Snakelike nonautonomous solitons in a graded-index grating waveguide

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We present a series of analytical solutions which describe nonautonomous solitons in a planar waveguide with an additional periodical structure, that is, a long-period grating. The explicit functions which describe the evolution of the width, peak, and trajectory of the soliton's wave center are presented exactly. The gain parameter has no effects on the motion of the soliton's wave center or its width; it affects just the evolution of the soliton's peak. The grating term affects the motion of the soliton's wave center without changing its shape. The evolution of the soliton under the propagation-distance-dependent gain term is investigated too. It is reported that an arbitrary additional structure can be added on the graded-index waveguide to control the motion of the soliton without affecting its shape.

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I. INTRODUCTION

The discovery of optical solitons in nonlinear optics has attracted great interest in experimental and theoretical studies [1–14]. Especially, solitons in a nonlinear waveguide have been widely studied. For example, Agrawal and Ponomarenko presented an exact analytical solution inside planar graded-index waveguide amplifiers with Kerr nonlinearity using a symmetry-group analysis method [3]. Li *et al.* addressed the dynamics of fundamental and high-order dark solitons on the intense parabolic background in a planar graded-index waveguide with self-defocusing nonlinearity [4]. Serkin and Hasegawa presented a broad class of self-similar solitary wave solutions of the nonlinear equation model with varying dispersion, nonlinearity, and gain or absorption [5]. Nonlinearity management in optics has been done experimentally in [14]. However, a planar nonlinear waveguide with an additional periodic structure in the direction of propagation has drawn less attention, perhaps due to a lack of experimental realizations. It is, nevertheless, an interesting and potentially useful geometry where optical solitons can propagate. In this grating waveguide, the dynamics of optical solitons are governed by the nonautonomous nonlinear Schödinger equation (see the following) due to the management of nonlinearity and the presence of dissipation or gain [2,10,12].

In this article, we study the properties of solitons in a planar long-period-grating waveguide. In particular, investigations have been made to understand the properties of nonautonomous solitons under variation of the Kerr nonlinear parameter, appropriate gain or loss terms, modulation of the refractive index, and so on. We find that the grating term affects the motion of the soliton's wave center without changing its shape. A certain additional structure can be added on the graded-index waveguide to control the soliton's motion, preserving its shape. When the parameter $\lambda \to 0$, which relates the refractive index and Kerr nonlinear parameter, the width of the soliton can be stable. The gain parameter affects only the evolution of the soliton's peak and has no effects on the motion of the soliton's wave center or its width. Especially, if the gain is a constant γ, the peak will be stable with the condition $\lambda = 2\gamma$. The evolution of nonautonomous solitons under the propagation distance-dependent gain term are investigated too.

II. THE NONLINEAR EQUATION AND BRIGHT NONAUTONOMOUS SOLITON SOLUTION

We start by considering the propagation of a continuous-wave optical beam inside a planar, graded-index nonlinear waveguide amplifier with the refractive index

$$n = n_0 + n_1[f(z)x^2 + 2\tilde{l}x\cos(\tilde{\omega}z)] + n_2 r(z)I(x,z),$$

where $I(x,z)$ is the optical intensity and x and z are the spatial coordinate and propagation distance, respectively. Here the first two terms describe the linear part of the refractive index, $\tilde{l}x\cos(\tilde{\omega}z)$ stands for a long-period grating, and the last term represents a Kerr-type nonlinearity of the waveguide amplifier. For convenience, we assume $n_1 > 0$, and the dimensionless function $f(z)$ can be negative or positive, corresponding to acting as a focusing or a defocusing lens. The Kerr parameter $n_2 r(z)$ can be positive (negative) for a nonlinear self-focusing (self-defocusing) medium.

It is well known that exactly self-similar waves have been found in optical fibers whose dispersion, nonlinearity, and gain profile are allowed to change with the propagation distance, but the function cannot be chosen independently [12,15]. The nonautonomous nonlinear wave equation governing beam propagation in such a waveguide can be written as

$$i\frac{\partial u}{\partial z} + \frac{1}{2k_0}\frac{\partial^2 u}{\partial x^2} + \frac{k_0 n_1}{n_0}V(x,z)u + \frac{k_0 n_2}{n_0}r(z)|u|^2 u + \frac{ig(z)}{2}u = 0, \tag{1}$$

where $V(x,z) = f(z)x^2 + 2\tilde{l}x\cos(\tilde{\omega}z)$, $g(z)$ is the gain [$g(z) < 0$] or loss [$g(z) > 0$] coefficient, and $k_0 = 2\pi n_0/\lambda_0$ is the wave number, with λ_0 being the wavelength of the optical source generating the beam. Introducing the normalized variables $U = \sqrt{(k_0|n_2|L_D/n_0)}u, X = \sqrt{2}x/\omega_0$, $l = \sqrt{2}\tilde{l}\omega_0, Z = z/L_D$, and $\omega = \tilde{\omega}L_D, G(Z) = g(z)L_D$, where $\omega_0 = (2k_0^2 n_1/n_0)^{-1/4}$ and $L_D = k_0\omega_0^2$ represent the

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characteristic transverse scale and the diffraction length, respectively. Thus Eq. (1) can be rewritten in a dimensionless form:

$$i\frac{\partial U}{\partial Z}+\frac{\partial^2 U}{\partial X^2}+\frac{1}{4}FX^2U+lX\cos(\omega Z)U + \sigma R|U|^2U+\frac{iG}{2}U=0, \tag{2}$$

where $\sigma = n_2/|n_2| = \pm 1$ corresponds to self-focusing (+) and self-defocusing (−) nonlinearity of the waveguide, respectively, and $F(Z)$, $R(Z)$, and $G(Z)$ are functions of the normalized distance Z. In this case, we assume

$$F=\lambda^2,\qquad R=2g\exp\left[\int G(Z)dZ+\lambda Z\right]. \tag{3}$$

By performing the Darboux transformation method [16], the analytical solution of Eq. (2) is presented as

$$U[X,Z]=\frac{4\alpha A_c\exp\theta}{\sqrt{g}\left[1+A_c^2\exp\varphi\right]}, \tag{4}$$

where

$$\theta = A+B+C+D+E,$$

$$\varphi=\frac{8\alpha\beta}{\lambda}e^{2\lambda Z}-4\alpha X e^{\lambda Z}-8l\alpha e^{\lambda Z}\cos(\omega Z)/(\lambda^2+\omega^2),$$

where

$$A=\frac{\lambda l X}{2\omega}\left(\frac{e^{-i\omega Z}}{i\omega+\lambda}+\frac{e^{i\omega Z}}{i\omega-\lambda}\right)-2(\alpha-i\beta)Xe^{\lambda Z} -\frac{\lambda^2l^2}{8\omega^3}\left(\frac{e^{-i2\omega Z}}{(i\omega+\lambda)^2}-\frac{e^{i2\omega Z}}{(-i\omega+\lambda)^2}\right),$$

$$B=\frac{i\lambda^2l^2Z}{2\omega^2(\omega^2+\lambda^2)}+\frac{i2\lambda l(\alpha-i\beta)}{\omega(\lambda^2+\omega^2)}(e^{\lambda Z+i\omega Z}-e^{\lambda Z-i\omega Z}) +i2(\alpha-i\beta)^2e^{2\lambda Z}/\lambda,$$

$$C=\frac{\lambda l^2}{4\omega^3}\left(\frac{e^{-i2\omega Z}}{i\omega+\lambda}+\frac{e^{i2\omega Z}}{i\omega-\lambda}\right)+\frac{i\lambda l^2}{2\omega^2} \times\left(\frac{Z}{-i\omega+\lambda}+\frac{Z}{i\omega+\lambda}\right),$$

$$D=\frac{i2l(\alpha-i\beta)}{\omega}\left(\frac{e^{\lambda Z-i\omega Z}}{-i\omega+\lambda}-\frac{e^{\lambda Z+i\omega Z}}{i\omega+\lambda}\right) -\frac{il^2Z}{2\omega^2}+\frac{il^2\sin(2\omega Z)}{4\omega^3},$$

$$E=(-i\lambda X^2/4)+(\lambda Z/2)-\int[G(Z)/2dZ] +[ilX\sin(\omega Z)/\omega].$$

Here A_c, α, and β are arbitrary real constants. In our solution, the gain term can be chosen arbitrarily, as long as the function can be integrated. It is convenient to study the property of nonautonomous solitons under different gain media in the planar graded-index grating waveguide. This is one of the main features discussed here. When $l = 0$, the properties of solitons in the planar graded-index waveguide without grating can be studied conveniently, which is similar to the work in [3] and [5].

III. THE FUNCTION OF EACH SYSTEM PARAMETER

Assume that the peak of the soliton corresponds to the central position of the envelope, so we can present the explicit expression of the central position, which satisfies the condition $1 - A_c^2e^{\varphi} = 0$. In this case, the evolution of its width can be given as follows (we define the half-value corresponding width as the width of a bright nonautonomous soliton):

$$W(Z)=\frac{e^{-\lambda Z}}{4\alpha}\ln\frac{2+\sqrt{3}}{2-\sqrt{3}}. \tag{5}$$

This means that the grating and gain parameters have no effects on the soliton's width; one can control the evolution of width via the parameter λ. It is clear that the width decreases with propagation distance for $\lambda > 0$ and increases for $\lambda < 0$. This is because the soliton can exist only if nonlinear effects balance diffraction or dispersion effects; if the balance is destroyed, its width will change. When $\lambda > 0$, the nonlinearity increases with the propagation distance [shown in Eq. (3)], yet the diffraction term is constant, so the soliton will be compressed and its width will become smaller. On the contrary, for $\lambda < 0$, the width will be broadened since the nonlinearity becomes weak but the diffraction term remains invariant. Obviously, the width will be stable at $\lambda \to 0$, which means that the two sides will nearly balance each other.

The wave central position of the soliton is given by

$$X_c=\frac{\ln A_c}{2\alpha}e^{-\lambda Z}+\frac{2\beta}{\lambda}e^{\lambda Z}-\frac{2l\cos(\omega Z)}{\lambda^2+\omega^2}, \tag{6}$$

which shows that the grating affects the motion of the soliton effectively and the gain parameter has no effects on the trajectory of the soliton's wave center. We plot the trajectory of the wave center with $\beta = 0$ in Fig. 1. It is obvious that the soliton oscillates, and it approaches the central axis ($x = 0$) when $\beta = 0$ and $\lambda > 0$. Interestingly, they approach the central axis and oscillate around the central axis while increasing the distance Z.

In the contrail equation for the soliton's wave center, the oscillation term is $-2l\cos(\omega Z)/(\lambda^2+\omega^2)$. It depends on the system parameters ω, l, and λ. The oscillating period along the x direction is $2\pi/\omega$; ω is called the oscillating frequency.

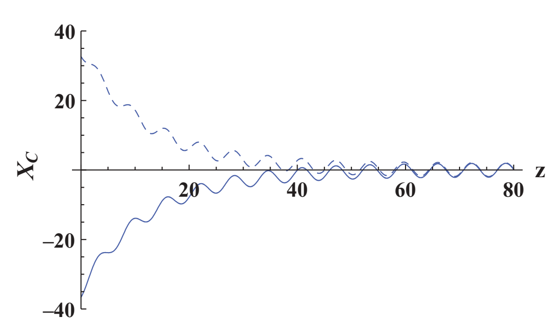

FIG. 1. (Color online) Evolution of a nonautonomous soliton's wave center for $\alpha = 0.01$ (dashed line) and $\alpha = -0.01$ (solid line) under the conditions $A_c = 2$, $\lambda = 0.08$, $\beta = 0$, $l = 1$, and $\omega = 1$.

The amplitude along the x direction of the oscillation is $2l/(\lambda^2+\omega^2)$. Since λ relates to the Kerr nonlinear parameter, which has a great influence on the shape of the soliton, one can change the amplitude along the x direction without changing its shape through the grating parameters l and ω. Thus, the bright nonautonomous soliton can oscillate with the addition of a grating, and its periodicity and amplitude can be controlled by adjusting ω and l.

From the preceding discussion, it can also be seen that the gain parameter has no effects on the trajectory of the soliton's wave center or its width. But the gain should affect the evolution of the soliton's peak. In the rest of this section, we study the effect of the gain parameter in detail.

First, we discuss the dynamics of a nonautonomous soliton with gain term $G(Z)=2\gamma$, which is a constant. The matter wave density from the solution [Eq. (4)] reads

$$|U[X,Z]|^2=\frac{16\alpha^2A_c^2\exp(\varphi+\lambda Z-2\gamma Z)}{g\left[1+A_c^2\exp(\varphi)\right]^2}. \tag{7}$$

The evolution of its peak can be described by the function

$$|U|^2_{\max}=4\alpha^2\exp(\lambda Z-2\gamma Z)/|g|. \tag{8}$$

It is clear that the grating does not affect the peak of the soliton. Consequently, it does not affect the soliton's shape at all in our system. We can make the bright soliton increase or decrease or remain stable by adjusting the two coefficients λ and γ. This provides a physical way to control the soliton's peak. Especially, when $\lambda=2\gamma$, its peak is a constant $(4\alpha^2/g)$ and is shown in Fig. 2. Of course, we can also design some other particular forms of gain parameter to control the evolution of the peak. Here we choose a Z-dependent form of gain parameter and observe the peak's evolution from the analytical solution.

The particular profile of the gain function is chosen as $G(z)=2l'\cos(\omega' Z)$; the solution can be presented from the normal solution (4). The evolution of its peak is given as follows:

$$|U|^2_{\max}=\frac{4\alpha^2}{|g|}\exp\left[\lambda Z-\frac{2l'\sin(\omega' Z)}{\omega'}\right]. \tag{9}$$

From Eq. (9), we find that the period of the peak's oscillation increases when the value of the gain frequency ω' decreases, and the amplitude of the peak's oscillation can be increased as l' increases. On the contrary, for $A_c=1$ and $\beta=0$, the nonautonomous soliton will oscillate around the central axis $x=0$. The trajectory of its wave center is $X_c=-2l\cos(\omega Z)/(\lambda^2+\omega^2)$ and the oscillation along the x direction of the soliton can be designed well by the grating. Therefore, we can design the evolution of the nonautonomous soliton's peak and the wave center's motion through the parameters l,ω and l',ω'. When $\lambda\to 0$, solitons will become similar to the one in Fig. 3 ($\omega'=\pm\omega$) or the one in Fig. 4 ($\omega'\neq\pm\omega$). Through modulation of both the grating parameter

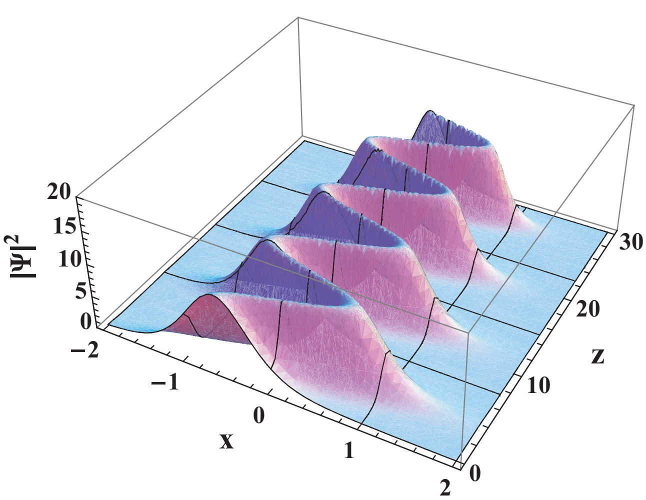

FIG. 2. (Color online) Dynamics of a bright nonautonomous soliton when $\lambda=2\gamma$. Parameters are $\alpha=1$, $\beta=0$, $\lambda=0.02$, $l=0.2$, $\omega=0.8$, $g=0.35$, $A_c=1$, and $\gamma=0.01$. It is shown that the peak is stable.

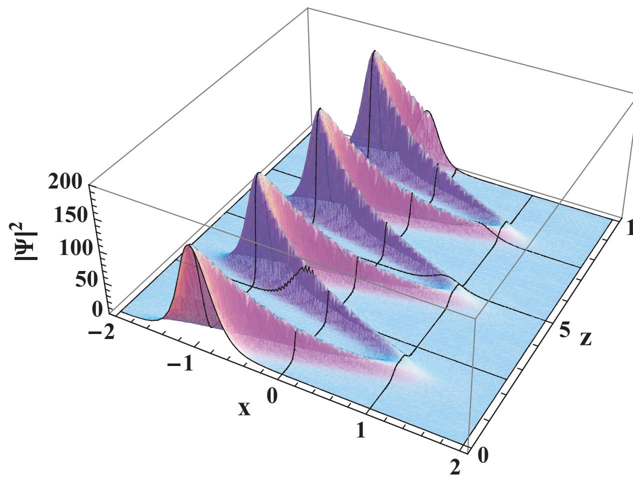

FIG. 3. (Color online) Dynamics of a bright nonautonomous soliton with different Z-dependent gain parameters for $\alpha=2$, $\beta=0$, $\lambda=0.02$, $g=0.35$, $A_c=1$, $l=2$, $\omega=2$, $l'=1.2$, and $\omega'=-2$.

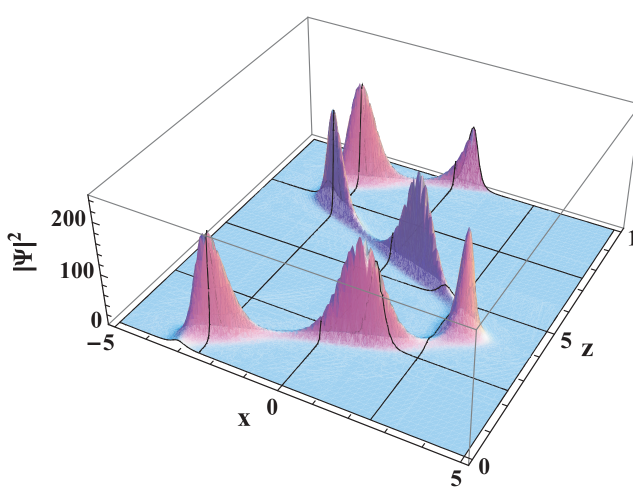

FIG. 4. (Color online) Dynamics of a bright nonautonomous soliton with different Z-dependent gain parameters for $\alpha=2$, $\beta=0$, $\lambda=0.02$, $g=0.35$, $A_c=1$, $l=1$, $\omega=0.8$, $l'=3$, and $\omega'=4$.

and the gain coefficients concurrently, we can get many kinds of solitons with different shapes.

Based on this property of nonautonomous solitons, one can obtain the expected soliton by controlling the grating's structure. For example, we replace the grating $l\cos(\omega Z)$ with $H(Z)$. In this grating waveguide, the contrail equation of the soliton takes the form

$$X_c = \frac{\ln A_c}{2\alpha}e^{-\lambda Z} + \frac{2\beta}{\lambda}e^{\lambda Z} + 2\frac{\int[\int e^{\lambda Z}H(Z)dZ]dZ}{e^{\lambda Z}}, \quad (10)$$

and the evolution of its width and peak remains invariant. Excitingly, the additional structure does not affect the soliton's shape and can be used to control its trajectory. Therefore, we can design the refractive index of the waveguide to control its motion without changing its shape, which lies in the wide potential application of spatial solitons.

Numerous dark soliton solutions with a negative Kerr nonlinear parameter have been presented in [4] and [17]. However, it should be noted that dark solitons cannot be obtained in our model by performing the Darboux transformation from a trivial seed, even if the Kerr nonlinear parameter is negative. It should be interesting to investigate dark solitons in this graded-index grating waveguide.

IV. CONCLUSION

We have deduced the exact bright nonautonomous soliton solutions of a nonlinear equation with an arbitrary gain profile. Having studied the evolution equations of the soliton's main quantities, including its width, its peak, and the motion of its center, we find that long-period grating can change the motion of a soliton and preserve its shape. The gain parameter has no effects on the motion of the soliton or its width; it affects only the peak. The peak can be controlled finely via λ and the gain parameter. However, the width can be controlled by the parameter λ, which relates to the refractive index and Kerr nonlinear parameter. This provides a particular way to control the evolution of solitons in the waveguide. Moreover, an arbitrary additional structure can be added on the graded-index waveguide to control the motion of nonautonomous solitons without affecting their shape.

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