

Location-dependent communications using quantum entanglement

Robert A. Malaney*

School of Electrical Engineering and Telecommunications, University of New South Wales, New South Wales 2052, Australia

(Received 25 October 2009; published 23 April 2010)

The ability to unconditionally verify the location of a communication receiver would lead to a wide range of new security paradigms. However, it is known that unconditional location verification in classical communication systems is impossible. In this work we show how unconditional location verification can be achieved with the use of quantum communication channels. Our verification remains unconditional irrespective of the number of receivers, computational capacity, or any other physical resource held by an adversary. Quantum location verification represents an application of quantum entanglement that delivers a feat not possible in the classical-only channel. It gives us the ability to deliver real-time communications viable only at specified geographical coordinates.

DOI: [10.1103/PhysRevA.81.042319](https://doi.org/10.1103/PhysRevA.81.042319)

PACS number(s): 03.67.Hk, 03.67.Bg, 03.67.Dd, 03.67.Mn

The ability to offer a real-time communication channel whose viability is unconditionally a function of the receiver location would offer a range of new information security paradigms and applications. In particular, there are a range of industries and organizations that clearly would be interested in delivering information content in the sure knowledge a recipient receiver is at an *a priori* agreed upon location (e.g., see discussions in [1–4]). The ability to guarantee location-sensitive communications requires unconditional (independent of the physical resources held by an adversary) location verification. However, in the classical-only channel, such unconditional location verification is impossible. The finite speed of light can only be used to bound the minimum (but not the maximum) range a receiver is located from some reference station. Add to the mix that classical information can be copied, and that an adversary can possess unlimited receivers (each of which can be presumed to possess unlimited computational capacity), and it is straightforward to see why classical-only unconditional location verification is impossible. It is the purpose of this work to show how the introduction of quantum entanglement into the communication channel overcomes the above concerns, providing an unconditional location verification protocol.

Quantum teleportation [5], the transfer of unknown quantum state information, is now experimentally verified through a host of experiments (e.g., [6,7]). In addition, the key resource underpinning teleportation, quantum entanglement, has been experimentally verified over very large ranges. An entanglement measurement over 144 km, achieved recently using optical free-space communications between two telescopes [8], proves the validity of ground-station-to-satellite quantum communications and is widely seen as a major step in the path toward a global quantum communications network. In such a network it is envisaged that a combination of satellite and fiber optic links will interconnect a multitude of quantum nodes, quantum devices, and quantum computers. In optical fiber, transmission of entangled photons is limited to about 100 km by losses and decoherence effects (e.g., [9]). Communications over fiber beyond this range will make use of either quantum repeaters [10] or the trusted relay paradigm used in a recent deployment of an eight-node quantum network [11].

Experimental verification of quantum superdense coding [12] has also been achieved through a series of experiments (e.g., [13,14]). In superdense coding, two bits of classical information can be transferred at the cost of only one qubit.

Teleportation and superdense coding are strongly related, and indeed they are often considered protocols which are the inverse of each other, differing only in how and when they utilize quantum entanglement. Quantum location verification can be considered a protocol that differs again in how and when it uses quantum entanglement.

The principal condition for unconditional location verification is that only a device at *one* unique location (the authorized location) is able to, immediately and correctly, respond to signals received from multiple reference stations. In the classical-only channel, this condition can never be unconditionally guaranteed. However, as we now show, with the introduction of quantum communication channels the condition necessary for unconditional location verification can in fact be guaranteed.

Consider some reference stations at publicly known locations, and a device which is not a reference station (Cliff) that is to be verified at a publicly known location (x_v, y_v) . Let us assume that processing times, such as those due to local quantum measurements, are negligible (we discuss later the minor impact of this assumption). We also assume that the reference stations are authenticated and share secure communication channels between each other via quantum key distribution [15,16] and that all classical communication between Cliff and the reference stations occurs via wireless channels. The use of wireless communications is important since we require the time delay of all classical communications to be set by the line-of-sight distance between transceivers divided by c (light speed in a vacuum).

For two-dimensional location verification, we require a minimum of three reference stations. Consider N maximally entangled multipartite systems available to a network that possesses k reference stations. Consider also that each of the multipartite systems comprises k qubits, with each reference station initially holding one qubit from each of the N systems. The 2^k orthogonal basis states of each multipartite state can be written

$$|S_b\rangle = \frac{1}{\sqrt{2}}(|a\rangle_1 \otimes |a\rangle_2 \otimes \cdots \otimes |a\rangle_k) \pm |a\rangle_1 \otimes |a\rangle_2 \otimes \cdots \otimes |a\rangle_k, \quad (1)$$

*r.malaney@unsw.edu.au

where $b = 1, \dots, 2^k$, and the states $|a\rangle$ represent $|0\rangle$ or $|1\rangle$ with the index on the state labeling the location (ignoring any null state).

Transformation between the basis states can be achieved by a set of 2^k unitary transformations induced on the locally held qubits. By this means a k -bit message, per entangled state, can be transferred from the stations to Cliff. This transfer is achieved using superdense coding in which the stations encode each message to a specific basis state $|S_b\rangle$, with Cliff decoding the message via a quantum measurement that deterministically discriminates all possible basis states (the state $|S_b\rangle$ is sent directly to Cliff via quantum channels connected to the reference stations).

Quantum location verification builds on this concept of state encoding with one key addition. It must be the case that deterministic discrimination among the encoded states is possible, within a *predescribed* time bound at only one location. This can be achieved if the 2^k states which encode the k -bit messages are made *nonorthogonal* by the introduction of an additional local unitary transformation at each reference station. Let these additional transformations be labeled U_i^r , where $r = 1, \dots, k$ indexes the reference station, and $i = 1, \dots, N$ references the specific multipartite state to which the local transformation is applied.

Consider the i th encoded multipartite state in which a k -bit message is encoded as $|S_b\rangle$. Then on application of the additional transformations a new state, $|\Upsilon_i\rangle = U_i^1 \otimes U_i^2 \otimes \dots \otimes U_i^k |S_b\rangle$, is produced. Our requirement is that $\langle \Upsilon_i | \Upsilon_j \rangle \neq 0$ when $|\Upsilon_i\rangle \neq |\Upsilon_j\rangle$. Ideally, the unitary matrices U_i^r are chosen so that, upon measurement of $|\Upsilon_i\rangle$ in a measurement basis $|S_1\rangle, |S_2\rangle, \dots, |S_{2^k}\rangle$, the probability of collapse to each basis state is approximately equal ($1/2^k$).

For quantum location verification to be unconditional it must be impossible for an adversary to map the values of U_i^r to specific k -bit messages. (In our protocol all matrices U_i^r and all k -bit messages are ultimately sent over a classical channel.) This means that there must be some form of randomness applied to the selection of each U_i^r . One strategy that provides for both a random selection mechanism and the required nonorthogonal behavior between the states $|\Upsilon_i\rangle$ is to allow the U_i^r to be constructed from four random real parameters $(\alpha, \beta, \gamma, \phi)$. The unitary matrix at each reference station can then be implemented as

$$U = e^{i\phi} R_z(\alpha) R_y(\beta) R_z(\gamma), \quad (2)$$

where the rotations R are given by

$$R_y(\theta) = e^{-i\theta\sigma_y/2} \quad \text{and} \quad R_z(\theta) = e^{-i\theta\sigma_z/2},$$

and the σ 's represent the Pauli operators. Classical communication of the additional matrices can be achieved in many ways, such as passing of experimental instructions (e.g., duration of laser pulses), indexing of a large number of matrices, or as a transfer of matrix element information. The last, which we adopt here, involves the transmission of the values $(\alpha, \beta, \gamma, \phi)$ adopted for each U_i^r . Although finite bandwidth of the classical channel limits the precision of this information transfer, required precision is available at the cost of additional bandwidth. In actual deployment, any global phase can be ignored. We discuss later a pragmatic implementation strategy

leading to an outcome effectively the same as the outcome derived from Eq. (2).

The location verification proceeds by the encoding of a secret sequence onto a set of N entangled systems $|\Upsilon_{i=1, \dots, N}\rangle$, transmission of each $|\Upsilon_i\rangle$ to Cliff via quantum channels, followed by transmission of the unitary matrices U_i^r [i.e., the set $(\alpha, \beta, \gamma, \phi)$] to Cliff by classical channels. Upon receiving this quantum and classical information, Cliff can decode and broadcast the decoded sequence via the classical channel. Given that information transfer over the classical channel proceeds at a velocity c , location information becomes unconditionally verifiable (as explained later). Ultimately, the verification is based on the inability to clone deterministically the set $|\Upsilon_i\rangle$ with fidelity 1. Although cloning with lower fidelities is possible, confidence levels on the location verification can be increased to any arbitrary level by increasing N .

We now outline the protocol in more detail using well-known maximally entangled states. For clarity, we proceed with a one-dimensional location verification using just two reference stations, which we henceforth refer to as Alice and Bob. A geometrical constraint for one-dimensional location verification is that the device to be located must lie between Alice and Bob. That is, $\tau_{AC} + \tau_{BC} = \tau_{AB}$, where τ_{AC} (τ_{BC}) is the light travel time between Alice (Bob) and Cliff, and where τ_{AB} is the light travel time between Alice and Bob.

Let Alice share with Bob a set of N maximally entangled qubit pairs $|\Omega_i^{AB}\rangle$, where the subscript $i = 1, \dots, N$ labels the entangled pairs. Let each of the pairs be described by one of the Bell states $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, with the first qubit being held by Alice and the second by Bob. We assume an encoding ($00 \rightarrow \Phi^+$, etc.) that is public.

Without loss of generality we can assume all pairs are initially in the state $|\Phi^+\rangle$. After the encoding of a sequence onto a series of entangled pairs, Alice and Bob apply an additional random unitary transformation U_i^A and U_i^B , respectively, to their local qubit from each pair. As a consequence, the entangled pairs held by Alice and Bob now form a nonorthogonal set,

$$|\Upsilon_i^{AB}\rangle = U_i^A \otimes U_i^B |\Omega_i^{AB}\rangle. \quad (3)$$

For example, for $|\Phi^+\rangle$, Eq. (3) leads to a state $\frac{1}{\sqrt{2}}(U_i^A|0\rangle_A \otimes U_i^B|0\rangle_B + U_i^A|1\rangle_A \otimes U_i^B|1\rangle_B)$. A step-by-step exposition of the protocol follows.

Step 1. Via a secure channel, Alice and Bob agree on a mutual random bit sequence S_{ab} that is to be encoded. The encoding is achieved via superdense coding in which two classical bits are encoded using local unitary operators as described by $I|\Phi^+\rangle = |\Phi^+\rangle$, $\sigma_x|\Phi^+\rangle = |\Psi^+\rangle$, $i\sigma_y|\Phi^+\rangle = |\Psi^-\rangle$, and $\sigma_z|\Phi^+\rangle = |\Phi^-\rangle$. For each pair of entangled qubits, Alice and Bob also agree who will induce the necessary unitary operation on their local qubit in order to encode sequential two-bit segments of S_{ab} .

Step 2. Prior to the transmission of any qubit, the transformation $|\Omega_i^{AB}\rangle \rightarrow |\Upsilon_i^{AB}\rangle$ as described by Eq. (3) is induced. This set is then transmitted by Alice and Bob to Cliff via two separate quantum channels.

Step 3. Alice and Bob communicate to Cliff, via separate classical channels, the random matrices U_i^A and U_i^B used to form the set $|\Upsilon_i^{AB}\rangle$. This classical information is transmitted

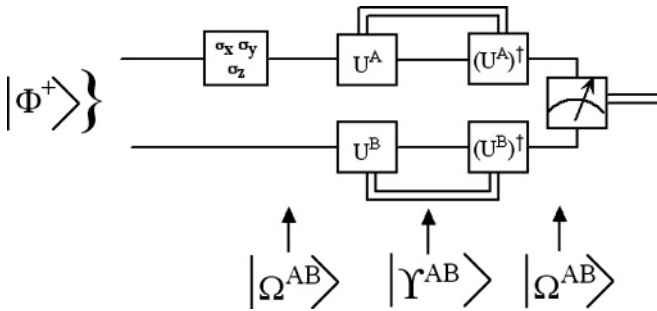


FIG. 1. Quantum circuit for location verification.

in a synchronized manner to Cliff such that for each value of i the U_i^A sent by Alice and the U_i^B sent by Bob arrive simultaneously at Cliff's publicly announced location (x_v, y_v) . It is also ensured that this classical information is received by Cliff *after* the arrival of the corresponding qubit pair of $|\Upsilon_i^{AB}\rangle$.

Step 4. Upon receipt of each matrix pair U_i^A, U_i^B , Cliff undertakes the transform $(U_i^A \otimes U_i^B)^\dagger |\Upsilon_i^{AB}\rangle \rightarrow |\Omega_i^{AB}\rangle$ before taking a Bell state measurement (BSM) in order to determine the two-bit segment encoded in the entangled pair. Cliff then immediately broadcasts (classically) the decoded two-bit segment back to Alice and Bob.

Step 5. Alice checks that the sequence returned to her by Cliff is correctly decoded and notes the round-trip time for the process. Likewise Bob. Alice and Bob can then compare their round-trip times to Cliff ($2\tau_{AC}$ and $2\tau_{BC}$) in order to verify consistency with Cliff's publicly reported location (x_v, y_v) .

The quantum circuit for the one-dimensional quantum location verification just described is given in Fig. 1.

Quantum location verification is independent of the physical resources an adversary may possess. In the classical-only channel, an adversary can place cooperating devices closer to reference stations and then delay responses in order to defeat any location verification (e.g., [2,4]). Attempts to remedy this problem by making devices unique and tamper-proof are clearly limited (see discussion in [2]) and cannot provide for unconditional security.

However, in quantum verification, multiple devices are of no value. In order to decode immediately, Cliff's receiver must possess *all* the qubits that comprise each entangled state. Cliff cannot distribute copies of his local qubits to other devices due to the no-cloning theorem [17]. The key point is that, for any given location (x_v, y_v) that is to undergo a verification process, one can always find placements for the reference stations such that no other location can be *simultaneously* closer to all of the reference stations than (x_v, y_v) (e.g., recall the geometrical constraint in our one-dimensional verification). This being the case, an adversary with no device at the location being verified cannot pass the verification test. Even if the adversary possesses multiple receivers, an additional round-trip communication time between his devices is required for decoding. This will result in a round-trip time between at least one reference station and the location (x_v, y_v) being larger than expected. In classical verification, the round-trip communication between the adversary's devices is not required.

Extension of the one-dimensional location verification protocol to two-dimensional verification could be a straightforward

application of additional bipartite entanglement between Alice and some third reference station, say Dan. This can be achieved by introduction of a new set of Bell states shared between Alice and Dan, with the protocol following a similar exposition to that given. However, perhaps a more elegant solution is the use of multipartite entangled states. For example, consider a Green-Horne-Zeilinger (GHZ) [18,19] state in which three qubits are maximally entangled, such as $|S\rangle^+ = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. Transformation from this GHZ basis state to one of the eight basis states is achieved by the set of transforms $U_{\text{GHZ}} = (\sigma_z \otimes \sigma_z, I_2 \otimes \sigma_z, i\sigma_y \otimes \sigma_z, \sigma_x \otimes \sigma_z, I_2 \otimes \sigma_x, \sigma_z \otimes \sigma_x, \sigma_x \otimes \sigma_x, i\sigma_y \otimes \sigma_x)$, where the first (second) operator acts on the first (second) qubit [20]. A step-by-step quantum location verification using such tripartite states proceeds in similar manner to the bipartite protocol.

Clearly, a security threat to the protocol is the potential ability of an adversary who is in possession of an optimal cloning machine, redistributing the set $|\Upsilon_i\rangle$ to other devices. If cloning were exact, the verification test would fail because the *round-trip* communication between the devices (needed to decode) would not be required. However, optimal cloning of the set $|\Upsilon_i\rangle$ can be described by the fidelity, F_c , between this set and a cloned set. This is known to be upper bounded by $F_c \approx 0.7$ for bipartite entanglement and $F_c \approx 0.6$ for tripartite entanglement [21,22]. As such, for a series of two-bit messages encoded in $N = 100$ bipartite states, an optimal cloning machine would have a probability of 1 in 10^{16} of passing the verification system even though not at the authorized location. For 100 three-bit messages encoded in tripartite states, this decreases to a probability of 1 in 10^{22} . Arbitrary smaller probabilities are achieved exponentially in N .

A key aspect of our protocol is rapid implementation of the random unitary matrices, U_i , at the reference stations. One pragmatic strategy that provides for both a random selection mechanism and the required nonorthogonal behavior between the states $|\Upsilon_i\rangle$ is to allow the U_i to be constructed from random permutations of the Hadamard gate H , and the $\pi/8$ gate T . It is known that any single-qubit unitary operation can be approximated to arbitrary accuracy from H and T gates (e.g., [23]), and that standard optical devices can be deployed to induce such gates on polarized photons. In simulations we have explored permutations of the T and H gates as a means of producing the random transforms needed to remove the orthogonality of the original basis. A series of random permutations leading to gates of the form $TTHTHTHTT\dots$ were performed, and the average orthogonality of the set $|\Upsilon_i\rangle$ measured. It was found that even with gates using only five random combinations (e.g., $THHTH$) the required nonorthogonal properties between the states $|\Upsilon_i\rangle$ was achieved, with the average fidelity between any two states being $F \sim 0.3$. Similar fidelities were found using the random matrix formulation of Eq. (2).

The quantum protocol we have outlined is aimed at networks in which the quantum channel utilizes fiber and the classical channel utilizes wireless communications. The protocol requires the use of random transformations at the reference stations and the presence of efficient millisecond quantum memory at the receiver (see [24] for state-of-the-art implementation of quantum memory at telecommunication

wavelengths). However, implementation of our protocol is simpler when it is assumed that qubits in the quantum channel move with velocity c , as no additional transformations are required, and the need for quantum memory is negated (in many setups). In such a circumstance, the one-dimensional verification protocol would follow a setup similar to that utilized in recent experiments on entanglement swapping [25]. In [25], a BSM via linear optics is conducted on a series of entangled photons arriving from different synchronized pulsed sources. Coincidence counting is achieved within the nanosecond range. Using similar techniques, an implementation of location verification over tens of kilometers, to an accuracy of meters, is currently possible. Any relaxation of our initial assumptions, such as zero processing time, will manifest itself in a (determinable) reduction in the accuracy of the location being verified. Note that, even though we have described our protocol under the assumption that all four Bell states can be discriminated in the BSM, this is not a requirement. When using linear optics for BSM, only two Bell states can be discriminated (deterministically). In this case our encoding scheme would need to be adjusted to a three-message encoding. This has the minor effect of a drop in the channel capacity.

Clearly there are many variants on our protocol, such as the use of teleportation, the use of other entanglement degrees of freedom, and the use of entanglement swapping between the reference stations and the device. For example, a modified verification protocol that uses entanglement swapping can be constructed that entirely negates the requirement for direct transfer of qubits between the reference stations and the device.

Location verification would then be possible in a satellite-to-device communications system, provided the satellite and the device shared *a priori* an entangled resource stored in quantum memory.

Quantum location verification could greatly assist in the authentication of devices within large-scale multihop quantum networks [26]. Current quantum authentication techniques require the distribution of secret keys distributed *a priori* among potential users [27]. However, such keys, whether classical bits or entangled qubits, are subject to unauthorized redistribution. We also note that quantum location verification can be used within other data-delivery protocols in which real-time data transfer can be communicated to a device successfully only if that device is at a specific location. The location verification can be monitored continuously in real time, halting any real-time data transfer upon violation of the verification procedures. An adversary could not continue to receive real-time data without one of his devices being at the specified location.

Quantum location verification represents an application in the emerging field of quantum communications. It delivers an outcome not possible in the classical-only channel. We possess the ability to unconditionally authenticate a communication channel based on the geographical coordinates of a receiver.

ACKNOWLEDGMENT

This work has been supported by the University of New South Wales.

-
- [1] L. Scott and D. E. Denning, *GPS World*, April 2003, pp. 38–47, <http://faculty.nps.edu/dedennin/publications/GPSWorld.pdf>.
 - [2] S. Capkun and J.-P. Hubaux, *IEEE J. Sel. Area. Commun.* **24**, 221 (2006).
 - [3] R. A. Malaney, *IEEE International Conference on Communications* (Institute of Electrical and Electronics Engineers Inc., Piscataway, NJ, United States, 2007), pp. 1558–1563.
 - [4] J. T. Chiang, J. J. Haas, and Y.-C. Hu, in *WiSec '09, Proceedings of the Second ACM Conference on Wireless Network Security Zurich, Switzerland* (Association for Computing Machinery, New York, NY, USA, 2009), pp. 181–192.
 - [5] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993).
 - [6] D. Bouwmeester *et al.*, *Nature* **390**, 575 (1997).
 - [7] R. Ursin *et al.*, *Nature* **430**, 849 (2004).
 - [8] R. Ursin *et al.*, *Nature Physics* **3**, 481 (2007).
 - [9] H. Takesue *et al.*, *New J. Phys.* **7**, 232 (2005).
 - [10] H. J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **81**, 5932 (1998).
 - [11] A. Poppe, M. Peev, and O. Maurhart, *Int. J. Quantum Inform.* **6**, 209 (2008).
 - [12] C. H. Bennett and S. J. Wiesner, *Phys. Rev. Lett.* **69**, 2881 (1992).
 - [13] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, *Phys. Rev. Lett.* **76**, 4656 (1996).
 - [14] J. T. Barreiro, T. C. Wei, and P. G. Kwiat, *Nature Physics* **4**, 282 (2008).
 - [15] C. H. Bennett and G. Brassard, *Proceedings of IEEE International Conference on Computers Systems and Signal Processing* (Bangalore, India, 1984), pp. 175–179.
 - [16] A. K. Ekert, *Phys. Rev. Lett.* **67**, 661 (1991).
 - [17] W. K. Wootters and W. H. Zurek, *Nature* **299**, 802 (1982).
 - [18] D. M. Greenberger, M. A. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989), p. 69.
 - [19] D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, *Am. J. Phys.* **58**, 1131 (1990).
 - [20] H. J. Lee, D. Ahn, and S. W. Hwang, *Phys. Rev. A* **66**, 024304 (2002).
 - [21] L. P. Lamoureux, P. Navez, J. Fiurášek, and N. J. Cerf, *Phys. Rev. A* **69**, 040301(R) (2004).
 - [22] E. Karpov, P. Navez, and N. J. Cerf, *Phys. Rev. A* **72**, 042314 (2005).
 - [23] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
 - [24] B. Lauritzen *et al.*, *Phys. Rev. Lett.* **104**, 080502 (2010).
 - [25] R. Kaltenbaek, R. Prevedel, M. Aspelmeyer, and A. Zeilinger, *Phys. Rev. A* **79**, 040302(R) (2009).
 - [26] R. Van Meter, T. D. Ladd, W. J. Munro, and K. Nemoto, *IEEE/ACM Transactions on Networking* **17**(3), 1002 (2009).
 - [27] K. G. Paterson, F. Piper, and R. Schack, e-print [arXiv:quant-ph/0406147](https://arxiv.org/abs/quant-ph/0406147).