Analog Landau-He-McKellar-Wilkens quantization due to noninertial effects of the Fermi-Walker reference frame

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We will show that when a neutral particle with permanent electric dipole moment interacts with a specific field configuration when the local reference frames of the observers are Fermi-Walker transported, the Landau quantization analog to the He-McKellar-Wilkens setup arises in the nonrelativistic quantum dynamics of the neutral particle due the noninertial effects of the Fermi-Walker reference frame. We show that the noninertial effects do not break the infinity degeneracy of the energy levels, but in this case, the cyclotron frequency depends on the angular velocity.

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I. INTRODUCTION

The quantization of the motion of a charged particle in discrete orbits when it interacts with a uniform magnetic field is known in the literature as the Landau quantization [1] and has been studied in several systems [2–7]. Interesting works about the Landau quantization for charged particles were done in the presence of topological defects [8–11], where it was shown that the degeneracy of the Landau levels is broken due to the presence of a topological defect [10] and when it is considered the continuum elastic medium with the presence of a density of screw dislocations [11].

For neutral particles, interesting discussions about the behavior of the magnetic and electric dipoles in interferometry experiments were made in [12–14], but the analog of the Landau quantization was proposed by Ericsson and Sjöqvist in [15] for the Aharonov-Casher (AC) setup [16]. This analog to the Landau quantization is achieved when the permanent dipole moment of the neutral particle interacts with an external electric field in such a way that there is no torque on the dipole moment and the electric field configuration must satisfy the electrostatic conditions. Another condition established in [15] is that there is an effective uniform magnetic field given by $\vec{B}_{\text{AC}} = \vec{\nabla} \times [\vec{n} \times \vec{E}]$ (where $\vec{n}$ is the direction of the magnetic dipole moment). Following the conditions established in [15], Ribeiro *et al.* [17] studied the Landau quantization for a neutral particle in the He-McKellar-Wilkens (HMW) setup [18,19] assuming the existence of magnetic charge density. In [20], the Landau quantization for neutral particles was discussed in the presence of a linear topological defect in the Aharonov-Casher setup and, through the duality transformations, the Landau quantization in the HMW setup was discussed. They showed that the infinite degeneracy of energy levels in the Landau quantization for neutral particles is broken due to to the presence of a linear topological defect. Without the hypothesis of magnetic charges, Wei *et al.* [21] studied topological effects involving induced electric dipoles and following this an interesting work of Landau quantization for neutral particles was done for an induced electric dipole moment interacting with crossed electric and magnetic fields [22] and in the presence of topological defects the Landau quantization for induced electric dipole was discussed in [23].

The most known noninertial effect in interferometry experiments is the Sagnac effect [24] and it has been a source of wide discussions in the literature [25–33]. Another important noninertial effect in nonrelativistic quantum mechanics is the coupling between the spin of the particles with the angular velocity of the rotating frame, which is called the Mashhoon effect [34]. In the relativistic quantum dynamics the Sagnac-type effects and rotation-spin coupling were discussed in [35]. In [36] the origin of the Sagnac and Mashhoon effects is related to the application of Lorentz transformations. In the presence of a gravitational field, the Sagnac effect and the spin-rotation coupling are derived in [37] as in the relativistic and nonrelativistic dynamics of a spin-half particle through the weak field approximation. In [38], the appearance of the geometric quantum phases in the cosmic string space-time in rotating frames is studied.

In this paper, we study the bound states for a neutral particle with permanent electric dipole moment which arise due to the noninertial effects when the permanent electric dipole moment of the neutral particle interacts with external fields when the reference frames of the observers are Fermi-Walker transported. We show that we can obtain an analog Landau quantization for the He-McKellar-Wilkens setup provided by the noninertial effects of the Fermi-Walker reference frame, where the energy levels are infinitely degenerated and the cyclotron frequency depends on the angular velocity and the intensity of the electric field.

The structure of this paper is as follows. In Sec. II, we show the mathematical features and the field configuration when the reference frame of the observers are Fermi-Walker transported. In Sec. III, we discuss the Landau quantization in the nonrelativistic quantum dynamics of the neutral particle with permanent electric dipole moment. In Sec. IV, we present our conclusions.

II. FERMI-WALKER REFERENCE FRAME AND FIELD CONFIGURATION

In the HMW setup [18,19], the wave function of a neutral particle with permanent electric dipole moment acquires a phase shift due to the interaction between the electric dipole

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moment with a radial magnetic field generated by a linear distribution of magnetic charges. Several discussions about the topological nature of the quantum phase of permanent electric dipoles were done [39–42] in the last decade. Several experiments have been proposed [43–47] to reproduce the same field configuration of the HMW setup. In [43–45] the magnetic field configuration is generated by two concentric cylindrical magnets and in [46] a cylindrical solenoid with a surface current density where the radial magnetic field is achieved in the external region of the solenoid is considered. In [47], an interferometer experiment using ferromagnetic wire is proposed, where the magnetization of the ferromagnetic wire is considered parallel to the wire direction and where the magnitude of magnetization changes linearly along the wire. However, bound states which correspond to the Landau quantization in the HMW setup were obtained in [17] using a radial magnetic field generated by a linear distribution of magnetic charges. In this section, we show the mathematical features to describe the field configuration and spinors when we consider that the local reference frame of the observers are transported via Fermi-Walker transport. We show that, in this reference frame, we can obtain a field configuration with a radial magnetic field where we can observe bound states analogous to the Landau quantization in the HMW setup. We work out in the flat space-time and in the units where $\hbar = c = 1$. Thus, we write the line element as

$$ds^2 = -d\mathcal{T}^2 + d\mathcal{R}^2 + \mathcal{R}^2 d\Phi^2 + d\mathcal{Z}^2, \tag{1}$$

We consider that there is a uniform electric field along the z axis of the space-time in the rest frame of the observers, that is,

$$E_{\rm rf}^{\mathcal{Z}} = \mathbf{E}_0, \tag{2}$$

However, we are interested in studying the noninertial effects of a noninertial frame on the quantum dynamics of the neutral particle. Thus, we realize the following coordinate transformation:

$$\mathcal{T} = t; \quad \mathcal{R} = \rho; \quad \Phi = \varphi + \omega t; \quad \mathcal{Z} = z, \tag{3}$$

where ω is the constant angular velocity of the rotating frame and must satisfy $\omega\rho \ll 1$. With this transformation, the line element (1) becomes

$$ds^2 = -(1-\omega^2\rho^2)\, dt^2 + 2\omega\rho^2 d\varphi dt + d\rho^2 + \rho^2 d\varphi^2 + dz^2. \tag{4}$$

With the line element given in the expression (4), it is convenient to treat spinors as in curved space-time [48,49]. In curved space-time, we define the spinors through the local reference frame for the observers. We can build the local reference frame through a noncoordinate basis $\hat{\theta}^a = e^a{}_\mu(x)\, dx^\mu$, where its components are $e^a{}_\mu(x)$ and satisfy the following relation [49,50]:

$$g_{\mu\nu}(x) = e^a{}_\mu(x)\, e^b{}_\nu(x)\, \eta_{ab}, \tag{5}$$

where $\eta^{ab} = \text{diag}(-+++)$ is the Minkowisky tensor and the indices $(i,j,k = 1,2,3)$ are the spatial index of the local reference frame. The components of the noncoordinate basis $e^a{}_\mu(x)$ are called *tetrads* or *Vierbein* and they form our local reference frame. The tetrads have an inverse defined as $dx^\mu = e^\mu{}_a(x)\,\hat{\theta}^a$, where they are related by $e^a{}_\mu(x)\, e^\mu{}_b(x) = \delta^a{}_b$ and $e^\mu{}_a(x)\, e^a{}_\nu(x) = \delta^\mu{}_\nu$.

We have a freedom to choose the local reference frame for the observers. If we wish to observe noninertial effects due to the action of external forces without any effects due to arbitrary rotations of the local spatial axis of the reference frame of the observers, we need to build a nonrotating frame or Fermi-Walker reference frame [48]. A Fermi-Walker reference frame can be built with the components of the noncoordinate basis given in the rest frame of the observers at each instant, that is, $\hat{\theta}^0 = e^0{}_t(x)\, dt$, and where the spatial components of this noncoordinate basis $\hat{\theta}^i$, $i = 1,2,3$, do not rotate [48]. Interesting effects due to noninertial effects can be observed in this frame as the Mashhoon effect [34] and the Page-Werner *et al.* term [32,33,35], where a coupling between the angular momentum of the quantum particle with the angular velocity arises in the nonrelativistic dynamics of a quantum particle. As pointed out in [36], the Sagnac effect cannot be observed in the Fermi-Walker reference frame due to the absence of precession effects or dragging effects on the local spacial axis, but the Mashhoon effect can be observed in the Fermi-Walker reference frame due the behavior of quantum particles as gyroscopes when the presence of a rotational motion exists. In that way, we can write the local reference frame of the observers in the form [48]

$$\hat{\theta}^0 = dt; \quad \hat{\theta}^1 = d\rho; \quad \hat{\theta}^2 = \omega\rho\, dt + \rho\, d\varphi; \quad \hat{\theta}^3 = dz. \tag{6}$$

With the information about the choice of the local reference frame, we can obtain the one-form connection $\omega^a{}_b = \omega_\mu{}^a{}_b\, dx^\mu$ through the Maurer-Cartan structure equation [50]. In the absence of the torsion field, the Maurer-Cartan structure equation may be written as $d\hat{\theta}^a + \omega^a{}_b \wedge \hat{\theta}^b = 0$, where the operator d is the exterior derivative and the symbol $\wedge$ means the wedge product [50]. Solving the Maurer-Cartan structure equations, we have that the non-null components of the one-form connection are $\omega_t{}^1{}_2 = -\omega_t{}^2{}_1 = -\omega$ and $\omega_\varphi{}^1{}_2 = -\omega_\varphi{}^2{}_1 = -1$.

We must note that the electric field given in (2) corresponds to the field configuration in the rest frame of the observer, which we can write as $E^3 = e^3{}_{\mathcal{Z}}\, E^{\mathcal{Z}} = \mathbf{E}_0$. In the Fermi-Walker reference frame (6), the fields are given by [51–53]

$$F^{\mu\nu} = e^\mu{}_a(x)\, e^\nu{}_b(x)\, F^{ab}. \tag{7}$$

Thus, we obtain that there are non-null components of the electric and magnetic fields when the local reference frames of the observers are Fermi-Walker transported

$$E^z = E^3 = \mathbf{E}_0; \quad B^\rho = \omega\rho\, E^3 = \omega\, \mathbf{E}_0\, \rho. \tag{8}$$

We can see, considering that the permanent electric dipole moment of the neutral particle is initially aligned with the z axis, that there is no torque on the permanent electric dipole moment of the neutral particle generated by the field configuration (8). Moreover, the electrostatic conditions are satisfied and we have a uniform effective magnetic field given by $\vec{B}_{\rm eff} = \vec{\nabla} \times [\vec{n} \times \vec{B}] = 2\omega E_0$ (where $\vec{n}$ is the direction of the electric dipole moment) which shows us that all conditions for the Landau-He-McKellar-Wilkens quantization given in [17] are satisfied. In that way, with the field configuration given in (8), we will see that we can obtain a Landau quantization for a neutral particle without assuming the existence of a radial magnetic field generated by a magnetic charge density as done in Ref. [17], when the Landau quantization for the He-McKellar-Wilkens setup [18,19] was proposed.

Hence, with the local reference frames of the observers (6) and the field configuration given in this frame (8), we are able to study the Landau quantization in the nonrelativistic quantum dynamics of the neutral particle with permanent electric dipole moment induced by the noninertial effects of the reference frame of the observers.

III. BOUND STATES FOR A NEUTRAL PARTICLE IN THE NONINERTIAL FRAME

In this section, we study the bound states which arise in the nonrelativistic quantum dynamics of the neutral particles when the local reference frames of the observers are Fermi-Walker transported. We consider that the neutral particle has a permanent electric dipole moment and it interacts with the external electric and magnetic fields. The quantum dynamics of the neutral particle with permanent magnetic dipole moment interacting with external magnetic and electric field is described by the introduction of the following nonminimal coupling into the Dirac equation [12–14]:

$$i\gamma^{\mu}\nabla_{\mu} \rightarrow i\gamma^{\mu}\nabla_{\mu} - i\frac{d}{2}\Sigma^{\mu\nu}\gamma^{5}F_{\mu\nu}, \tag{9}$$

where d is the permanent electric dipole moment of the neutral particle, $F_{\mu\nu}$ is the electromagnetic field tensor whose non-null components are $F_{0i} = -F_{0i} = E_i$, $F_{ij} = -F_{ji} = -\epsilon_{ijk}B^k$ and $\Sigma^{ab} = \frac{i}{2}[\gamma^a,\gamma^b]$, with the indices $(a,b,c = 0,1,2,3)$ indicating the local reference frame of the observers. The γ^a matrices are defined in the local reference frame and are the Dirac matrices given in the flat space-time [49,54], i.e.,

$$\gamma^0 = \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \gamma^i = \hat{\beta}\,\hat{\alpha}^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix};$$
$$\gamma^5 = \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}; \quad \Sigma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}, \tag{10}$$

with I being the 2×2 identity matrix, $\vec{\Sigma}$ being the spin vector, and σ^i the Pauli matrices which satisfy the relation $(\sigma^i\sigma^j + \sigma^j\sigma^i) = 2\eta^{ij}$. The γ^{μ} matrices are related to the γ^a matrices via $\gamma^{\mu} = e^{\mu}_{\ a}(x)\gamma^a$. Since we are in curvilinear coordinates, we can write the Dirac equation in the same formulation given in curved space-time [49,50,55–57]. In that way, the partial derivative must be changed in the expression (9) by the covariant derivative of a spinor [49,56,57] given by $\nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu}$, with $\Gamma_{\mu} = \frac{i}{4}\omega_{\mu ab}\Sigma^{ab}$ being the spinorial connection [49,50]. Hence, the Dirac equation in curved space-time with the interaction of the permanent electric dipole moment of the neutral particle with external fields (8) is given by the following expression:

$$i\,\gamma^a e^{\mu}_{\ a}(x)\,\partial_{\mu}\psi + i\,\gamma^{\mu}\Gamma_{\mu}\psi - i\frac{d}{2}\Sigma^{\mu\nu}\gamma^5 F_{\mu\nu}\psi = m\psi. \tag{11}$$

Taking the expression for the one-form connections $\omega_{t\ 2}^{\ 1} = -\omega_{t\ 1}^{\ 2} = -\omega$ and $\omega_{\varphi\ 2}^{\ 1} = -\omega_{\varphi\ 1}^{\ 2} = -1$, we can calculate the spinorial connection Γ_{μ} and obtain that $\gamma^{\mu}\Gamma_{\mu} = \frac{\gamma^1}{2\rho}$ [56]. After some manipulations, we can rewrite the Dirac Eq. (11) in the form

$$i\frac{\partial\psi}{\partial t} = m\hat{\beta}\psi + i\omega\frac{\partial\psi}{\partial\varphi} - i\hat{\alpha}^1\left(\frac{\partial}{\partial\rho} + \frac{1}{2\rho}\right)\psi - i\frac{\hat{\alpha}^2}{\rho}\frac{\partial\psi}{\partial\varphi}$$
$$- i\hat{\alpha}^3\frac{\partial\psi}{\partial z} + id\hat{\beta}\vec{\alpha}\cdot\vec{B}\psi + d\hat{\beta}\vec{\Sigma}\cdot\vec{E}\psi, \tag{12}$$

where the electric and magnetic field in the Dirac Eq. (12) are given in the expression (8). Our interest in this work is to study the bound states which arise in the nonrelativistic quantum dynamics of the neutral particle. In that way, we can obtain the nonrelativistic dynamics of the neutral particle when we extract the temporal dependence of the wave function due the rest energy [54]. So, we write the Dirac spinor in the form

$$\psi = e^{-imt}\begin{pmatrix} \phi \\ \chi \end{pmatrix}, \tag{13}$$

where ϕ and χ are two-spinors. Thus, substituting (13) into the Dirac Eq. (12), we obtain two coupled equations of ϕ and χ. The first coupled equation is

$$i\frac{\partial\phi}{\partial t} - i\omega\frac{\partial\phi}{\partial\varphi} - d\mathbf{E}_0\sigma^3\phi = \left[-i\sigma^1\frac{\partial}{\partial\rho} - \frac{i\sigma^1}{2\rho} + id\omega\mathbf{E}_0\rho\sigma^1 - \frac{i\sigma^2}{\rho}\frac{\partial}{\partial\varphi} - i\sigma^3\frac{\partial}{\partial z}\right]\chi, \tag{14}$$

while the second coupled equation is

$$i\frac{\partial\chi}{\partial t} + 2m\chi - i\omega\frac{\partial\chi}{\partial\varphi} + d\mathbf{E}_0\sigma^3\chi = \left[-i\sigma^1\frac{\partial}{\partial\rho} - \frac{i\sigma^1}{2\rho} - id\omega\mathbf{E}_0\rho\sigma^1 - \frac{i\sigma^2}{\rho}\frac{\partial}{\partial\varphi} - i\sigma^3\frac{\partial}{\partial z}\right]\phi. \tag{15}$$

Considering χ being the "small" component of the wave function, we can consider that $|2m\chi| \gg |\frac{\partial\chi}{\partial t}|$, $|2m\chi| \gg |\omega\frac{\partial\chi}{\partial\varphi}|$, and $|2m\chi| \gg |d\mathbf{E}_0\sigma^3\chi|$, thus, we can relate the "small" component with the "large" component ϕ as

$$\chi \approx \frac{1}{2m}\left[-i\sigma^1\frac{\partial}{\partial\rho} - \frac{i\sigma^1}{2\rho} - id\omega\mathbf{E}_0\rho\sigma^1 - \frac{i\sigma^2}{\rho}\frac{\partial}{\partial\varphi} - i\sigma^3\frac{\partial}{\partial z}\right]\phi. \tag{16}$$

Substituting χ in the expression (16) into (15), we obtain a second-order differential equation given by

$$i\frac{\partial\phi}{\partial t} = -\frac{1}{2m}\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}\right]\phi + \frac{i}{2m}\frac{\sigma^3}{\rho^2}\frac{\partial\phi}{\partial\varphi} + i\frac{d\omega\mathbf{E}_0}{m}\sigma^3\frac{\partial\phi}{\partial\varphi} + \frac{1}{8m\rho^2}\phi - \frac{d\omega\mathbf{E}_0}{2m}\phi + \frac{d^2\omega^2\mathbf{E}_0^2}{2m}\rho^2\phi + i\omega\frac{\partial\phi}{\partial\varphi} + d\mathbf{E}_0\sigma^3\phi, \tag{17}$$

which corresponds to the Schrödinger-Pauli equation for a neutral particle with permanent electric dipole moment interacting with external electric and magnetic fields when the local reference frames of the observers are Fermi-Walker transported. We can see in the Eq. (17) that ϕ is an eigenfunction of σ^3, whose eigenvalues are $s = \pm 1$. Thus, we must write $\sigma^3 \phi_s = \pm \phi_s = s\phi_s$. Let us take the solutions of the Eq. (17) in the form

$$\phi_s = e^{-i\mathcal{E}t}\, e^{i(l+\frac{1}{2})\varphi}\, e^{ikz}\, R_s(\rho), \tag{18}$$

where $l = 0, \pm 1, \pm 2, \ldots$ and k is a constant. Substituting the solution (18) into the the Schrödinger-Pauli Eq. (17), we obtain

$$\left[\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{\zeta_s^2}{\rho^2} - \delta^2\rho^2 - \beta_s\right] R_s(\rho) = 0, \tag{19}$$

where we defined the parameters

$$\zeta_s = l + \frac{1}{2}(1-s); \quad \delta = d\omega\mathbf{E}_0;$$
$$\beta_s = 2m\left[\mathcal{E} + \omega(l+1/2) - sd\mathbf{E}_0 + s\frac{\delta}{m}\zeta_s + \frac{\delta}{m}\right]. \tag{20}$$

Let us make a change of variables $\xi = \delta\rho^2$. In this way, the second-order differential Eq. (19) becomes

$$\left[\xi\frac{d^2}{d\xi^2} + \frac{d}{d\xi} - \frac{\zeta_s^2}{4\xi} - \frac{\xi}{4} + \frac{\beta_s}{4\delta}\right] R_s(\xi) = 0. \tag{21}$$

The solution of the radial Eq. (21) must be given in the form

$$R_s(\xi) = e^{-\frac{\xi}{2}}\, \xi^{\frac{|\zeta_s|}{2}}\, F_s(\xi). \tag{22}$$

Substituting the solution (22) into the Eq. (21), we obtain the following equation:

$$\xi\frac{d^2F_s}{d\xi^2} + [(|\zeta_s|+1) - \xi]\frac{dF_s}{d\xi} + \left[\frac{\beta_s}{4\delta} - \frac{|\zeta_s|}{2} - \frac{1}{2}\right]F_s = 0, \tag{23}$$

where the Eq. (23) is the equation of the confluent hypergeometric function $F_s(\xi) = F[\frac{|\zeta_s|}{2} + \frac{1}{2} - \frac{\beta_s}{4\delta}, |\zeta_s|+1, \xi]$. To obtain the bound states in this nonrelativistic quantum dynamics of the neutral particle, we must impose a condition that the parameter $\frac{|\zeta_s|}{2} + \frac{1}{2} - \frac{\beta_s}{4\delta}$ be a nonpositive integer number, making the wave function normalizable. Thus, we have that

$$\frac{|\zeta_s|}{2} + \frac{1}{2} - \frac{\beta_s}{4\delta} = -n, \tag{24}$$

where $n = 0,1,2,\ldots$. Taking the expression for the parameters ζ_s, β, and δ given in (20), we obtain

$$\mathcal{E}_{n,l} = \omega_c\left[n + \frac{\left|l + \frac{1}{2}(1-s)\right|}{2} - s\frac{\left[l + \frac{1}{2}(1-s)\right]}{2}\right] + s\,d\mathbf{E}_0 - \omega(l+1/2). \tag{25}$$

The energy levels obtained in the expression (25) correspond to the bound states generated by noninertial effects in the nonrelativistic quantum dynamics of a neutral particle with permanent electric dipole moment interacting with external electric and magnetic fields when the local reference frame of the observers are Fermi-Walker transported. We can see, in the Fermi-Walker reference frame, that we can generate a field configuration, with a radial magnetic field, where there exists no torque on the electric dipole moment of the neutral particle in a similar way to the He-McKellar-Wilkens system without assuming the existence of magnetic charge density. We must observe that these bound states are infinitely degenerated, but if we take $\omega = 0$ the bound states vanish because there are no more inertial effects which provide the field configuration given in (8). Thus, the energy levels (20) provided by the noninertial effects correspond to an analog of the Landau-He-McKellar-Wilkens quantization, where the cyclotron frequency in this case is given by

$$\omega_c = 2\frac{d\omega\mathbf{E}_0}{m}. \tag{26}$$

Comparing with the cyclotron frequency obtained in [17] in the He-McKellar-Wilkens setup where the cyclotron frequency depends on the magnetic charge density, we can see through the expression (26) that we can build a system which provides an analog Landau-He-McKellar-Wilkens quantization where the cyclotron frequency depends on the intensity of the electric field and the angular velocity of the reference frame. We can also see that the noninertial effects provide a new term for the bound states given in the last term of (25) that corresponds to the coupling between the angular velocity ω and the quantum number l which is known as the Page-Werner *et al.* term [32,33,35].

Hence, when the local reference frames of the observers are Fermi-Walker transported, the field configuration (8) provides no torque on the dipole moment of the neutral particle and give us the Landau quantization for a neutral particle with permanent electric dipole moment. We can see that we obtain the Landau quantization for a neutral particle with permanent electric dipole moment without assuming the existence of magnetic charges as done in [17] with the He-McKellar-Wilkens setup. Here, the noninertial effects provide the Landau quantization in the nonrelativistic quantum dynamics of the neutral particle.

IV. CONCLUSIONS

In this paper, we study the nonrelativistic quantum dynamics of a neutral particle with permanent electric dipole moment interacting with external fields when the local reference frame of the observers are Fermi-Walker transported. We considered that in the rest frame of the observers there was a uniform electric field along the z axis of the space-time. Thus, when the local reference frame of the observers was Fermi-Walker transported, an alternate field configuration arose where there was no torque on the electric dipole moment of the neutral particle. In this alternate field configuration, a radial magnetic field was generated which provided us with another way to obtain bound states for a neutral particle with permanent electric dipole moment without considering the field configuration of the Landau-He-McKellar-Wilkens setup as done in [17].

Hence, we saw that bound states could arise in the nonrelativistic quantum dynamic of the neutral particle due to the noninertial effects of the reference frame of the observers. The energy levels obtained were infinitely degenerate which corresponded to another way to obtain the Landau quantization for neutral particles with permanent electric dipole moment. When we compared the cyclotron frequency given in the inertial frame of the Landau-He-McKellar-Wilkens setup [17], which depends on the magnetic charge density, we saw that the noninertial effects provided a cyclotron frequency which depends on the electric field and the angular velocity. However, when we took $\omega = 0$, we saw that the bound states vanished because there was no longer a field configuration which provides the possibility of bound states in the nonrelativistic quantum dynamics of the neutral particle.

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