Spectral representation of the three-body Coulomb problem. II. Autoionizing doubly excited states of unnatural parity in helium

Johannes Eiglsperger

Physik Department, Technische Universität München, D-85747 Garching, Germany

Bernard Piraux

Laboratoire de Physique Atomique, Moléculaire et Optique (PAMO), Université catholique de Louvain, B-1348 Louvain-la Neuve, Belgium

Javier Madroñero

Physik Department, Technische Universität München, D-85747 Garching, Germany and Laboratoire de Physique Atomique, Moléculaire et Optique (PAMO), Université catholique de Louvain, B-1348 Louvain-la Neuve, Belgium (Received 15 January 2010; revised manuscript received 25 March 2010; published 30 April 2010)

A spectral approach of configuration interaction type is used to evaluate energies and widths for a wide range of singlet and triplet P^e resonance states of helium up to the eighth single ionization threshold. While the present data are in excellent agreement with existing theoretical results (below the N = 3-5 ionization threshold) obtained within an explicitly correlated approach, there are substantial differences with the energies, the widths, and the number of resonances obtained with the stabilization method.

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I. INTRODUCTION

Since the discovery of strong electron-electron correlation effects in doubly excited states (DES) of helium in the pioneering experiment by Madden and Codling [1], these states have attracted the continuous interest of both theoreticians and experimentalists. Most of the investigations have been focused on the states that are accessible through dipole transitions from the ground state of helium, that is states of a given angular momentum L with parity $\pi = (-1)^L$. States of unnatural parity $\pi = (-1)^{L+1}$ below the second single ionization threshold (SIT) do not autoionize (within a nonrelativistic approach). The existence of these nonautoionizing doubly excited states has been confirmed in a series of experiments [2–5]. Since the conventional variation procedure does apply in a straightforward way to nonautoionizing DES [6], it is not surprising that the large majority of the calculations are based on variational approaches (see Saha and Mukherjee [7] and references therein).

Above the second SIT, unnatural parity states are resonances. Energies and widths of ${}^{3}P^{e}$ resonances up to the seventh SIT have recently been evaluated using the stabilization method by Saha and Mukherjee [7]. The data are in good agreement with the few available low-lying resonances of the series converging to the N = 3-5 SIT obtained with the help of an approach that combines an explicitly correlated expansion in Hylleraas-type wave functions with complex rotation by Ho and Bathia [8]. In the present contribution, we evaluate the energy and the width of rather high-lying ${}^{1}P^{e}$ and ${}^{3}P^{e}$ resonances of unnatural parity of the series below the N = 3-8 SIT. Notice that nonautoionizing states of unnatural parity have already been considered in our recent paper [9]. The calculations are performed by means of a spectral approach of the configuration interaction (CI) type where the atomic wave function is expanded in a basis of products of Coulomb-Sturmian functions of the electron radial coordinates and the bipolar spherical harmonics of the angular coordinates. The resonances are extracted by using complex

rotation. Although low-lying resonances are found to be in perfect agreement with the results of Ho and Bathia [8], we find substantial differences with the results of Saha and Mukherjee [7] for high excitations of the outer electron. Moreover, starting from the series below the sixth SIT, we do not find any agreement at all with the results of Saha and Mukherjee [7]. After a brief description of our approach in Sec. II, we present our results in Sec. III and our conclusions in Sec. IV. Atomic units (a.u.) are used throughout this contribution unless stated otherwise.

II. THEORETICAL APPROACH

A detailed description of our approach has already been given elsewhere [10-13]. We will thus give only a brief review of its most relevant aspects.

The eigenstate wave function of the helium atom with a total angular momentum *L* of projection *M* and total energy E_{α} satisfies the time-independent Schrödinger equation

$$(H - E_{\alpha})\Psi_{\alpha}^{L,M}(\vec{r}_1, \vec{r}_2) = 0.$$
(1)

The nonrelativistic Hamiltonian *H*—under the assumption of an infinitely heavy nucleus—reads

$$H = \frac{\vec{p}_1^2}{2} + \frac{\vec{p}_2^2}{2} - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_{12}},$$
 (2)

where Z = 2, \vec{p}_i , r_i , and r_{12} are, respectively, the nuclear charge, the momentum and the distance of the electron *i* to the nucleus, and the distance between the two electrons.

In our CI approach [10–13], the solutions for Eq. (1) are expanded in Coulomb-Sturmian functions $S_{n_i,l_i}^{(k_i)}(r_i)$ [14,15] with independent dilation parameters k_i for both electrons (i = 1, 2) coupled with bipolar spherical harmonics

$$\begin{split} \Lambda_{l_1,l_2}^{L,M}(\hat{r}_1,\hat{r}_2) \ [16], \\ \Psi_{\alpha}^{L,M}(\vec{r}_1,\vec{r}_2) &= \sum_{l_1,l_2} \sum_{s} \sum_{n_1,n_2} \psi_{k_{1s},k_{2s},n_1,n_2,\alpha}^{l_1,l_2,L,M} \\ &\times \mathcal{A} \frac{S_{n_1,l_1}^{(k_{1s})}(r_1)}{r_1} \frac{S_{n_2,l_2}^{(k_{2s})}(r_2)}{r_2} \Lambda_{l_1,l_2}^{L,M}(\hat{r}_1,\hat{r}_2). \end{split}$$
(3)

The value of $\psi_{k_{1s},k_{2s},n_{1},n_{2},\alpha}^{l_{1},l_{2},L,M}$ is the expansion coefficient, and \mathcal{A} is an operator for the projection on singlet or triplet states.

The fact that we allow the dilation parameters and the number of the Coulomb-Sturmian functions associated with electrons one and two to be different leads to the introduction of a set of Coulomb-Sturmian functions $\{S_{n_1,l_1}^{(k_{1s})}(r_1), S_{n_2,l_2}^{(k_{2s})}(r_2)\}$ associated with electrons one and two, which is characterized by the combination $[k_{1,s}, N_{1,s}^{\min}, N_{1,s}^{\max}, k_{2,s}, N_{2,s}^{\min}, N_{2,s}^{\max}]$ with $l_1 + N_{1,s}^{\min} \leq n_1 \leq l_1 + N_{1,s}^{\max}$ and $l_2 + N_{2,s}^{\min} \leq n_2 \leq l_2 + N_{2s}^{\max}$. Moreover, more than one and different sets—labeled by the subscript *s*—may be selected for any angular configuration (l_1, l_2) . To avoid redundancies in the expansion equation (3), the orbital angular momenta are restricted to $l_1 \leq l_2$, and if $l_1 = l_2$ and $k_{1s} = k_{2s}$, are restricted to $n_1 \leq n_2$.

By choosing appropriate sets of Coulomb-Sturmian functions, the description of a given energy regime, that is, below a certain ionization threshold, is possible with a rather small number of basis functions compared with other state-of-the-art approaches [13].

The resolution of the resonances in the complex plane is achieved with the help of the complex rotation method [17-22], consisting of a rotation of the coordinates by a suitable angle θ into the complex plane: $\vec{r} \to \vec{r} \exp(i\theta)$ and $\vec{p} \to \vec{p} \exp(-i\theta)$. This nonunitary transformation leads to a complex symmetric basis representation of the eigenvalue problem (1). The spectrum of the rotated Hamiltonian is thus complex, and it has the following important properties [18,20,22]: (a) The bound spectrum of H is invariant under the complex rotation. (b) The continuum states are located on half-lines rotated by an angle -2θ around the ionization thresholds of the unrotated Hamiltonian into the lower half of the complex plane. In the specific case of the unperturbed three-dimensional helium Hamiltonian (2), the continuum states are rotated around the SIT $I_N = -2/N^2$ [23], with $N \in \mathbb{N}$. (c) There are isolated complex eigenvalues $E_{i,\theta} = E_i - \iota \Gamma_i/2$ in the lower halfplane, corresponding to resonance states. These are stationary under changes of θ , provided the dilation angle is large enough to uncover their positions on the Riemannian sheets of the associated resolvent [23,24]. The associated resonance eigenfunctions are square integrable [21], in contrast to the resonance eigenfunctions of the unrotated Hamiltonian. The latter are asymptotically diverging, outgoing waves [21,25,26].

After substituting $\Psi_{\alpha}^{L,M}$ in Eq. (1) by its expansion [Eq. (3)] and using the complex rotation method, we obtain the generalized eigenvalue problem

$$\mathbf{H}_{\theta} \mathbf{\Psi}_{i,\theta} = E_{i,\theta} \mathbf{S} \mathbf{\Psi}_{i,\theta}, \tag{4}$$

where $\Psi_{i,\theta}$ is the vector representation of the wave function $|\Psi_{i,\theta}\rangle$, **S** is the matrix representing the overlap (the Sturmian basis is nonorthogonal), and \mathbf{H}_{θ} is the matrix associated with the rotated Hamiltonian. The corresponding matrix elements are calculated with the help of Gauss-Laguerre quadrature

techniques combined with the generalized Wigner-Eckart theorem and recurrence relations. An accurate and stable calculation of these recurrence relations requires the use of quadruple precision [13]. However, the final matrix elements are obtained in double precision. The matrix elements of the electron-electron repulsion require, in addition, the use of the multipole expansion

$$\frac{1}{r_{12}} = \sum_{q=0}^{\infty} \sum_{p=-q}^{q} \frac{4\pi}{2q+1} \frac{r_{<}^{q}}{r_{>}^{q+1}} Y_{q,p}^{*}(\hat{r}_{1}) Y_{q,p}(\hat{r}_{2}), \qquad (5)$$

with $r_{<} = \min(r_1, r_2)$ and $r_{>} = \max(r_1, r_2)$.

The inclusion of many sets of Coulomb-Sturmian functions with different dilation parameters in our basis makes it numerically overcomplete, which means that some eigenvalues of the overlap matrix (which must be positive definite) can be numerically zero. This results from a loss of numerical independence due to finite-precision arithmetic. This problem is solved by rejecting the eigenvectors of **S** associated with the eigenvalues that are numerically zero. This is achieved by an appropriate orthogonal transformation that finally reduces the dimensions $n \times n$ of the original matrices \mathbf{H}_{θ} and **S** to $p \times p$, where n - p is the number of rejected eigenvalues [12,13]. The numerical diagonalization of the reduced eigenvalue problem is performed by an efficient implementation of the Lanczos algorithm [27–29].

III. RESULTS

Helium states of unnatural parity are characterized by the condition $(-1)^{L+1} = (-1)^{l_1+l_2}$, where *L* is the total angular momentum, and l_1 and l_2 are the individual angular momenta of the electrons. As a consequence of this relation and of the triangular condition for the addition of angular momenta, the single-particle angular momenta cannot be zero $(l_i \neq 0, i = 1, 2)$, the total angular momentum is at least 1, and the inner electron excitation must be at least $n_i = 2$. Therefore, there are no unnatural parity states below the first ionization threshold, and unnatural parity states converging to the second ionization threshold are nonautoionizing states. These states have already been considered in our recent paper [9]. Here we concentrate on the autoionizing *P* states of unnatural parity.

TABLE I. Sets $[k_{1,s}, N_{1,s}^{\min}, N_{1,s}^{\max}, k_{2,s}, N_{2,s}^{\min}, N_{2,s}^{\max}]$ of Coulomb-Sturmian functions used for the description of the spectrum between the seventh and the eighth SITs. This energy regime contains states of the Rydberg series converging with the N = 8 and the N = 9 SITs. k_1 is thus chosen to be 2/8 = 0.25 and $2/9 \approx 0.222$. The dilation parameters of the outer electron are chosen to span an adequate energy region.

s	$k_{1,s}$	$k_{2,s}$	$N_{1,s}^{\min}$	$N_{1,s}^{\max}$	$N_{2,s}^{\min}$	$N_{2,s}^{\max}$
1	0.250	0.250	1	25	1	25
2	0.250	0.167	1	13	1	20
3	0.250	0.118	1	13	5	25
4	0.250	0.080	1	13	10	40
5	0.222	0.222	1	15	1	15
6	0.222	0.181	1	13	1	20

TABLE II. ${}^{3}P^{e}$ resonances of helium below the third (N = 3) and the fourth (N = 4) SITs: Our results are compared with data from Refs. [7] and [8]. Only converged digits are displayed for our results. The results of Ref. [8] have been converted from Rydberg (Ry) to a.u. The data of Ref. [7] are presented without comment on the actual precision. The states marked with (*) can also be identified with states given in Ref. [7]. The dimensions of the largest matrices used to obtain these data were n = 28520, p = 16911 for N = 3 and n = 27000, p = 16010 for N = 4.

				N = 3			
This work		Saha and Mukherjee [7]		Ho and Bhatia [8]		This work (continued)	
$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$
0.336 09	0.00225	0.33607	0.002 27	0.336 087 9	0.002 244 35	0.225 113 89	0.000 013 30
0.291 157 9	0.000 036 9	0.29116	0.000 035 0	0.291 158 225	0.000 037 0	0.224750367	0.0000004740
0.271 557	0.000 894	0.271 56	0.000 885	0.271 557 15	0.000 894 35	0.22471977	0.000 010 65
0.253 574 58	0.000 011 74	0.25357	0.000 010 0	0.253 574 65	0.000 011 761 5	0.224 425 756	0.000 000 389
0.250932	0.000 428	0.25093	0.000 425	0.250 931 5	0.000 427 5	0.224 401 04	0.000 008 66
0.241 958 25	0.000 007 21	0.241 96	0.000 005 0	0.241 958 3	0.000 007 2	0.2241598887	0.0000003224
0.240 960 3	0.000 227 6	0.24096	0.000 225	0.24096	0.000 225	0.224 139 63	0.000 007 14
0.235 894 22	0.000 004 69	0.235 89	0.000 004 67	0.235 893 5	0.000 004 0	0.223 929 297 7	0.0000002704
0.235 396 3	0.000 133 6	0.235 39	0.000 135			0.223 922 58	0.000 005 96
0.232 270 95	0.000 003 18	0.23227	0.000 003 16			0.223 754 513	0.000 000 229
0.231 982 8	0.000 084 6	0.23198	0.000 085			0.223 740 40	0.000 005 02
0.229 923 82	0.000 002 25	0.22992	0.000 002 245			0.223 597 959 3	0.000 000 195 5
0.229 740 7	0.000 056 8	0.22974	0.000 055 0			0.223 585 99	0.000 004 27
0.228 314 28	0.000 001 64	0.22831	0.000 001 625			0.223 464 30	0.000 000 17
0.228 190 2	0.000 040 0	0.228 19	0.000 040 0			0.223 453 99	0.000 003 66
0.227 161 96	0.000 001 23	0.227 16	0.000 001 205			0.223 349 1	0.000 000 2
0.227 073 7	0.000 029 2	0.227 07	0.000 035 0			0.223 340	0.000 003
0.226 308 471	0.000 000 940	0.22631	0.000 000 825				
0.226 243 35	0.000 021 92	0.22624	0.000 025 0				
0.225 658 634	0.000 000 736	0.22565	0.000 003 93				
0.225 609 15	0.000 016 90	0.22561	0.000 020 0				
0.225 152 407	0.000 000 586	0.225 14	0.00001				
				N = 4			
0.194 442	0.001 653	0.19738	0.001 685	0.194 442	0.001 652 5	0.128 821 702	0.000 004 491
0.178 257	0.002402	0.17878	0.002 295	0.178 257	0.002 403 5	0.128 803 68	$0.00001041^{(*)}$
0.161 229 7	0.0009514	0.161 36	0.000 945	0.161 223	0.000 951 5	0.128 274 538	0.000 047 145
0.155 176 7	0.000 129 1	0.155 20	0.000135	0.155 176 8	0.000 129 25	0.128 234 933	0.000 004 126
0.151 550	0.000 883	0.15154	0.00077	0.151 549 5	0.000 883	0.128 229 787	0.000 004 085
0.148 032 3	0.000 524 5	0.14808	0.000 58	0.148 031 5	0.000 525	0.127 804 35	0.000 036 55
0.142 341 3	0.000 164 1	0.14235	0.000 255	0.142 341 5	0.000 164 25	0.127 783 669	0.000 002 192
0.142 341 1	0.000 419 2	0.14207	0.000370	0.142341	0.00041895	0.127 768 101	0.000 002 356
0.1408404	0.0002882	0.14090	0.000400	0.14084	0.000 235	0.127 428 137	$0.000028487^{(*)}$
0.137 293 3	0.0003112	0.137 42	0.000 290	0.137 295	0.000 306	0.127 422 760 1	0.0000012725
0.137 153 11	0.00005671	0.137 16	0.000 055	0.137 15	0.000 055	0.127 398 134	0.000 001 730
0.136 522 5	0.000 164 0	0.13655	0.000 240			0.127 129 079 9	0.000 001 206 3
0.134 182 94	0.000 215 26					0.127 122 185	0.000 022 666
0.134 061 30	0.00002841	0.13405	0.0000200			0.127 097 627	0.000 001 314
0.133 744 5	0.000 096 2	0.13375	0.000 090			0.1268886070	0.000 001 397 4
0.132 126 6	0.000 151 7	0.13202	0.000 134 5			0.126870516	0.000018768
0.132 028 688	0.000016275					0.126 850 250 0	0.000 001 021 2
0.131 858 58	0.00005724	0.13182	0.000 089			0.126 682 106	0.000 002 031
0.130 692 96	0.000 109 68	0.13077	0.000 120 5			0.126 661 287 6	0.0000158575
0.130614425	0.000010054	0.13062	0.000135			0.126 644 181 8	0.000 000 808 3
0.130 522 59	0.000 033 86	0.13047	0.000 051 5			0.126 509 105	0.000 001 939
0.129 652 551	0.000081231					0.1264853783	0.0000135071
0.129 589 107	0.000006557					0.1264707128	0.0000006502
0.129 542 91	0.00001942	0.12952	0.00002405			0.126 361 036	0.000 003 841
0.128 873 334	0.000 061 402					0.1263360130	0.0000115748

TABLE III. ³ P^e resonances of helium below the fifth SIT: Our results on the left are compared with data from Refs. [7] and [8]. Only converged digits are displayed for our results. The results of Ref. [8] have been converted from Ry to a.u. The data of Ref. [7] are presented without comment on the actual precision. The state marked with (*) can be also identified with states given in Ref. [7]. The dimension of the largest matrices used to obtain these data was n = 27000 and p = 15380.

N = 5								
This work		Saha and Mukherjee [7]		Ho and Bhatia [8]		This work (continued)		
$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	
0.11930	0.001 77	0.11937	0.000 985	0.11930	0.001 77	0.0849144	0.000 0887(*)	
0.109 463	0.001 561	0.10996	0.000 925	0.109463	0.001 561	0.08467477	0.000 013 60	
0.107 265	0.000765	0.10765	0.000 50	0.107 265	0.000765	0.084 463 5	0.000 144 9	
0.102 096	0.001 134	0.10299	0.001 02	0.102 096	0.001 134 5	0.084 249 56	0.00000624	
0.098 603 1	0.000 451 7	0.09828	0.000414	0.098 602	0.000 453	0.0840879	0.000 060 3	
0.096 641	0.000687	0.09656	0.000453	0.096 640 5	0.000687	0.083 964 29	0.000 054 12	
0.095 254 2	0.000 044 2	0.09526	0.000 201	0.095 253 85	0.00004425	0.08374283	0.000 110 13	
0.095 136	0.000744			0.095 143	0.00074	0.083 564 41	0.000 004 23	
0.093 440 9	0.000 221 2	0.093 65	0.000 226 5	0.093 435	0.000 205	0.083 448 9	0.000 043 0	
0.091 509 3	0.0004702	0.09166	0.000 327	0.091 51	0.00047	0.083 380 9	0.0000792	
0.091 108	0.000 435	0.09112	0.000 368 5	0.091 105	0.000 425	0.083 184 1	0.0000827	
0.09072496	0.000 021 25	0.09072	0.00001725	0.0907245	0.000 021	0.08303258	0.000 002 81	
0.090 236 5	0.0000532	0.09026	0.000127			0.0829466	0.0000322	
0.088 632	0.000 340	0.08908	0.000 299 5			0.0829094	0.0000856	
0.088 503 2	0.000 218 9	0.08850	0.000 009 95			0.0827411	0.000 061 5	
0.088 501 5	0.000 058 3					0.08261147	0.000 001 83	
0.088 120 97	0.00001786	0.08810	0.000067			0.0825452	0.000 024 8	
0.087 296	0.000 209	0.08744	0.0000110			0.0825278	0.000 081 9	
0.0867708	0.0000967	0.08696	0.000 101 5			0.0823837	0.000 045 5	
0.0867355	0.0002446	0.08602	0.0001795			0.08227232	0.000 001 19	
0.086 382 63	0.000 013 08					0.08221996	0.000 019 55	
0.085 989 9	0.000 139 2	0.08593	0.0000775			0.082 215 2	0.0000737	
0.085 588 9	0.000 013 0					0.082 091 19	0.000 033 70	
0.085 419 5	0.000 188 9	0.085 43	0.000112			0.081 995 15	0.00000077	
0.085 153 58	0.000 009 08					0.081 955 9	0.000 064 1	

We present energies and widths for ${}^{1}P^{e}$ and ${}^{3}P^{e}$ resonances below the third to eighth SIT. The data for the third up to the seventh threshold of ${}^{3}P^{e}$ resonances and for the third up to the fifth threshold of ${}^{1}P^{e}$ resonances are compared to existing data [7,8], while the energies and the widths for ${}^{3}P^{e}$ resonances below the eighth threshold and for ${}^{1}P^{e}$ resonances below the sixth up to the eighth threshold are reported for the first time. The results have been computed using up to twenty-one angular configurations and up to seven sets of Coulomb-Sturmian functions for each angular configuration. Since the Sturmian function $S_{n,l}^{(k)}(r)$ for k = Z/n is the radial eigenfunction of principal quantum number n and angular momentum l of a hydrogenic atom with nuclear charge Z, the choice of $k_{1s} \approx 2/n_1$ and $k_{2s} \approx 2/n_2$ provides an approximate description of a helium state with electronic excitations n_1 and n_2 , respectively. Although there is not a well-defined rule for the choice of the optimal parameter sets, this simple argument gives a hint for an appropriate choice of them in order to obtain a description of a rather large energy regime. As an illustration, Table I summarizes typical parameter sets for each angular configuration used for the description of the spectrum below the eighth SIT. These include two sets with $k_1 = k_2 \approx 2/N$, N = 8,9, for the description of the low-lying states of the series converging to I_8 and of the intruder states of

the series converging to I_9 . For the other sets, we have chosen $k_1 \approx 2/N$, N = 8,9, and $k_2 = 2/n_2$, $n_2 > N$ to account for higher excitations of the outer electron.

Our results have been tested for convergence with respect to variation of the basis size—including the number of angular configurations and Coulomb-Sturmian functions used, variation of the real dilation parameters, and variation of the complex rotation angle θ , with $\theta \in \{0.1, 0.12, 0.14, 0.16, 0.18, 0.2\}$. Tables II–IV contain the data for ³*P*^e resonances, while the data for ¹*P*^e resonances are presented in Tables V and VI. The tables are limited to at most fifty entries even if more converged resonances have been identified.

For symmetric excitation of both electrons, our approach suffers from the influence of the Kato cusp [30], which is a discontinuity of the derivative of the wave function at $r_{12} = 0$ that is not resolvable within our approach. The effect can easily be spotted in our tables as the precision of our results increases with increasing excitation of the outer electron.

The agreement with the results for ${}^{3}P^{e}$ (Tables II and III) and for ${}^{1}P^{e}$ (Table V) of Ref. [8], where an explicitly correlated approach together with complex rotation has been used to extract energy and the width of resonances, is, in general, excellent. Nevertheless, we obtain a significantly larger amount of additional converged resonances and are

TABLE IV. ${}^{3}P^{e}$ resonances of helium below the sixth (N = 6), the seventh (N = 7), and the eighth (N = 8) SITs: Our results on the left are compared with data from Ref. [7]. Only converged digits are displayed for our results. The data of Ref. [7] are presented without comment on the actual precision. The dimensions of the largest matrices used to obtain these data were n = 27000, p = 14570 for N = 6; n = 27000, p = 14020 for N = 7; and n = 29180, p = 14257 for N = 8.

N = 6							
This work		Saha and Mukherjee [7]		This work (continued)		This work (continued)	
$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$
0.076443	0.000 596	0.07721	0.000 369	0.062 138 9	0.000 155 5	0.058 821 9	0.000 105 1
0.073 661	0.000871	0.07079	0.000 483	0.061 869 1	0.000 211 0	0.058 798 52	0.000 014 22
0.073 387 0	0.0007392	0.06825	0.000735	0.061 300 50	0.00000546	0.058 636 24	0.00008777
0.0707545	0.000 358 1	0.06704	0.000275	0.061 275	0.000 329	0.0584702	0.000 057 9
0.070218	0.000 841	0.06665	0.000149	0.061 051 0	0.000 082 6	0.058 390 002	0.000007444
0.068 631 2	0.0005865	0.065 45	0.000 274 5	0.06097248	0.000 018 12	0.058 354 6	0.000 123 4
0.067 072 5	0.000 105 0	0.065 22	0.0000705	0.060 666 9	0.000 181 3	0.058 352 48	0.000 005 17
0.066 527 2	0.000 368 3	0.06449	0.000 005 1	0.060 361 9	0.000 221 3	0.058 225 98	0.00005642
0.066 190	0.000655	0.063 83	0.000 152	0.060 256 92	0.000 006 81	0.058 076 16	0.000 039 78
0.065 516 9	0.000 234	0.063 58	0.000 003 425	0.060 114 66	0.000 025 53	0.058 011 62	0.000 004 71
0.065 294 9	0.0000941	0.06272	0.000 000 336 5	0.060 0951	0.000 0488	0.057 997 917 8	0.000 000 580 3
0.064 495 12	0.000 008 92	0.06187	0.0001735	0.0597971	0.000 153 8	0.057 979 4	0.000 142 1
0.063 937	0.000256	0.061 15	0.00 012 55	0.059 582 5	0.000 134 9		
0.063 814 2	0.000 388 9	0.06075	0.000 092	0.059 470 995	0.000 012 354		
0.063 613 8	0.000 257 2	0.06028	0.000 163	0.059 402 6	0.000 073 9		
0.063 594 605	0.000 004 761	0.05973	0.000 234 5	0.059 365 2	0.000 027 9		
0.062 721 69	0.000 003 40	0.05913	0.000 267	0.059 144 9	0.000 122 5		
0.062 337 2	0.000 207 3	0.05774	0.000 168 5	0.058 962 0	0.000 086 5		
0.062 326	0.000 145	0.05632	0.000 127	0.058 864 15	0.00001062		
				N = 7			
0.053 47	0.00070	0.05363	0.000125	0.046 881 8	0.000 089 3	0.044 569 6	0.0000784
0.053 402 9	0.000 280 3	0.05190	0.000280	0.0467502	0.000 103 7	0.044 520 85	0.000 033 66
0.052 228 8	0.000 367 7	0.05085	0.0000450	0.046 576 57	0.000 012 32	0.044 381 00	0.000 004 08
0.052 015	0.000 430	0.04965	0.000 020 0	0.046 240 2	0.000 186 2	0.044 188	0.000116
0.051 024	0.000 048 2	0.04860	0.000 105	0.046 110 8	0.000 204 0	0.044 180 1	0.000 108 4
0.050 847 80	0.00001202	0.047 52	0.000 295	0.0459701	0.000 096 6	0.044 167 7	0.000 095 6
0.050 394	0.000 530	0.04655	0.0000150	0.045 826 8	0.000 031 4	0.044 051 93	0.000 006 79
0.049773	0.000241	0.04573	0.000075	0.04581448	0.00000527	0.043 971 97	0.000 067 18
0.049 702 6	0.000 062 3	0.045 19	0.00 025 0	0.045 580 6	0.000 093 1	0.043 859 62	0.000 006 15
0.048 578	0.000458	0.044 84	0.000110	0.045 348 2	0.000 385 7	0.043 762 2	0.0000627
0.048 461 1	0.000 156 7	0.04409	0.000 275	0.045 313 7	0.000 087 3	0.043 705 0	0.000 095 8
0.048 404 8	0.000 157 6	0.04337	0.000 165	0.045 169 12	0.000 008 14	0.043 692 8	0.0000567
0.048 397	0.000 174			0.045 070 11	0.000 000 94	0.043 629 21	0.00000448
0.048 275	0.000 193			0.045 033 07	0.000 001 80	0.043 522 5	0.000 090 8
0.047 323	0.000366			0.044 785 9	0.000 218 4	0.043 442 86	0.000 007 69
0.047 154 83	0.00001855			0.0447759	0.000 097 4	0.043 377 0	0.000 036 8
0.047 070 7	0.000 285 4			0.044 600 0	0.000 034 4		
				N = 8			
This	work	This wor	k (continued)	This work	(continued)	This work (continued)	
$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$
0.040 86	0.000 32	0.037 168 8	0.000 179 3	0.035 428 934	0.000 010 562	0.034 435 779	0.000 012 427
0.04000562	0.00000504	0.0370676	0.000 317 8	0.035407698	0.000000352	0.034 408 53	0.000 035 55
0.039 768	0.000 413	0.03673341	0.000 061 63	0.035 341 3	0.0000808	0.034 401 12	0.000 020 60
0.039 188 412	0.000 003 812	0.03670367	0.000 039 08	0.034 979 65	0.000 059 16	0.034 267 095	0.000022262
0.039 173 6	0.0002594	0.0363853	0.0004327	0.034 979 1	0.000 084 5	0.034 195 95	0.000 126 31
0.038 991 7	0.0003175	0.03627663	0.000 024 18	0.034 968 1	0.000 043 5	0.034 077 29	0.000 071 49
0.038 552 56	0.000 161 79	0.036 208 60	0.000 086 45	0.034 866 08	0.000 032 74	0.034 032 27	0.000 011 04
0.038 484 3	0.000 502 5	0.0360819	0.000 231 4	0.034 847 195	0.000 001 719	0.034 019 28	0.000 004 55
0.038 239 94	0.000 031 34	0.035 931 87	0.000 013 34	0.034 820	0.000 497	0.034 015 479	0.000 006 518
0.038 200 8	0.0003181	0.035 692 53	0.000 048 20	0.034 764 50	0.000 011 78	0.033 947 73	0.000 062 12
0.037 843 9	0.0001367	0.035 690 0	0.0001755	0.034 595 1	0.000 165 1	0.033 853 77	0.000 035 12
0.037 221 5	0.000 121 3	0.035 484 2	0.0001532	0.034 456 3	0.000 036 8		
0.037 208 7	0.0002426	0.035 436 1	0.0001282	0.034 438 4	0.000 202 3		

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TABLE V. ¹ P^{e} resonances of helium below the N = 3-5 SIT: Our results are compared with data from Ref. [8]. Only converged digits are displayed for our results. The results of Ref. [8] have been converted from Ry to a.u. The dimensions of the largest matrices used to obtain these data were n = 27720, p = 16372 for N = 3; n = 26200, p = 15491 for N = 4; and n = 26200, p = 14865 for N = 5.

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	N = 3								
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	This work		Ho and Bhatia [8]		This work (continued)		This work (continued)		
0.278.925 0.000.021 0.278.925 0.000.000 0.222 4172.5969 0.000.000.013 0.255.688 0.000.0013 0.235.638 0.000.000.015 0.2224.127.5969 0.0000.000.015 0.242.137.18 0.000.0013 0.242.137 0.000.000.015 0.2224.172.5969 0.0000.000.015 0.242.137.18 0.000.00175 0.2224.171.515 0.0000.00548 0.223.900.007 0.0000.0015.90 0.232.4223 0.000.00175 0.222.517.610 0.0000.00558 0.223.97.645.02 0.0000.001.145 0.232.410.750 0.0000.0133 0.222.517.601 0.0000.000.520 0.223.665.652.3 0.0000.001.145 0.230.02.023 0.0000.0133 0.222.517.601 0.0000.002.50 0.223.476.7530 0.0000.001.145 0.230.02.023 0.000.001.01 0.224.476.9233 0.0000.002.50 0.223.476.7531 0.0000.001.145 0.230.02.023 0.000.001.01 0.155.653.3 0.000.001.53 0.223.476.7531 0.0000.001.153 0.230.02.023 0.000.001.01 0.223.476.7530 0.0000.001.55 0.223.476.7531 0.000000.01.55 0.145	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	
0.259 352 043 0.000 000 628 0.259 352 0.000 000 629 5 0.227 437 119 1 0.000 000 000 000 75 0 0.224 172 596 0 0.000 000 281 9 0.245 638 72 0.000 000 435 0.242 237 48 0.000 000 437 5 0.225 613 435 2 0.000 000 684 9 0.223 950 000 7 0.000 000 235 6 0.242 237 48 0.000 000 475 0.222 347 149 0 0.000 000 644 2 0.223 950 000 7 0.000 000 235 6 0.235 048 7 0.000 000 197 1 0.225 581 54 67 0 0.000 000 642 0 0.223 636 66 2 0.000 000 145 0 0.234 140 753 0.000 001 197 1 0.225 176 001 0.000 000 000 023 0 0.223 650 61 2 0.000 000 145 0 0.234 67 0558 0 0.000 000 10 1 0.224 48 139 0 0.000 000 23 0 0.223 470 751 0 0.000 000 145 0 0.236 400 558 0 0.000 023 0 0.224 68 139 0 0.000 000 145 0 0.224 970 751 0 0.000 000 145 0 0.236 400 558 0 0.000 023 0 0.224 769 293 0 0.000 000 23 0 0.124 759 75 0 0.000 000 15 1 0.156 018 0 0.000 023 0 0.124 592 14 0 0.000 01 29 0 0.129 023 144 0 0.000 01 15 1 0.129	0.278 992 5	0.000 022 1	0.278 992 5	0.000 022 15	0.228 383 750	0.000 001 520	0.224 441 163 5	0.000 000 341 3	
0.253 638 72 0.000 01320 0.235 6387 0.000 00131915 0.222 131 35 0.0000 0011 1 0.224 513 61 0.0000 0000 159 0.242 237 48 0.000 00759 0.242 237 0.000 0000 281 0.0226 347 159 0.000 0006 80 0.223 935 607 1 0.000 0000 0159 0.232 407 535 0.000 000 171 0.225 688 535 0.000 0000 0198 0.223 763 4503 0.000000 0198 0.232 410 753 0.000 000 171 0.225 688 535 0.000 0000 028 0 0.223 470 753 0.000000 0198 0.232 410 753 0.000 000 173 0.222 176 1501 0.000000028 0 0.223 470 751 0.000000 0198 0.232 410 753 0.000 000 1183 0.0165 5193 0.0000 0241 0.125 5193 0.000000145 0.224 470 239 0.00000028 0 0.0000 0153 0.232 669 596 0.000 004210 0.145 5193 0.0000153 0.130 571 662 0.000 00145 0.224 470 239 0.121 575 907 5 0.000001153 0.145 232 16 0.000 0142 1 0.145 518 0.130 571 662 0.000 0142 5 0.121 575 907 5 0.000001153 0.145 225 16 0.000 0124 1 0.145 518	0.259 352 043	0.000 000 628	0.259 352	0.000 000 629 5	0.227 437 119 1	0.0000000745	0.224 225 013 4	0.000 000 019 1	
0.244 1316 16 0.000 000435 0.242 0374 0.000 000 437 0.226 3471 0000 0.000 000 0153 0.242 0374 80 0.000 000475 0.226 3471 000 0.000 000 0153 0.226 3471 0000 0.223 00000 153 0.233 114407 20 0.000 000 1971 0.225 815 467 0 0.000 000 000 020 0.223 656 016 20 0.000 000 1145 0.234 76 058 6 0.000 001 1971 0.225 751 151 0.000 000 000 22 0.223 656 016 20 0.000 000 1145 0.234 76 058 6 0.000 001 1071 0.225 751 151 0.000 000 12 0.000 000 1145 0.000 000 1145 0.234 76 058 6 0.000 001 101 0.224 586 015 0 0.000 000 12 0.000 000 153 0.234 76 058 6 0.000 001 101 0.224 585 012 5 0.000 001 023 0.000 001 153 0.154 084 9 0.000 0421 0 0.145 0519 0 0.000 0423 0 0.127 157 590 75 0.000 001 153 0.154 084 9 0.000 0422 0 0.145 0223 0 0.000 023 0 0.128 523 144 0000 012 139 0.127 01 148 2000 01 123 0.127 01 148 2000 01 123 0.127 01 148 2000 01 123 0.127 01 148 2000 01 123 0.127 01 148 2000 0000 123 0.0000 0058 1 0.128 52	0.253 638 72	0.000 013 20	0.253 638 7	0.0000131915	0.227 213 135	0.000 001 121	0.224 172 596 9	0.000 000 281 9	
0.242 237 48 0.000 007 69 0.242 237 0.000 000 785 0.225 437 159 0.000 000 44 0.223 800 4645 0.000 000 135 0.235 087 88 0.000 001 77 0.000 000 171 0.225 685 533 0.000 000 550 0.223 605 563 0.000 000 0198 0.232 410 753 0.000 000 133 0.225 75 1151 0.000 000 2250 0.223 605 562 0.000 000 0193 0.230 407 753 0.000 000 133 0.022 757 1151 0.000 000 225 0.223 470 737 0.000 000 1693 0.230 407 753 0.000 001 001 0.224 769 293 0.000 000 225 0.223 470 737 0.000 000 1693 0.230 407 923 0.000 001 001 0.224 769 293 0.000 000 420 0.125 5193 0.0000 001 15 0.155 5192 K 0.000 003 619 0.135 514 662 0.000 001 219 0.126 924 2493 0.0000 001 153 0.0000 000 129 0.126 5191 20 0.0000 000 000 000 000 000 129 0.126 5191 20 0.000 000 000 000 000 000 000 000 000 0	0.244 513 616	0.000 000 435	0.244 513 6	0.000 000 437 5	0.226 513 436 2	0.000 000 056 8	0.223 993 560 71	0.000 000 015 93	
0.237 S22 423 0.000 000 000 290 0.225 815 467 0 0.000 000 424 2 0.223 800 464 5 0.000 000 0134 5 0.236 098 88 0.000 001 197 1 0.225 815 467 0 0.000 000 520 0.223 605 563 2 0.000 000 114 5 0.234 174 073 0.000 000 133 3 0.225 815 470 1 0.000 000 228 0.223 605 563 3 0.000 000 098 8 0.230 476 058 6 0.000 001 133 3 0.224 4769 293 0.000 000 228 0.223 605 563 3 0.000 000 098 8 0.230 476 058 6 0.000 001 101 0.224 4769 293 0.000 000 228 0.223 470 751 0.000 001 153 1 0.156 408 49 0.000 024 1 0.156 5193 0.000 00360 0.130 574 062 0.000 002 139 0.127 157 590 75 0.000 001 153 1 0.145 6408 49 0.000 005 64 0.145 222 2 0.000 002 245 12 0.000 002 249 0.126 892 492 2 0.000 000 376 8 0.144 232 40 0.000 001 758 0.141 578 0.000 0175 8 0.129 521 45 0.000 002 719 9 0.126 893 449 0.000 000 898 39 0.144 232 40 0.000 01 673 8 0.141 578 0.000 0175 8 0.129 521 450 5001 000 0173 8 0.126 689 714 3	0.242 237 48	0.000 007 69	0.242237	0.000 008	0.226 347 159	0.000 000 849	0.223 950 000 7	0.000 000 235 6	
0.236 098 88 0.000 0004 75 0.222 568 553 0.000 0006 580 0.223 763 4803 0.000 000 104 0.232 410 753 0.000 000 133 0.225 275 115 1 0.000 000 022 0 0.223 666 662 0 0.000 000 1693 0.234 107 508 6 0.000 000 11 3 0.224 418 199 3 0.000 000 22 0 0.223 407 313 0 0.000 000 1693 0.230 202 923 0.000 000 11 0 0.224 448 199 3 0.000 000 22 0 0.223 407 313 0 0.000 000 1693 0.224 669 659 6 0.000 001 10 1 0.224 505 021 5 0.000 000 22 0 0.223 407 313 0 0.000 001 153 0.165 519 28 0.000 024 1 0.165 65193 0.000 0024 0 0.129 797 973 7 0.000 001 201 9 0.126 891 422 2 0.000 000 521 0 0.126 891 422 2 0.000 000 521 0 0.126 891 422 2 0.000 000 521 0 0.126 891 423 2 0.000 000 521 0 0.126 891 423 2 0.000 000 521 0 0.126 891 423 2 0.000 000 521 0 0.126 891 423 2 0.000 000 627 1 0.126 891 436 1 0.000 000 627 2 0.126 681 486 1 0.000 000 627 2 0.126 591 430 0 0.000 000 571 4 0.128 597 517 4 0.000 000 600 0 0.128 597 517 4 0.000 000 617 3	0.237 282 423	0.000 000 290			0.225 815 467 0	0.000 000 044 2	0.223 800 046 45	0.000 000 013 45	
0.233 114 407 2 0.000 000 107 1 0.225 275 115 1 0.000 000 035 0 0.223 636 602 9 0.000 000 010 4 0.232 4107 53 0.000 000 138 3 0.224 769 203 0.000 000 129 0.000 000 120 0.230 476 058 6 0.000 001 13 3 0.224 769 203 0.000 000 123 9 0.000 000 120 0.230 476 058 6 0.000 01 101 0.224 769 203 0.000 000 123 9 0.000 000 123 9 0.230 476 058 6 0.000 01 141 0.165 5193 0.000 00360 0.123 9 0.127 157 590 75 0.000 01 153 18 0.156 408 49 0.000 026 41 0.146 60 985 0.000 024 0 0.129 279 793 7 0.000 003 095 0.126 981 48 2 0.000 000 292 4 0.126 981 48 2 0.000 000 292 4 0.126 984 48 2 0.000 000 000 88 1 0.144 232 54 0.000 017 78 0.141 473 55 0.000 017 78 0.128 977 666 6 0.000 027 72 4 0.126 683 724 4 0.000 000 88 13 0.138 778 490 0.000 001 404 0.128 237 719 7 0.000 007 173 3 0.126 638 714 3 0.000 001 637 1 0.138 718 490 40 0.000 0175 8 0.128 217 91 71 7 0.0000 0163 7 0.126 638 714 3	0.236 098 88	0.000 004 75			0.225 688 553	0.000 000 658	0.2237634503	0.000 000 198 8	
0.232410753 0.0000013104 0.225176001 0.000000222 0.2236955623 0.000000193 0.2300476586 0.0000001133 0.2248481993 0.000000228 0.2234757339 0.000000098 0.230042923 0.000001101 0.224769293 0.000000148 0.223477575 0.000000153 0.15551928 0.00004201 0.1554085 0.000026 0.12979737 0.000001235 0.12715759075 0.000001293 0.14966102 0.00002280 0.1442322 0.00002285 0.12979737 0.000003190 0.1268934292 0.0000002552 0.145221766 0.00000524 0.14522225 0.0000055 0.129755425 0.000003754 0.1268934292 0.0000002579 0.141579530 0.000175 0.1289776666 0.00000274 0.1266350249 0.0000002571 0.13877499 0.00001464 0.128802370 0.00000274 0.12663501667 0.0000002717 0.136934328 0.00001755 0.128475730 0.000002774 0.126528440305 0.0000002717 0.136894348 0.000001755 0.1283507197 0.00000015431 <	0.233 114 407 2	0.000 000 197 1			0.225 275 115 1	0.000 000 035 0	0.223 636 606 29	0.000 000 011 45	
0.230 476 058 6 0.000 000 1183 0.224 848 1993 0.000 000 002 12 0.223 470 7113 0.000 000 000 145 0.230 020 923 0.000 000 1219 0.224 769 293 0.000 000 148 0.223 470 751 0.000 000 145 0.228 696 059 6 0.000 001 41 0.155 5193 0.000 0034 81 0.155 5193 0.000 001 213 0.127 157 590 75 0.000 000 123 0.156 4084 9 0.000 002 641 0.149 660 985 0.000 002 0.129 528 144 0.000 000 123 9 0.121 01482 0.000 000 008 52 0.144 232 24 0.000 0017 58 0.141 572 52 0.000 0017 55 0.128 977 666 6 0.000 000 224 0.126 899 4292 0.000 000 878 1 0.144 232 24 0.000 0017 58 0.141 572 0.000 001 755 0.128 802 707 0.000 000 213 0 0.126 682 148 661 0.000 000 818 0 0.138 231 81 0.000 011 765 0.128 802 707 0.000 000 716 3 0.126 685 1143 0.000 000 616 6 0.133 694 494 0 0.000 000 175 8 0.128 591 79 7 0.000 000 717 4 0.128 802 707 0.000 000 716 3 0.126 685 0143 0.000 000 616 6 0.133 691 49 0 0.000 001	0.232 410 753	0.000 003 104			0.225 176 001	0.000 000 520	0.223 605 562 3	0.000 000 169 3	
0.230 020 923 0.000 000 129 0.224 769 293 0.000 000 13 0.223 470 751 0.000 000 145 0.155 519 28 0.000 004 201 0.155 408 5 0.000 004 201 0.155 408 5 0.000 004 201 0.153 518 0.156 408 49 0.000 004 201 0.156 408 5 0.000 004 201 0.156 408 5 0.000 004 201 0.127 901 048 2 0.000 000 300 5 0.149 661 02 0.000 005 624 0.148 522 25 0.000 002 88 5 0.000 002 88 5 0.000 002 754 0.148 522 25 0.000 000 809 5 0.126 894 492 2 0.000 000 809 5 0.000 000 809 5 0.128 692 485 1 0.000 000 809 5 0.000 000 809 5 0.000 000 875 1 0.126 894 492 1 0.000 000 809 5 0.129 854 14 0.000 000 716 3 0.126 584 402 51 0.000 000 805 2 0.128 823 70 70 118 0.000 000 809 3 0.128 585 719 7 0.000 000 717 3 0.126 581 403 05 0.000 000 81 35 0.126 581 403 05 0.000 000 81 35 0.128 517 143 0.000 000 81 35 0.128 517 143 0.000 000 81 35 0.128 517 143 0.000 000 81 35 0.128 517 143 0.000 000 81 35 0.128 517 143 0.000 000 81 35 0.128 517 517 143 0.000 000 81 35	0.230 476 058 6	0.000 000 138 3			0.224 848 199 3	0.000 000 028 2	0.223 497 313 9	0.000 000 009 8	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	0.230 020 923	0.000 002 129			0.224769293	0.000 000 418	0.223 470 751	0.000 000 145	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	0.228 696 059 6	0.000 000 100 1			0.224 505 021 5	0.000 000 023 0			
0.165 519 28 0.000 034 81 0.165 519 3 0.000 0420 0.130 574 062 0.000 04235 0.127 157 590 75 0.000 001 133 18 0.156 408 49 0.000 04201 0.156 408 5 0.000 0264 0.129 797 937 0.000 003 609 5 0.126 932 498 39 0.000 0003 07 68 0.144 232 54 0.000 002 624 0.145 222 25 0.000 00264 0.129 797 937 0.000 003 190 0.126 839 4929 0.000 0000 856 2 0.141 575 35 0.000 017 58 0.141 578 0.000 0175 0.128 807 370 0.000 007 163 0.126 639 7143 0.000 000 861 2 0.138 94328 0.000 017 55 0.128 807 197 0.000 007 163 0.126 639 7143 0.000 000 717 2 0.133 941 318 0.000 003 229 8 0.128 350 7197 0.000 003 763 0.000 000 616 6 0.133 941 318 0.000 003 217 0.0126 059 066 0.000 000 616 6 0.000 000 616 6 0.133 941 318 0.000 003 774 0.127 757 822 8 0.000 001 733 0.126 532 7440 0 0.000 000 522 4 0.131 962 967 0.000 001 637 8 0.127 757 822 8 0.000 000 733 0.126 321 282 0.0000 000 520 4 <td></td> <td></td> <td></td> <td></td> <td>N = 4</td> <td></td> <td></td> <td></td>					N = 4				
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0.109 927 09 0.000 036 33 0.109 927 05 0.000 036 33 0.086 223 04 0.000 009 04 0.082 799 549 7 0.000 002 868 9 0.105 179 08 0.000 064 12 0.105 179 15 0.000 064 1 0.085 734 152 0.000 002 996 0 0.082 690 61 0.000 000 05 08 0.099 587 38 0.000 040 30 0.099 588 0.000 040 5 0.085 549 555 4 0.000 002 996 0 0.082 690 61 0.000 002 611 0.096 353 44 0.000 050 97 0.096 355 0.000 10 0.085 047 016 0.000 067 95 0.082 422 356 0.000 002 293 0.092 842 00 0.000 027 89 0.096 355 0.000 10 0.084 669 095 0.000 026 399 0.082 334 626 0.000 002 138 0.092 085 843 0.000 016 52 0.008 4433 498 0.000 010 356 0.082 244 211 0.000 002 134 0.091 596 61 0.000 016 52 0.084 473 498 0.000 010 356 0.082 244 211 0.000 001 862 7 0.098 398 33 0.000 117 0 0.083 775 500 0.000 004 761 0.082 075 055 0.000 011 82 0.088 983 560 0.000 012 856 0.083 507 802 0.000 004 761 0.081 865 5993 0.000 001 765 5 0.088 683 11					N = 5				
0.105 179 08 0.000 064 12 0.105 179 15 0.000 00641 0.085 734 152 0.000 008 757 0.082 736 551 0.000 001 637 0.099 587 38 0.000 032 20 0.099 588 0.000 032 0.085 549 555 4 0.000 002 996 0 0.082 690 61 0.000 002 611 0.099 131 81 0.000 050 97 0.096 355 0.000 10 0.085 047 016 0.000 006 795 0.082 276 812 0.000 002 293 0.093 797 57 0.000 023 74 0.096 355 0.000 10 0.084 669 095 0.000 006 6360 0.082 371 490 0.000 001 388 0.092 842 00 0.000 001 652 0.084 433 498 0.000 010 356 0.822 242 11 0.000 002 134 0.091 596 61 0.000 016 652 0.084 433 498 0.000 010 356 0.082 116 730 2 0.000 001 862 7 0.098 398 33 0.000 017 00 0.083 775 500 0.000 002 276 0.082 045 009 0.000 001 862 7 0.088 683 11 0.000 028 56 0.083 699 897 0.000 008 022 0.081 971 944 9 0.000 001 755 5 0.088 683 11 0.000 012 278 0.083 507 802 0.000 004 074 0.081 865 5993 0.000 001 011 2 0.087 7220 161 0.000 012 2482 0.083 193 307<	0.109 927 09	0.000 036 33	0.109 927 05	0.000 036 33	0.086 223 04	0.000 009 04	0.0827995497	0.000 002 868 9	
0.099 587 380.000 032 200.099 5880.000 0320.085 549 555 40.000 002 996 00.082 690 610.000 005 080.099 131 810.000 040 300.099 131 50.000 040 50.085 411 1250.000 013 6540.082 576 8120.000 002 2930.093 797 570.000 023 740.096 3550.000 100.085 047 0160.000 006 3600.082 371 4900.000 001 3880.092 842 000.000 002 7890.000 016 520.084 669 0950.000 010 3560.082 244 2110.000 002 1340.091 596 610.000 016 520.084 433 4980.000 010 3560.082 244 2110.000 002 1340.092 17 820.000 017 000.083 877 6520.000 004 7610.082 075 0550.000 001 862 70.088 938 330.000 018 440.083 775 5000.000 002 2760.082 045 0090.000 003 4090.088 683 110.000 025 520.083 699 8970.000 008 0220.081 971 944 90.000 001 755 50.088 683 110.000 012 2780.083 272 6930.000 003 6560.081 831 019 30.000 001 533 90.087 720 1610.000 012 4820.083 193 3070.000 003 6560.081 831 019 30.000 002 8420.086 554 5880.000 003 1690.083 135 0070.000 003 310.081 746 287 60.000 001 476 10.086 756 130.000 018 430.082 988 8950.000 003 2370.000 003 237	0.105 179 08	0.000 064 12	0.105 179 15	0.0000641	0.085734152	0.000 008 757	0.082736551	0.000 001 637	
0.099 131 81 0.000 040 30 0.099 131 5 0.000 040 5 0.085 411 125 0.000 013 654 0.082 576 812 0.000 002 611 0.096 353 44 0.000 050 97 0.096 355 0.000 10 0.085 047 016 0.000 006 795 0.082 422 356 0.000 002 293 0.092 842 00 0.000 027 89 0.000 01652 0.084 669 095 0.000 002 639 0.082 371 490 0.000 002 134 0.091 596 61 0.000 036 09 0.084 433 498 0.000 010 356 0.082 244 211 0.000 002 134 0.099 217 82 0.000 017 00 0.083 877 652 0.000 004 761 0.082 075 055 0.000 001 862 7 0.088 983 560 0.000 002 856 0.083 877 550 0.000 002 276 0.82 045 009 0.000 001 862 7 0.088 983 560 0.000 002 856 0.083 699 897 0.000 008 022 0.81 971 944 9 0.000 001 765 5 0.088 683 11 0.000 012 278 0.083 507 802 0.000 004 074 0.081 865 599 3 0.000 001 765 5 0.087 7220 161 0.000 012 2482 0.083 193 307 0.000 003 656 0.081 831 019 3 0.000 001 242 0.086 954 588 0.000 003 169 0.082 988 895 0.0000 003 237 0.000 001 476 1 <	0.099 587 38	0.000 032 20	0.099 588	0.000032	0.085 549 555 4	0.000 002 996 0	0.08269061	0.000 005 08	
0.096 353 44 0.000 050 97 0.096 355 0.000 10 0.085 047 016 0.000 006 795 0.082 422 356 0.000 002 293 0.093 797 57 0.000 023 74 0.084 669 095 0.000 006 360 0.082 371 490 0.000 001 388 0.092 842 00 0.000 001 652 0.084 534 149 0.000 002 639 0.082 244 211 0.000 002 134 0.091 596 61 0.000 016 52 0.084 433 498 0.000 005 213 0.082 116 730 2 0.000 001 862 7 0.090 217 82 0.000 017 00 0.083 877 652 0.000 004 761 0.082 045 009 0.000 001 862 7 0.088 983 360 0.000 002 856 0.083 699 897 0.000 008 022 0.081 971 944 9 0.000 001 765 5 0.088 683 11 0.000 012 278 0.083 507 802 0.000 004 074 0.081 865 5993 0.000 001 533 9 0.097 257 284 0.000 012 278 0.083 193 307 0.000 001 933 0.081 806 201 0.000 002 842 0.086 954 588 0.000 003 169 0.082 988 895 0.000 003 237 0.081 746 287 6 0.000 001 476 1	0.099 131 81	0.000 040 30	0.099 131 5	0.000 040 5	0.085 411 125	0.000 013 654	0.082 576 812	0.000 002 611	
0.093 797 570.000 023 740.084 669 0950.000 006 3600.082 371 4900.000 001 3880.092 842 000.000 027 890.084 534 1490.000 002 6390.082 334 6260.000 004 1360.092 085 8430.000 011 6520.084 433 4980.000 010 3560.082 244 2110.000 002 1340.091 596 610.000 036 090.084 173 6280.000 005 2130.082 116 730 20.000 001 862 70.090 217 820.000 017 000.083 877 6520.000 004 7610.082 075 0550.000 001 1820.088 983 3600.000 018 440.083 775 5000.000 008 0220.081 971 944 90.000 001 765 50.088 683 110.000 025 520.083 507 8020.000 004 0740.081 865 599 30.000 001 765 50.097 857 2840.000 012 2780.083 193 3070.000 003 6560.081 831 019 30.000 001 2420.086 954 5880.000 003 1690.083 135 0070.000 006 3310.081 746 287 60.000 001 476 10.086 756 130.000 018 430.082 988 8950.000 003 2370.001 014 76 1	0.096 353 44	0.000 050 97	0.096 355	0.00010	0.085 047 016	0.000 006 795	0.082 422 356	0.000 002 293	
0.092 842 000.000 027 890.084 534 1490.000 002 6390.082 334 6260.000 004 1360.092 085 8430.000 001 6520.084 433 4980.000 010 3560.082 244 2110.000 002 1340.091 596 610.000 036 090.084 173 6280.000 005 2130.082 116 730 20.000 001 862 70.090 217 820.000 017 000.083 877 6520.000 004 7610.082 075 0550.000 001 1820.089 398 330.000 018 440.083 775 5000.000 002 2760.082 045 0090.000 003 4090.088 983 5600.000 002 8560.083 699 8970.000 008 0220.081 971 944 90.000 001 765 50.088 683 110.000 012 5520.083 507 8020.000 004 0740.081 865 599 30.000 001 533 90.097 857 2840.000 012 2780.083 193 3070.000 003 6560.081 831 019 30.000 001 24220.086 954 5880.000 003 1690.083 135 0070.000 006 3310.081 746 287 60.000 001 476 10.086 756 130.000 018 430.082 988 8950.000 003 2370.000 001 2476	0.093 797 57	0.000 023 74			0.084 669 095	0.000 006 360	0.082371490	0.000 001 388	
0.092 085 8430.000 001 6520.084 433 4980.000 010 3560.082 244 2110.000 002 1340.091 596 610.000 036 090.084 173 6280.000 005 2130.082 116 730 20.000 001 862 70.090 217 820.000 017 000.083 877 6520.000 004 7610.082 075 0550.000 001 1820.088 983 330.000 018 440.083 775 5000.000 002 2760.082 045 0090.000 003 4090.088 983 5600.000 002 8560.083 699 8970.000 008 0220.081 971 944 90.000 001 765 50.088 683 110.000 012 5520.083 507 8020.000 004 0740.081 865 599 30.000 001 533 90.097 857 2840.000 012 2780.083 193 3070.000 003 6560.081 831 019 30.000 001 011 20.087 220 1610.000 012 4820.083 135 0070.000 006 3310.081 746 287 60.000 001 476 10.086 756 130.000 018 430.082 988 8950.000 003 2370.001 012 237	0.092 842 00	0.000 027 89			0.084 534 149	0.000 002 639	0.082334626	0.000 004 136	
0.091 596 610.000 036 090.084 173 6280.000 005 2130.082 116 730 20.000 001 862 70.090 217 820.000 017 000.083 877 6520.000 004 7610.082 075 0550.000 001 1820.089 398 330.000 018 440.083 775 5000.000 002 2760.082 045 0090.000 003 4090.088 983 5600.000 002 8560.083 699 8970.000 008 0220.081 971 944 90.000 001 765 50.088 683 110.000 012 5520.083 507 8020.000 004 0740.081 865 599 30.000 001 533 90.097 857 2840.000 012 2780.083 193 3070.000 003 6560.081 831 019 30.000 001 011 20.087 220 1610.000 012 4820.083 135 0070.000 006 3310.081 746 287 60.000 001 476 10.086 756 130.000 018 430.082 988 8950.000 003 2370.001 001 2370.000 001 476 1	0.092 085 843	0.000 001 652			0.084 433 498	0.000 010 356	0.082 244 211	0.000 002 134	
0.090 217 820.000 017 000.083 877 6520.000 004 7610.082 075 0550.000 001 1820.089 398 330.000 018 440.083 775 5000.000 002 2760.082 045 0090.000 003 4090.088 983 5600.000 002 8560.083 699 8970.000 008 0220.081 971 944 90.000 001 765 50.088 683 110.000 025 520.083 507 8020.000 004 0740.081 865 599 30.000 001 533 90.097 857 2840.000 012 2780.083 193 3070.000 003 6560.081 831 019 30.000 001 011 20.087 220 1610.000 012 4820.083 135 0070.000 006 3310.081 806 2010.000 002 8420.086 954 5880.000 003 1690.082 988 8950.000 003 2370.001 013 237	0.091 596 61	0.000 036 09			0.084 173 628	0.000 005 213	0.0821167302	0.000 001 862 7	
0.089 398 330.000 018 440.083 775 5000.000 002 2760.082 045 0090.000 003 4090.088 983 5600.000 002 8560.083 699 8970.000 008 0220.081 971 944 90.000 001 765 50.088 683 110.000 025 520.083 507 8020.000 004 0740.081 865 599 30.000 001 533 90.097 857 2840.000 012 2780.083 193 3070.000 003 6560.081 831 019 30.000 001 011 20.087 220 1610.000 012 4820.083 135 0070.000 006 3310.081 806 2010.000 002 8420.086 954 5880.000 003 1690.082 988 8950.000 003 2370.000 003 237	0.090 217 82	0.000 017 00			0.083 877 652	0.000 004 761	0.082075055	0.000 001 182	
0.088 983 5600.000 002 8560.083 699 8970.000 008 0220.081 971 944 90.000 001 765 50.088 683 110.000 025 520.083 507 8020.000 004 0740.081 865 599 30.000 001 533 90.097 857 2840.000 012 2780.083 272 6930.000 003 6560.081 831 019 30.000 001 011 20.087 220 1610.000 012 4820.083 193 3070.000 001 9330.081 806 2010.000 002 8420.086 954 5880.000 003 1690.082 988 8950.000 003 2370.001 013 237	0.089 398 33	0.000 018 44			0.083 775 500	0.000 002 276	0.082 045 009	0.000 003 409	
0.088 683 110.000 025 520.083 507 8020.000 004 0740.081 865 599 30.000 001 533 90.097 857 2840.000 012 2780.083 272 6930.000 003 6560.081 831 019 30.000 001 011 20.087 220 1610.000 012 4820.083 193 3070.000 001 9330.081 806 2010.000 002 8420.086 954 5880.000 003 1690.082 988 8950.000 003 2370.001 746 287 60.000 001 476 1	0.088 983 560	0.000 002 856			0.083 699 897	0.000 008 022	0.081 971 944 9	0.000 001 765 5	
0.097 857 2840.000 012 2780.083 272 6930.000 003 6560.081 831 019 30.000 001 011 20.087 220 1610.000 012 4820.083 193 3070.000 001 9330.081 806 2010.000 002 8420.086 954 5880.000 003 1690.083 135 0070.000 006 3310.081 746 287 60.000 001 476 10.086 756 130.000 018 430.082 988 8950.000 003 2370.000 003 237	0.088 683 11	0.000 025 52			0.083 507 802	0.000 004 074	0.081 865 599 3	0.000 001 533 9	
0.087 220 161 0.000 012 482 0.083 193 307 0.000 001 933 0.081 806 201 0.000 002 842 0.086 954 588 0.000 003 169 0.083 135 007 0.000 006 331 0.081 746 287 6 0.000 001 476 1 0.086 756 13 0.000 018 43 0.082 988 895 0.000 003 237 0.000 001 476 1	0.097 857 284	0.000 012 278			0.083 272 693	0.000 003 656	0.081 831 019 3	0.000 001 011 2	
0.086 954 588 0.000 003 169 0.083 135 007 0.000 006 331 0.081 746 287 6 0.000 001 476 1 0.086 756 13 0.000 018 43 0.082 988 895 0.000 003 237 0.000 003 237	0.087 220 161	0.000 012 482			0.083 193 307	0.000 001 933	0.081 806 201	0.000 002 842	
0.08675613 0.00001843 0.082988895 0.000003237	0.086 954 588	0.000 003 169			0.083 135 007	0.000 006 331	0.081 746 287 6	0.000 001 476 1	
	0.08675613	0.000 018 43			0.082 988 895	0.000 003 237			

TABLE VI. ¹ P^e resonances of helium below the sixth (N = 6), the seventh (N = 7), and the eighth (N = 8) SITs. Only converged digits are displayed. The dimensions of the largest matrices used to obtain these data were $n = 26\,200$, $p = 14\,058$ for N = 6; $n = 26\,200$, $p = 13\,511$ for N = 7; and $n = 28\,360$, $p = 13\,748$ for N = 8.

N = 6		Ν	= 7	Ν	N = 8	
$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	$-\mathrm{Re}\left(E_{i,\theta}\right)$	$-\mathrm{Im}\left(E_{i,\theta}\right)$	
0.078 422 83	0.000 034 70	0.05498563	0.000 074 29	0.041 114 9	0.000 053 0	
0.075 674 10	0.000 061 56	0.05379687	0.000 034 40	0.040 024 52	0.000 069 60	
0.072 249 47	0.000 073 79	0.052 440 29	0.000 063 47	0.03965467	0.000 045 67	
0.071 369 29	0.000 034 24	0.052 425 98	0.000 056 34	0.03964537	0.000 029 74	
0.069 315 97	0.000 056 75	0.050 850 99	0.00007015	0.038 878 998	0.000 046 634	
0.068 046 70	0.000 029 51	0.050 627 00	0.000 029 05	0.03879548	0.000 070 29	
0.067 133 78	0.000 027 26	0.049 569 44	0.000 046 69	0.038 028 00	0.000 060 44	
0.066 949 10	0.000 058 92	0.049 438 68	0.000 028 63	0.037 941 778	0.000 023 810	
0.065 631 72	0.000 044 20	0.049 047 12	0.000 041 63	0.037 565 069	0.000 039 814	
0.064 484 32	0.000 024 21	0.048 472 274	0.000 022 801	0.037 331 038	0.000 037 500	
0.064 373 63	0.000 020 59	0.048 398 038	0.000 057 117	0.037 282 784 8	0.000 011 997 0	
0.063 983 06	0.000 043 17	0.047 657 60	0.000 036 62	0.037 098 091	0.000 056 651	
0.063 509 277	0.000 000 322	0.047 338 00	0.000 029 19	0.0367510629	0.000 008 898 8	
0.063 266 38	0.000 033 22	0.047 106 953	0.000 028 058	0.036 666 14	0.000 049 01	
0.062 480 20	0.00001541	0.046 946 18	0.00001716	0.036 547 503 5	0.000 010 974 2	
0.062 395 99	0.000 018 80	0.046 777 44	0.000 044 75	0.036 235 803	0.000 033 929	
0.062 090 55	0.000 031 72	0.046 398 218 3	0.000 000 100 4	0.036 226 863	0.000 025 302	
0.061 854 177	0.000 000 995	0.046 312 48	0.000 028 00	0.036 025 107 7	0.000 016 380 1	
0.061 650 78	0.000 024 98	0.046 001 749	0.000 024 303	0.035950619	0.000 040 618	
0.061 129 523	0.000 011 573	0.045 880 46	0.000 005 63	0.035 797 877 9	0.000 002 858 8	
0.060 997 89	0.000 014 39	0.045 837 00	0.000 021 76	0.0357415165	0.000 016 447 5	
0.060 789 409	0.000 023 612	0.045 681 944	0.000 008 656	0.035 690 77	0.000 037 26	
0.060 650 304	0.000 001 608	0.045 639 315	0.000 034 766	0.035 376 674 02	0.000 000 047 61	
0.060 496 58	0.000 018 99	0.045 428 747 6	0.000 000 304 9	0.035 332 76	0.000 026 54	
0.060 135 459	0.000 008 693	0.045 334 24	0.000 020 42	0.035 324 86	0.000 028 04	
0.060 000 138	0.000 011 071	0.045 058 74	0.000 019 50	0.035 172 746	0.000 019 904	
0.059 849 171	0.000 017 862	0.044 961 036	0.000 011 745	0.035 127 772	0.000 026 941	
0.059757586	0.000 001 966	0.044 936 141	0.000 017 547	0.035 028 755	0.000 012 623	
0.059 642 86	0.000 014 62	0.044 806 26	0.000 026 81	0.035 018 480	0.000 013 207	
0.059 387 022	0.000 006 243	0.044 681 170	0.000 002 718	0.03479384	0.000 014 10	
0.059 258 30	0.000 008 62	0.0446750807	0.0000005270	0.0347548968	0.000 000 140 9	
0.059 144 754	0.000 013 756	0.044 857 747	0.000 014 471	0.034703614	0.000 020 698	
0.059 079 707	0.000 002 080	0.044 359 353	0.000 015 631	0.034656412	0.000 023 140	
0.058 993 69	0.000 011 35	0.044 315 742	0.000 009 102	0.034 546 30	0.000 020 49	
0.058 867 727 5	0.000 000 516 9	0.044 265 80	0.000 014 33	0.034 510 834	0.000 017 374	
0.058 727 64	0.000 005 72	0.044 175 595	0.000020766	0.034470248	0.000 009 508	
0.058 689 952	0.000 006 813	0.044 085 465 8	0.0000006741	0.034 354 198	0.000027905	
0.058 602 152	0.000010788	0.043 987 367	0.000015535	0.03425506932	0.000 000 247 64	
0.058 553 488	0.000 002 031	0.043 823 270	0.000 012 646	0.034 203 523	0.000 016 100	
0.058 487 751	0.000 009 006	0.043 806 797	0.000 007 182	0.034 146 526	0.000 019 309	
0.058 323 987	0.000 005 157	0.043 750 863	0.000 011 800	0.034076654	0.000 018 621	
0.058 244 147	0.000 005 463	0.043 692 093	0.000 014 920	0.034038254	0.000007958	
0.058 174 779	0.000008607	0.043 618 591 9	0.000 000 746 1	0.034035005	0.000005440	
0.058 137 002	0.000 001 892	0.035 430 29	0.000 012 24	0.033 923 963	0.000022555	
0.058 086 914	0.000 007 123	0.043 402 328	0.000010416	0.033 849 915 26	0.000 000 320 44	
0.057 959 583	0.000 004 030	0.043 399 909	0.000005738	0.033 829 442	0.000002015	
0.057887672	0.000 004 441	0.043 354 373	0.000 006 963	0.033 803 874	0.000 012 300	
0.057 831 903	0.000 006 976	0.043 333 045	0.000006771	0.033 797 584 1	0.000 001 358 6	
0.057 801 762	0.000001715	0.043 243 942 18	0.00000072520	0.033742379	0.000014370	
0.057 763 951	0.000005597	0.043 183 398	0.000 009 805	0.033 668 198 9	0.000 005 965 5	



FIG. 1. (Color online) Energies and half-widths of ${}^{3}P^{e}$ states of unnatural parity below the sixth (left) and the seventh (right) SITs. As expected, the complex energies calculated with the help of the spectral approach described in Sec. II (solid circles) are organized in Rydberg-like converging to the respective SITs. In contrast, the data obtained with the stabilization method (open red circles) [7] exhibit a scattered pattern, and no agreement with our results can be seen.

able to provide high accuracy for asymmetrically excited states.

In contrast, the comparison with the recently published results for ${}^{3}P^{e}$ resonances obtained within the stabilization technique [7] yields significant disagreements. Already, the data for resonances below the third SIT (Table II) show significant differences for $Im(E_{i,\theta})$ of higher excited resonances. Below the fourth SIT (Table II), the approach by Saha and Mukherjee [7] seems not to be able to resolve the close-lying resonances $E_{\theta} = -0.1423413 - \iota 0.0001641$ and $E_{\theta} = -0.1423411 - \iota 0.0004192$. Moreover, the part of the spectrum lying closest to the fourth SIT is not completely resolved. Table III, which presents the data for the fifth SIT, already shows pronounced deviations for the first two states of the table, and again, the spectrum for the higher excited states misses some resonances. The disagreement for the close-lying resonances E = -0.0952542 - i 0.0000442 and $E = -0.095136 - \iota 0.000744$ has already been addressed in Ref. [7]. However, we do not agree with their interpretation that the sudden decrease in the gap between resonances favors their results over those of Ref. [8]. Instead, we speculate that the stabilization method does have problems resolving resonances lying close to each other (see Tables II and III). Moreover, the statement by Saha and Mukherjee that the stabilization method allows them to obtain more results than the complex rotation method contradicts our experience: A large number of states calculated within our approach below the fourth and the fifth SITs is not obtained with the help of the stabilization method. Table IV presents a comparison of the data by Saha and Mukherjee [7] with our results obtained below the sixth and the seventh SITs. We are unable to associate any result by Saha and Mukherjee [7] with our results, as there is no agreement at all.

For moderate excitation of the inner electron—which is the case here—resonances are organized in Rydberg-like series converging to the SIT. Figure 1 displays the energies and the half-widths of ${}^{3}P^{e}$ states of unnatural parity below the sixth (left) and the seventh (right) SITs. Our data are compared with the results of Saha and Mukherjee. Note that these plots contain all our converged results for the sixth and the seventh SITs (around 100 for each threshold, although only fifty entries are given in Table IV). In addition to the small number of states given by Saha and Mukherjee [7], their results are organized

in an unexpectedly irregular pattern, which does not coincide at all with our calculations.

Table IV also presents our results for ${}^{3}P^{e}$ resonances below the eighth SIT, while Table VI lists our results obtained for ${}^{1}P^{e}$ resonances below the sixth, the seventh, and the eighth SITs, respectively. Data for the combination of these symmetries and energy regimes are reported for the first time. Given the huge difficulties of resolving close-lying resonances in the data of Saha and Mukherjee [7], in general, and the nonexistent series structure for their data below the sixth and the seventh SITs for ${}^{3}P^{e}$ resonances, our results should be viewed as benchmark results below the sixth, the seventh, and the eighth SITs for singlet as well as for triplet symmetry.

IV. SUMMARY

A CI expansion of the two-electron wave function in terms of Sturmian functions with independent nonlinear parameters has been used to calculate the energies of autoionizing singlet and triplet doubly excited P helium states of unnatural parity up to the eighth SIT. Our results are in perfect agreement with existing data by Ho and Bhatia [8], while significant disagreement is found with the recently published results of Saha and Mukherjee [7] calculated using the stabilization method based on the Ritz variational method. From the structure of the spectrum in the complex plane and the low number of resonances obtained in Ref. [7], we conclude that their data are incorrect in most cases.

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