Spectral representation of the three-body Coulomb problem. II. Autoionizing doubly excited states of unnatural parity in helium

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A spectral approach of configuration interaction type is used to evaluate energies and widths for a wide range of singlet and triplet P^e resonance states of helium up to the eighth single ionization threshold. While the present data are in excellent agreement with existing theoretical results (below the $N = 3$–5 ionization threshold) obtained within an explicitly correlated approach, there are substantial differences with the energies, the widths, and the number of resonances obtained with the stabilization method.

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I. INTRODUCTION

Since the discovery of strong electron-electron correlation effects in doubly excited states (DES) of helium in the pioneering experiment by Madden and Codling [1], these states have attracted the continuous interest of both theoreticians and experimentalists. Most of the investigations have been focused on the states that are accessible through dipole transitions from the ground state of helium, that is states of a given angular momentum L with parity $\pi = (-1)^L$. States of unnatural parity $\pi = (-1)^{L+1}$ below the second single ionization threshold (SIT) do not autoionize (within a nonrelativistic approach). The existence of these nonautoionizing doubly excited states has been confirmed in a series of experiments [2–5]. Since the conventional variation procedure does apply in a straightforward way to nonautoionizing DES [6], it is not surprising that the large majority of the calculations are based on variational approaches (see Saha and Mukherjee [7] and references therein).

Above the second SIT, unnatural parity states are resonances. Energies and widths of $^3P^e$ resonances up to the seventh SIT have recently been evaluated using the stabilization method by Saha and Mukherjee [7]. The data are in good agreement with the few available low-lying resonances of the series converging to the $N = 3$–5 SIT obtained with the help of an approach that combines an explicitly correlated expansion in Hylleraas-type wave functions with complex rotation by Ho and Bathia [8]. In the present contribution, we evaluate the energy and the width of rather high-lying $^1P^e$ and $^3P^e$ resonances of unnatural parity of the series below the $N = 3$–8 SIT. Notice that nonautoionizing states of unnatural parity have already been considered in our recent paper [9]. The calculations are performed by means of a spectral approach of the configuration interaction (CI) type where the atomic wave function is expanded in a basis of products of Coulomb-Sturmian functions of the electron radial coordinates and the bipolar spherical harmonics of the angular coordinates. The resonances are extracted by using complex rotation. Although low-lying resonances are found to be in perfect agreement with the results of Ho and Bathia [8], we find substantial differences with the results of Saha and Mukherjee [7] for high excitations of the outer electron. Moreover, starting from the series below the sixth SIT, we do not find any agreement at all with the results of Saha and Mukherjee [7]. After a brief description of our approach in Sec. II, we present our results in Sec. III and our conclusions in Sec. IV. Atomic units (a.u.) are used throughout this contribution unless stated otherwise.

II. THEORETICAL APPROACH

A detailed description of our approach has already been given elsewhere [10–13]. We will thus give only a brief review of its most relevant aspects.

The eigenstate wave function of the helium atom with a total angular momentum L of projection M and total energy E_α satisfies the time-independent Schrödinger equation

$$(H - E_\alpha)\Psi_\alpha^{L,M}(\vec{r}_1,\vec{r}_2) = 0. \tag{1}$$

The nonrelativistic Hamiltonian H—under the assumption of an infinitely heavy nucleus—reads

$$H = \frac{\vec{p}_1^{\,2}}{2} + \frac{\vec{p}_2^{\,2}}{2} - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_{12}}, \tag{2}$$

where $Z = 2$, $\vec{p}_i$, r_i, and r_{12} are, respectively, the nuclear charge, the momentum and the distance of the electron i to the nucleus, and the distance between the two electrons.

In our CI approach [10–13], the solutions for Eq. (1) are expanded in Coulomb-Sturmian functions $S^{(k_i)}_{n_i,l_i}(r_i)$ [14,15] with independent dilation parameters k_i for both electrons ($i = 1,2$) coupled with bipolar spherical harmonics

$\Lambda^{L,M}_{l_1,l_2}(\hat{r}_1,\hat{r}_2)$ [16],

$$\Psi^{L,M}_{\alpha}(\vec{r}_1,\vec{r}_2) = \sum_{l_1,l_2}\sum_{s}\sum_{n_1,n_2} \psi^{l_1,l_2,L,M}_{k_{1s},k_{2s},n_1,n_2,\alpha} \times \mathcal{A}\frac{S^{(k_{1s})}_{n_1,l_1}(r_1)}{r_1}\frac{S^{(k_{2s})}_{n_2,l_2}(r_2)}{r_2}\Lambda^{L,M}_{l_1,l_2}(\hat{r}_1,\hat{r}_2). \quad (3)$$

The value of $\psi^{l_1,l_2,L,M}_{k_{1s},k_{2s},n_1,n_2,\alpha}$ is the expansion coefficient, and $\mathcal{A}$ is an operator for the projection on singlet or triplet states.

The fact that we allow the dilation parameters and the number of the Coulomb-Sturmian functions associated with electrons one and two to be different leads to the introduction of a set of Coulomb-Sturmian functions $\{S^{(k_{1s})}_{n_1,l_1}(r_1), S^{(k_{2s})}_{n_2,l_2}(r_2)\}$ associated with electrons one and two, which is characterized by the combination $[k_{1,s}, N^{\min}_{1,s}, N^{\max}_{1,s}, k_{2,s}, N^{\min}_{2,s}, N^{\max}_{2,s}]$ with $l_1 + N^{\min}_{1,s} \leqslant n_1 \leqslant l_1 + N^{\max}_{1,s}$ and $l_2 + N^{\min}_{2,s} \leqslant n_2 \leqslant l_2 + N^{\max}_{2s}$. Moreover, more than one and different sets—labeled by the subscript s—may be selected for any angular configuration (l_1,l_2). To avoid redundancies in the expansion equation (3), the orbital angular momenta are restricted to $l_1 \leqslant l_2$, and if $l_1 = l_2$ and $k_{1s} = k_{2s}$, are restricted to $n_1 \leqslant n_2$.

By choosing appropriate sets of Coulomb-Sturmian functions, the description of a given energy regime, that is, below a certain ionization threshold, is possible with a rather small number of basis functions compared with other state-of-the-art approaches [13].

The resolution of the resonances in the complex plane is achieved with the help of the complex rotation method [17–22], consisting of a rotation of the coordinates by a suitable angle θ into the complex plane: $\vec{r} \to \vec{r}\exp(\imath\theta)$ and $\vec{p} \to \vec{p}\exp(-\imath\theta)$. This nonunitary transformation leads to a complex symmetric basis representation of the eigenvalue problem (1). The spectrum of the rotated Hamiltonian is thus complex, and it has the following important properties [18,20,22]: (a) The bound spectrum of H is invariant under the complex rotation. (b) The continuum states are located on half-lines rotated by an angle -2θ around the ionization thresholds of the unrotated Hamiltonian into the lower half of the complex plane. In the specific case of the unperturbed three-dimensional helium Hamiltonian (2), the continuum states are rotated around the SIT $I_N = -2/N^2$ [23], with $N \in \mathbb{N}$. (c) There are isolated complex eigenvalues $E_{i,\theta} = E_i - \imath\Gamma_i/2$ in the lower half-plane, corresponding to resonance states. These are stationary under changes of θ, provided the dilation angle is large enough to uncover their positions on the Riemannian sheets of the associated resolvent [23,24]. The associated resonance eigenfunctions are square integrable [21], in contrast to the resonance eigenfunctions of the unrotated Hamiltonian. The latter are asymptotically diverging, outgoing waves [21,25,26].

After substituting $\Psi^{L,M}_{\alpha}$ in Eq. (1) by its expansion [Eq. (3)] and using the complex rotation method, we obtain the generalized eigenvalue problem

$$\mathbf{H}_\theta \mathbf{\Psi}_{i,\theta} = E_{i,\theta}\mathbf{S}\mathbf{\Psi}_{i,\theta}, \quad (4)$$

where $\mathbf{\Psi}_{i,\theta}$ is the vector representation of the wave function $|\Psi_{i,\theta}\rangle$, $\mathbf{S}$ is the matrix representing the overlap (the Sturmian basis is nonorthogonal), and $\mathbf{H}_\theta$ is the matrix associated with the rotated Hamiltonian. The corresponding matrix elements are calculated with the help of Gauss-Laguerre quadrature techniques combined with the generalized Wigner-Eckart theorem and recurrence relations. An accurate and stable calculation of these recurrence relations requires the use of quadruple precision [13]. However, the final matrix elements are obtained in double precision. The matrix elements of the electron-electron repulsion require, in addition, the use of the multipole expansion

$$\frac{1}{r_{12}} = \sum_{q=0}^{\infty}\sum_{p=-q}^{q}\frac{4\pi}{2q+1}\frac{r^q_<}{r^{q+1}_>}Y^*_{q,p}(\hat{r}_1)Y_{q,p}(\hat{r}_2), \quad (5)$$

with $r_< = \min(r_1,r_2)$ and $r_> = \max(r_1,r_2)$.

The inclusion of many sets of Coulomb-Sturmian functions with different dilation parameters in our basis makes it numerically overcomplete, which means that some eigenvalues of the overlap matrix (which must be positive definite) can be numerically zero. This results from a loss of numerical independence due to finite-precision arithmetic. This problem is solved by rejecting the eigenvectors of $\mathbf{S}$ associated with the eigenvalues that are numerically zero. This is achieved by an appropriate orthogonal transformation that finally reduces the dimensions $n \times n$ of the original matrices $\mathbf{H}_\theta$ and $\mathbf{S}$ to $p \times p$, where $n - p$ is the number of rejected eigenvalues [12,13]. The numerical diagonalization of the reduced eigenvalue problem is performed by an efficient implementation of the Lanczos algorithm [27–29].

III. RESULTS

Helium states of unnatural parity are characterized by the condition $(-1)^{L+1} = (-1)^{l_1+l_2}$, where L is the total angular momentum, and l_1 and l_2 are the individual angular momenta of the electrons. As a consequence of this relation and of the triangular condition for the addition of angular momenta, the single-particle angular momenta cannot be zero ($l_i \neq 0$, $i = 1,2$), the total angular momentum is at least 1, and the inner electron excitation must be at least $n_i = 2$. Therefore, there are no unnatural parity states below the first ionization threshold, and unnatural parity states converging to the second ionization threshold are nonautoionizing states. These states have already been considered in our recent paper [9]. Here we concentrate on the autoionizing P states of unnatural parity.

TABLE I. Sets $[k_{1,s}, N^{\min}_{1,s}, N^{\max}_{1,s}, k_{2,s}, N^{\min}_{2,s}, N^{\max}_{2,s}]$ of Coulomb-Sturmian functions used for the description of the spectrum between the seventh and the eighth SITs. This energy regime contains states of the Rydberg series converging with the $N = 8$ and the $N = 9$ SITs. k_1 is thus chosen to be $2/8 = 0.25$ and $2/9 \approx 0.222$. The dilation parameters of the outer electron are chosen to span an adequate energy region.

s	$k_{1,s}$	$k_{2,s}$	$N^{\min}_{1,s}$	$N^{\max}_{1,s}$	$N^{\min}_{2,s}$	$N^{\max}_{2,s}$
1	0.250	0.250	1	25	1	25
2	0.250	0.167	1	13	1	20
3	0.250	0.118	1	13	5	25
4	0.250	0.080	1	13	10	40
5	0.222	0.222	1	15	1	15
6	0.222	0.181	1	13	1	20

TABLE II. $^3P^e$ resonances of helium below the third ($N = 3$) and the fourth ($N = 4$) SITs: Our results are compared with data from Refs. [7] and [8]. Only converged digits are displayed for our results. The results of Ref. [8] have been converted from Rydberg (Ry) to a.u. The data of Ref. [7] are presented without comment on the actual precision. The states marked with (∗) can also be identified with states given in Ref. [7]. The dimensions of the largest matrices used to obtain these data were $n = 28\,520$, $p = 16\,911$ for $N = 3$ and $n = 27\,000$, $p = 16\,010$ for $N = 4$.

$N = 3$							
This work		Saha and Mukherjee [7]		Ho and Bhatia [8]		This work (continued)	
$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$
0.336 09	0.002 25	0.336 07	0.002 27	0.336 087 9	0.002 244 35	0.225 113 89	0.000 013 30
0.291 157 9	0.000 036 9	0.291 16	0.000 035 0	0.291 158 225	0.000 037 0	0.224 750 367	0.000 000 474 0
0.271 557	0.000 894	0.271 56	0.000 885	0.271 557 15	0.000 894 35	0.224 719 77	0.000 010 65
0.253 574 58	0.000 011 74	0.253 57	0.000 010 0	0.253 574 65	0.000 011 761 5	0.224 425 756	0.000 000 389
0.250 932	0.000 428	0.250 93	0.000 425	0.250 931 5	0.000 427 5	0.224 401 04	0.000 008 66
0.241 958 25	0.000 007 21	0.241 96	0.000 005 0	0.241 958 3	0.000 007 2	0.224 159 888 7	0.000 000 322 4
0.240 960 3	0.000 227 6	0.240 96	0.000 225	0.240 96	0.000 225	0.224 139 63	0.000 007 14
0.235 894 22	0.000 004 69	0.235 89	0.000 004 67	0.235 893 5	0.000 004 0	0.223 929 297 7	0.000 000 270 4
0.235 396 3	0.000 133 6	0.235 39	0.000 135			0.223 922 58	0.000 005 96
0.232 270 95	0.000 003 18	0.232 27	0.000 003 16			0.223 754 513	0.000 000 229
0.231 982 8	0.000 084 6	0.231 98	0.000 085			0.223 740 40	0.000 005 02
0.229 923 82	0.000 002 25	0.229 92	0.000 002 245			0.223 597 959 3	0.000 000 195 5
0.229 740 7	0.000 056 8	0.229 74	0.000 055 0			0.223 585 99	0.000 004 27
0.228 314 28	0.000 001 64	0.228 31	0.000 001 625			0.223 464 30	0.000 000 17
0.228 190 2	0.000 040 0	0.228 19	0.000 040 0			0.223 453 99	0.000 003 66
0.227 161 96	0.000 001 23	0.227 16	0.000 001 205			0.223 349 1	0.000 000 2
0.227 073 7	0.000 029 2	0.227 07	0.000 035 0			0.223 340	0.000 003
0.226 308 471	0.000 000 940	0.226 31	0.000 000 825				
0.226 243 35	0.000 021 92	0.226 24	0.000 025 0				
0.225 658 634	0.000 000 736	0.225 65	0.000 003 93				
0.225 609 15	0.000 016 90	0.225 61	0.000 020 0				
0.225 152 407	0.000 000 586	0.225 14	0.000 01				
				$N = 4$			
0.194 442	0.001 653	0.197 38	0.001 685	0.194 442	0.001 652 5	0.128 821 702	0.000 004 491
0.178 257	0.002 402	0.178 78	0.002 295	0.178 257	0.002 403 5	0.128 803 68	0.000 010 41(*)
0.161 229 7	0.000 951 4	0.161 36	0.000 945	0.161 223	0.000 951 5	0.128 274 538	0.000 047 145
0.155 176 7	0.000 129 1	0.155 20	0.000 135	0.155 176 8	0.000 129 25	0.128 234 933	0.000 004 126
0.151 550	0.000 883	0.151 54	0.000 77	0.151 549 5	0.000 883	0.128 229 787	0.000 004 085
0.148 032 3	0.000 524 5	0.148 08	0.000 58	0.148 031 5	0.000 525	0.127 804 35	0.000 036 55
0.142 341 3	0.000 164 1	0.142 35	0.000 255	0.142 341 5	0.000 164 25	0.127 783 669	0.000 002 192
0.142 341 1	0.000 419 2	0.142 07	0.000 370	0.142 341	0.000 418 95	0.127 768 101	0.000 002 356
0.140 840 4	0.000 288 2	0.140 90	0.000 400	0.140 84	0.000 235	0.127 428 137	0.000 028 487(*)
0.137 293 3	0.000 311 2	0.137 42	0.000 290	0.137 295	0.000 306	0.127 422 760 1	0.000 001 272 5
0.137 153 11	0.000 056 71	0.137 16	0.000 055	0.137 15	0.000 055	0.127 398 134	0.000 001 730
0.136 522 5	0.000 164 0	0.136 55	0.000 240			0.127 129 079 9	0.000 001 206 3
0.134 182 94	0.000 215 26					0.127 122 185	0.000 022 666
0.134 061 30	0.000 028 41	0.134 05	0.000 020 0			0.127 097 627	0.000 001 314
0.133 744 5	0.000 096 2	0.133 75	0.000 090			0.126 888 607 0	0.000 001 397 4
0.132 126 6	0.000 151 7	0.132 02	0.000 134 5			0.126 870 516	0.000 018 768
0.132 028 688	0.000 016 275					0.126 850 250 0	0.000 001 021 2
0.131 858 58	0.000 057 24	0.131 82	0.000 089			0.126 682 106	0.000 002 031
0.130 692 96	0.000 109 68	0.130 77	0.000 120 5			0.126 661 287 6	0.000 015 857 5
0.130 614 425	0.000 010 054	0.130 62	0.000 135			0.126 644 181 8	0.000 000 808 3
0.130 522 59	0.000 033 86	0.130 47	0.000 051 5			0.126 509 105	0.000 001 939
0.129 652 551	0.000 081 231					0.126 485 378 3	0.000 013 507 1
0.129 589 107	0.000 006 557					0.126 470 712 8	0.000 000 650 2
0.129 542 91	0.000 019 42	0.129 52	0.000 024 05			0.126 361 036	0.000 003 841
0.128 873 334	0.000 061 402					0.126 336 013 0	0.000 011 574 8

TABLE III. $^3P^e$ resonances of helium below the fifth SIT: Our results on the left are compared with data from Refs. [7] and [8]. Only converged digits are displayed for our results. The results of Ref. [8] have been converted from Ry to a.u. The data of Ref. [7] are presented without comment on the actual precision. The state marked with (∗) can be also identified with states given in Ref. [7]. The dimension of the largest matrices used to obtain these data was $n = 27\,000$ and $p = 15\,380$.

$N = 5$							
This work		Saha and Mukherjee [7]		Ho and Bhatia [8]		This work (continued)	
$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$
0.119 30	0.001 77	0.119 37	0.000 985	0.119 30	0.001 77	0.084 914 4	0.000 0887(*)
0.109 463	0.001 561	0.109 96	0.000 925	0.109 463	0.001 561	0.084 674 77	0.000 013 60
0.107 265	0.000 765	0.107 65	0.000 50	0.107 265	0.000 765	0.084 463 5	0.000 144 9
0.102 096	0.001 134	0.102 99	0.001 02	0.102 096	0.001 134 5	0.084 249 56	0.000 006 24
0.098 603 1	0.000 451 7	0.098 28	0.000 414	0.098 602	0.000 453	0.084 087 9	0.000 060 3
0.096 641	0.000 687	0.096 56	0.000 453	0.096 640 5	0.000 687	0.083 964 29	0.000 054 12
0.095 254 2	0.000 044 2	0.095 26	0.000 201	0.095 253 85	0.000 044 25	0.083 742 83	0.000 110 13
0.095 136	0.000 744			0.095 143	0.000 74	0.083 564 41	0.000 004 23
0.093 440 9	0.000 221 2	0.093 65	0.000 226 5	0.093 435	0.000 205	0.083 448 9	0.000 043 0
0.091 509 3	0.000 470 2	0.091 66	0.000 327	0.091 51	0.000 47	0.083 380 9	0.000 079 2
0.091 108	0.000 435	0.091 12	0.000 368 5	0.091 105	0.000 425	0.083 184 1	0.000 082 7
0.090 724 96	0.000 021 25	0.090 72	0.000 017 25	0.090 724 5	0.000 021	0.083 032 58	0.000 002 81
0.090 236 5	0.000 053 2	0.090 26	0.000 127			0.082 946 6	0.000 032 2
0.088 632	0.000 340	0.089 08	0.000 299 5			0.082 909 4	0.000 085 6
0.088 503 2	0.000 218 9	0.088 50	0.000 009 95			0.082 741 1	0.000 061 5
0.088 501 5	0.000 058 3					0.082 611 47	0.000 001 83
0.088 120 97	0.000 017 86	0.088 10	0.000 067			0.082 545 2	0.000 024 8
0.087 296	0.000 209	0.087 44	0.000 011 0			0.082 527 8	0.000 081 9
0.086 770 8	0.000 096 7	0.086 96	0.000 101 5			0.082 383 7	0.000 045 5
0.086 735 5	0.000 244 6	0.086 02	0.000 179 5			0.082 272 32	0.000 001 19
0.086 382 63	0.000 013 08					0.082 219 96	0.000 019 55
0.085 989 9	0.000 139 2	0.085 93	0.000 077 5			0.082 215 2	0.000 073 7
0.085 588 9	0.000 013 0					0.082 091 19	0.000 033 70
0.085 419 5	0.000 188 9	0.085 43	0.000 112			0.081 995 15	0.000 000 77
0.085 153 58	0.000 009 08					0.081 955 9	0.000 064 1

We present energies and widths for $^1P^e$ and $^3P^e$ resonances below the third to eighth SIT. The data for the third up to the seventh threshold of $^3P^e$ resonances and for the third up to the fifth threshold of $^1P^e$ resonances are compared to existing data [7,8], while the energies and the widths for $^3P^e$ resonances below the eighth threshold and for $^1P^e$ resonances below the sixth up to the eighth threshold are reported for the first time. The results have been computed using up to twenty-one angular configurations and up to seven sets of Coulomb-Sturmian functions for each angular configuration. Since the Sturmian function $S^{(k)}_{n,l}(r)$ for $k = Z/n$ is the radial eigenfunction of principal quantum number n and angular momentum l of a hydrogenic atom with nuclear charge Z, the choice of $k_{1s} \approx 2/n_1$ and $k_{2s} \approx 2/n_2$ provides an approximate description of a helium state with electronic excitations n_1 and n_2, respectively. Although there is not a well-defined rule for the choice of the optimal parameter sets, this simple argument gives a hint for an appropriate choice of them in order to obtain a description of a rather large energy regime. As an illustration, Table I summarizes typical parameter sets for each angular configuration used for the description of the spectrum below the eighth SIT. These include two sets with $k_1 = k_2 \approx 2/N$, $N = 8,9$, for the description of the low-lying states of the series converging to I_8 and of the intruder states of the series converging to I_9. For the other sets, we have chosen $k_1 \approx 2/N$, $N = 8,9$, and $k_2 = 2/n_2$, $n_2 > N$ to account for higher excitations of the outer electron.

Our results have been tested for convergence with respect to variation of the basis size—including the number of angular configurations and Coulomb-Sturmian functions used, variation of the real dilation parameters, and variation of the complex rotation angle θ, with $\theta \in \{0.1,0.12,0.14,0.16,0.18,0.2\}$. Tables II–IV contain the data for $^3P^e$ resonances, while the data for $^1P^e$ resonances are presented in Tables V and VI. The tables are limited to at most fifty entries even if more converged resonances have been identified.

For symmetric excitation of both electrons, our approach suffers from the influence of the Kato cusp [30], which is a discontinuity of the derivative of the wave function at $r_{12} = 0$ that is not resolvable within our approach. The effect can easily be spotted in our tables as the precision of our results increases with increasing excitation of the outer electron.

The agreement with the results for $^3P^e$ (Tables II and III) and for $^1P^e$ (Table V) of Ref. [8], where an explicitly correlated approach together with complex rotation has been used to extract energy and the width of resonances, is, in general, excellent. Nevertheless, we obtain a significantly larger amount of additional converged resonances and are

TABLE IV. $^3P^e$ resonances of helium below the sixth ($N = 6$), the seventh ($N = 7$), and the eighth ($N = 8$) SITs: Our results on the left are compared with data from Ref. [7]. Only converged digits are displayed for our results. The data of Ref. [7] are presented without comment on the actual precision. The dimensions of the largest matrices used to obtain these data were $n = 27\,000$, $p = 14\,570$ for $N = 6$; $n = 27\,000$, $p = 14020$ for $N = 7$; and $n = 29\,180$, $p = 14\,257$ for $N = 8$.

$N = 6$

This work		Saha and Mukherjee [7]		This work (continued)		This work (continued)	
$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$
0.076 443	0.000 596	0.077 21	0.000 369	0.062 138 9	0.000 155 5	0.058 821 9	0.000 105 1
0.073 661	0.000 871	0.070 79	0.000 483	0.061 869 1	0.000 211 0	0.058 798 52	0.000 014 22
0.073 387 0	0.000 739 2	0.068 25	0.000 735	0.061 300 50	0.000 005 46	0.058 636 24	0.000 087 77
0.070 754 5	0.000 358 1	0.067 04	0.000 275	0.061 275	0.000 329	0.058 470 2	0.000 057 9
0.070 218	0.000 841	0.066 65	0.000 149	0.061 051 0	0.000 082 6	0.058 390 002	0.000 007 444
0.068 631 2	0.000 586 5	0.065 45	0.000 274 5	0.060 972 48	0.000 018 12	0.058 354 6	0.000 123 4
0.067 072 5	0.000 105 0	0.065 22	0.000 070 5	0.060 666 9	0.000 181 3	0.058 352 48	0.000 005 17
0.066 527 2	0.000 368 3	0.064 49	0.000 005 1	0.060 361 9	0.000 221 3	0.058 225 98	0.000 056 42
0.066 190	0.000 655	0.063 83	0.000 152	0.060 256 92	0.000 006 81	0.058 076 16	0.000 039 78
0.065 516 9	0.000 234	0.063 58	0.000 003 425	0.060 114 66	0.000 025 53	0.058 011 62	0.000 004 71
0.065 294 9	0.000 0941	0.062 72	0.000 000 336 5	0.060 0951	0.000 0488	0.057 997 917 8	0.000 000 580 3
0.064 495 12	0.000 008 92	0.061 87	0.000 173 5	0.059 797 1	0.000 153 8	0.057 979 4	0.000 142 1
0.063 937	0.000 256	0.061 15	0.00 012 55	0.059 582 5	0.000 134 9		
0.063 814 2	0.000 388 9	0.060 75	0.000 092	0.059 470 995	0.000 012 354		
0.063 613 8	0.000 257 2	0.060 28	0.000 163	0.059 402 6	0.000 073 9		
0.063 594 605	0.000 004 761	0.059 73	0.000 234 5	0.059 365 2	0.000 027 9		
0.062 721 69	0.000 003 40	0.059 13	0.000 267	0.059 144 9	0.000 122 5		
0.062 337 2	0.000 207 3	0.057 74	0.000 168 5	0.058 962 0	0.000 086 5		
0.062 326	0.000 145	0.056 32	0.000 127	0.058 864 15	0.000 010 62		
				$N = 7$			
0.053 47	0.000 70	0.053 63	0.000 125	0.046 881 8	0.000 089 3	0.044 569 6	0.000 078 4
0.053 402 9	0.000 280 3	0.051 90	0.000 280	0.046 750 2	0.000 103 7	0.044 520 85	0.000 033 66
0.052 228 8	0.000 367 7	0.050 85	0.000 045 0	0.046 576 57	0.000 012 32	0.044 381 00	0.000 004 08
0.052 015	0.000 430	0.049 65	0.000 020 0	0.046 240 2	0.000 186 2	0.044 188	0.000 116
0.051 024	0.000 048 2	0.048 60	0.000 105	0.046 110 8	0.000 204 0	0.044 180 1	0.000 108 4
0.050 847 80	0.000 012 02	0.047 52	0.000 295	0.045 970 1	0.000 096 6	0.044 167 7	0.000 095 6
0.050 394	0.000 530	0.046 55	0.000 015 0	0.045 826 8	0.000 031 4	0.044 051 93	0.000 006 79
0.049 773	0.000 241	0.045 73	0.000 075	0.045 814 48	0.000 005 27	0.043 971 97	0.000 067 18
0.049 702 6	0.000 062 3	0.045 19	0.00 025 0	0.045 580 6	0.000 093 1	0.043 859 62	0.000 006 15
0.048 578	0.000 458	0.044 84	0.000 110	0.045 348 2	0.000 385 7	0.043 762 2	0.000 062 7
0.048 461 1	0.000 156 7	0.044 09	0.000 275	0.045 313 7	0.000 087 3	0.043 705 0	0.000 095 8
0.048 404 8	0.000 157 6	0.043 37	0.000 165	0.045 169 12	0.000 008 14	0.043 692 8	0.000 056 7
0.048 397	0.000 174			0.045 070 11	0.000 000 94	0.043 629 21	0.000 004 48
0.048 275	0.000 193			0.045 033 07	0.000 001 80	0.043 522 5	0.000 090 8
0.047 323	0.000 366			0.044 785 9	0.000 218 4	0.043 442 86	0.000 007 69
0.047 154 83	0.000 018 55			0.044 775 9	0.000 097 4	0.043 377 0	0.000 036 8
0.047 070 7	0.000 285 4			0.044 600 0	0.000 034 4		

$N = 8$

This work		This work (continued)		This work (continued)		This work (continued)	
$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$
0.040 86	0.000 32	0.037 168 8	0.000 179 3	0.035 428 934	0.000 010 562	0.034 435 779	0.000 012 427
0.040 005 62	0.000 005 04	0.037 067 6	0.000 317 8	0.035 407 698	0.000 000 352	0.034 408 53	0.000 035 55
0.039 768	0.000 413	0.036 733 41	0.000 061 63	0.035 341 3	0.000 080 8	0.034 401 12	0.000 020 60
0.039 188 412	0.000 003 812	0.036 703 67	0.000 039 08	0.034 979 65	0.000 059 16	0.034 267 095	0.000 022 262
0.039 173 6	0.000 259 4	0.036 385 3	0.000 432 7	0.034 979 1	0.000 084 5	0.034 195 95	0.000 126 31
0.038 991 7	0.000 317 5	0.036 276 63	0.000 024 18	0.034 968 1	0.000 043 5	0.034 077 29	0.000 071 49
0.038 552 56	0.000 161 79	0.036 208 60	0.000 086 45	0.034 866 08	0.000 032 74	0.034 032 27	0.000 011 04
0.038 484 3	0.000 502 5	0.036 081 9	0.000 231 4	0.034 847 195	0.000 001 719	0.034 019 28	0.000 004 55
0.038 239 94	0.000 031 34	0.035 931 87	0.000 013 34	0.034 820	0.000 497	0.034 015 479	0.000 006 518
0.038 200 8	0.000 318 1	0.035 692 53	0.000 048 20	0.034 764 50	0.000 011 78	0.033 947 73	0.000 062 12
0.037 843 9	0.000 136 7	0.035 690 0	0.000 175 5	0.034 595 1	0.000 165 1	0.033 853 77	0.000 035 12
0.037 221 5	0.000 121 3	0.035 484 2	0.000 153 2	0.034 456 3	0.000 036 8		
0.037 208 7	0.000 242 6	0.035 436 1	0.000 128 2	0.034 438 4	0.000 202 3		

TABLE V. $^1P^e$ resonances of helium below the $N = 3$–5 SIT: Our results are compared with data from Ref. [8]. Only converged digits are displayed for our results. The results of Ref. [8] have been converted from Ry to a.u. The dimensions of the largest matrices used to obtain these data were $n = 27\,720$, $p = 16\,372$ for $N = 3$; $n = 26\,200$, $p = 15\,491$ for $N = 4$; and $n = 26\,200$, $p = 14\,865$ for $N = 5$.

This work		Ho and Bhatia [8]		This work (continued)		This work (continued)	
$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$
				$N = 3$			
0.278 992 5	0.000 022 1	0.278 992 5	0.000 022 15	0.228 383 750	0.000 001 520	0.224 441 163 5	0.000 000 341 3
0.259 352 043	0.000 000 628	0.259 352	0.000 000 629 5	0.227 437 119 1	0.000 000 074 5	0.224 225 013 4	0.000 000 019 1
0.253 638 72	0.000 013 20	0.253 638 7	0.000 013 191 5	0.227 213 135	0.000 001 121	0.224 172 596 9	0.000 000 281 9
0.244 513 616	0.000 000 435	0.244 513 6	0.000 000 437 5	0.226 513 436 2	0.000 000 056 8	0.223 993 560 71	0.000 000 015 93
0.242 237 48	0.000 007 69	0.242 237	0.000 008	0.226 347 159	0.000 000 849	0.223 950 000 7	0.000 000 235 6
0.237 282 423	0.000 000 290			0.225 815 467 0	0.000 000 044 2	0.223 800 046 45	0.000 000 013 45
0.236 098 88	0.000 004 75			0.225 688 553	0.000 000 658	0.223 763 450 3	0.000 000 198 8
0.233 114 407 2	0.000 000 197 1			0.225 275 115 1	0.000 000 035 0	0.223 636 606 29	0.000 000 011 45
0.232 410 753	0.000 003 104			0.225 176 001	0.000 000 520	0.223 605 562 3	0.000 000 169 3
0.230 476 058 6	0.000 000 138 3			0.224 848 199 3	0.000 000 028 2	0.223 497 313 9	0.000 000 009 8
0.230 020 923	0.000 002 129			0.224 769 293	0.000 000 418	0.223 470 751	0.000 000 145
0.228 696 059 6	0.000 000 100 1			0.224 505 021 5	0.000 000 023 0		
				$N = 4$			
0.165 519 28	0.000 034 81	0.165 5193	0.000 0363	0.130 574 062	0.000 004 235	0.127 157 590 75	0.000 001 153 18
0.156 408 49	0.000 042 01	0.156 408 5	0.000 042 0	0.129 928 314 4	0.000 001 213 9	0.127 091 048 2	0.000 001 029 3
0.149 661 02	0.000 026 41	0.149 660 985	0.000 0264	0.129 799 793 7	0.000 003 609 5	0.126 932 498 39	0.000 000 307 68
0.145 222 196	0.000 005 624	0.145 222 25	0.000 0056	0.129 562 145	0.000 003 190	0.126 899 429 2	0.000 000 958 1
0.144 232 54	0.000 022 89	0.144 2325	0.000 022 885	0.129 075 542 5	0.000 000 922 4	0.126 844 866 1	0.000 000 856 2
0.141 579 53	0.000 017 58	0.141 578	0.000 017 5	0.128 977 666 6	0.000 002 772 4	0.126 712 724 36	0.000 000 257 19
0.138 778 490	0.000 004 604			0.128 802 370	0.000 002 457	0.126 685 028 49	0.000 000 804 39
0.138 231 81	0.000 014 04			0.128 426 940 1	0.000 000 716 3	0.126 639 714 3	0.000 000 719 7
0.136 894 328	0.000 011 765			0.128 350 719 7	0.000 002 173 3	0.126 528 440 305	0.000 000 217 152
0.135 069 449 4	0.000 003 229 8			0.128 217 530	0.000 001 931	0.126 505 016 67	0.000 000 681 75
0.134 717 582	0.000 009 398			0.127 921 991 7	0.000 000 566 9	0.126 466 960 6	0.000 000 610 6
0.133 941 318	0.000 008 109			0.127 861 501 1	0.000 001 733 8	0.126 372 390 94	0.000 000 184 99
0.132 698 041 4	0.000 002 270 9			0.127 757 822 8	0.000 001 543 1	0.126 352 406 06	0.000 000 582 74
0.132 456 357 2	0.000 006 606 6			0.127 521 115 6	0.000 000 456 1	0.126 320 128 2	0.000 000 522 4
0.131 962 967	0.000 005 774			0.127 472 320 9	0.000 001 404 6	0.126 239 086	0.000 000 159
0.131 081 749 9	0.000 001 637 8			0.127 389 968	0.000 001 252	0.126 221 900	0.000 000 502
0.130 908 403 8	0.000 004 813 6			0.127 197 511 8	0.000 000 372 2		
				$N = 5$			
0.109 927 09	0.000 036 33	0.109 927 05	0.000 036 33	0.086 223 04	0.000 009 04	0.082 799 549 7	0.000 002 868 9
0.105 179 08	0.000 064 12	0.105 179 15	0.000 0641	0.085 734 152	0.000 008 757	0.082 736 551	0.000 001 637
0.099 587 38	0.000 032 20	0.099 588	0.000 032	0.085 549 555 4	0.000 002 996 0	0.082 690 61	0.000 005 08
0.099 131 81	0.000 040 30	0.099 131 5	0.000 040 5	0.085 411 125	0.000 013 654	0.082 576 812	0.000 002 611
0.096 353 44	0.000 050 97	0.096 355	0.000 10	0.085 047 016	0.000 006 795	0.082 422 356	0.000 002 293
0.093 797 57	0.000 023 74			0.084 669 095	0.000 006 360	0.082 371 490	0.000 001 388
0.092 842 00	0.000 027 89			0.084 534 149	0.000 002 639	0.082 334 626	0.000 004 136
0.092 085 843	0.000 001 652			0.084 433 498	0.000 010 356	0.082 244 211	0.000 002 134
0.091 596 61	0.000 036 09			0.084 173 628	0.000 005 213	0.082 116 730 2	0.000 001 862 7
0.090 217 82	0.000 017 00			0.083 877 652	0.000 004 761	0.082 075 055	0.000 001 182
0.089 398 33	0.000 018 44			0.083 775 500	0.000 002 276	0.082 045 009	0.000 003 409
0.088 983 560	0.000 002 856			0.083 699 897	0.000 008 022	0.081 971 944 9	0.000 001 765 5
0.088 683 11	0.000 025 52			0.083 507 802	0.000 004 074	0.081 865 599 3	0.000 001 533 9
0.097 857 284	0.000 012 278			0.083 272 693	0.000 003 656	0.081 831 019 3	0.000 001 011 2
0.087 220 161	0.000 012 482			0.083 193 307	0.000 001 933	0.081 806 201	0.000 002 842
0.086 954 588	0.000 003 169			0.083 135 007	0.000 006 331	0.081 746 287 6	0.000 001 476 1
0.086 756 13	0.000 018 43			0.082 988 895	0.000 003 237		

TABLE VI. $^1P^e$ resonances of helium below the sixth ($N = 6$), the seventh ($N = 7$), and the eighth ($N = 8$) SITs. Only converged digits are displayed. The dimensions of the largest matrices used to obtain these data were $n = 26\,200$, $p = 14\,058$ for $N = 6$; $n = 26\,200$, $p = 13\,511$ for $N = 7$; and $n = 28\,360$, $p = 13\,748$ for $N = 8$.

$N = 6$		$N = 7$		$N = 8$	
$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$	$-\mathrm{Re}\,(E_{i,\theta})$	$-\mathrm{Im}\,(E_{i,\theta})$
0.078 422 83	0.000 034 70	0.054 985 63	0.000 074 29	0.041 114 9	0.000 053 0
0.075 674 10	0.000 061 56	0.053 796 87	0.000 034 40	0.040 024 52	0.000 069 60
0.072 249 47	0.000 073 79	0.052 440 29	0.000 063 47	0.039 654 67	0.000 045 67
0.071 369 29	0.000 034 24	0.052 425 98	0.000 056 34	0.039 645 37	0.000 029 74
0.069 315 97	0.000 056 75	0.050 850 99	0.000 070 15	0.038 878 998	0.000 046 634
0.068 046 70	0.000 029 51	0.050 627 00	0.000 029 05	0.038 795 48	0.000 070 29
0.067 133 78	0.000 027 26	0.049 569 44	0.000 046 69	0.038 028 00	0.000 060 44
0.066 949 10	0.000 058 92	0.049 438 68	0.000 028 63	0.037 941 778	0.000 023 810
0.065 631 72	0.000 044 20	0.049 047 12	0.000 041 63	0.037 565 069	0.000 039 814
0.064 484 32	0.000 024 21	0.048 472 274	0.000 022 801	0.037 331 038	0.000 037 500
0.064 373 63	0.000 020 59	0.048 398 038	0.000 057 117	0.037 282 784 8	0.000 011 997 0
0.063 983 06	0.000 043 17	0.047 657 60	0.000 036 62	0.037 098 091	0.000 056 651
0.063 509 277	0.000 000 322	0.047 338 00	0.000 029 19	0.036 751 062 9	0.000 008 898 8
0.063 266 38	0.000 033 22	0.047 106 953	0.000 028 058	0.036 666 14	0.000 049 01
0.062 480 20	0.000 015 41	0.046 946 18	0.000 017 16	0.036 547 503 5	0.000 010 974 2
0.062 395 99	0.000 018 80	0.046 777 44	0.000 044 75	0.036 235 803	0.000 033 929
0.062 090 55	0.000 031 72	0.046 398 218 3	0.000 000 100 4	0.036 226 863	0.000 025 302
0.061 854 177	0.000 000 995	0.046 312 48	0.000 028 00	0.036 025 107 7	0.000 016 380 1
0.061 650 78	0.000 024 98	0.046 001 749	0.000 024 303	0.035 950 619	0.000 040 618
0.061 129 523	0.000 011 573	0.045 880 46	0.000 005 63	0.035 797 877 9	0.000 002 858 8
0.060 997 89	0.000 014 39	0.045 837 00	0.000 021 76	0.035 741 516 5	0.000 016 447 5
0.060 789 409	0.000 023 612	0.045 681 944	0.000 008 656	0.035 690 77	0.000 037 26
0.060 650 304	0.000 001 608	0.045 639 315	0.000 034 766	0.035 376 674 02	0.000 000 047 61
0.060 496 58	0.000 018 99	0.045 428 747 6	0.000 000 304 9	0.035 332 76	0.000 026 54
0.060 135 459	0.000 008 693	0.045 334 24	0.000 020 42	0.035 324 86	0.000 028 04
0.060 000 138	0.000 011 071	0.045 058 74	0.000 019 50	0.035 172 746	0.000 019 904
0.059 849 171	0.000 017 862	0.044 961 036	0.000 011 745	0.035 127 772	0.000 026 941
0.059 757 586	0.000 001 966	0.044 936 141	0.000 017 547	0.035 028 755	0.000 012 623
0.059 642 86	0.000 014 62	0.044 806 26	0.000 026 81	0.035 018 480	0.000 013 207
0.059 387 022	0.000 006 243	0.044 681 170	0.000 002 718	0.034 793 84	0.000 014 10
0.059 258 30	0.000 008 62	0.044 675 080 7	0.000 000 527 0	0.034 754 896 8	0.000 000 140 9
0.059 144 754	0.000 013 756	0.044 857 747	0.000 014 471	0.034 703 614	0.000 020 698
0.059 079 707	0.000 002 080	0.044 359 353	0.000 015 631	0.034 656 412	0.000 023 140
0.058 993 69	0.000 011 35	0.044 315 742	0.000 009 102	0.034 546 30	0.000 020 49
0.058 867 727 5	0.000 000 516 9	0.044 265 80	0.000 014 33	0.034 510 834	0.000 017 374
0.058 727 64	0.000 005 72	0.044 175 595	0.000 020 766	0.034 470 248	0.000 009 508
0.058 689 952	0.000 006 813	0.044 085 465 8	0.000 000 674 1	0.034 354 198	0.000 027 905
0.058 602 152	0.000 010 788	0.043 987 367	0.000 015 535	0.034 255 069 32	0.000 000 247 64
0.058 553 488	0.000 002 031	0.043 823 270	0.000 012 646	0.034 203 523	0.000 016 100
0.058 487 751	0.000 009 006	0.043 806 797	0.000 007 182	0.034 146 526	0.000 019 309
0.058 323 987	0.000 005 157	0.043 750 863	0.000 011 800	0.034 076 654	0.000 018 621
0.058 244 147	0.000 005 463	0.043 692 093	0.000 014 920	0.034 038 254	0.000 007 958
0.058 174 779	0.000 008 607	0.043 618 591 9	0.000 000 746 1	0.034 035 005	0.000 005 440
0.058 137 002	0.000 001 892	0.035 430 29	0.000 012 24	0.033 923 963	0.000 022 555
0.058 086 914	0.000 007 123	0.043 402 328	0.000 010 416	0.033 849 915 26	0.000 000 320 44
0.057 959 583	0.000 004 030	0.043 399 909	0.000 005 738	0.033 829 442	0.000 002 015
0.057 887 672	0.000 004 441	0.043 354 373	0.000 006 963	0.033 803 874	0.000 012 300
0.057 831 903	0.000 006 976	0.043 333 045	0.000 006 771	0.033 797 584 1	0.000 001 358 6
0.057 801 762	0.000 001 715	0.043 243 942 18	0.000 000 725 20	0.033 742 379	0.000 014 370
0.057 763 951	0.000 005 597	0.043 183 398	0.000 009 805	0.033 668 198 9	0.000 005 965 5

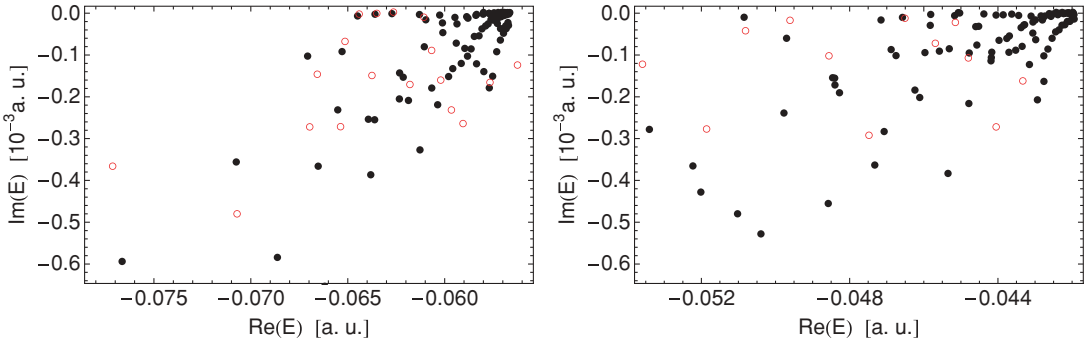

FIG. 1. (Color online) Energies and half-widths of $^3P^e$ states of unnatural parity below the sixth (left) and the seventh (right) SITs. As expected, the complex energies calculated with the help of the spectral approach described in Sec. II (solid circles) are organized in Rydberg-like converging to the respective SITs. In contrast, the data obtained with the stabilization method (open red circles) [7] exhibit a scattered pattern, and no agreement with our results can be seen.

able to provide high accuracy for asymmetrically excited states.

In contrast, the comparison with the recently published results for $^3P^e$ resonances obtained within the stabilization technique [7] yields significant disagreements. Already, the data for resonances below the third SIT (Table II) show significant differences for $\mathrm{Im}(E_{i,\theta})$ of higher excited resonances. Below the fourth SIT (Table II), the approach by Saha and Mukherjee [7] seems not to be able to resolve the close-lying resonances $E_\theta = -0.142\,3413 - \imath 0.000\,1641$ and $E_\theta = -0.142\,3411 - \imath 0.000\,4192$. Moreover, the part of the spectrum lying closest to the fourth SIT is not completely resolved. Table III, which presents the data for the fifth SIT, already shows pronounced deviations for the first two states of the table, and again, the spectrum for the higher excited states misses some resonances. The disagreement for the close-lying resonances $E = -0.095\,2542 - \imath 0.000\,0442$ and $E = -0.095\,136 - \imath 0.000\,744$ has already been addressed in Ref. [7]. However, we do not agree with their interpretation that the sudden decrease in the gap between resonances favors their results over those of Ref. [8]. Instead, we speculate that the stabilization method does have problems resolving resonances lying close to each other (see Tables II and III). Moreover, the statement by Saha and Mukherjee that the stabilization method allows them to obtain more results than the complex rotation method contradicts our experience: A large number of states calculated within our approach below the fourth and the fifth SITs is not obtained with the help of the stabilization method. Table IV presents a comparison of the data by Saha and Mukherjee [7] with our results obtained below the sixth and the seventh SITs. We are unable to associate any result by Saha and Mukherjee [7] with our results, as there is no agreement at all.

For moderate excitation of the inner electron—which is the case here—resonances are organized in Rydberg-like series converging to the SIT. Figure 1 displays the energies and the half-widths of $^3P^e$ states of unnatural parity below the sixth (left) and the seventh (right) SITs. Our data are compared with the results of Saha and Mukherjee. Note that these plots contain all our converged results for the sixth and the seventh SITs (around 100 for each threshold, although only fifty entries are given in Table IV). In addition to the small number of states given by Saha and Mukherjee [7], their results are organized in an unexpectedly irregular pattern, which does not coincide at all with our calculations.

Table IV also presents our results for $^3P^e$ resonances below the eighth SIT, while Table VI lists our results obtained for $^1P^e$ resonances below the sixth, the seventh, and the eighth SITs, respectively. Data for the combination of these symmetries and energy regimes are reported for the first time. Given the huge difficulties of resolving close-lying resonances in the data of Saha and Mukherjee [7], in general, and the nonexistent series structure for their data below the sixth and the seventh SITs for $^3P^e$ resonances, our results should be viewed as benchmark results below the sixth, the seventh, and the eighth SITs for singlet as well as for triplet symmetry.

IV. SUMMARY

A CI expansion of the two-electron wave function in terms of Sturmian functions with independent nonlinear parameters has been used to calculate the energies of autoionizing singlet and triplet doubly excited P helium states of unnatural parity up to the eighth SIT. Our results are in perfect agreement with existing data by Ho and Bhatia [8], while significant disagreement is found with the recently published results of Saha and Mukherjee [7] calculated using the stabilization method based on the Ritz variational method. From the structure of the spectrum in the complex plane and the low number of resonances obtained in Ref. [7], we conclude that their data are incorrect in most cases.

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