Efficient quantum entanglement distribution over an arbitrary collective-noise channel

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We present an efficient quantum entanglement distribution over an arbitrary collective-noise channel. The basic idea in the present scheme is that two parties in quantum communication first transmit the entangled states in the frequency degree of freedom which suffers little from the noise in an optical fiber. After the two parties share the photon pairs, they add some operations and equipments to transfer the frequency entanglement of pairs into the polarization entanglement with the success probability of 100%. Finally, they can get maximally entangled polarization states with polarization independent wavelength division multiplexers and quantum frequency up-conversion which can erase distinguishability for frequency. Compared with conventional entanglement purification protocols, the present scheme works in a deterministic way in principle. Surprisingly, the collective noise leads to an additional advantage.

I. INTRODUCTION

Entanglement between two distant locations is an essential resource for quantum information and communication [1–3]. Many quantum information processes cannot be realized perfectly without maximally entangled states. For instance, quantum teleportation [4], quantum dense coding [5,6], and quantum-state sharing [7] require entangled states to set up a quantum channel between Alice and Bob, the two parties in quantum communication. Also, Alice and Bob can exploit entangled photon pairs to create a private key efficiently [8–13], in particular in long-distance quantum communication with quantum repeater [14–16]. As photons are the best physical systems for long-distance transmission of quantum states, people always choose their entangled states in the polarization degree of freedom to fulfill these tasks discussed previously. However, during a practical transmission, the polarization degree of freedom of photons is incident to be influenced by the thermal fluctuation, vibration, and the imperfection of an optical fiber. That is, they suffer from the channel noise inevitably whether they are single photons or entangled photon pairs. Thus, various error correction and error-rejection processes are proposed. For example, with decoherent-free subspaces, Walton *et al.* [17] proposed a scheme for rejecting the errors introduced by a collective noise. Quantum redundancy-code is also introduced to solve this problem [3]. For the faithful transmission of a single-photon polarization state over a collective-noise channel, Yamamoto *et al.* [18] proposed an error-rejecting scheme with an additional single photon in 2005. The success probability is in principle 1/16 without two-qubit operations. Subsequently, Kalamidas [19] proposed two schemes to reject and correct arbitrary qubit errors without additional particles, but fast polarization modulators. In 2007, Li *et al.* [20] also proposed a faithful qubit transmission scheme against a collective noise without ancillary qubits. Its success probability is 50% with only linear optical elements in a passive way. Also, they

presented another faithful single-qubit transmission scheme with a success probability of 50% based on the frequency degree of freedom of photons, resorting to an additional qubit [21].

For entangled quantum systems, there is another kind of processes which can be used to decrease the influence arising from the noise, named entanglement purification. For purifying a Werner state [22], Bennett *et al.* [23] proposed an original entanglement purification protocol (EPP) based on quantum controlled-NOT (CNOT) gates in 1996. Subsequently, several EPPs based on similar quantum logic operations have been introduced. At present, a perfect CNOT gate based on linear optical elements is very difficult to implement experimentally with current technology. In 2001, Pan *et al.* [24] proposed an EPP based on linear optics, without resorting to CNOT gates, which is feasible in experiment. We also proposed an EPP based on cross-Kerr nonlinearity [25]. However, entanglement purification is essentially used to distill some high-fidelity entangled states from less-entangled ones by sacrificing several qubits. In other words, all conventional EPPs [23–26] cannot get perfect maximally entangled photon pairs by far as they work probabilistically in principle. Thus, the faithful distribution of maximally pure entangled states between two distant locations is valuable for the realization of long-distance quantum communication.

The polarization entanglement of photon pairs is easily disturbed by the noise in quantum channel, so it is not a good way to transmit the polarization entanglement of photons directly over a noisy channel. There are some other degrees of freedom of photons, which suffer little from the channel noise over an optical fiber, such as the spatial degree of freedom and the frequency degree of freedom of photons. With present technology, the entanglement of photons in the frequency degree of freedom is not difficult to be prepared with spontaneous parametric down-conversion [27,28]. When a light propagates through an optically nonlinear medium with second-order nonlinearity (χ^2), we can produce a pair of photons in the "idler" and the "signal" modes. Also the conservation in energy and momentum can give rise to entanglement in various degrees of freedom, such as

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polarization entanglement, time-energy entanglement, and position-momentum entanglement.

In this paper, we present an efficient entanglement distribution scheme over an arbitrary collective-noise channel. The basic idea of the present scheme is that the two parties, say Alice and Bob, first transmit an entangled state in the frequency degree of freedom which suffers little from the channel noise in an optical fiber. After Alice and Bob share an entangled photon pair, they add some operations and equipments to transfer the frequency entanglement of the photon pair into the polarization entanglement with a success probability of 100%. Compared with conventional entanglement purification, this scheme does not require quantum resources largely. Our protocol has several advantages. First, the noise channel can be an arbitrarily collective one. Second, the two parties can get a perfect maximally entangled state in polarization in principle. In a practical transmission, Alice and Bob can also obtain perfect maximally entangled states in polarization by controlling the distances between the entangled source and the users. Moreover, this protocol can be generalized to distribute a multipartite entangled quantum system and can be easily realized in current experimental conditions.

II. EFFICIENT QUANTUM ENTANGLEMENT DISTRIBUTION OF TWO-QUBIT SYSTEMS

Cross-Kerr nonlinearity is a powerful tool for us to construct nondestructive quantum nondemoliton detectors (QND) [29,30]. The cross-Kerr nonlinearity has been used to prepare CNOT gates [29] and complete a local Bell-state analysis [30]. Also it can be used to fulfill the quantum entanglement purification and entanglement concentration protocols [25,31]. The Hamiltonian of the cross-Kerr nonlinearity is $H_{ck} = \hbar \chi a_s^\dagger a_s a_p^\dagger a_p$ [29,30]. Here $a_s^\dagger$ and $a_p^\dagger$ are the creation operations and a_s and a_p are the destruction operations. Suppose a signal state $|\Psi\rangle_s = c_0|0\rangle_s + c_1|1\rangle_s$ ($|0\rangle_s$ and $|1\rangle_s$ denote that there are no photon and one photon, respectively, in this state) and a coherent probe beam in the state $|\alpha\rangle$ couple with a cross-Kerr nonlinearity medium, the whole system evolves as

$$U_{ck}|\Psi\rangle_s|\alpha\rangle_p = e^{iH_{ck}t/\hbar}[c_0|0\rangle_s + c_1|1\rangle_s]|\alpha\rangle_p$$
$$= c_0|0\rangle_s|\alpha\rangle_p + c_1|1\rangle_s|\alpha e^{i\theta}\rangle_p, \quad (1)$$

where $\theta = \chi t$ and t is the interaction time. The coherent beam picks up a phase shift θ directly proportional to the number of the photons in the Fock state $|\Psi\rangle_s$, which can be read out with a general homodyne-heterodyne measurement. So one can exactly check the number of photons in the Fock state but not destroy them.

Now let us explain the principle of our entanglement distribution protocol over an arbitrary collective-noise channel. We suppose that the center, say Carl prepares an entangled photon pair ab in the following state:

$$|\Psi\rangle_{ab} = \frac{1}{\sqrt{2}}|H\rangle_a|H\rangle_b(|\omega_1\rangle|\omega_2\rangle + |\omega_2\rangle|\omega_1\rangle). \quad (2)$$

Here we denote the state of a horizontally polarized photon by $|H\rangle$ and the state of a vertically polarized photon by $|V\rangle$. $|\omega_1\rangle|\omega_2\rangle$ and $|\omega_2\rangle|\omega_1\rangle$ are two different frequency modes of the
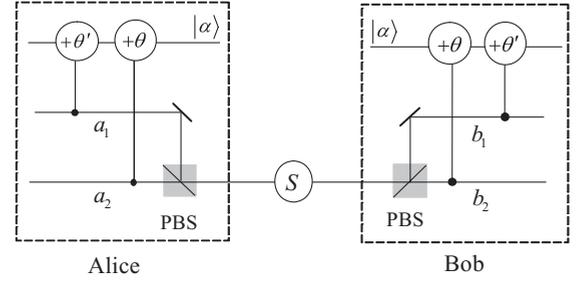

FIG. 1. Schematic drawing of quantum entanglement distribution over a collective-noise channel with QND. The entanglement source (S) produces two entangled photons which are transmitted to two parties in quantum communication, say Alice and Bob, respectively. The two photons suffer from the noise during the transmission. PBS presents a polarization beam splitter. Alice and Bob can check the phase shifts of their coherent beams to judge which state they obtain in a deterministic way. $+\theta$ and $+\theta'$ represent two cross-Kerr nonlinear media with the phase shifts $+\theta$ and $+\theta'$, respectively.

two photons. The subscripts a and b mean that the two photons are distributed to Alice and Bob, respectively. Suppose the collective noises in the two channels have the same form but different noise parameters which alter with time in principle, i.e.,

$$|H\rangle_a \rightarrow \alpha|H\rangle + \beta|V\rangle,$$
$$|H\rangle_b \rightarrow \delta|H\rangle + \gamma|V\rangle, \quad (3)$$

where

$$|\alpha|^2 + |\beta|^2 = 1, \quad |\delta|^2 + |\gamma|^2 = 1. \quad (4)$$

The photon pair in the input modes of the channels will suffer from two collective noises, shown in Fig. 1, that is, the evolution of its state through the noisy channels can be written as

$$|\Psi\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle_{\omega_1}|H\rangle_{\omega_2} + |H\rangle_{\omega_2}|H\rangle_{\omega_1}) \xrightarrow{\text{noises}}$$

$$|\Psi\rangle'_{ab} = \frac{1}{\sqrt{2}}[(\alpha|H\rangle_{\omega_1} + \beta|V\rangle_{\omega_1})(\delta|H\rangle_{\omega_2} + \gamma|V\rangle_{\omega_2})$$
$$+ (\alpha|H\rangle_{\omega_2} + \beta|V\rangle_{\omega_2})(\delta|H\rangle_{\omega_1} + \gamma|V\rangle_{\omega_1})]$$

$$= \frac{1}{\sqrt{2}}[\alpha\delta|H\rangle_{\omega_1}|H\rangle_{\omega_2} + \alpha\gamma|H\rangle_{\omega_1}|V\rangle_{\omega_2} \quad (5)$$
$$+ \beta\delta|V\rangle_{\omega_1}|H\rangle_{\omega_2} + \beta\gamma|V\rangle_{\omega_1}|V\rangle_{\omega_2}$$
$$+ \alpha\delta|H\rangle_{\omega_2}|H\rangle_{\omega_1} + \alpha\gamma|H\rangle_{\omega_2}|V\rangle_{\omega_1}$$
$$+ \beta\delta|V\rangle_{\omega_2}|H\rangle_{\omega_1} + \beta\gamma|V\rangle_{\omega_2}|V\rangle_{\omega_1}].$$

After the noisy channel, the photon a (b) will pass through a polarization beam splitter (PBS) which transmits the horizontal polarization mode $|H\rangle$ and reflects the vertical polarization mode $|V\rangle$. If Alice and Bob combine their photons and their coherent probe beams ($|\alpha\rangle_A$ and $|\alpha\rangle_B$) with cross-Kerr nonlinearity media (shown in Fig. 1), the state of whole quantum system becomes

$$\rightarrow \frac{1}{\sqrt{2}}[\alpha\delta(|H\rangle_{\omega_1}|H\rangle_{\omega_2} + |H\rangle_{\omega_2}|H\rangle_{\omega_1})|\alpha e^{i\theta}\rangle_A|\alpha e^{i\theta}\rangle_B$$
$$+ \alpha\gamma(|H\rangle_{\omega_1}|V\rangle_{\omega_2} + |H\rangle_{\omega_2}|V\rangle_{\omega_1})|\alpha e^{i\theta}\rangle_A|\alpha e^{i\theta'}\rangle_B$$
$$+ \beta\delta(|V\rangle_{\omega_1}|H\rangle_{\omega_2} + |V\rangle_{\omega_2}|H\rangle_{\omega_1})|\alpha e^{i\theta'}\rangle_A|\alpha e^{i\theta}\rangle_B$$
$$+ \beta\gamma(|V\rangle_{\omega_1}|V\rangle_{\omega_2} + |V\rangle_{\omega_2}|V\rangle_{\omega_1})|\alpha e^{i\theta'}\rangle_A|\alpha e^{i\theta'}\rangle_B].$$

Here $|\alpha e^{i\theta}\rangle_A$ means that the coherent probe beam in Alice's hand picks up a phase shift θ. The other terms are analogical with it.

After X homodyne measurements on their coherent beams independently, Alice and Bob will get some different phase shifts and the photon pair will appear at some different output modes. In detail, if Alice and Bob have the same phase shift θ, the photon pair ab collapses to the state $|\phi_1\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle_{\omega_1}|H\rangle_{\omega_2} + |H\rangle_{\omega_2}|H\rangle_{\omega_1})_{ab}$ and they will appear at the lower output modes $a_2 b_2$. If Alice and Bob have the same phase shift θ', the photon pair ab collapses to the state $|\phi_2\rangle_{ab} = \frac{1}{\sqrt{2}}(|V\rangle_{\omega_1}|V\rangle_{\omega_2} + |V\rangle_{\omega_2}|V\rangle_{\omega_1})_{ab}$ and they will appear at the upper output modes $a_1 b_1$. The state $|\phi_3\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle_{\omega_1}|V\rangle_{\omega_2} + |H\rangle_{\omega_2}|V\rangle_{\omega_1})_{ab}$ will be in the output modes a_2 and b_1, which leads the phase shift θ in Alice and θ' in Bob. $|\phi_4\rangle_{ab} = \frac{1}{\sqrt{2}}(|V\rangle_{\omega_1}|H\rangle_{\omega_2} + |V\rangle_{\omega_2}|H\rangle_{\omega_1})_{ab}$ leads the phase shift θ' in Alice and θ in Bob, and the photon pair will appear at the output modes $a_1 b_2$. That is, with X homodyne measurements Alice and Bob can distinguish the four entangled states $|\phi_i\rangle_{ab}$ ($i = 1, 2, 3, 4$).

The second step of our entanglement distribution protocol is to convert the frequency-entangled states $|\phi_i\rangle_{ab}$ to polarization-entangled ones. We take $|\phi_1\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle_{\omega_1}|H\rangle_{\omega_2} + |H\rangle_{\omega_2}|H\rangle_{\omega_1})_{ab}$ as an example to describe the principle of this step, shown in Fig. 2. WDM represents a polarization independent wavelength division multiplexer. It can be used to guide photons to different spatial modes according to their frequencies. For Alice (Bob), the photons with the frequencies ω_1 and ω_2 will be guided to the spatial modes c_1 (d_1) and c_2 (d_2), respectively. Two wave plates $R_{90°}$ are used to rotate the horizontal polarization H and the vertical polarization V by $90°$. That is, they complete the transformation $|H\rangle \rightarrow |V\rangle$ and $|V\rangle \rightarrow |H\rangle$. This task can be accomplished with a half-wave plate whose orientation is $45°$. After the photon pair ab is coupled by the two PBSs, its state becomes

$$\rightarrow |\phi_1'\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle_{\omega_1}|V\rangle_{\omega_2} + |V\rangle_{\omega_2}|H\rangle_{\omega_1}) \quad (6)$$

and will be in the output modes c_2 and d_2. Following the similar way, Alice and Bob can obtain the other three entangled states
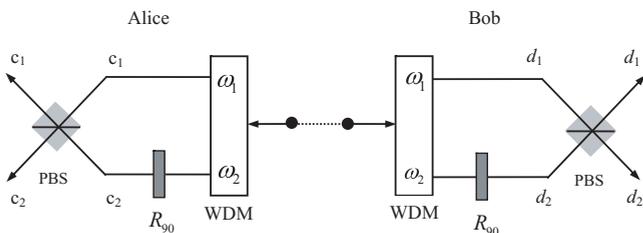

FIG. 2. Schematic drawing of converting frequency entanglements to polarization entanglements. WDM guides photons to different spatial modes according to their different frequencies. R_{90} represents a rotation by $90°$, which acts as a bit-flip operation and completes the transformation between the horizontal polarization H and the vertical polarization V.

$|\phi_2'\rangle, |\phi_3'\rangle$, and $|\phi_4'\rangle$ in the output modes $c_1 d_1, c_2 d_1$, and $c_1 d_2$, respectively. Here

$$|\phi_2'\rangle = \frac{1}{\sqrt{2}}(|V\rangle_{\omega_1}|H\rangle_{\omega_2} + |H\rangle_{\omega_2}|V\rangle_{\omega_1}), \quad (7)$$

$$|\phi_3'\rangle = \frac{1}{\sqrt{2}}(|H\rangle_{\omega_1}|H\rangle_{\omega_2} + |V\rangle_{\omega_2}|V\rangle_{\omega_1}), \quad (8)$$

$$|\phi_4'\rangle = \frac{1}{\sqrt{2}}(|V\rangle_{\omega_1}|V\rangle_{\omega_2} + |H\rangle_{\omega_2}|H\rangle_{\omega_1}). \quad (9)$$

Each of the four states $\{|\phi_1'\rangle, |\phi_2'\rangle, |\phi_3'\rangle, |\phi_4'\rangle\}$ is a maximally entangled one in both polarization and frequency degrees of freedom. Alice and Bob can erase the distinguishability for the frequency of their photons with the help of quantum frequency up-conversion [32] and turn them into a standard Bell state $|\phi^+\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$ with local unitary operations. Moreover, the success probability of this entanglement distribution scheme is in principle 100% over an arbitrary collective-noise channel as it is independent of the noise parameters α, β, δ, and γ, which is different from single-photon error-rejecting protocols [17–21].

III. EFFICIENT QUANTUM ENTANGLEMENT DISTRIBUTION OF MULTIQUBIT SYSTEMS

This scheme can be generalized for distribution of n-qubit system ($n > 2$) in a Greenberger-Horne-Zeilinger (GHZ) state over an arbitrary collective-noise channel. Let us use the distribution of a four-qubit system as an example to describe its principle. The other cases are similar to it with or without a little of modification.

Suppose that the initial state of a four-qubit system is

$$|\Phi_4\rangle_{ABCD} = \frac{1}{\sqrt{2}}|0000\rangle(|\omega_1\omega_2\omega_1\omega_2\rangle + |\omega_2\omega_1\omega_2\omega_1\rangle)_{ABCD}$$

and the collective noises in the four channels have the same form but different noise parameters which alter with time t in principle, i.e.,

$$|0\rangle_i \rightarrow \beta_0^i|0\rangle + \beta_1^i|1\rangle, \quad (10)$$

where $i = A, B, C, D$ represent the four photons which are sent to Alice, Bob, Charlie, and Daniel, respectively. Here $|0\rangle \equiv |H\rangle$ and $|1\rangle \equiv |V\rangle$. After passing through the noisy channels, the four-qubit system evolves as

$$|\Phi_4\rangle_{ABCD} \xrightarrow{\text{noises}}$$

$$|\Phi_4\rangle'_{ABCD} = \frac{1}{\sqrt{2}}\left(\sum_{jklm} \beta_j^A \beta_k^B \beta_l^C \beta_m^D |j\rangle_A |k\rangle_B |l\rangle_C |m\rangle_D\right)$$

$$\cdot (|\omega_1\omega_2\omega_1\omega_2\rangle + |\omega_2\omega_1\omega_2\omega_1\rangle)_{ABCD}, \quad (11)$$

where $j, k, l, m \in \{0, 1\}$. Similar to Fig. 1, Alice, Bob, Charlie, and Daniel use their QNDs to check the polarization states of their photons. That is, if one obtains the phase shift of his coherent beam θ, his photon is in the polarization state $|0\rangle = |H\rangle$; otherwise, the photon is in $|1\rangle = |V\rangle$. With their outcomes of their X homodyne measurements and some local unitary operations, the four users can obtain the state $|\Phi_4\rangle''_{ABCD} = \frac{1}{\sqrt{2}}|0000\rangle(|\omega_1\omega_2\omega_1\omega_2\rangle + |\omega_2\omega_1\omega_2\omega_1\rangle)_{ABCD}$. With the setups

similar to Fig. 2, the four users can obtain the entangled state in polarization $|\Psi_4\rangle''_{ABCD} = \frac{1}{\sqrt{2}}(|H_{\omega_1}V_{\omega_2}H_{\omega_1}V_{\omega_2}\rangle + V_{|\omega_2}H_{\omega_1}V_{\omega_2}H_{\omega_1}\rangle)_{ABCD}$. Alice, Bob, Charlie, and Daniel can erase the distinguishability for the frequencies of their photons with the help of quantum frequency up-conversion [32] and turn their system into a GHZ state $|\Psi\rangle'_{ABCD} = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle|H\rangle|V\rangle + |V\rangle|H\rangle|V\rangle|H\rangle)_{ABCD}$. With two bit-flip operations on the photons B and D, respectively, they will obtain a standard GHZ state $|\Psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle|H\rangle|H\rangle + |V\rangle|V\rangle|V\rangle|V\rangle)_{ABCD}$.

IV. DISCUSSION AND SUMMARY

We have discussed our quantum entanglement distribution scheme in the case that the frequency degree of freedom of photon pairs is insensitive to channel noise. The previous experiments showed that the polarization entanglement is quite unsuitable for transmission over distances of more than a few kilometers in an optical fiber [10]. For example, Naik *et al.* demonstrated the Ekert protocol [8] by only a few meters [10,33]. Also, they observed the quantum bit error rate (QBER) increase to 33% in the experiment implementation of the six-state protocol [34,35]. For frequency coding [36–41], for example, the Besancon group performed a key distribution over a 20-km single-mode optical-fiber spool. They recorded a $QBER_{opt}$ contribution of approximately 4%, and estimated that 2% could be attributed to the transmission of the central frequency by the Fabry-Perot cavity [41]. That is, on one hand, the channel noise less affects the entanglement in the frequency degree of freedom. On the other hand, the optical fibers used to transmit photons will introduce a relative phase on the entanglement as there are two different frequencies in each photon. That is, the entangled state in the frequency degree of freedom will become $\frac{1}{\sqrt{2}}(|\omega_1\omega_2\rangle + e^{i\Delta\phi_f}|\omega_2\omega_1\rangle)$ after the two photons a and b are sent to Alice and Bob, respectively. Here $\Delta\phi_f \equiv \frac{1}{v}[(\omega_2 - \omega_1)L_A + (\omega_1 - \omega_2)L_B]$. v and L_A (L_B) represent the velocity of photons in an optical fiber and the distance between the entangled source and Alice (Bob), respectively. When $L_A = L_B$, $\Delta\phi_f = 0$. That is, Alice and Bob can obtain a perfect entangled state in the frequency degree of freedom after their transmission if they can control their distances between them and the entangled source. Also, Alice and Bob can compensate the relative phase $\Delta\phi_f$ after their transmission if $L_A \neq L_B$, as $\Delta\phi_f$ is in general invariable and can be detected. In this case, the relative phase $\Delta\phi_f$ in frequency will be transferred into the entanglement in polarization. That is, Alice and Bob will obtain the maximally entangled state in the polarization degree of freedom with the form $|\phi'^+\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + e^{i\Delta\phi_f}|V\rangle|V\rangle)$. With some unitary operations by wave plates, they will obtain the standard Bell state $|\phi^+\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$.

Let us compare this distribution scheme with conventional entanglement purification protocols [23–26]. In the latter, the two parties transmit the entangled photon pairs in the polarization degree of freedom directly over a noisy channel. The photon pair transmitted suffers from the channel noise and its state becomes a mixed entangled one. In Ref. [24], the two sources produce two pairs of entangled photons and one photon from each pair is distributed to Alice and the other one to Bob.

The two photons in each side overlap at a PBS. By selecting the four-mode instances, Alice and Bob can thus obtain a subset of high-fidelity entangled photon pairs. In order to get the entangled states with a higher fidelity, Alice and Bob should repeat this protocol and consume more less-entangled states. Ref. [26] presented a more practical polarization entanglement purification using spatial entanglement. In their protocol, the parametric down-conversion source produces an entangled photon pair in both polarization and spatial degrees of freedom. By selecting those events where photons are both in the upper mode or in the lower mode, the two parties can purify the bit-flip error. However, both of these two protocols can not get perfect maximally entangled pairs and they can only improve the fidelity of an ensemble in a mixed entangled state by consuming the quantum resource exponentially. The present scheme exploit the entanglement in the frequency degree of freedom to create the entanglement in the polarization degree of freedom perfectly. After the homodyne detectors, the entanglement in the frequency degree of freedom does not degrade in principle, which makes the present scheme work in a deterministic way. This is different from the entanglement purification protocols as the polarization entanglement is degraded in the noise channel in the latter. So the yield of entanglement purification protocols is far lower than the present scheme. This result is kept for the case with entanglement concentration protocols [31,42] as the latter also needs to sacrifice the less-entangled states largely to obtain a maximally entangled one. Compared with the deterministic entanglement purification protocol [43], the present scheme requires less entanglement resource as the former resorts to hyperentanglement in three degrees of freedom (such as polarization, spatial mode, and frequency) while the latter only resorts to the entanglement in the frequency degree of freedom. Compared with the faithful distribution of single-qubit scheme with linear optics [20], the success probability of the present scheme is 100% while that of single-photon error-rejecting protocol [20] is only 50%. That is, the present scheme may more practical for distribution of entanglement in quantum communication with the development of techniques.

We should point out that cross-Kerr effect is yet not easy to implement in current experiment. The largest natural cross-Kerr nonlinearities are extremely weak ($\chi^{(3)} \approx 10^{-22}$ m^2 V^{-2}) [44]. In Ref. [45], Kok *et al.* showed that operating in the optical single-photon regime, the Kerr phase shift is only $\tau \approx 10^{-18}$. With electromagnetically induced transparent materials, cross-Kerr nonlinearities of $\tau \approx 10^{-5}$ can be obtained. The weak cross-Kerr nonlinearity will make the phase shift θ and θ' of the coherent state became extremely small, which will be hard to detect. That is to say, using homodyne detector, it is difficult to determine the phase shift due to the impossible discrimination of two overlapping coherent states, which will decrease the success probability of the present scheme to 1/4 at worst. In 2003, Hofmann *et al.* showed that a phase shift of π can be achieved with a single two-level atom in a one-sided cavity [46]. In 2010, Wittmann *et al.* investigated quantum measurement strategies capable of discriminating two coherent states using a homodyne detector and a photon number resolving (PNR) detector [47]. In order to lower the error probability, the postselection strategy is applied to the measurement data of homodyne detector as well

as a PNR detector. They indicated that the performance of the new displacement controlled PNR is better than homodyne receiver.

In summary, we have presented an efficient entanglement distribution scheme over an arbitrary collective-noise channel. Compared with conventional entanglement purification protocols [17–21], the present scheme does not consume a great deal of less-entangled resources and it works in a determinate way. In essence, it is the entanglement transformation between two different degrees of freedom of photons. We exploit the feature that the frequency of photons suffers little from the channel noise to generate the entanglement in the polarization degree of freedom. If other degrees of freedom are robust to the channel noise, they also can be used to implement our protocol, and the frequency degree of freedom is not unique. We believe that the present scheme for the distribution of entangled states in the polarization degree of freedom may be a vital ingredient in the realization of long-distance quantum communication in the future.

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