a - \overline{\theta}_b + \overline{\theta}_c) \\ &\times\{G - 2\sin(\widetilde{\theta}_a + \widetilde{\theta}_b - \widetilde{\theta}_c)\} + (-1 + C_{31} - C_{12} \\ &+ C_{23})\sin(\overline{\theta}_a - \overline{\theta}_b - \overline{\theta}_c)\{G - 2\sin(\widetilde{\theta}_a + \widetilde{\theta}_b + \widetilde{\theta}_c)\}\} \\ = \{&-\sin\Sigma + \sin\Sigma_a + \sin\Sigma_b + \sin\Sigma_c\} \\ &+ C_{13}\{\sin\Sigma + \sin\Sigma_a - \sin\Sigma_b + \sin\Sigma_c\} \\ &+ C_{12}\{\sin\Sigma - \sin\Sigma_a + \sin\Sigma_b + \sin\Sigma_c\} \\ &+ C_{23}\{\sin\Sigma + \sin\Sigma_a + \sin\Sigma_b - \sin\Sigma_c\} \end{aligned} \tag{24}$$

$$S(\psi_w) \equiv 4(p_1 + p_2 C_{13} + p_3 C_{12} + p_4 C_{23}), \tag{25}$$

$$\begin{aligned} G = \{&\sin(\widetilde{\theta}_a + \widetilde{\theta}_b + \widetilde{\theta}_c) + \sin(\widetilde{\theta}_a + \widetilde{\theta}_b - \widetilde{\theta}_c) \\ &+ \sin(\widetilde{\theta}_a - \widetilde{\theta}_b + \widetilde{\theta}_c) + \sin(\widetilde{\theta}_a - \widetilde{\theta}_b - \widetilde{\theta}_c)\}. \end{aligned} \tag{26}$$

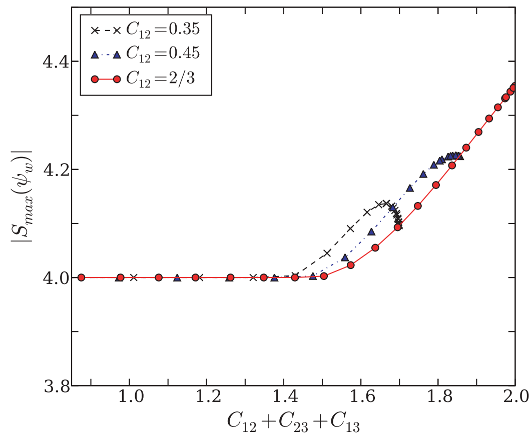

FIG. 2. (Color online) Maximum of the Svetlichny operator for varying sum $(C_{12}+C_{23}+C_{31}) \leqslant 2$, for three values of $C_{12} = \{0.35, 0.45, \frac{2}{3}\}$.

and where the second equality (24) is achieved when $\overline{\theta}_a = \overline{\theta}_b = \overline{\theta}_c = \pi/2$. By symmetry, the global maximum of $S_{\max}(\psi_w)$ occurs when $C_{31} = C_{12} = C_{23} = 2/3$, for which $\widetilde{\theta}_a = \widetilde{\theta}_b = \widetilde{\theta}_c = \widetilde{\theta}$. Then, $|S_{\max}(\psi_w)| = \sin 3\widetilde{\theta} + 5\sin\widetilde{\theta}$. The maximum occurs at $\widetilde{\theta} = 54.736^\circ$ giving $S_{\max}(\psi_w) = 4.354$ [15], which can be obtained when the measurement directions are $\vec{a} = \vec{b} = \vec{c} = \hat{x}\cos\widetilde{\theta} + \hat{z}\sin\widetilde{\theta}$, and $\vec{a}' = \vec{b}' = \vec{c}' = \hat{x}\cos\widetilde{\theta} - \hat{z}\sin\widetilde{\theta}$. As expected, $S_{\max}(\psi_w)$ is 4 for the tri-separable states [then only the first term survives in (25)], and for the bi-separable states when the first two terms ($C_{13} \neq 0$) survives in (25). For arbitrary tripartite entangled states (see Fig. 2), the only state which violates SI is when $(p_1 + p_2C_{13} + p_3C_{12} + p_4C_{23}) \geqslant 1$.

III. CONCLUSION

In this paper we explicitly quantified the genuine tripartite NL of the subclass of three-qubit pure states. We showed that by construction SI is a more suitable measure of genuine tripartite nonlocality for the *W*-class states, since SI for the GHZ-class states reduces to Mermin's inequality, and thus gives counter intuitive results. Moreover, we showed that a large number of states in both classes do not violate the inequality, which strongly suggests that the Svetlichny kind of hybrid local-nonlocal theory is too strong by assumption; therefore, in order to identify and quantify the genuine tripartite NL of entangled states one needs a new BTI which is *weaker* than SI, but stronger than Mermin's inequality.

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