

## Proposed method for direct measurement of the non-Markovian character of the qubits coupled to bosonic reservoirs

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Non-Markovianity is a recently proposed characterization of the non-Markovian behavior in an open quantum system, based on which we first present a practical idea for directly measuring the non-Markovian character of a single qubit coupled to a zero-temperature bosonic reservoir, and then extend to investigate the dynamics of two noninteracting qubits subject to two reservoirs, respectively, with a lower bound of non-Markovianity. Our scheme, with no need for optimization procedures and quantum state tomography, is helpful for experimental implementation.

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Realistic quantum systems are fragile due to an unavoidable interaction with the environment [1,2]. For this reason, the dynamics of open quantum systems have attracted much attention in the investigations of modern quantum theory, particularly of quantum-information processing (QIP) [3]. Over the past decades, the conventionally employed Markovian approximation with the assumption of an infinitely short correlation time of the environment has experienced more and more challenges due to the advance of experimental techniques [1], and non-Markovian features have been observed in some physical systems (e.g., atomic and molecular systems [4], high- $Q$  cavity systems [5], and solid state systems [6]). To understand these non-Markovian effects, there have been various kinds of analytical and numerical methods developed so far [1,7–10], such as non-Markovian quantum trajectories [7], pseudomodes [8], non-Markovian quantum jumps [9], and quantum semi-Markov processes [10].

Among the recent investigations for non-Markovian behavior [11–14], a particular approach using non-Markovianity contains the exact characterization of the non-Markovian behavior of a quantum process without making any approximation for the dynamics [12]. The non-Markovianity is defined to quantify the total amount of information flowing from the environment back to the system through a quantum process  $\Phi(t)$  with  $\rho(t) = \Phi(t)\rho(0)$ , where  $\rho(0)$  and  $\rho(t)$  are the density operators of the system at an initial time and at an arbitrary time. In Ref. [12], the information is characterized by the trace distance  $\mathcal{D}(\rho_1, \rho_2) = \frac{1}{2}\text{tr}|\rho_1 - \rho_2|$  of two quantum states  $\rho_1$  and  $\rho_2$ , describing the distinguishability between the two states and satisfying  $0 \leq \mathcal{D} \leq 1$  [3].  $\mathcal{D}$  maximally reaches 1 when the two states are totally distinguishable and approaches 0 for two identical states [3]. The direction of information flow is dependent on the slope of  $\mathcal{D}(\rho_1(t), \rho_2(t))$  [i.e., when  $\partial_t \mathcal{D}(\rho_1(t), \rho_2(t)) < 0$ ] the information dissipates to the environment and vice versa. Therefore, the non-Markovianity can

be calculated by

$$\mathcal{N}(\Phi) = \max_{\rho_{1,2}(0)} \sum_n [\mathcal{D}(\rho_1(\tau_n^{\max}), \rho_2(\tau_n^{\max})) - \mathcal{D}(\rho_1(\tau_n^{\min}), \rho_2(\tau_n^{\min}))], \quad (1)$$

with  $\tau_n^{\min}$  ( $\tau_n^{\max}$ ) the time point when  $\mathcal{D}(\rho_1(t), \rho_2(t))$  reaches the  $n$ th local minimum (maximum). Equation (1) can be carried out by summing up the amount of the increase of the trace distance over each time interval  $[\tau_n^{\min}, \tau_n^{\max}]$  for any pair of initial states  $\rho_1(0)$  and  $\rho_2(0)$ , where the maximum is considered as the non-Markovianity  $\mathcal{N}(\Phi)$ . Since this is a problem of optimization, however, we have to consider all pairs of initial states in our calculation, which is inconvenient and impractical, especially from the viewpoint of experimental exploration.

In this work, we show that the optimization problem of  $\mathcal{N}(\Phi)$  can be simplified to an effectively computable expression in the case of a single qubit coupled to a zero-temperature bosonic reservoir. Moreover, for two independent qubits coupled to two bosonic reservoirs, respectively, we investigate a lower bound of the non-Markovianity. The favorable feature of our method is the possibility to connect the non-Markovianity to the population of the qubit in the excited state, which can be detected directly in experiments without the requirement of tomographic reconstruction of the density matrix. In addition, our result also makes it possible to have an easy evaluation of non-Markovianity even without any information about the interaction between the qubit and the reservoir.

We first consider a single qubit coupled to a zero-temperature bosonic reservoir. The Hamiltonian in units of  $\hbar = 1$  is given by

$$H = \omega_0 |e\rangle\langle e| + \sum_l \omega_l a_l^\dagger a_l + \sum_l (g_l |e\rangle\langle g| a_l + g_l^* |g\rangle\langle e| a_l^\dagger), \quad (2)$$

where  $\omega_0$  is the resonant transition frequency of the qubit between the excited state  $|e\rangle$  and the ground state  $|g\rangle$ .  $\omega_l$  and  $a_l$  ( $a_l^\dagger$ ) are, respectively, the frequency and the annihilation (creation) operator of the  $l$ th mode of the reservoir with the

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coupling constant  $g_l$  to the qubit. The dynamics of the single qubit can be represented by the reduced density matrix [1]

$$\rho^S(t) = \begin{bmatrix} \rho_{ee}^S(0)|b(t)|^2 & \rho_{eg}^S(0)b(t) \\ \rho_{ge}^S(0)b^*(t) & 1 - \rho_{ee}^S(0)|b(t)|^2 \end{bmatrix}, \quad (3)$$

in the qubit basis  $\{|e\rangle, |g\rangle\}$ , where the superscript  $S$  of  $\rho$  represents the single-qubit case.  $b(t)$  can be interpreted as the amplitude of the upper level  $|e\rangle$  of the qubit initially prepared with  $\rho_{ee}^S(0) = 1$  and  $b(0) = 1$ , and is given by the inverse Laplace transform

$$b(t) = \mathcal{L}^{-1} \left[ \frac{1}{s + F(s)} \right], \quad (4)$$

where the parameter  $s$  is a complex number and  $F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t) \exp(-st) dt$  with the correlation function  $f(t) = \sum_l |g_l|^2 e^{i\delta_l t} = \int d\omega J(\omega) e^{i\delta_l t}$  and  $\delta_{(l)} = \omega_0 - \omega_{(l)}$ . The explicit form of  $b(t)$  depends on the specific spectral density of the reservoir [1].

As the density matrix should be of Hermiticity, normalization, and semipositivity, any pair of initial states can be written as

$$\begin{aligned} \rho_1^S(0) &= \begin{pmatrix} \alpha & \beta \\ \beta^* & 1 - \alpha \end{pmatrix}, \\ \rho_2^S(0) &= \begin{pmatrix} \mu & \nu \\ \nu^* & 1 - \mu \end{pmatrix}, \end{aligned} \quad (5)$$

with  $|\beta|^2 \leq \alpha(1 - \alpha)$ ,  $|\nu|^2 \leq \mu(1 - \mu)$ ,  $(\beta, \nu) \in \mathbb{C}$  and  $0 \leq \alpha$  ( $\mu$ )  $\leq 1$ ,  $(\alpha, \mu) \in \mathbb{R}$ . So we have

$$\begin{aligned} \rho_1^S(t) &= \begin{bmatrix} \alpha|b(t)|^2 & \beta b(t) \\ \beta^* b^*(t) & 1 - \alpha|b(t)|^2 \end{bmatrix}, \\ \rho_2^S(t) &= \begin{bmatrix} \mu|b(t)|^2 & \nu b(t) \\ \nu^* b^*(t) & 1 - \mu|b(t)|^2 \end{bmatrix}. \end{aligned} \quad (6)$$

Using the definition of the trace distance, we obtain

$$\mathcal{D}^S(\rho_1^S(t), \rho_2^S(t)) = |b(t)| \sqrt{|b(t)|^2(\alpha - \mu)^2 + |\beta - \nu|^2}. \quad (7)$$

In what follows, we consider the case that the qubit interacts resonantly with a reservoir with Lorentzian spectral distribution

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \Gamma^2}{(\omega_0 - \omega)^2 + \Gamma^2}, \quad (8)$$

with  $\gamma_0$  the Markovian decay rate and  $\Gamma$  the spectral width of the coupling [1], which has been widely employed in quantum optics [15]. We may distinguish the Markovian and the non-Markovian regimes using  $\gamma_0$  and  $\Gamma$ :  $\gamma_0 < \Gamma/2$  means the Markovian regime and  $\gamma_0 > \Gamma/2$  corresponds to the non-Markovian regime. Therefore, using Eq. (4), we have  $b(t) = \exp(-\Gamma t/2) [\cosh(\kappa t/2) + (\Gamma/\kappa) \sinh(\kappa t/2)]$ ,  $\gamma_0 < \Gamma/2$  and  $b(t) = \exp(-\Gamma t/2) [\cos(\kappa t/2) + (\Gamma/\kappa) \sin(\kappa t/2)]$ ,  $\gamma_0 > \Gamma/2$ , with  $\kappa = \sqrt{|\Gamma^2 - 2\gamma_0\Gamma|}$  [1]. We may check that, in the non-Markovian regime, all local minima of  $|b(t)|$  approach zeros at  $\tau_n^{\min} = 2[n\pi - \arctan(\kappa/\Gamma)]/\kappa$  with  $n = 1, 2, 3, \dots$  [i.e.,  $|b(\tau_n^{\min})| = 0$ ]. According to Eq. (7), all nontrivial trace distances own the same monotonicity, so  $\mathcal{D}^S(\rho_1^S(\tau_n^{\min}), \rho_2^S(\tau_n^{\min})) = 0$ . Therefore, Eq. (1) is reduced to  $\mathcal{N}^S = \max_{\rho_{1,2}(0)} \sum_n \mathcal{D}^S(\rho_1^S(\tau_n^{\max}), \rho_2^S(\tau_n^{\max}))$  and the

maximum taken over all pairs of initial states is equivalent to finding a trace distance whose local maxima are larger than those of others. In what follows, we adopt  $\mathcal{D}_{\mathcal{N}}^S(\rho_1^S(t), \rho_2^S(t))$  for such a trace distance and

$$\mathcal{N}^S = \sum_n \mathcal{D}_{\mathcal{N}}^S(\rho_1^S(\tau_n^{\max}), \rho_2^S(\tau_n^{\max})), \quad (9)$$

for the non-Markovianity of a single qubit case, where the summation is over all local maxima of  $\mathcal{D}_{\mathcal{N}}^S(\rho_1^S(t), \rho_2^S(t))$ . We first prove the theorem below.

*Theorem.* There exists a maximum trace distance  $|b(t)|$  at any instant time in Eq. (7) when  $\alpha = \mu = 1/2$ ,  $|\beta| = |\nu| = 1/2$  and  $|\beta - \nu| = 1$ .

*Proof.* Suppose  $\mathcal{D}^S(\rho_1^S(t), \rho_2^S(t)) = |b(t)|d(t)$  with  $d(t) = \sqrt{|b(t)|^2(\alpha - \mu)^2 + |\beta - \nu|^2}$  where  $d(t)$  can be taken as the distance between the points  $P_1(|b(t)|\alpha, \beta)$  and  $P_2(|b(t)|\mu, \nu)$ , and  $P_i(x, y)$  ( $i = 1, 2$ ) denotes the points with  $x \in \mathbb{R}$  and  $y \in \mathbb{C}$ . It is convenient to check that  $[|b(t)|\alpha - |b(t)|/2]^2 / [|b(t)|/2]^2 + |\beta|^2 / (1/2)^2 - 1 = 4[|\beta|^2 - \alpha(1 - \alpha)] \leq 0$ , which implies that the point  $P_1[|b(t)|\alpha, \beta]$  or  $P_2[|b(t)|\mu, \nu]$  is in (or on the circumference of) the ellipse

$$[x - |b(t)|/2]^2 / [|b(t)|/2]^2 + |y|^2 / (1/2)^2 = 1, \quad (10)$$

with  $x \in \mathbb{R}$  and  $y \in \mathbb{C}$ . We know that the maximum distance between the two points in an ellipse is in between the two ends of the major axis. Since  $|b(t)| < 1$  ( $t > 0$ ),  $d(t)$  reaches the maximum 1 only in the case of  $|b(t)|\alpha = |b(t)|\mu = |b(t)|/2$  (i.e.,  $\alpha = \mu = 1/2$  and  $|\beta| = |\nu| = 1/2$  and  $|\beta - \nu| = 1$ ). Consequently,  $\mathcal{D}^S(\rho_1^S(t), \rho_2^S(t))$  will also reach the maximum  $|b(t)|$ . ■

It is easy to check that any pair of initial states satisfying the conditions in the above theorem definitely owns the same trace distance  $|b(t)|$ . Since the maximum of the trace at any instant time is  $|b(t)|$ , for any pair of the initial states not meeting the conditions in the above theorem, the local maxima of  $|b(t)|$  should never be larger than those of the initial pairs meeting the conditions. In what follows, we will employ  $\mathcal{D}_{\mathcal{N}}^S = |b(t)|$  as the trace distance for measuring non-Markovianity. As a result, the calculation of non-Markovianity can be simplified to an easily computable expression

$$\mathcal{N}^S = \sum_n |b(\tau_n^{\max})|, \quad (11)$$

with  $\tau_n^{\max}$  the time point when  $|b(t)|$  reaches the  $n$ th local maximum.

Straightforwardly, we can find the relationship between the population of a single qubit initially in the excited state  $|e\rangle$  and the maximum trace distance  $\mathcal{P}_{|e\rangle} = (\mathcal{D}_{\mathcal{N}}^S)^2$ , which yields

$$\mathcal{N}^S = \sum_n \sqrt{\mathcal{P}_{|e\rangle}(\tau_n^{\max})}, \quad (12)$$

from which the non-Markovianity of the qubit coupled to the reservoir can be measured from the population of the upper level of the qubit. The requirements for this implementation are (i) the bosonic reservoir is initially in a vacuum state and (ii) the initial state of the qubit is prepared in the upper level.

Figure 1(a) shows a comparison between the non-Markovianity [the dark (blue) bars] and  $\mathcal{N}_{\rho_{|e\rangle}, \rho_{|g\rangle}}^S$  [the light (blue) bars] in the case of a non-Markovian regime ranging

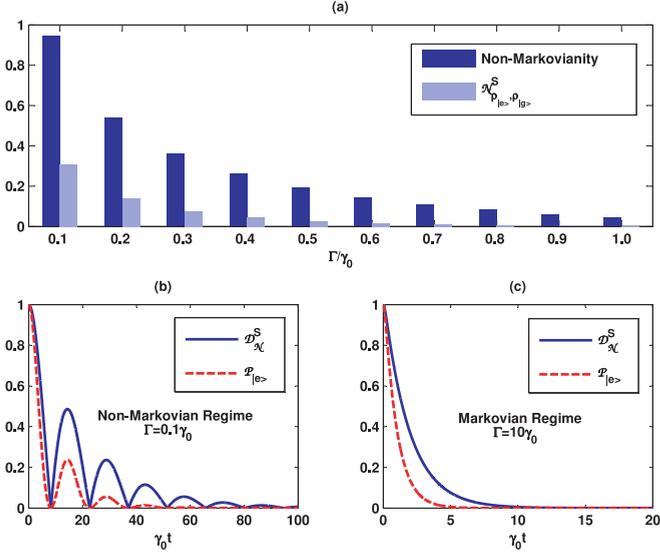


FIG. 1. (Color online) (a) Blue bars (black in the printed version): The non-Markovianity of a single qubit coupled to a zero-temperature bosonic reservoir with a Lorentzian spectrum. Light blue bars (gray in the printed version): The total growth of trace distance with the initial pair of states  $\rho_{|e\rangle}(0) = |e\rangle\langle e|$  and  $\rho_{|g\rangle}(0) = |g\rangle\langle g|$ . The maximum trace distance  $\mathcal{D}_{\mathcal{N}}^S$  (blue solid lines) characterizes the decay process (red dashed lines) of a single qubit in (b) non-Markovian regime (e.g.,  $\Gamma = 0.1\gamma_0$ ) and (c) Markovian regime (e.g.,  $\Gamma = 10\gamma_0$ ), respectively.

from  $\Gamma = 0.1\gamma_0$  to  $\Gamma = \gamma_0$ , where  $\mathcal{N}_{\rho_{|e\rangle}, \rho_{|g\rangle}}^S$  is measured by the trace distance  $\mathcal{D}^S(\rho_{|e\rangle}(t), \rho_{|g\rangle}(t)) = |b(t)|^2$  with the initial pair of states  $\rho_{|e\rangle}(0) = |e\rangle\langle e|$  ( $\alpha = 1, \beta = 0$ ) and  $\rho_{|g\rangle}(0) = |g\rangle\langle g|$  ( $\mu = 0, \nu = 0$ ). Due to  $|b(t)| < 1$  and  $|b(t)|^2 < |b(t)|$ , the light (blue) bars are always shorter than the dark (blue) ones. In addition, it can be seen that the indistinguishability of the states grows with the increase of the reservoir bandwidth  $\Gamma$ . This might be interpreted as the non-Markovian character becoming less evident when the coupling between the qubit and the reservoir decreases.

We studied in Fig. 1(b) the population of the excited state  $\mathcal{P}_{|e\rangle}$  and  $\mathcal{D}_{\mathcal{N}}^S$  in the non-Markovian regime (e.g.,  $\Gamma = 0.1\gamma_0$ ), which shows the population  $\mathcal{P}_{|e\rangle}$  reviving with the increase of  $\mathcal{D}_{\mathcal{N}}^S$ . This can be explained as the non-Markovian effect of the bosonic reservoir: The process with  $\mathcal{D}_{\mathcal{N}}^S$  being larger corresponds to the case that information lost by the qubit flows back from the reservoir, increasing the distinguishability. Therefore, the population revives for several times during this period. In contrast to the non-Markovian regime, no revival of the population occurs in the weak-coupling Markovian regime (e.g.,  $\Gamma = 10\gamma_0$ ) as shown in Fig. 1(c) because  $\mathcal{D}_{\mathcal{N}}^S$  monotonously approaches zero with  $b(t) \simeq \exp[-(\Gamma - \kappa)t/2]$ . This implies no information flowing back to the qubit.

The above analysis can be conveniently extended to other spectral densities, other than Lorentzian spectral distribution, for the non-Markovian characterization of a qubit coupled to a bosonic reservoir [16], as long as the condition  $|b(\tau_n^{\min})| = 0$  ( $n = 1, 2, 3, \dots$ ) is fulfilled [17].

Let us take a brief look at the non-Markovianity of two identical noninteracting qubits  $A$  and  $B$  locally interacting with two independent zero-temperature bosonic reservoirs,

respectively. We noticed that taking the maximum over any pair of initial states according to Eq. (1) is nearly intractable in the two-qubit case, although numerical simulation might be employed for this job [12]. However, since any growth of the trace distance is a clear illustration of non-Markovian character, in the following, instead of finding the maximum, we consider the trace distance

$$\mathcal{D}^T(\rho_{|++\rangle}(t), \rho_{|--\rangle}(t)) = |b(t)|\sqrt{2 - 2|b(t)|^2 + |b(t)|^4}, \quad (13)$$

as the measurement of the non-Markovian character of the two qubits where the initial pair of states  $\rho_{|++\rangle}(0) = |+\rangle_A\langle +| \otimes |+\rangle_B\langle +|$  and  $\rho_{|--\rangle}(0) = |-\rangle_A\langle -| \otimes |-\rangle_B\langle -|$  with  $|\pm\rangle_i = (|g\rangle_i \pm |e\rangle_i)/\sqrt{2}$  and  $i = A, B$ . Similar to the single-qubit case, the trace distance approaches zeros [i.e.,  $\mathcal{D}^T(\rho_{|++\rangle}(\tau_n^{\min}), \rho_{|--\rangle}(\tau_n^{\min})) = 0$  when  $|b(\tau_n^{\min})| = 0$ ]. Therefore, a lower bound of non-Markovianity can be calculated by

$$\mathcal{N}_{\rho_{|++\rangle}, \rho_{|--\rangle}}^T = \sum_n |b(\tau_n^{\max})|\sqrt{2 - 2|b(\tau_n^{\max})|^2 + |b(\tau_n^{\max})|^4}, \quad (14)$$

or by the equivalent form

$$\mathcal{N}_{\rho_{|++\rangle}, \rho_{|--\rangle}}^T = \sum_n \sqrt{2\mathcal{P}_{|e\rangle}(\tau_n^{\max}) - 2\mathcal{P}_{|e\rangle}(\tau_n^{\max})^2 + \mathcal{P}_{|e\rangle}(\tau_n^{\max})^3}. \quad (15)$$

Similar to the single-qubit case, the non-Markovian character lowers with the increase of the spectral width of the coupling, as shown in Fig. 2(a).

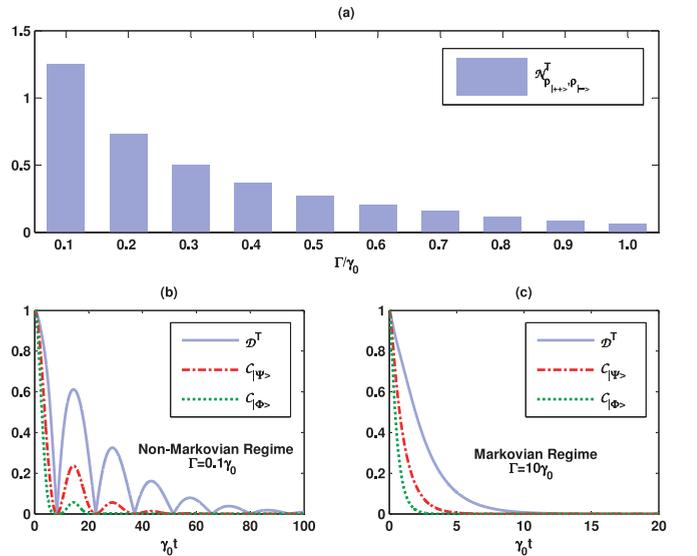


FIG. 2. (Color online) (a) A lower bound of non-Markovianity corresponding to the trace distance  $\mathcal{D}^T$  with the initial pair of states  $\rho_{|++\rangle}(0) = |+\rangle_A\langle +| \otimes |+\rangle_B\langle +|$  and  $\rho_{|--\rangle}(0) = |-\rangle_A\langle -| \otimes |-\rangle_B\langle -|$  ( $|\pm\rangle_i = (|g\rangle_i \pm |e\rangle_i)/\sqrt{2}, i = A, B$ ). The trace distance  $\mathcal{D}^T$  (blue solid lines) characterizes the dynamics of two noninteracting qubits coupling to two independent reservoirs in (b) the non-Markovian regime (e.g.,  $\Gamma = 0.1\gamma_0$ ) and (c) the Markovian regime (e.g.,  $\Gamma = 10\gamma_0$ ). While the red dot-dashed lines and green dotted lines represent the concurrence of two qubits initially prepared in Bell states  $|\Psi\rangle = (|ge\rangle + |eg\rangle)/\sqrt{2}$  and  $|\Phi\rangle = (|gg\rangle + |ee\rangle)/\sqrt{2}$ , respectively.

Conventionally, the dynamics of two entangled qubits can be measured by the Wootters concurrence [18]. Using the method in Ref. [19], we obtain the concurrence of the two qubits as

$$\begin{aligned} C_{|\Psi\rangle} &= \max\{0, |b(t)|^2\}, \\ C_{|\Phi\rangle} &= \max\{0, |b(t)|^4\}, \end{aligned} \quad (16)$$

when the initial states are prepared in Bell states  $|\Psi\rangle = (|ge\rangle + |eg\rangle)/\sqrt{2}$  and  $|\Phi\rangle = (|gg\rangle + |ee\rangle)/\sqrt{2}$ , respectively [19].

The trace distance  $\mathcal{D}^T$  [Eq. (13)] measuring a lower bound for non-Markovianity can well characterize the non-Markovian behavior of the dynamics of two qubits. In Fig. 2(b), the dynamics of the qubits initially prepared in Bell states  $|\Psi\rangle$  [(red) dot-dashed line] and  $|\Phi\rangle$  [(green) dotted line], respectively, in the non-Markovian regime (e.g.,  $\Gamma = 0.1\gamma_0$ ) is investigated. The entanglement measured by concurrence periodically vanishes in accordance with the trace distance  $\mathcal{D}^T$ . In contrast, the trace distance  $\mathcal{D}^T$ , in the Markovian regime (e.g.,  $\Gamma = 10\gamma_0$ ), asymptotically approaches zero, as shown in Fig. 2(c), which implies that no entanglement revival exists in the Markovian regime.

In practice, our scheme will be very preferable for experimental implementation. According to Eqs. (12) and (15),

we can study the non-Markovian effect on coherence and entanglement of the qubits coupled to bosonic reservoirs by directly measuring the population of a qubit without resorting to tomographic reconstruction of the density matrix. Besides, our proposal requires no specific information about the interaction between the qubit and the reservoir. These favorable features make our scheme feasible in experiments under real environments (e.g., using two-level atoms confined in optical microcavities [20]) or under simulated reservoirs [21] (e.g., using a spin-reservoir model with spectral densities ranging from subohmic to superohmic cases simulated by trapped ions [22]).

To summarize, we presented a simple method for measuring the non-Markovian character of the qubits coupled to bosonic reservoirs. We believe that this easily operated measure for non-Markovianity will be very useful for further understanding non-Markovian behavior and also for experimental exploration, particularly in realistic experimental situations without knowing much about the interaction between a qubit and the environment.

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