Cooling a quantum circuit via coupling to a multiqubit system

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The cooling effects of a quantum *LC* circuit coupled inductively with an ensemble of artificial qubits are investigated. The particles may decay independently or collectively through their interaction with the environmental vacuum electromagnetic field reservoir. For appropriate bath temperatures and the resonator's quality factors, we demonstrate an effective cooling well below the thermal background. In particular, we found that for larger samples the cooling efficiency is better for independent qubits. However, the cooling process can be faster for collectively interacting particles.

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I. INTRODUCTION

The ability to cool interacting quantum systems below the values imposed by the thermal fluctuations of the environmental reservoir of each subsystem is actually of great interest [1]. For instance, a laser cooling technique for trapped particles exploiting quantum interference, or electromagnetically induced transparency, in a three-level atom was presented in [2]. There, by appropriately designing the absorption profile with a strong coupling laser, the cooling transitions induced by a cooling laser are enhanced while heating by resonant absorption is strongly suppressed. The experimental demonstration of ground-state laser cooling with electromagnetically induced transparency was performed in [3]. Recently, this idea was adopted to cool a nanomechanical resonator [4]. Further, the collective-emission-induced cooling of atoms in an optical cavity was also observed [5]. Mechanical effects of light in optical resonators were studied in [6] while cavity-assisted nondestructive laser cooling of atomic qubits was analyzed in [7]. A laser cooling method that can be used at large detuning and low saturation to cool particles inside an optical cavity was proposed in Ref. [8]. A significant speed-up of the cooling process was found in [9] while fast cooling of trapped ions using the dynamical Stark shift was described in [10].

By engineering superconducting elements as artificial atoms and coupling them to a photon field of a resonator or to vibrational states of a nanomechanical resonator, one can demonstrate interesting related phenomena, such as single artificial atom lasing or cooling. In particular, schemes to ground-state cooling of mechanical resonators were proposed in [11]. A flux qubit was experimentally cooled [12] using techniques somewhat related to the well-known optical sideband cooling methods (see, e.g., Ref. [1] and references therein). Lasing effects of a Josephson-junction charge qubit, embedded in a superconducting resonator, was experimentally demonstrated in [13]. Single-qubit lasing and cooling at the Rabi frequency was proposed in [14], while a mechanism of simultaneously cooling of an artificial atom and its neighboring quantum system was analyzed in [15]. Few-qubit lasing in circuit QED was discussed in Ref. [16]. A *LC* oscillator can be cooled via its nonlinear coupling to a Josephson flux qubit [17]. The cooling of a nanomechanical resonator via a Cooper pair box qubit has been recently suggested in Ref. [18] while cooling carbon nanotubes to the phononic ground state with a constant electron current was achieved in [19]. Further interesting works on cooling micro- and nanomechanical resonators were presented in Refs. [20–29].

Here, we describe a cooling scheme via coupling a pumped multiparticle ensemble (i.e., artificial atoms or qubits) to a single mode of a quantum *LC* circuit (see Fig. 1). Our motivation is to present an efficient method allowing for a rapid cooling of the resonator mode. The multiqubit system can be formed by an independent N-particle sample or by collectively interacting N particles. By independent, we mean that each particle spontaneously decays individually and all of them are maximally coupled with the oscillator mode and with the same phase. Collectively interacting particles means that their interactions are mediated by the environmental electromagnetic field reservoir such that their decay is of a collective nature. In this case, the particles are close to each other on a scale smaller than the emission wavelength and coupled with the same strength to the quantum oscillator mode. The advantages or disadvantages regarding the interparticle interactions to the cooling phenomena of the quantum oscillator degree of freedoms will be discussed in detail. In particular, we found that the cooling phenomenon is better for independently interacting qubits if the quantum dynamics of the *LC* oscillator is slower than that of the qubits. However, the cooling effects may occur faster for collectively interacting qubits. Apart from a fundamental interest, these systems have a great feature in various applications such as novel quantum sources of light (single photon sources, for instance), quantum processing of information, or entanglement. However, at MHz frequency ranges thermal fluctuations affect considerably the *LC* oscillators, that is, populate their energy levels and induce additional decoherences. Therefore, a suitable method of cooling these systems can be very useful.

The article is organized as follows. In Sec. II, we introduce the system of interest. Section III describes the obtained results. We finalize the article with conclusions presented in Sec. IV.

II. APPROACH

We describe the cooling effects of a quantum oscillator mode, that is, a quantum *LC* circuit coupled inductively with a collection of two-level Josephson flux qubits (see

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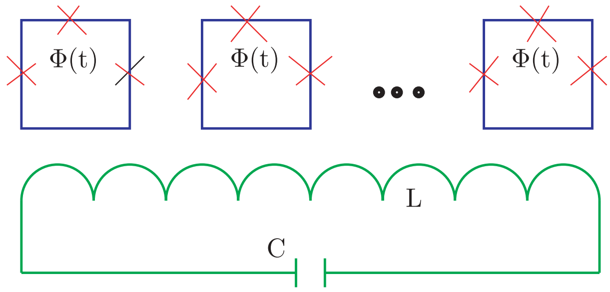

FIG. 1. (Color online) An ensemble of N independent three-junction flux qubits coupled to a LC quantum circuit by their mutual inductances. An ac magnetic flux $\Phi(t)$ drives the qubits.

Fig. 1). The two-level particles are pumped with a moderately intense magnetic flux and damped spontaneously via their interactions with the environmental electromagnetic field reservoir. The single-particle spontaneous decay rate is γ. Both subsystems interact with thermostats at effective temperatures T_1 and T_2. We shall consider that the particles are independent or collectively interacting. The frequency of the oscillator is much lower than the qubit's tunnel splitting; that is, $\omega_c \ll \Delta$. Therefore, the qubit is driven with Rabi frequencies near resonance with the oscillator frequency that affects the oscillator, increasing its oscillation amplitude [30]. Near the symmetry point (i.e., the energy bias ϵ between the flux states is small) and after transformation to the qubit's eigenbasis, the Hamiltonian describing the multiqubit systems is

$$H = \sum_{i=1}^{N}\{\Delta E\sigma_{zi}/2 + \Omega_0 \sin 2\theta\sigma_{xi}\cos(\omega t)\} + \omega_c a^\dagger a - g\sum_{i=1}^{N}(\cos 2\theta\sigma_{zi} - \sin 2\theta\sigma_{xi})(a+a^\dagger), \quad (1)$$

where the first term describes the qubits, each with the transition frequency $\Delta E = \sqrt{\Delta^2+\epsilon^2}$, while the second one considers their driving by an applied ac magnetic flux with amplitude $\tilde{\Omega}_0 = \Omega_0 \sin 2\theta$ and frequency ω. Here, $\cot 2\theta = \epsilon/\Delta$ with $\cos 2\theta = \epsilon/\Delta E$ and $\sin 2\theta = \Delta/\Delta E$. The last two terms describe the oscillator with frequency $\omega_c = 1/\sqrt{LC}$ as well as the qubit-oscillator interaction, respectively. Here, $g \approx MI_pI_{c0}$, where M is the mutual inductance, I_p the magnitude of the persistent current in the qubit, and $I_{c0} = \sqrt{\omega_c/2L}$, the amplitude of the vacuum fluctuations of the current in the LC oscillator. $a^\dagger$ and a are the creation and annihilation operators corresponding to the oscillator degrees of freedom, while σ_α ($\alpha \in \{x,y,z\}$) are the Pauli matrices operating in the dressed flux basis of the qubit subsystem. As $\Delta \gg \omega_c$, the transverse coupling in the Hamiltonian (1) is transformed into a second-order longitudinal coupling by employing a Schrieffer-Wolf-type transformation [14,31]; that is, $U_S = \exp(iS)$, with

$$S = (g/\Delta E)\sin 2\theta(a+a^\dagger)\sum_{i=1}^{N}\sigma_{yi}.$$

By further using the rotating wave approximation with respect to ω and diagonalizing the qubit term, as well as applying the secular approximation, that is, omitting terms oscillating with the generalized Rabi frequency, one arrives at the following Hamiltonian describing the interaction between the multiqubit system and the LC oscillator:

$$H = \omega_c a^\dagger a + \sum_{i=1}^{N}\{\Omega R_{zi}/2 + \tilde{g}(R_{+-}^{(i)}a + a^\dagger R_{-+}^{(i)}) + g_0(aa^\dagger + a^\dagger a)R_{zi}/2\}. \quad (2)$$

Here, we have further assumed that the generalized Rabi frequency is of the order of ω_c; that is, $\Omega \approx \omega_c$. In Eq. (2), $\tilde{g} = g\cos 2\theta \sin 2\xi$ gives the qubit-oscillator coupling strength, while $g_0 = 2g^2\cos 2\xi \sin^2 2\theta/\Delta E$ accounts for a small frequency shift of the qubit's frequency. Further,

$$\cot 2\xi = \delta\omega/\tilde{\Omega}_0, \quad (3a)$$
$$\cos^2\xi = [1+\delta\omega/\Omega]/2, \quad (3b)$$
$$\sin^2\xi = [1-\delta\omega/\Omega]/2, \quad (3c)$$

where $\delta\omega = \Delta E - \omega$ and where $\Omega = \sqrt{(\delta\omega)^2 + \tilde{\Omega}_0^2}$ stands for the generalized Rabi frequency. The dressed-state qubit operators $R_{\alpha\beta}^{(i)} = |\alpha\rangle_{ii}\langle\beta|$ describe the internal transition in the ith particle between the dressed state $|\beta\rangle$ and $|\alpha\rangle$ for $\alpha \neq \beta$ and population for $\alpha = \beta$, $\{\alpha,\beta \in +,-\}$ and obey the standard commutation relations of su(2) algebra; that is,

$$[R_{\alpha\beta}^{(j)}, R_{\alpha'\beta'}^{(l)}] = \delta_{jl}[\delta_{\beta\alpha'}R_{\alpha\beta'}^{(j)} - \delta_{\beta'\alpha}R_{\alpha'\beta}^{(j)}],$$

where $\alpha,\beta \in \{+,-\}$. $R_{zi} = R_{++}^{(i)} - R_{--}^{(i)}$ is the dressed-state inversion operator for the ith particle.

In the mean-field, dipole, Born-Markov and secular approximations, the combined system is characterized by the following master equation:

$$\frac{d}{dt}\rho + i[H,\rho] = -\Lambda_a\rho - \Lambda_c\rho. \quad (4)$$

The quantum dissipation due to spontaneous emission into the surrounding electromagnetic field reservoir is described by the $\Lambda_a\rho$ term, which for N independent qubits can be represented as follows:

$$\Lambda_a\rho = \sum_{i=1}^{N}\{\gamma_0[R_{zi}, R_{zi}\rho] + \gamma_+[R_{+-}^{(i)}, R_{-+}^{(i)}\rho] + \gamma_-[R_{-+}^{(i)}, R_{+-}^{(i)}\rho]\} + \text{H.c.} \quad (5)$$

For N nonindependent radiators, that is, for collectively interacting particles, the corresponding damping is

$$\Lambda_a\rho = \sum_{i,j=1}^{N}\{\gamma_0[R_{zi}, R_{zj}\rho] + \gamma_+[R_{+-}^{(i)}, R_{-+}^{(j)}\rho] + \gamma_-[R_{-+}^{(i)}, R_{+-}^{(j)}\rho]\} + \text{H.c.} \quad (6)$$

The damping rates are given by the following expressions:

$$\gamma_+ = \gamma\cos^4\xi, \quad (7a)$$
$$\gamma_- = \gamma\sin^4\xi, \quad (7b)$$
$$\gamma_0 = \gamma\sin^2 2\xi/4. \quad (7c)$$

The last term in Eq. (4) characterizes the damping of the quantum oscillator mode and is given as follows:

$$\Lambda_c\rho = \kappa[1+\bar{n}(\omega_c)][a^\dagger, a\rho] + \kappa\bar{n}(\omega_c)[a, a^\dagger\rho] + \text{H.c.} \quad (8)$$

Here, $\bar{n}(\omega_c)$ is the mean thermal photon number corresponding to the resonator frequency ω_c while κ is the resonator decay rate. We have omitted the coherent part of the dipole-dipole interaction in Eq. (4), which is justified if the Rabi frequency dominates over the dipole-dipole induced energy shifts.

A general analytical solution of Eq. (4) is not evident. However, one can obtain its solution for different regimes of interest, namely in the bad or the good cavity limit. Therefore, in the next section, we proceed by investigating the properties of Eq. (4) when the qubit's quantum dynamics is faster than the one of the quantum oscillator, that is, in the good cavity limit [32].

III. RESULTS AND DISCUSSIONS

We assume a moderately intense pumping field, that is, $\Omega \gg \{\gamma, \tilde{g}\sqrt{N}\}$ and a high-quality resonator such that $\gamma \gg \tilde{g}\sqrt{N} \gg \kappa$ for N independent particles or $\Omega \gg \{N\gamma, \tilde{g}\sqrt{N}\}$, and $N\gamma \gg \tilde{g}\sqrt{N} \gg \kappa$ for N collectively interacting particles. Therefore, in this case, the qubit subsystem achieves its steady-state on a time scale faster than the resonator field and, thus, the qubit variables can be eliminated to arrive at a master equation for the resonator field mode alone:

$$\dot{\rho} = -\Gamma_-\{a^\dagger a\rho - a\rho a^\dagger\} - \Gamma_+\{aa^\dagger\rho - a^\dagger\rho a\} + \text{H.c.}, \quad (9)$$

where an overdot means differentiation with respect to time and

$$\Gamma_- = \kappa[1+\bar{n}(\omega_c)] + B, \quad (10a)$$

$$\Gamma_+ = \kappa\bar{n}(\omega_c) + A. \quad (10b)$$

The physical meaning of the parameters in Eq. (9) is as follows: $\Gamma_+(\Gamma_-)$ describes the process of increasing (decreasing) of the photon number in the resonator mode. The interplay between Γ_+ and Γ_- leads to lasing or cooling of the quantum LC circuit.

For an independent N particle system, one has

$$A = \frac{\tilde{g}^2 N}{\Gamma_\perp}\langle R_{++}\rangle, \quad \text{and} \quad B = \frac{\tilde{g}^2 N}{\Gamma_\perp}\langle R_{--}\rangle, \quad (11)$$

where

$$\Gamma_\perp = 4\gamma_0 + \gamma_+ + \gamma_-. \quad (12)$$

We have considered here that $\langle R^{(i)}_{\alpha\beta}\rangle$ are identical for all $i \in \{1,2,\ldots,N\}$ and, thus, $\langle R^{(i)}_{\alpha\beta}\rangle \equiv \langle R_{\alpha\beta}\rangle$, where $\{\alpha,\beta \in +,-\}$. The expectation values for $\langle R_{\alpha\beta}\rangle$ are calculated in the absence of the resonator mode. Therefore, from Eq. (5), we have

$$\langle R_{++}\rangle = \frac{\gamma_-}{\gamma_- + \gamma_+} \quad \text{and} \quad \langle R_{--}\rangle = \frac{\gamma_+}{\gamma_- + \gamma_+}. \quad (13)$$

The expressions (11) are valid for any N satisfying the restrictions imposed in the beginning of the section, including $N = 1$.

For an ensemble of collectively interacting N particles, we obtain the following relations for A and B:

$$A = \frac{\tilde{g}^2}{\tilde{\Gamma}_\perp}\langle R_{+-}R_{-+}\rangle, \quad \text{and} \quad B = \frac{\tilde{g}^2}{\tilde{\Gamma}_\perp}\langle R_{-+}R_{+-}\rangle, \quad (14)$$

where the collective decay rate is given as

$$\tilde{\Gamma}_\perp = \Gamma_\perp + (\gamma_- - \gamma_+)\langle R_z\rangle. \quad (15)$$

One can observe here that the decay rate $\tilde{\Gamma}_\perp$ has a contribution arising from all particles, that is, the term proportional to $\langle R_z\rangle$. Here, in contrast to independent qubits, collective operators were introduced, that is, $R_{\alpha\beta} = \sum_{i=1}^N R^{(i)}_{\alpha\beta}$. Note that to obtain Eqs. (14), we decoupled the involved multiparticle correlators—an approximation valid for larger N, that is, $N \gg 1$. However, the corresponding expressions for $N = 1$ are identical to Eqs. (11) but with the single-particle decay rate $\Gamma_\perp$ instead of collective ones. The steady-state expectation values for the collective correlators entering into the preceding expressions [Eqs. (14)] can be estimated from the steady-state solution of the master equation Eq. (6) describing the strongly driven particles in the absence of the resonator [33,34],

$$\langle R_z\rangle = \frac{N(1-f^{2+N}) + f(N+2)(f^N-1)}{(f-1)(f^{N+1}-1)}, \quad (16a)$$

$$\langle R_{+-}R_{-+}\rangle = \frac{1}{1-f}\langle R_z\rangle, \quad (16b)$$

$$\langle R_{-+}R_{+-}\rangle = \frac{f}{1-f}\langle R_z\rangle, \quad (16c)$$

where $f = \gamma_+/\gamma_-$. For $N = 1$ one obtains Eqs. (13).

The Eq. (9) has an exact steady-state solution. For instance, the steady-state expectation values for the diagonal elements of Eq. (9) can be obtained from the relation

$$\rho = Z^{-1}\exp[-\alpha a^\dagger a], \quad (17)$$

where $\alpha = \ln(\eta)$, with $\eta = \Gamma_-/\Gamma_+$ and Z is determined by the requirement $\text{Tr}\{\rho\} = 1$. Evidently, the expectation values of the operators needed for evaluating the properties of the quantum oscillator are obtained from Eq. (17). In particular, the oscillator mean photon number, that is, $\langle n\rangle \equiv \langle a^\dagger a\rangle = \text{Tr}\{a^\dagger a\rho\}$, and its second-order correlations can be determined from the expressions

$$\langle a^\dagger a\rangle = \frac{1}{\eta - 1}, \quad (18a)$$

$$\langle a^{\dagger 2} a^2\rangle = \frac{2}{(\eta-1)^2}, \quad (18b)$$

to such an extent that the photon second-order correlation function, that is,

$$g^{(2)}(0) = \frac{\langle a^{\dagger 2}a^2\rangle}{\langle a^\dagger a\rangle^2},$$

is equal to 2, which means that the photon statistics is always super-Poissonian. Note that the mean photon numbers obtained with the help of Eq. (9) or Eq. (17) should be below the photon saturation number n_0, which for N interacting qubits reads approximately as

$$n_0 = \Gamma_\perp(\gamma_+ + \gamma_-)/(\tilde{g}^2 N). \quad (19)$$

Figure 2 depicts the mean photon number in the oscillator mode which is coupled with N independent qubits. We have used typical parameters here (see, for instance, [30]). To elucidate the role of many particles regarding the cooling

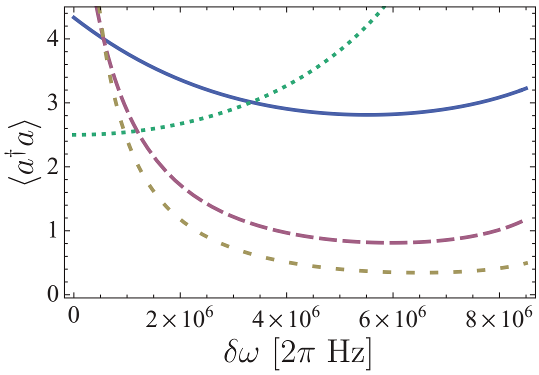

FIG. 2. (Color online) The mean photon number $\langle n \rangle$ into the quantum circuit as function of $\delta\omega$ and different numbers of independent qubits. The solid line is for $N = 1$, the long-dashed line stands for $N = 10$, and the short-dashed curve corresponds to $N = 30$. The dotted curve shows the saturation photon number n_0 for $N = 30$ qubits. Here, $\bar{n}(\omega_c) = 4$, $\Delta/2\pi = 3 \times 10^9$ Hz, $\epsilon = 0.01\Delta$, $g/2\pi = 10^6$ Hz, $\omega_c/2\pi = 10^7$ Hz, $\gamma/2\pi = 10^5$ Hz, $\kappa/2\pi = 10^3$ Hz, and $\tilde{\Omega}_0 = \sqrt{\Omega^2 - (\delta\omega)^2}$.

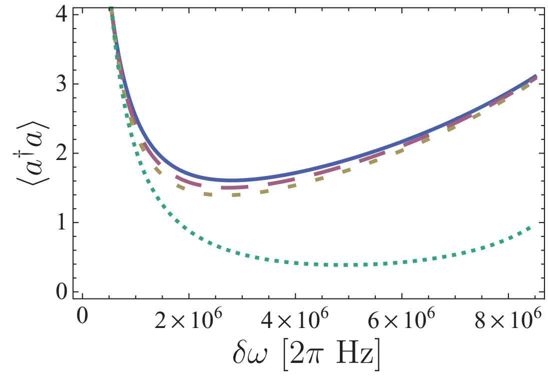

FIG. 3. (Color online) The mean photon number $\langle a^\dagger a \rangle$ of the quantum oscillator as the function of $\delta\omega$ and different numbers of collectively interacting qubits. The solid line is for $N = 10$, while the long-dashed line stands for $N = 15$. The short-dashed curve corresponds to $N = 30$, while the dotted one corresponds to $N = 30$, but $\kappa/2\pi = 10^2$ Hz. Other parameters are the same as in Fig. 2.

issue, we fix the involved parameters and change the number of qubits. Already for $N = 10$ particles, the cooling efficiency is significantly improved in comparison to the single-qubit case, that is, $N = 1$. Better cooling can be achieved, that is, $\langle n \rangle \ll \bar{n}(\omega_c)$, by increasing further the number of qubits (see the short-dashed curve in Fig. 2). Evidently, the qubits are in their lower dressed state when cooling occurs, that is, $\langle R_{--} \rangle > \langle R_{++} \rangle$. The diagram showing the energy levels of the qubit and oscillator indicating the cooling cycle with photon emission and absorption can be found in Refs. [25,26].

Further, we turn to cooling effects via collectively interacting particles. Figure 3 shows the mean photon number in the quantum oscillator mode as the function of $\delta\omega$. The mean photon number $\langle n \rangle$ is well below the thermal mean photon number $\bar{n}(\omega_c)$; however, the cooling is not so significant as for independent qubits (compare the short-dashed curves in Figs. 2 and 3). The reason is that the decay rate $\tilde{\Gamma}_\perp$ is dependent on the number of qubits and, thus, $\langle n \rangle = \Gamma_+/(\Gamma_- - \Gamma_+)$ is smaller than the corresponding one for independent qubits since $\Gamma_- - \Gamma_+ = \kappa - \tilde{g}^2\langle R_z \rangle/\tilde{\Gamma}_\perp$, where $\langle R_z \rangle$ is given by Eq. (16) (in other words, for collectively interacting particles, we do not have a factor N in the denominator). However, adjusting the involved parameters, one can improve the cooling efficiency in general (see the dotted line in Fig. 3). Note that the coupling of qubits can be controlled [35].

Finally, we discuss the time scaling for the cooling phenomenon. We observe that cooling rates depend on the number of qubits and, therefore, the cooling may occur faster in both schemes. However, the faster decay rate in our approach is the qubit spontaneous emission. Thus, the cooling phenomena cannot occur faster than γ^{-1} for independent qubits or $(N\gamma)^{-1}$ for collectively interacting particles, respectively. Therefore, in general, the cooling processes are faster for collectively interacting particles.

IV. SUMMARY

In summary, we described a scheme that is able to cool a quantum *LC* circuit coupled inductively to externally pumped artificial particles (Josephson flux qubits) and damped through their interaction with the environmental electromagnetic field reservoir. The qubits may interact collectively or they are independent. If the qubit's dynamics is faster than that of the *LC* oscillator, the cooling of the oscillators degrees of freedom occurs when controlling the qubit quantum dynamics. We found that the cooling phenomenon is better for an ensemble of independent qubits. However, in general, the cooling processes are faster for collectively interacting particles.

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