Linear optical quantum computation with imperfect entangled photon-pair sources and inefficient non–photon-number-resolving detectors

Yan-Xiao Gong,[1,2,*] Xu-Bo Zou,[2,†] Timothy C. Ralph,[3] Shi-Ning Zhu,[1] and Guang-Can Guo[2]

[1]*National Laboratory of Solid State Microstructures and Department of Physics,*
Nanjing University, Nanjing, 210093, People's Republic of China

[2]*Key Laboratory of Quantum Information, University of Science and Technology of China, CAS,*
Hefei, 230026, People's Republic of China

[3]*Centre for Quantum Computer Technology and Physics Department, University of Queensland,*
Queensland 4072, Brisbane, Australia

We propose a scheme for efficient cluster state quantum computation by using imperfect polarization-entangled photon-pair sources, linear optical elements, and inefficient non-photon-number-resolving detectors. The efficiency threshold for loss tolerance in our scheme requires the product of source and detector efficiencies should be $> 1/2$, the best known figure. This figure applies to uncorrelated loss. We further find that the loss threshold is unaffected by correlated loss in the photon pair source. Our approach sheds new light on efficient linear optical quantum computation with imperfect experimental conditions.

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I. INTRODUCTION

Linear optical quantum computation (LOQC) has been known to be possible since Knill, Laflamme, and Milburn (KLM) proposed a scheme by employing measurement-induced nonlinearities [1]. Although scalable in principle, their scheme requires an unacceptably large number of operations. Much progress has been made in recent years to simplify LOQC [2]. One of the approaches is cluster state quantum computation [3,4]. This model, differing from the circuit computation model, starts from an entangled state, known as a cluster state, and requires only adaptive single-qubit measurements and feed-forward operations. This model has been the subject of numerous LOQC investigations, such as proposals for scalable cluster state generation [5–9], experimental realizations of small-scale cluster quantum computation [10–15], and fault-tolerant thresholds [16–19].

Error correction is essential for large-scale quantum computation. A major error source in LOQC is photon loss, arising from imperfect photon sources, inefficient photon detectors and nonideal optical circuits. For the circuit computation model, a loss error threshold of between 1.78% and 11.5% was found using the seven-qubit Calderbank-Shor-Steane (CSS) code [20] and up to 18% with parity encoding [21]. Thresholds for loss are considerably higher than for errors such as bit flips because loss in LOQC is a locatable error that removes the qubit rather than changing its logical value. Even higher loss thresholds have been found for cluster-state quantum computation. Recently, by building tree-cluster states, and assuming the loss error is in an independent degraded (ID) form, i.e., each qubit in the cluster state has the same and uncorrelated loss error rate, Vanarva, Browne, and Rudolph proposed a scheme [22] for loss-tolerant LOQC by using imperfect single-photon sources, inefficient photon-number-resolving detectors, and linear optical elements, providing the product of the detector efficiency and source efficiency greater than 2/3. This is the current highest known loss threshold for any LOQC scheme. In this article, we consider a different technique for building the tree clusters that removes the requirement for photon-number-resolving detectors and leads to an improved loss threshold of 1/2. Furthermore, the ID source model considered in Ref. [22] assumes no correlated loss occurs for the sources of entangled photons. This would seem unlikely to be true as for a generic deterministic model any loss in the pumping process of the entangled state source will lead to a correlated two-photon loss in the output. It is usually assumed that such correlated noise will be detrimental. Here, in contrast, we find that our construction method is fail safe to such correlated noise. We consider a situation in which both correlated and uncorrelated loss errors occur to a polarization-entangled photon-pair source. We find that thresholds are strongly improved compared with that in Ref. [22]. In particular, if no uncorrelated loss occurs to either photon, we obtain that the detector efficiency is allowed to be as small as a value larger than 1/2.

II. A SCHEME FOR LINEAR OPTICAL QUANTUM COMPUTATION

The mixed state from the inefficient source is assumed to be in the form

$$\rho_s = (1 - \eta_s - \eta_a - \eta_b)|\text{vac}\rangle\langle\text{vac}| + \eta_s|\Phi^+\rangle_{a,b}\langle\Phi^+|_{a,b}$$
$$+ \frac{\eta_a}{2}(|H\rangle_a\langle H|_a + |V\rangle_a\langle V|_a)$$
$$+ \frac{\eta_b}{2}(|H\rangle_b\langle H|_b + |V\rangle_b\langle V|_b), \tag{1}$$

with the target entangled state

$$|\Phi^+\rangle_{a,b} = \frac{1}{\sqrt{2}}(|HH\rangle_{a,b} + |VV\rangle_{a,b}), \tag{2}$$

*gongyanxiao@gmail.com

†xbz@ustc.edu.cn

where η_s, η_a, η_b represent the rates of emitting a photon-pair, only one photon in beam a, b, respectively, with $|H\rangle$ ($|V\rangle$) denoting the horizontal (vertical) polarization state and $|vac\rangle$ the vacuum state. Note that, the source model we consider has the property that it emits one and only one photon-pair, one or both of which may be lost with some probability. As in Ref. [22], we do not consider the possibility of higher photon number emission, that means, the source we consider does not emit more than two photons once it emits photons.

Another way to understand the source is in terms of loss error rate. A perfect Bell state given by Eq. (2) is assumed to suffer a correlated loss rate f_c and uncorrelated loss rates f_a and f_b in modes a and b, respectively. Then the state given by Eq. (1) can be equivalently rewritten under the following relations

$$\eta_s = (1 - f_c)(1 - f_a)(1 - f_b),$$
$$\eta_a = (1 - f_c)(1 - f_a)f_b, \tag{3}$$
$$\eta_b = (1 - f_c)(1 - f_b)f_a.$$

The photon detectors we consider are the realistic detectors commonly used in photonic experiments. The detector cannot resolve the number of photons detected but instead tell us whether photons exist in a detection event with nonunit probability η_d. The dark count of this kind of detector is usually very low and hence here is neglected. The POVM describing a non-photon-number-resolving detector is [23]

$$E_{\text{click}} = \sum_{n=0}^{\infty} [1 - (1 - \eta_d)^n]|n\rangle\langle n|, \tag{4}$$

$$E_{\text{off}} = \sum_{n=0}^{\infty} (1 - \eta_d)^n |n\rangle\langle n|. \tag{5}$$

According to the strategy for creating tree clusters in Ref. [22], we first need to prepare a four-photon GHZ state. Our scheme for generating a four-photon polarization-entangled GHZ state using four photon-pair sources is shown in Fig. 1, which can be regarded as two steps. The first step is fusing two sources with a polarizing beam splitter (PBS) followed by a photon detection at one output port in the basis of $+/-$. Here $+$ and $-$ denote $45°$ and $-45°$ polarization bases, respectively. For example, based on successful detections at $D_{1c}(D_{1c}^+, D_{1c}^-)$, the unnormalized state in modes $1a$, $2a$, and $1d$ can be written as

$$\frac{1}{2}\eta_s^2\eta_d|\text{GHZ}_3\rangle_{1a,2a,1d}\langle\text{GHZ}_3|_{1a,2a,1d}$$
$$+ \frac{1}{4}\eta_s\eta_b\eta_d(|HH\rangle_{1a,1d}\langle HH|_{1a,1d} + |VV\rangle_{1a,1d}\langle VV|_{1a,1d}$$
$$+ |HH\rangle_{2a,1d}\langle HH|_{2a,1d} + |VV\rangle_{2a,1d}\langle VV|_{2a,1d})$$
$$+ \frac{1}{4}\eta_b^2\eta_d(|H\rangle_{1d}\langle H|_{1d} + |V\rangle_{1d}\langle V|_{1d}) + \cdots, \tag{6}$$

where

$$|\text{GHZ}_3\rangle_{1a,2a,1d} = \frac{1}{\sqrt{2}}(|HHH\rangle_{1a,2a,1d} + |VVV\rangle_{1a,2a,1d}). \tag{7}$$

Here, for simplicity, we neglect the amplitudes containing no photon in beam $1d$. Note that, if the detector D_{1c}^- clicks a correction of σ_z operation on beam $1d$ ($1a$, or $2a$) is needed (analogous calculations can be found in Refs. [24,25]). It is
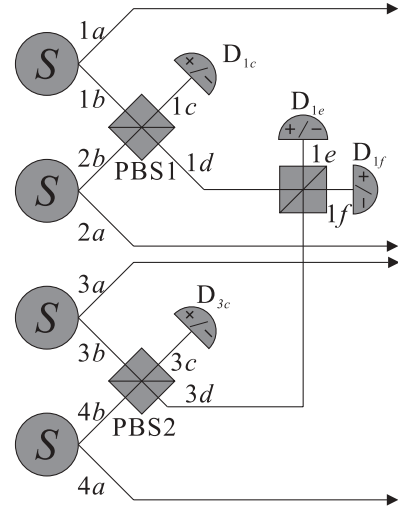

FIG. 1. Scheme for generating a four-photon GHZ state from four photon-pair sources (represented by S). Lowercase letters and numbers label the beams. Polarizing beam splitter (PBS) transmits horizontally polarized ($|H\rangle$) photons and reflects vertically polarized ($|V\rangle$) photons. $+/-$ denotes $\pm 45°$ polarization basis.

clear that the state in modes $3a$, $4a$, and $3d$ has the same form as Eq. (6).

The second step is fusing the two mixed states obtained in the first step with a PBS followed by a photon detection in each output port. Based on twofold coincidence detection at detectors $D1e(D1e^+, D1e^-)$ and $D1f(D1f^+, D1f^-)$, the normalized state in modes $1a$, $2a$, $3a$, and $4a$ is in the following form

$$\rho_r = (1 - \epsilon)^4 \rho_0 + (1 - \epsilon)^3\epsilon\rho_1 + (1 - \epsilon)^2\epsilon^2\rho_2$$
$$+ (1 - \epsilon)\epsilon^3\rho_3 + \epsilon^4|vac\rangle\langle vac|, \tag{8}$$

where ρ_0 is a perfect four-photon GHZ state,

$$|\text{GHZ}_4\rangle_{1a,2a,3a,4a}$$
$$= \frac{1}{\sqrt{2}}(|HHHH\rangle_{1a,2a,3a,4a} + |VVVV\rangle_{1a,2a,3a,4a}), \tag{9}$$

with ρ_1, ρ_2, and ρ_3 representing the mixed states evolved from a GHZ state by losing one, two, and three photons, respectively. Explicitly,

$$\rho_1 = \sigma_{1a,2a,3a} + \sigma_{1a,2a,4a} + \sigma_{1a,3a,4a} + \sigma_{2a,3a,4a},$$
$$\rho_2 = \varrho_{1a,2a} + \varrho_{1a,3a} + \varrho_{1a,4a} + \varrho_{2a,3a} + \varrho_{2a,4a} + \varrho_{3a,4a},$$
$$\rho_3 = \varsigma_{1a} + \varsigma_{2a} + \varsigma_{3a} + \varsigma_{4a}, \tag{10}$$

where

$$\sigma_{i,j,k} = \frac{1}{2}(|HHH\rangle_{i,j,k}\langle HHH|_{i,j,k} + |VVV\rangle_{i,j,k}\langle VVV|_{i,j,k}),$$
$$\varrho_{i,j} = \frac{1}{2}(|HH\rangle_{i,j}\langle HH|_{i,j} + |VV\rangle_{i,j}\langle VV|_{i,j}),$$
$$\varsigma_i = \frac{1}{2}(|H\rangle_i\langle H|_i + |V\rangle_i\langle V|_i), \tag{11}$$

with $i, j, k = 1a, 2a, 3a, 4a$.

From the above four equations, we can see the resource state produced, ρ_r, has an ID form as in Ref. [22], with the ID loss probability

$$\epsilon = \frac{\eta_b}{\eta_b + \eta_s} = f_a. \quad (12)$$

The success probability to obtain ρ_r is

$$P_{\text{success}} = \frac{1}{8}\eta_d^4(\eta_b + \eta_s)^4 = \frac{1}{8}\eta_d^4(1 - f_c)^4(1 - f_b)^4. \quad (13)$$

Note that when calculating the success probability we have considered the feed-forward corrections depending on different detection results (for details on analogous calculations, see Refs. [24,25]).

Next, with ρ_r as a resource, using the method introduced by Varnava-Browne-Rudolph [22] (see Fig. 1 in Ref. [22]), arbitrarily large tree-cluster states can be built with local Hadamard gates and type II fusion gates. Therefore, based on Varnava-Browne-Rudolph's error threshold [26], to realize loss-tolerant LOQC we require $(1 - \epsilon)\eta_d > 1/2$, which implies $\eta_s\eta_d/(\eta_b + \eta_s) > 1/2$. If we define a ratio $\kappa \equiv \eta_b/\eta_s$, then it is clear that the requirement becomes $\eta_d > (\kappa + 1)/2$, which makes sense when $\kappa < 1$, i.e., $\eta_b < \eta_s$. A consequent result is that the threshold for the detector improves as κ decreases and a max value is $1/2$ when κ equals 0. In the perspective of loss error rate, the requirement becomes

$$(1 - f_a)\eta_d > \frac{1}{2}, \quad (14)$$

which makes sense as long as $f_a < 1/2$. Note that not only is the requirement better than the previous threshold of 2/3 but also that it does not depend at all on any correlated loss or loss in arm b of the source. Moreover, as shown in Fig. 2, the detector efficiency requirement gets looser as f_a become smaller. If $f_a = 0$, i.e., photon a does not suffer any uncorrelated loss, we get the loosest requirement, $\eta_d > 1/2$. Note that f_c and f_b are only required to be smaller than 1 to make sure $P_{\text{success}} > 0$.

To further compare our threshold with that obtained in Ref. [22], recall that their threshold is $\eta_s\eta_d > 2/3$, implying that $\eta_d > 2/3$ is necessary. While in our scheme, in the case of $\kappa < 1/3$ or equivalently $f_a < 1/4$, the threshold for the detector is certainly improved compared with their threshold.
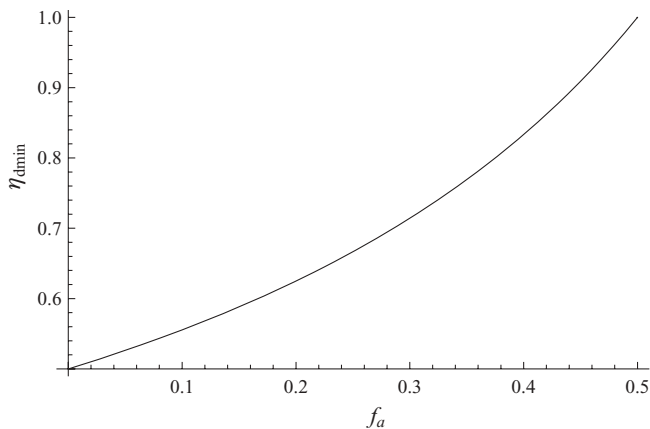

FIG. 2. Threshold of the detection efficiency ($\eta_{d\min}$) against the uncorrelated loss rate f_a in mode a. The detection efficiency η_d requires larger than $\eta_{d\min}$.

Moreover, photon-number-resolving detectors required in their scheme are not necessary in our scheme.

Consider the case of $\eta_b = 0$ or, equivalently, $f_a = 0$, in which we have the best threshold as just discussed. From Eq. (8) we can see that in this case we can produce a pure four-photon polarization-entangled GHZ state heralded by a fourfold coincidence detection at detectors $D1c$, $D3c$, $D1e$, and $D1f$. As the type II fusion gate is inherently loss tolerant and does not require photon-number-resolving detectors [5,22], we can create an arbitrarily large heralded pure tree-cluster state. Hence, our scheme also provides a way to produce "event-ready" multiphoton polarization-entangled state [27–29].

Equation (12) shows that the ID loss rate is exactly the uncorrelated loss occurring to the photon in mode a. This implies that this uncorrelated loss f_a can be shifted to the detector. In this respect, we get an equivalent circuit in which the sources have no uncorrelated loss in either photon and the detectors have an effective efficiency η_d' satisfying $\eta_d' = (1 - f_a)\eta_d$. Consequently, under detection with these "new" detectors, pure GHZ states and tree-cluster states can be produced and the threshold is obtained as $\eta_d' > 1/2$. Even in the absence of correlated loss this is a superior threshold to that of Vanarva et al. [22].

A surprising result is that only the ratio of the generation probability of photon a to that of the photon-pair contributes to the threshold requirement. From the loss error perspective, only the loss rate of either photon affects the threshold. This result may facilitate the experimental realization, as the loss rates on each arm are usually not equal, and thus, to achieve a better threshold for the detector, we can select the modes to satisfy $\eta_b \leqslant \eta_a$ or, equivalently, $f_a \leqslant f_b$.

III. CONCLUSIONS AND DISCUSSIONS

Finally, we would like to make some comments on the photon-pair sources we employ. Our source is a bit stricter than the single-photon source usually assumed in LOQC and has not been realized in experiment yet. However, entangled photon pairs are not only a vital resource in quantum optics [30] but also of importance in quantum information processing [31]. Much progress has been made in generating polarization-entangled photon pairs with the required characteristics, for example, in quantum dot systems [32–37]. We believe that the proof-in-principle experimental demonstration of our scheme is possible with current experimental conditions. In addition, from our analysis above, we can see that the photon-pair sources in some sense relax the requirement of the photon detectors on number resolving ability and efficiency. From this point of view, our assumption on the source should make sense, as at present photon-number-resolving detectors are more challenging than non-photon-number-resoling detectors. We guess that the source may also find applications in some other LOQC model, for example, in parity-encoded quantum computation [38,39].

In summary, we have presented a scheme for loss-tolerant LOQC by using imperfect entangled photon-pair sources, inefficient non-photon-number-resolving detectors and linear optical elements. We obtained a better efficiency threshold for loss tolerance of 1/2 compared to the current best threshold

of 2/3. We have also discussed the roles of correlated and uncorrelated loss errors in the threshold for the photon detector. Our approach opens a door to efficient LOQC with imperfect experimental conditions.

Recently, we became aware of a related article [40], in which the authors proposed to prepare a three-photon GHZ state using a similar approach to ours. Since building tree-cluster states requires four-photon GHZ states as resource states, they have to fuse two three-photon GHZ states to obtain a four-photon GHZ state with type II fusion gates, and, therefore, our scheme should be more economical and more efficient.

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