Single-particle machine for quantum thermalization

Jie-Qiao Liao (廖洁桥), H. Dong (董辉), and C. P. Sun (孙昌璞)
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
(Received 28 April 2009; revised manuscript received 11 April 2010; published 28 May 2010)

The long time accumulation of the *random* actions of a single particle "reservoir" on its coupled system can transfer some temperature information of its initial state to the coupled system. This dynamic process can be referred to as a quantum thermalization in the sense that the coupled system can reach a stable thermal equilibrium with a temperature equal to that of the reservoir. We illustrate this idea based on the usual micromaser model, in which a series of initially prepared two-level atoms randomly pass through an electromagnetic cavity. It is found that, when the randomly injected atoms are initially prepared in a thermal equilibrium state with a given temperature, the cavity field will reach a thermal equilibrium state with the same temperature as that of the injected atoms. As in two limit cases, the cavity field can be cooled and "coherently heated" as a maser process, respectively, when the injected atoms are initially prepared in ground and excited states. Especially, when the atoms in equilibrium are driven to possess some coherence, the cavity field may reach a higher temperature in comparison with the injected atoms. We also point out a possible experimental test for our theoretical prediction based on a superconducting circuit QED system.

DOI: 10.1103/PhysRevA.81.052121 PACS number(s): 03.65.Yz, 05.30.−d, 85.25.−j

I. INTRODUCTION

A small system in contact with a large reservoir (or so-called heat bath) in thermal equilibrium of temperature T will dynamically approach an equilibrium state with the same temperature T [1]. This irreversible process from a nonequilibrium state into a stable one is conventionally referred to as quantum thermalization. Most recently, another kind of thermalization, called canonical thermalization (e.g., Refs. [2–5]), investigated in the meaning of typicality that almost all pure states in the universe (the system plus its bath) are entangled, and thus the system can reach an approximately canonical thermal state by averaging over the bath. Here, the temperature appears as an "emergent" concept.

In conventional thermalization, the heat bath consists of a very large number of degrees of freedom (for example, a set of harmonic oscillators for the bosonic heat bath), and the coupling strengths of the thermalized system with the degrees of freedom of its bath are *randomly* distributed. According to the viewpoint in statistical mechanics that an average over an ensemble is equivalent to the time average in some sense [6], a natural question is if a series of *random* actions of a single-particle "reservoir" injected randomly in a time domain can transfer some temperature information of its initial state to the coupled system at a steady state as a thermalization process? To answer this question, in this paper we study the steady state of a quantum system which is controlled to have a randomly "multipulse" type interaction with a single-particle system initially prepared in thermal equilibrium with a temperature. If the steady state of the quantum state is a thermal one with the same temperature as that of the single-particle system, we think that this quantum system has been thermalized by the single-particle system through a randomly "multipulse" type interaction.

Since the randomly "multipulse" type interaction can be realized by random injections, in this paper we will illustrate our idea based on the usual micromaser model (e.g., Refs. [7–20]), in which a series of initially prepared atoms pass through an electromagnetic cavity. Here, the single-mode cavity field is the system to be thermalized and the randomly injected atoms play the role of the single-particle reservoir [21]. Under some conditions we will clarify, if the injected atoms are initially prepared in thermal equilibrium, that the conventional thermalization enables the cavity field to transit from any initial state to a thermal state with the same temperature as that of the atoms. We also find that the temperature of the cavity field in thermal equilibrium depends on the initial state of the injected atoms. As in two limit cases, such quantum thermalization can describe the cooling [22] and masering processes [7–20,23], which respectively correspond to the cases where the injected atoms are initially prepared in ground and excited states.

It is worth noting that when the atoms initially possess some quantum coherence [24,25], the "thermalized state" of the cavity field will carry the information of this coherence. Actually, quantum coherence has been proved to be a kind of resource to enhance quantum information processing. Most recently, some studies have shown that physical processes with quantum coherence usually possess some novel effect for energy transfer [26,27]. For example, quantum heat engines using quantum matter (and even with the assistance of Maxwell's demon) as a working substance can improve work extraction as well as the working efficiency in the thermodynamics cycle [28–30]. In the present study, it is expected that, when the injected two-level atoms possess some coherence in some situations, the cavity field will reach a steady state with higher temperature than that for the incoherent case.

Though we calculate the steady-state photon number in the cavity of the micromaser, we still emphasize that the motivation of this paper is not to simply study the statistical properties of the cavity field, but to study the quantum thermalization of a quantum system randomly coupled to a series of single-particle reservoirs in a time domain. Therefore our present work is different from other previous papers on quantum statistical properties of a micromaser (e.g., Refs. [7–18]). Here we employ the micromaser model only for convenience. The micromaser involves the process of a quantum system (the single-mode cavity field) randomly coupled with a series of single-particle reservoirs (these injected atoms). In other words, the micromaser model is the platform to show our idea of thermalization. More importantly, we focus on the *temperature* of the cavity field at a steady state. If the temperature of the cavity field at a steady state is equal to that of the injected atoms, we consider that the cavity field of the micromaser has been thermalized by these injected atoms. For the case of random injections, the steady state of the cavity field can be naturally identified as a thermal state with the same temperature as that of the injected atoms.

In addition, from the viewpoint of experimental implementation, this paper also provides a possibility for the examination of thermodynamics with a cavity QED system. As we know, the cavity QED system has became a mature candidate for the implementation of experiments in quantum physics and quantum information processing [31]. Therefore the present work can also be considered an example for the experimental examination of thermodynamics with a cavity QED system.

The paper is organized as follows. In Sec. II we present our thermalization model of a single-mode cavity field interacting with a series of atoms injected randomly. A quantum master equation is derived to describe the dynamics of the single-mode cavity field. In Sec. III we show that the present quantum thermalization model can give a unified description of cooling, masering, and thermalization processes. In Sec. IV we study the quantum thermalization when the initial state of the injected two-level systems possesses some quantum coherence. In Sec. V we propose an experimental implementation of our quantum thermalized model with superconducting circuit-QED. We also show that the dynamics of the cavity field in the micromaser is equivalent to the dynamics of the transmission line resonator in the circuit QED. Finally, we conclude this paper with some discussions in Sec. VI.

II. CAVITY QED MODEL FOR THERMALIZATION WITH SINGLE-PARTICLE RESERVOIR

The cavity QED model [as illustrated in Fig. 1(a)] for thermalization contains a single-mode cavity field of frequency ω and a series of injected two-level systems (TLSs) with excited state $|e\rangle$, ground state $|g\rangle$, and energy separation ω_0. These TLSs pass through the single-mode cavity one by one randomly. Here, the single-mode cavity field is considered as

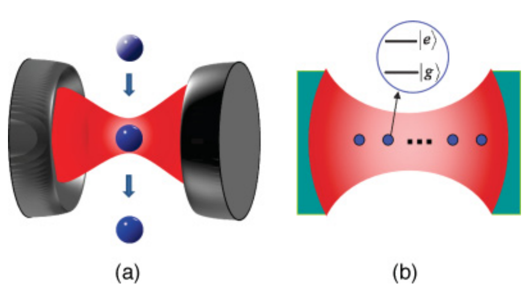

FIG. 1. (Color online) (a) Schematic diagram of our single particle thermalization model that a series of prepared TLSs randomly pass through a single-mode cavity one by one, in equilibrium it is equivalent to the conventional reservoir model, (b) where many identical atoms with spatially random distribution thermalize the single-mode cavity field.

the system to be thermalized, while the TLSs are considered as the single particle reservoir. The injections of the TLSs into the cavity are *random* and there is a limit of one TLS in the cavity each time. According to the viewpoint in statistical mechanics, the average over an ensemble is equivalent to the time average in some sense. It is expected that the single-model cavity field will approach a steady state equilibrium with a temperature as that of the injected atoms, since this system is equivalent to the conventional thermalization model, as shown in Fig. 1(b), where many identical atoms (reservoir) with spatially random distribution thermalize the single-mode cavity field.

A single TLS interacting with the single-mode cavity field is described by the Jaynes-Cummings (JC) Hamiltonian

$$\hat{H} = \frac{\omega_0}{2}\hat{\sigma}_z + \omega\hat{a}^\dagger\hat{a} + g(\hat{a}\hat{\sigma}_+ + \hat{\sigma}_-\hat{a}^\dagger), \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are, respectively, the annihilation and creation operators of the single-mode cavity field, they satisfy the usual bosonic commutation relation $[\hat{a},\hat{a}^\dagger] = 1$. Hereafter we set $\hbar = 1$. The operators of the TLS are defined as

$$\hat{\sigma}_+ = \hat{\sigma}_-^\dagger = |e\rangle\langle g|, \quad \hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|. \tag{2}$$

The parameter g is the coupling strength of the cavity field with a TLS.

In the rotating picture with respect to

$$\hat{H}_0 = \frac{\omega}{2}\hat{\sigma}_z + \omega\hat{a}^\dagger\hat{a}, \tag{3}$$

the Hamiltonian becomes

$$\hat{V}_I = \frac{\delta}{2}\hat{\sigma}_z + g(\hat{a}\hat{\sigma}_+ + \hat{\sigma}_-\hat{a}^\dagger), \tag{4}$$

where

$$\delta \equiv \omega_0 - \omega \tag{5}$$

is the detuning of the cavity frequency ω with the energy separation ω_0 of the TLS. In the resonant case, namely $\delta = 0$, the unitary evolution operator governed by the Hamiltonian (4) of the cavity QED reads [9]

$$\hat{U}(\tau) \equiv \exp(-i\hat{V}_I\tau) = \begin{pmatrix} \cos(g\tau\sqrt{\hat{a}\hat{a}^\dagger}) & -i\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^\dagger})}{\sqrt{\hat{a}\hat{a}^\dagger}}\hat{a} \\ -i\hat{a}^\dagger\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^\dagger})}{\sqrt{\hat{a}\hat{a}^\dagger}} & \cos(g\tau\sqrt{\hat{a}^\dagger\hat{a}}) \end{pmatrix}, \tag{6}$$

which is written in the Hilbert subspace of the TLS with the basis states

$$|e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{7}$$

To thermalize the cavity field, a series of TLSs are randomly injected into the cavity for a fixed time interval τ. All of the TLSs are initially prepared in the density matrix

$$\hat{\rho}_{\text{TLS}} = p_e|e\rangle\langle e| + p_g|g\rangle\langle g| + \lambda|e\rangle\langle g| + \lambda^*|g\rangle\langle e|, \tag{8}$$

where λ is the parameter describing the coherence of the TLSs. The state preparation of the TLSs can be realized by using a pumping field to excite the TLSs. We assume that the jth TLS is injected into the cavity at time t_j. After an interaction of time τ, the state of the cavity field becomes

$$\hat{\rho}(t_j+\tau) = \mathrm{Tr}_{\mathrm{TLS}}[\hat{U}(\tau)\hat{\rho}(t_j)\otimes\hat{\rho}_{\mathrm{TLS}}U^{\dagger}(\tau)] \equiv \mathcal{M}(\tau)\hat{\rho}(t_j), \tag{9}$$

where $\mathrm{Tr}_{\mathrm{TLS}}$ means tracing over the degree of freedom of the TLS. The superoperator $\mathcal{M}(\tau)$ introduced in Eq. (9) can be expressed as follows:

$$\begin{aligned}\mathcal{M}(\tau)\hat{\rho}(t_j) =\; & p_e\cos(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})\hat{\rho}(t_j)\cos(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}}) + p_e\hat{a}^{\dagger}\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})}{\sqrt{\hat{a}\hat{a}^{\dagger}}}\hat{\rho}(t_j)\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})}{\sqrt{\hat{a}\hat{a}^{\dagger}}}\hat{a}\\ &+ p_g\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})}{\sqrt{\hat{a}\hat{a}^{\dagger}}}\hat{a}\hat{\rho}(t_j)\hat{a}^{\dagger}\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})}{\sqrt{\hat{a}\hat{a}^{\dagger}}} + p_g\cos(g\tau\sqrt{\hat{a}^{\dagger}\hat{a}})\hat{\rho}(t_j)\cos(g\tau\sqrt{\hat{a}^{\dagger}\hat{a}})\\ &+ i\lambda\cos(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})\hat{\rho}(t_j)\hat{a}^{\dagger}\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})}{\sqrt{\hat{a}\hat{a}^{\dagger}}} - i\lambda\hat{a}^{\dagger}\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})}{\sqrt{\hat{a}\hat{a}^{\dagger}}}\hat{\rho}(t_j)\cos(g\tau\sqrt{\hat{a}^{\dagger}\hat{a}})\\ &+ i\lambda^{*}\cos(g\tau\sqrt{\hat{a}^{\dagger}\hat{a}})\hat{\rho}(t_j)\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})}{\sqrt{\hat{a}\hat{a}^{\dagger}}}\hat{a} - i\lambda^{*}\frac{\sin(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}})}{\sqrt{\hat{a}\hat{a}^{\dagger}}}\hat{a}\hat{\rho}(t_j)\cos(g\tau\sqrt{\hat{a}\hat{a}^{\dagger}}).\end{aligned} \tag{10}$$

In fact, in addition to the action of the injected TLSs, the cavity inevitably couples with an external environment through the cavity wall. Within the quantum noise theory, we model the external environment of the cavity as a heat bath. When the coupling of the cavity field with the heat bath is weak, the decay of the cavity field can be described by [10]

$$\begin{aligned}\mathcal{L}\hat{\rho} =\; & \tfrac{1}{2}\kappa(\bar{n}_{\mathrm{th}}+1)(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a})\\ &+ \tfrac{1}{2}\kappa\bar{n}_{\mathrm{th}}(2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{a}\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^{\dagger}),\end{aligned} \tag{11}$$

where κ is the decay rate of the cavity. The thermal average photon number is

$$\bar{n}_{\mathrm{th}} = \frac{1}{e^{\beta_b\omega}-1} \tag{12}$$

with $\beta_b = 1/(k_B T)$ being the inverse temperature of the heat bath. Hereafter we denote $\hat{\rho}(t)$ as $\hat{\rho}$ to be concise.

Since the TLSs are injected at *random*, we can introduce a rate r of a Poisson process to depict the arrival of the TLSs. In a time interval of $(t, t+\delta t)$, the probability of a TLS arrival is $r\delta t$. Hence the density matrix of the cavity field at time $t+\delta t$ can be written as [32]

$$\hat{\rho}(t+\delta t) = (1-r\delta t)[\hat{\rho}(t) + \mathcal{L}\hat{\rho}(t)\delta t] + r\delta t\mathcal{M}(\tau)\hat{\rho}(t). \tag{13}$$

Here the first term on the right-hand side of Eq. (13) describes the density matrix of the cavity field at time $t+\delta t$ when a TLS does not pass through the cavity, with the probability $1-r\delta t$. In this case, the cavity field evolves under the action of $\mathcal{L}$. Additionally, the last term on the right-hand side of Eq. (13) describes the density matrix of the cavity field at time $t+\delta t$ for the case of a TLS passing through the cavity, with the probability $r\delta t$. Notice that here we approximately neglect the action of the heat bath on the cavity field during the process of the TLS passing through the cavity, since the time spent by each TLS in the cavity is assumed to be much shorter than the mean time between two injections of the TLSs.

Taking the limit of $\delta t \to 0$, we can obtain the following quantum master equation [9–16]:

$$\dot{\hat{\rho}} = r(\mathcal{M}(\tau)-1)\hat{\rho} + \mathcal{L}\hat{\rho}, \tag{14}$$

to describe the evolution of the single-mode cavity field.

III. UNIFICATION OF COOLING, MASERING AND THERMALIZATION

The evolution of the quantum state of the cavity field is governed by the master Eq. (14), which depends on the initial state of the injected TLSs. Firstly, we consider the case where no coherence exists in the initial state of the TLSs, i.e., $\lambda = 0$ in Eq. (8). In the Fock state representation, the evolution equation for the diagonal elements $P_n = \langle n|\hat{\rho}|n\rangle$ of the density matrix $\hat{\rho}$ in the master Eq. (14) becomes

$$\begin{aligned}\dot{P}_n =\; & rp_e[\cos^2(g\tau\sqrt{n+1})P_n + \sin^2(g\tau\sqrt{n})P_{n-1}]\\ &+ rp_g[\cos^2(g\tau\sqrt{n})P_n + \sin^2(g\tau\sqrt{n+1})P_{n+1}]\\ &- rP_n + \frac{\kappa\bar{n}_{\mathrm{th}}}{2}[2nP_{n-1} - 2(n+1)P_n]\\ &+ \frac{\kappa(\bar{n}_{\mathrm{th}}+1)}{2}[2(n+1)P_{n+1} - 2nP_n].\end{aligned} \tag{15}$$

Using the relation $p_e + p_g = 1$ and after some simple collection, the above Eq. (15) becomes

$$\begin{aligned}\dot{P}_n =\; & -r\sin^2(g\tau\sqrt{n+1})(p_eP_n - p_gP_{n+1})\\ &- \kappa(n+1)[\bar{n}_{\mathrm{th}}P_n - (\bar{n}_{\mathrm{th}}+1)P_{n+1}]\\ &+ r\sin^2(g\tau\sqrt{n})(p_eP_{n-1} - p_gP_n)\\ &+ \kappa n[\bar{n}_{\mathrm{th}}P_{n-1} - (\bar{n}_{\mathrm{th}}+1)P_n].\end{aligned} \tag{16}$$

The steady state solution $\dot{P}_n = 0$ leads to the detailed balance condition and the relation

$$\begin{aligned}& r\sin^2(g\tau\sqrt{n})(p_eP_{n-1} - p_gP_n)\\ &+ \kappa n[\bar{n}_{\mathrm{th}}P_{n-1} - (\bar{n}_{\mathrm{th}}+1)P_n] = 0.\end{aligned} \tag{17}$$

Then the ratio $R_n = P_n/P_{n-1}$ between two neighboring photon number populations is obtained as

$$R_n = \frac{rp_e \sin^2(g\tau\sqrt{n}) + \kappa\bar{n}_{\mathrm{th}}n}{rp_g \sin^2(g\tau\sqrt{n}) + \kappa(\bar{n}_{\mathrm{th}}+1)n}. \tag{18}$$

We can understand such thermalization to the steady state with the definite population ratio (18) as a temperature information transfer process from the TLSs to the cavity field, namely, the curve of $-(\ln R_n)/\omega$ can explicitly reflect the information of the temperature of the TLSs.

Such a temperature information transfer process can result in various coherent manipulations for quantum state engineering. An example is the cooling of the cavity field as a generalized thermalization for all injected TLSs initially prepared in the ground state, i.e., $p_e = 0$ and $p_g = 1$. In this case, the TLSs on the ground state will take away the energy of the cavity field and then cool it to reach a lower temperature defined by the decreased photon population

$$P_n = P_0 \prod_{l=1}^{n} \frac{\bar{n}_{\mathrm{th}} l}{(\bar{n}_{\mathrm{th}}+1)l + \sin^2(g\tau\sqrt{l})r/\kappa}, \tag{19}$$

where P_0 is determined by the normalization condition $\sum_{n=0}^{\infty} P_n = 1$. This generalized thermalization mechanism was even used to cool the nanomechanical resonator by the pulse-driven charge qubit [22]. Another example with $p_e = 1$ and $p_g = 0$ shows the maser processes of the cavity field [7–20,23], which is represented by the amplified photon population

$$P_n = P_0 \prod_{l=1}^{n} \frac{\sin^2(g\tau\sqrt{l})r/\kappa + \bar{n}_{\mathrm{th}} l}{(\bar{n}_{\mathrm{th}}+1)l}, \tag{20}$$

where P_0 is determined by the normalization condition $\sum_{n=0}^{\infty} P_n = 1$.

In the absence of the cavity field dissipation, i.e., $\kappa = 0$, Eq. (18) becomes $R_n = p_e/p_g$, which is irrespective of both the index n and the average injection rate r. For this case, the temperature information of the TLS is perfectly transferred to the cavity field. For example, when the TLS is initially prepared in the thermal equilibrium with temperature T, that is

$$p_e(T) = \frac{\exp(-\beta\omega/2)}{2\cosh(\beta\omega/2)}, \quad p_g(T) = \frac{\exp(\beta\omega/2)}{2\cosh(\beta\omega/2)}, \tag{21}$$

where $T = 1/(k_B\beta)$, then the population ratio $R_n = \exp(-\beta\omega)$ of the cavity field is independent of the index n, thus a thermal equilibrium has the same temperature T as that of the TLS.

We give a physical explanation about the steady state of the cavity field in the absence of the cavity decay. When $\kappa = 0$, the evolution of the cavity is governed by the following quantum master equation:

$$\dot{\hat{\rho}} = r(\mathcal{M}(\tau) - 1)\hat{\rho}. \tag{22}$$

In the short τ case, we can make the short time approximation,

$$\cos(g\tau\sqrt{\hat{a}\hat{a}^\dagger}) \approx 1 - (g\tau)^2\hat{a}\hat{a}^\dagger/2, \tag{23a}$$

$$\cos(g\tau\sqrt{\hat{a}^\dagger\hat{a}}) \approx 1 - (g\tau)^2\hat{a}^\dagger\hat{a}/2, \tag{23b}$$

$$\sin(g\tau\sqrt{\hat{a}\hat{a}^\dagger}) \approx g\tau\sqrt{\hat{a}\hat{a}^\dagger}. \tag{23c}$$

Up to the second order of τ, the master Eq. (22) becomes

$$\dot{\hat{\rho}} \approx \frac{\alpha p_g}{2}(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}) + \frac{\alpha p_e}{2}(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{\rho}), \tag{24}$$

where $\alpha = r(g\tau)^2$. Now, the injected TLSs are prepared in a statistical mixture of the excited and ground states. From the above Eq. (24), we can see that the TLSs prepared in an *excited* state *excite* the cavity at an effective rate αp_e, while the TLSs prepared in a *ground* state *take away* the energy excitation of the cavity field at an effective rate αp_g [33]. By comparing Eq. (24) with Eq. (11), we can see that the long time accumulation of the actions of the injected TLSs is equivalent to an effective heat bath with the inverse temperature

$$\beta_{\mathrm{eff}}^{\mathrm{th}} = -\frac{1}{\omega}\ln\frac{p_e}{p_g}. \tag{25}$$

Therefore, the cavity field can reach a steady state even in the absence of the cavity decay through the walls. It is of interest that the inverse temperature $\beta_{\mathrm{eff}}^{\mathrm{th}}$ of the effective heat bath of the cavity can be controlled by changing the populations p_e and p_g of the injected TLSs.

In the presence of the cavity field dissipation, i.e., $\kappa \neq 0$, generally, it is impossible to define a temperature for the cavity field in the steady state, since in this case the ratio R_n given by Eq. (18) depends on n. In Fig. 2, we plot R_n versus photon number n for different $g\tau$. Clearly, for small $g\tau$, R_n shows the independence of the photon number n. Therefore, it is possible to define an effective temperature for the cavity field when $g\tau$ is small.

In the short interaction time τ limit, i.e., $g\tau\sqrt{n} \ll 1$ for all experimental accessible photon numbers n, we make an approximation $\sin^2(g\tau\sqrt{n}) \approx (g\tau)^2 n$, which results in an n-independent population ratio

$$R = \frac{\alpha p_e + \kappa\bar{n}_{\mathrm{th}}}{\alpha p_g + \kappa(\bar{n}_{\mathrm{th}}+1)}. \tag{26}$$

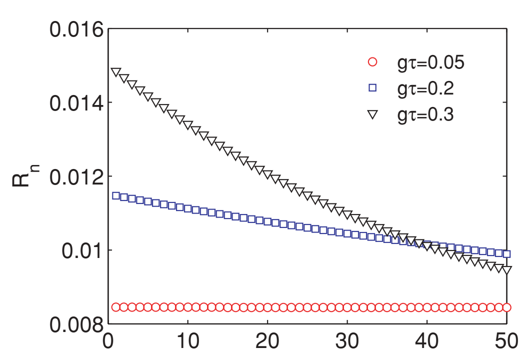

FIG. 2. (Color online) The ratio R_n versus the photon number n is plotted for $g\tau = 0.05, 0.2$, and 0.3. The injected TLSs are prepared in thermal equilibrium of inverse temperature $\beta = 2.898$ ($T = 200$ mK). Other parameters are set as $\omega = 1$, $\kappa/\omega = 10^{-4}$, $r = 2\times10^{-4}$, and $\bar{n}_{\mathrm{th}} = 0.008$ ($T_b = 100$ mK).

Thus for the TLS injection in thermal equilibrium, we can define an effective inverse temperature for the cavity field

$$\beta_{\rm eff} = -\frac{1}{\omega}\ln R, \tag{27}$$

which satisfies the relation

$$\min\{\beta_b,\beta\} < \beta_{\rm eff} < \max\{\beta_b,\beta\}. \tag{28}$$

It means that the cavity field will approach a thermal equilibrium with an intermediate inverse temperature $\beta_{\rm eff}$ between those for the TLSs and the heat bath. Additionally, for the case of $\beta_b = \beta$, the cavity field will approach a thermal equilibrium of $\beta_{\rm eff} = \beta$. This result is reasonable from the viewpoint of quantum noise. A system coupled with two heat baths with different temperatures will reach an equilibrium with intermediate temperatures between those of the two heat baths [34].

IV. QUANTUM COHERENCE ASSISTED THERMALIZATION

In the above section, we study the generalized thermalization for the incoherent case, in which the injected TLSs do not possess quantum coherence. In this section, we study the coherent case, i.e., $\lambda \neq 0$. In this case, up to the second order of τ, the master Eq. (14) can be reduced to

$$\dot{\hat\rho} \approx i[\hat\rho,\hat H_{\rm eff}] + \mathcal{J}\hat\rho \tag{29}$$

in the short τ limit, where the effective Hamiltonian reads

$$\hat H_{\rm eff} = \xi\hat a^\dagger + \xi^*\hat a, \tag{30}$$

with $\xi = rg\tau\lambda$. The superoperator $\mathcal{J}$ is defined as

$$\mathcal{J}\hat\rho = \tfrac{1}{2}\gamma_1(2\hat a^\dagger\hat\rho\hat a - \hat a\hat a^\dagger\hat\rho - \hat\rho\hat a\hat a^\dagger) + \tfrac{1}{2}\gamma_2(2\hat a\hat\rho\hat a^\dagger - \hat a^\dagger\hat a\hat\rho - \hat\rho\hat a^\dagger\hat a), \tag{31}$$

where we introduced two transition rates: the decay rate γ_1 and the excitation rate γ_2,

$$\gamma_1 = \alpha p_e + \kappa\bar n_{\rm th}, \tag{32a}$$

$$\gamma_2 = \alpha p_g + \kappa(\bar n_{\rm th}+1). \tag{32b}$$

During the derivation of the master Eq. (29), we have used the approximation given in Eqs. (23).

The above effective Hamiltonian $\hat H_{\rm eff}$ describes the role of the quantum coherence of the injected TLSs: the off-diagonal terms in the initial state offer nonvanishing atomic transition, which is added as a driving source of the cavity field. The generalized master Eq. (29) describes a driven cavity field in contact with an effective bath characterized by two rates. This effective bath consists of a TLS reservoir and a heat bath. It is worth pointing out that the properties of the TLS reservoir can be manipulated through changing the initial populations p_g and p_e.

From the master Eq. (29), we can obtain the following equation of motion of the average value of the creation, annihilation, and photon number operators:

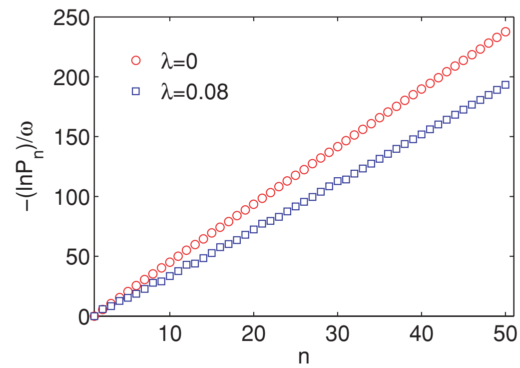

FIG. 3. (Color online) The $-(\ln P_n)/\omega$ versus the photon number n is plotted for both the coherent ($\lambda = 0.08$) and incoherent ($\lambda = 0$) cases. Here, $g\tau = 0.05$ and other parameters are set as those in Fig. 2.

$$\frac{d}{dt}\langle\hat a(t)\rangle = -\frac{1}{2}(\gamma_2-\gamma_1)\langle\hat a(t)\rangle - i\xi, \tag{33a}$$

$$\frac{d}{dt}\langle\hat a^\dagger(t)\rangle = -\frac{1}{2}(\gamma_2-\gamma_1)\langle\hat a^\dagger(t)\rangle + i\xi^*, \tag{33b}$$

$$\frac{d}{dt}\langle\hat n(t)\rangle = -(\gamma_2-\gamma_1)\langle\hat n(t)\rangle - i\xi\langle\hat a^\dagger(t)\rangle + i\xi^*\langle\hat a(t)\rangle + \gamma_1. \tag{33c}$$

The steady state solutions of the above equation are

$$\langle\hat a\rangle_{ss} = \langle\hat a^\dagger\rangle_{ss}^* = -\frac{2i\xi}{\gamma_2-\gamma_1}, \tag{34a}$$

$$\langle\hat n\rangle_{ss} = \frac{4|\xi|^2}{(\gamma_2-\gamma_1)^2} + \frac{\gamma_1}{\gamma_2-\gamma_1}. \tag{34b}$$

The above results show that the quantum coherence can increase the steady state average photon number [with the first term in Eq. (34b)] in the cavity field. When no TLS is injected, i.e., $r = 0$, the average photon number at steady state $\langle\hat n\rangle_{ss} = \bar n_{\rm th}$. On the other hand, for the case of thermal TLSs injection of inverse temperature β and a perfect cavity, i.e., $\kappa = 0$, the steady state average photon number $\langle\hat n\rangle_{ss} = 1/[\exp(\beta\omega)-1]$, which implies that the cavity approaches an equilibrium with the same temperature of the TLSs.

The above argument based on the short time approximation is only a heuristic analysis, thus we need to numerically solve the master Eq. (14) directly. In Fig. 3, we plot the $-(\ln P_n)/\omega$ versus the photon number n for the coherence and incoherence cases. It can be seen from Fig. 3 that the coherence in the initial state of the TLSs can increase the steady state temperature of the cavity. However, for the coherence case, the dependence of $-(\ln P_n)/\omega$ on the photon number n is approximately linear therefore it is an approximation to define an effective temperature for the cavity field.

V. EXPERIMENTAL IMPLEMENTATION WITH CIRCUIT QED SYSTEM

In this section, we present an experimental implementation of our quantum thermalization based on superconducting circuit QED system [35,36]. As shown in Fig. 4(a), a superconducting transmission line resonator (TLR) couples to

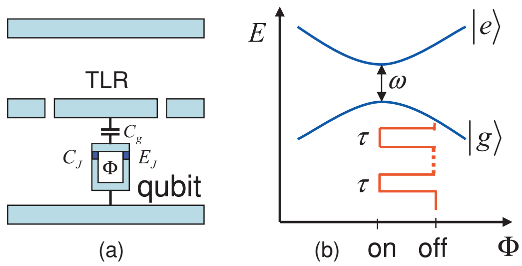

FIG. 4. (Color online) (a) Schematic diagram of the circuit QED system of a transmission line resonator (TLR) coupled with a charge qubit, which is controlled through an external magnetic flux Φ and a gate voltage V. (b) The energy levels of the charge qubit versus the magnetic flux Φ, the working status of the qubit is controlled by the magnetic flux "pulse series". By tuning the magnetic flux Φ, we can switch on or off the coupling between the TLR and the charge qubit.

a superconducting charge qubit. After the quantization of the electromagnetic field in the TLR, the Hamiltonian describing a single-model field in the TLR reads [35]

$$\hat{H}_{\mathrm{TLR}} = \omega\hat{a}^{\dagger}\hat{a}, \tag{35}$$

where ω is the resonant frequency of this mode. Here we only choose the single mode which is (near) resonant with the lowest two levels of the superconducting Cooper-pair box, i.e., the charge qubit. The Hamiltonian of the Cooper-pair box (CPB) superconducting circuit is [37]

$$\hat{H}_{\mathrm{CPB}} = 4E_c\sum_{n\in\mathbb{Z}}(n-n_g)^2|n\rangle\langle n| - E_J\cos\left(\frac{\pi\Phi}{\Phi_0}\right) \times \sum_{n\in\mathbb{Z}}(|n+1\rangle\langle n| + |n\rangle\langle n+1|), \tag{36}$$

where $E_c = e^2/(2C_\Sigma)$ is the Coulomb energy, with $C_\Sigma = 2C_J + C_g$ being the total capacitance connected with the superconducting island. E_J is the Josephson coupling energy of a single Josephson junction. An external magnetic flux Φ through the loop can tune the effective Josephson coupling energy of the CPB. The symbol Φ_0 is the magnetic flux quanta. In addition, we introduce the gate Cooper-pair number $n_g = C_gV/(2e)$ with the gate capacitance C_g and the gate voltage V. The state $|n\rangle$ ($n\in\mathbb{Z}$, where $\mathbb{Z}$ denotes the integer set) stands for n extra Cooper pairs on the island.

In the present scheme, since the existence of the TLR, the gate voltage contains two parts: a dc part V_g^{dc} and a quantum part V_q generated by the TLR. Then

$$n_g = \frac{C_g\left(V_g^{dc}+V_q\right)}{2e} = n_g^{dc} + \frac{C_gV_q}{2e}, \tag{37}$$

where $n_g^{dc} = C_gV_g^{dc}/(2e)$. The voltage generated by the TLR can be written as

$$V_q = \sqrt{\frac{\omega}{Lc}}(\hat{a}+\hat{a}^{\dagger}), \tag{38}$$

where L is the length of the TLR and c is the capacitance per unit length of the TLR.

With the substitution of Eqs. (37) and (38) into the Hamiltonian (36) and restriction into the subspace with basis states $|0\rangle$ and $|1\rangle$, we obtain

$$\hat{H}_{\mathrm{CPB}} = -2E_c\left(1-2n_g^{dc}\right)\hat{\tau}_z - E_J\cos\left(\frac{\pi\Phi}{\Phi_0}\right)\hat{\tau}_x - e\frac{C_g}{C_\Sigma}\sqrt{\frac{\omega}{Lc}}(\hat{a}+\hat{a}^{\dagger})\left(1-2n_g^{dc}-\hat{\tau}_z\right), \tag{39}$$

where we have introduced the Pauli operators

$$\hat{\tau}_x = |1\rangle\langle 0| + |0\rangle\langle 1|, \quad \hat{\tau}_z = |0\rangle\langle 0| - |1\rangle\langle 1|, \tag{40}$$

and discarded some constant terms. When the charge qubit is working at the optimal point $n_g^{dc} = 1/2$, the total Hamiltonian of the system becomes

$$\hat{H} = \omega\hat{a}^{\dagger}\hat{a} - E_J\cos\left(\frac{\pi\Phi}{\Phi_0}\right)\hat{\tau}_x + e\frac{C_g}{C_\Sigma}\sqrt{\frac{\omega}{Lc}}(\hat{a}+\hat{a}^{\dagger})\hat{\tau}_z. \tag{41}$$

By making a rotation

$$\hat{\tau}_x \to -\hat{\sigma}_z, \quad \hat{\tau}_z \to \hat{\sigma}_x, \tag{42}$$

the Hamiltonian becomes [35]

$$\hat{H} = \omega\hat{a}^{\dagger}\hat{a} + \frac{\omega_0}{2}\hat{\sigma}_z + g(\hat{a}+\hat{a}^{\dagger})\hat{\sigma}_x, \tag{43}$$

where the energy separation of the charge qubit is

$$\omega_0 = 2E_J\cos\left(\frac{\pi\Phi}{\Phi_0}\right), \tag{44}$$

and the coupling strength is

$$g = e\frac{C_g}{C_\Sigma}\sqrt{\frac{\omega}{Lc}}. \tag{45}$$

The Hamiltonian (43) reduces to the usual JC Hamiltonian given in Eq. (1) by making the rotation wave approximation.

From Eqs. (44) and (45), we can see that the energy separation ω_0 of the charge qubit is tunable by controlling the biasing magnetic flux Φ, and the coupling strength g is fixed once the superconducting circuit is fabricated, therefore the effective method to switch on and off the coupling of the resonator with the charge qubit is to tune Φ such that the qubit couples with the resonator in resonant and very largely detuned, respectively. We schematically plot the qubit energy level versus the magnetic flux Φ in Fig. 4(b). The working points "on" and "off" correspond respectively to the resonant coupling and decoupling of the qubit with the cavity. The magnetic flux "pulse serial" controls the interaction of the qubit with the resonator. Note that similar methods have been proposed to generate photon Fock states [38] and have recently been realized based on a circuit QED system consisting of a transmission line resonator coupled with a phase qubit [39,40].

However, it should be emphasized that, strictly speaking, the present circuit QED system is different from the micromaser system since a micromaser has many independent atoms, while the TLR has strictly one TLS. Therefore we need to know the conditions under which the dynamics of the single-mode field in the TLR is equivalent to that of the cavity field in the micromaser. Without loss of generality, in the following we study the dynamics of the circuit QED during a single cycle. We assume that the coupling of the charge qubit with the TLR is switched on at time t_i, and after an interaction of time τ, this coupling is switched off. We denote the density matrix of the circuit QED at time $t_i + \tau$ as

$$\hat{\rho}_{\text{cir-QED}} = \sum_{m,n=0}^{\infty} \sum_{r,s=\{e,g\}} \rho_{mnrs} |m\rangle_{\text{TLR}} \langle n|_{\text{TLR}} \otimes |r\rangle_q \langle s|_q, \tag{46}$$

where states $|m(n)\rangle_{\text{TLR}}$ and $|e(g)\rangle_q$ denote the states of the TLR and the charge qubit, respectively. Generally, this density matrix given in Eq. (47) is an entangled state due to the coupling between the charge qubit and the TLR. During the time interval from $t_i + \tau$ to t_{i+1}, the time of the $(i+1)$th turning on the coupling, the density matrix (47) evolves under the local actions of the environments of the charge qubit and the TLR.

In a micromaser, correspondingly, we only focus on the quantum state of the cavity by tracing over the atom. Therefore, if the total density matrix of the cavity and the ith injected atom is

$$\hat{\rho}_{\text{cav-QED}} = \sum_{m,n=0}^{\infty} \sum_{r,s=\{e,g\}} \rho_{mnrs} |m\rangle_{\text{cav}} \langle n|_{\text{cav}} \otimes |r\rangle_{a_i} \langle s|_{a_i} \tag{47}$$

at time $t_i + \tau$, where $|m(n)\rangle_{\text{cav}}$ and $|e(g)\rangle_{a_i}$ denote the states of the cavity and the ith injected atom in the cavity QED, respectively, then the reduced density matrix of the cavity at time t_{i+1} should be

$$\hat{\rho}_{\text{cav}} = \mathtt{Tr}_{a_i}[\hat{\rho}_{\text{cav-QED}}] = \sum_{m,n=0}^{\infty} \sum_{r=\{e,g\}} \rho_{mnrr} |m\rangle_{\text{cav}} \langle n|_{\text{cav}} \tag{48}$$

taking the trace over the ith injected atom. Notice where we have neglected the action from the environment of the cavity during the interval from $t_i + \tau$ to t_{i+1}. In addition, the $(i+1)$th atom is prepared in its initial state at time t_{i+1}. Therefore, to simulate the micromaser with the circuit QED system, it is required that the qubit should be disentangled from the resonator at time t_{i+1} to avoid the correlation between the qubit and the resonator at the beginning of the $(i+1)$th coupling. By comparing Eq. (47) with Eq. (48), we can see that the disentanglement condition is that, during the time interval from $t_i + \tau$ to a time $t_i + \tau + \tau_r$ before t_{i+1}, the qubit should be relaxed to its ground state as follows:

$$\begin{aligned} |e\rangle_q \langle e|_q &\to |g\rangle_q \langle g|_q, \\ |e\rangle_q \langle g|_q &\to 0, \\ |g\rangle_q \langle e|_q &\to 0, \\ |g\rangle_q \langle g|_q &\to |g\rangle_q \langle g|_q. \end{aligned} \tag{49}$$

Under this process the density matrix (47) becomes

$$\hat{\rho}_{\text{cir-QED}} = \hat{\rho}_{\text{TLR}} \otimes |g\rangle_q \langle g|_q, \tag{50}$$

where we denote

$$\hat{\rho}_{\text{TLR}} = \sum_{m,n=0}^{\infty} \sum_{r=\{e,g\}} \rho_{mnrr} |m\rangle_{\text{TLR}} \langle n|_{\text{TLR}}. \tag{51}$$

Clearly, the density matrices $\hat{\rho}_{\text{TLR}}$ of the TLR in the circuit QED and $\hat{\rho}_{\text{cav}}$ of the cavity in the cavity QED have the same form.

Before the beginning time t_{i+1} of the $(i+1)$th coupling, we need to prepare the qubit in its initial state, we denote the state-preparation time is τ_p. At time t_{i+1}, we switch on the coupling between the qubit and the TLR, repeating the process as described before, we can simulate the micromaser with the circuit QED system.

Now, there are several time scales in the circuit QED system: the interaction time τ, the relaxation time τ_r, and the state-preparation time τ_p. From the above discussions we can see that the requirement of these time scales is

$$\tau + \tau_r + \tau_p \leqslant t_{i+1} - t_i \tag{52}$$

for all i. In addition, since we neglect the relaxation of the qubit during the interaction time τ, then the relaxation time $\tau_r \gg \tau$. The preparation of the qubit's state can be realized by using a classical field. For example, a 2π pulse can transfer the qubit from its ground state $|g\rangle$ to excited state $|e\rangle$. The preparation time τ_p can be much smaller than other time scales through choosing a sufficiently strong field. Namely, we approximately have the relation $\tau + \tau_r + \tau_p \approx \tau + \tau_r$.

In the following, we give a simple estimation of the above time scales under the current experimental conditions. According to recent circuit QED experiments [36], we take the following parameters, the resonator frequency $\omega = 2\pi \times 10$ GHz, the coupling strength $g = 2\pi \times 50$ MHz, the cavity decay rate $\kappa = 2\pi \times 1$ MHz. The interaction time $\tau \sim 10^{-9}$ s for $g\tau \sim 0.05$. The rate to switch on the coupling $r \approx 2\pi \times 2$ MHz. Namely, the interaction between the resonator and the qubit takes place every $t \approx 1/r \sim 10^{-7}$ s. In other words, the average time $\overline{t_{i+1} - t_i} \approx 10^{-7}$ s, where the overline represents average value. Then the qubit relaxation time τ_r should be of the order of 10 ns to satisfy the conditions $\tau + \tau_r \leqslant t_{i+1} - t_i$ and $\tau_r \gg \tau$. Therefore, the requirements for the design of the random pulse are $\min\{t_{i+1} - t_i\} \geqslant \tau + \tau_r \sim 10^{-8}$ s and $\overline{t_{i+1} - t_i} \approx 10^{-7}$ s. We can decrease the average rate r to enlarge the variable space of the time scale τ_r. As an example, we take the temperatures of the TLS and the field heat bath as $T = 200$ mK ($\beta = 2.898$) and $T_b = 100$ mK ($\beta_b = 4.797$), respectively. We can calculate the temperature of the cavity field at thermal equilibrium as $T_{\text{eff}} = 100.54$ mK ($\beta_{\text{eff}} = 4.773$) for the case $g\tau = 0.05$.

VI. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have proposed a kind of generalized thermalization with a single-particle reservoir. This generalized thermalization uniquely describes the cooling, masering, and thermalization processes. We have shown our generalized thermalization based on a micromaser-like system, in which a series of well-prepared TLSs are injected *randomly* through the cavity. In the absence of the cavity decay, the cavity can reach an equilibrium with the same temperature as that of the TLSs. When the cavity is coupled with a heat bath, at a steady state the cavity can reach a thermal equilibrium with an intermediate temperature between those of the heat bath and the TLSs. We have also studied the effect of quantum coherence on the thermalization. It was found that the quantum coherence can increase the temperature of the system to be thermalized at a steady state. We have suggested an experimental implementation of our generalized thermalization with the superconducting circuit QED system.

We point out that the present investigations have some potential values in solid thermodynamical applications. For example, in circuit QED we can manipulate the steady state of the TLR by preparing the quantum state of the qubit. When the qubit is prepared in its ground state, the steady state of the TLR will be in its ground state. A similar idea has been used to cool a TLR [41]. In addition, by preparing the qubit in its excited state, the TLR can be used to realize a solid laser on-chip [42].

We give some discussions concerning the ergodicity during the quantum thermalization process. As mentioned in the Introduction, one of the motivations of our present investigation was to show the quantum thermalization of a system randomly coupled with a series of single-particle reservoirs in a time domain. Essentially, the physical principle at the background is the ergodicity, which involves the equivalence between the time average and ensemble average [43]. For the present case, it has been shown that the form of the quantum master Eq. (24) obtained in the short τ limit can guarantee the ergodicity [44,45]. Therefore, it is a correct result that these injected atoms can thermalize the single-mode cavity field in the micromaser model.

Finally, we emphasize that in this work the injected atoms are prepared in thermal equilibrium with positive temperature. As is known, the temperature of two-level atoms can be negative [46]. However, for a harmonic oscillator, it is impossible to define a negative temperature since the energy of the harmonic oscillator is finite. Therefore, the discussions in this paper are restricted within the case that the population p_e of the excited state is smaller than the population p_g of the ground state.

ACKNOWLEDGMENTS

This work is supported in part by NSFC Grants No. 10935010 and No. 10775048, NFRPC Grants No. 2006CB921205 and No. 2007CB925204.

[1] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).
[2] S. Popescu, A. J. Short, and A. Winter, Nat. Phys. **2**, 754 (2006).
[3] S. Goldstein, J. L. Lebowitz, R. Tumulka, and N. Zanghì, Phys. Rev. Lett. **96**, 050403 (2006).
[4] J. Gemmer and M. Michel, Europhys. Lett. **73**, 1 (2006).
[5] H. Dong, S. Yang, X. F. Liu, and C. P. Sun, Phys. Rev. A **76**, 044104 (2007).
[6] W. H. Louisell, *Quantum Statistical Properties of Radiation* (John Wiley & Sons, New York, 1973).
[7] M. O. Scully and W. E. Lamb, Phys. Rev. **159**, 208 (1967).
[8] P. Filipowicz, J. Javanainen, and P. Meystre, Phys. Rev. A **34**, 3077 (1986).
[9] M. Orszag, *Quantum Optics: Including Noise Reduction, Trapped Ions, Quantum Trajectories, and Decoherence* (Springer, Berlin, 2000).
[10] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).
[11] D. Meschede, H. Walther, and G. Müller, Phys. Rev. Lett. **54**, 551 (1985).
[12] F. Casagrande, M. Garavaglia, and A. Lulli, Opt. Commun. **151**, 395 (1998).
[13] J. D. Cresser, Phys. Rev. A **46**, 5913 (1992).
[14] J. D. Cresser and S. M. Pickles, Quantum Semiclass. Opt. **8**, 73 (1996).
[15] J. Bergou, L. Davidovich, M. Orszag, C. Benkert, M. Hillery, and M. O. Scully, Phys. Rev. A **40**, 5073 (1989); J. Bergou and P. Kálmán, *ibid.* **43**, 3690 (1991).
[16] E. S. Guerra, A. Z. Khoury, L. Davidovich, and N. Zagury, Phys. Rev. A **44**, 7785 (1991).
[17] C. Benkert and K. Rzazewski, Phys. Rev. A **47**, 1564 (1993).
[18] J. Bergou and M. Hillery, Phys. Rev. A **49**, 1214 (1994).
[19] M. Brune, J. M. Raimond, P. Goy, L. Davidovich, and S. Haroche, Phys. Rev. Lett. **59**, 1899 (1987).
[20] G. Rempe, F. Schmidt-Kaler, and H. Walther, Phys. Rev. Lett. **64**, 2783 (1990).
[21] S. Pielawa, G. Morigi, D. Vitali, and L. Davidovich, Phys. Rev. Lett. **98**, 240401 (2007).
[22] P. Zhang, Y. D. Wang, and C. P. Sun, Phys. Rev. Lett. **95**, 097204 (2005).
[23] J. Q. You, Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. B **75**, 104516 (2007).
[24] R. R. McGowan and W. C. Schieve, Phys. Rev. A **56**, 2373 (1997).
[25] J. Krause, M. O. Scully, and H. Walther, Phys. Rev. A **34**, 2032 (1986).
[26] H. Lee, Y. C. Cheng, and G. R. Fleming, Science **316**, 1462 (2007).
[27] G. S. Engel, T. R. Calhoun, E. L. Read, T. K. Ahn, T. Mancal, Y. C. Cheng, R. E. Blankenship, and G. R. Fleming, Nature (London) **446**, 782 (2007).
[28] M. O. Scully, M. S. Zubairy, G. S. Agarwal, and H. Walther, Science **299**, 862 (2003).
[29] H. T. Quan, P. Zhang, and C. P. Sun, Phys. Rev. E **73**, 036122 (2006).
[30] H. T. Quan, Y. D. Wang, Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. Lett. **97**, 180402 (2006).
[31] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).
[32] This equation (13) is valid only for the case of random injection. For other nonrandom injections, a more rigorous method is needed and we refer to Refs. [13,14].
[33] M. Sargent III, M. O. Scully, and W. E. Lamb Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1974).
[34] Nan Zhao, D. L. Zhou, J. L. Zhu, and C. P. Sun, e-print arXiv:0705.1964.
[35] A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **69**, 062320 (2004).

[36] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).
[37] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[38] Y. X. Liu, L. F. Wei, and F. Nori, Europhys. Lett. **67**, 941 (2004).
[39] M. Hofheinz, E. M. Weig, M. Ansmann, R. C. Bialczak, E. Lucero, M. Neeley, A. D. O'Connell, H. Wang, J. M. Martinis, and A. N. Cleland, Nature (London) **454**, 310 (2008).
[40] M. Hofheinz, H. Wang, M. Ansmann, R. C. Bialczak, E. Lucero, M. Neeley, A. D. O'Connell, D. Sank, J. Wenner, J. M. Martinis, and A. N. Cleland, Nature (London) **459**, 546 (2009).
[41] M. Grajcar, S. H. W. van der Ploeg, A. Izmalkov, E. Il'ichev, H.-G. Meyer, A. Fedorov, A. Shnirman, and Gerd Schön, Nature Phys. **4**, 612 (2008).
[42] O. Astafiev, K. Inomata, A. O. Niskanen, T. Yamamoto, Yu. A. Pashkin, Y. Nakamura, and J. S. Tsai, Nature (London) **449**, 588 (2007).
[43] L. E. Reichl, *A Modern Course in Statistical Physics*, 2nd ed. (John Wiley & Sons, New York, 1998).
[44] J. D. Cresser, in *Directions in Quantum Optics*, edited by H. J. Carmichael, R. J. Glauber, and M. O. Scully (Springer, Berlin, 2000), pp. 358–369.
[45] B. Kümmerer and H. Maassen, J. Phys. A: Math. Gen. **36**, 1 (2003).
[46] N. F. Ramsey, Phys. Rev. **103**, 20 (1956).