State transfer in static and dynamic spin chains with disorder

David Petrosyan, Georgios M. Nikolopoulos, and P. Lambropoulos
Institute of Electronic Structure and Laser, Foundation for Research and Technology–Hellas, GR-71110 Heraklion, Crete, Greece
(Received 15 January 2010; published 14 April 2010)

We examine the speed and fidelity of several protocols for state or single excitation transfer in finite spin chains subject to diagonal and off-diagonal disorder. We find that, for a given chain length and maximal achievable interspin exchange (XY) coupling strength, the optimal static spin-coupling protocol, implementing the fastest state transfer between the two ends of the chain, is more susceptible to off-diagonal (XY coupling) disorder, as compared to a much slower but robust adiabatic transfer protocol with time-dependent coupling strengths.

DOI: 10.1103/PhysRevA.81.042307 PACS number(s): 03.67.Hk, 75.10.Pq, 73.63.Kv, 03.67.Lx

I. INTRODUCTION

Faithful transfer of quantum states between physical qubits of an integrated quantum register is one of the important prerequisites for scalable quantum computation. Typically, qubit-qubit interactions are short range and implementing quantum logic gates between qubits located at distant subregisters would involve interconnecting them via quantum channels, or wires [1], which may consist of arrays of coupled quantum dots [2,3] or superconducting qubits [4–6], atoms in optical lattices [7,8], or other realizations of spin chains.

Quantum channels of permanently coupled spins would require no dynamical manipulations during the state transfer, but might be susceptible to noise and imperfections. Conversely, dynamically manipulated networks can be more robust with respect to certain kinds of disorder, but are more involved, requiring time-dependent external control. Here we reconsider critically several protocols for achieving efficient and dependable—ideally perfect—state transfer in disordered spin chains subject to physically constrained maximally achievable interspin coupling rates. The present work is an extension of our earlier studies [2,3] of perfect state transfer to more realistic scenarios with the aim of quantifying and neutralizing the influence of static (or slowly changing) noise inevitably present in any imperfect physical realization of the spin chain resulting in diagonal and off-diagonal disorder.

After outlining the model, we examine the speed and reliability of several state transfer protocols first for ideal and then for noisy spin chains, followed by conclusions.

II. THE MODEL

The Hamiltonian for a spin chain of length N has a general form [9]

$$H=\frac{1}{2}\sum_{j=1}^{N}h_j\hat{\sigma}_j^z-\frac{1}{2}\sum_{j=1}^{N-1}J_j\left(\hat{\sigma}_j^x\hat{\sigma}_{j+1}^x+\hat{\sigma}_j^y\hat{\sigma}_{j+1}^y+\Delta\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z\right), \tag{1}$$

where $\hat{\sigma}_j^{x,y,z}$ are the Pauli spin operators at position j, h_j determines the energy separation between the spin-up and spin-down states playing the role of the local "magnetic field," and J_j is the nearest-neighbor spin-spin interaction which can be static or time dependent. From now on we set the anisotropy parameter $\Delta=0$; Eq. (1) reduces then to the Hamiltonian of the XX model, which is isomorphic to the Hubbard Hamiltonian for spinless fermions or hard-core bosons [9],

$$H=\sum_{j=1}^{N}h_j\hat{a}_j^\dagger\hat{a}_j-\sum_{j=1}^{N-1}J_j(\hat{a}_j^\dagger\hat{a}_{j+1}+\hat{a}_{j+1}^\dagger\hat{a}_j), \tag{2}$$

where $\hat{a}_j^\dagger$ ($\hat{a}_j$) is the particle creation (annihilation) operator at site j with energy h_j and J_j now plays the role of tunnel couping between adjacent sites j and $j+1$.

Our objective here is to transfer an arbitrary single-qubit state $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$ between the two ends of the spin chain. To that end, we assume that all the spins can be prepared in the "ground" state $|\downarrow\rangle_j\equiv|0\rangle_j$ and at a certain initial time $t_{\rm in}=0$ the first site of the chain is initialized to $|\psi\rangle_1$. Ideal transfer would imply that at a well-defined final time $t_{\rm out}$ the last site of the chain is in state $|\psi\rangle_N$, up to a certain relative phase factor between the amplitudes of states $|0\rangle_N$ and $|1\rangle_N$ (see later in this article).

Since the Hamiltonian (1) [or (2)] preserves the number of spin [or particle] excitations, we need to consider only the zero $|\mathbf{0}\rangle\equiv\prod_{j=1}^{N}|0\rangle_j$ and single-excitation $|\boldsymbol{j}\rangle\equiv\hat{\sigma}_j^+|\mathbf{0}\rangle$ $[\hat{a}_j^\dagger|\mathbf{0}\rangle]$ subspaces of the total Hilbert space. Then the system initially in state $|\Psi_{\rm in}\rangle=\alpha|\mathbf{0}\rangle+\beta|\mathbf{1}\rangle$ evolves in time as $|\Psi(t)\rangle=U(t)|\Psi_{\rm in}\rangle=\alpha|\mathbf{0}\rangle+\beta\sum_{j=1}^{N}A_j(t)|\boldsymbol{j}\rangle$, where $U(t)=\mathcal{T}\exp[\frac{1}{i\hbar}\int_0^t H(t')dt']$ is the (time-ordered, $\mathcal{T}$) evolution operator. Apparently, only the states in the single-excitation subspace $\{|\boldsymbol{j}\rangle\}$ evolve in time with the corresponding amplitudes $A_j(t)\equiv\langle\boldsymbol{j}|U(t)|\mathbf{1}\rangle$, while the vacuum (or ground) state $|\mathbf{0}\rangle$ remains unchanged. Thus perfect state transfer would be achieved for the amplitude $|A_N(t_{\rm out})|=1$, provided its phase $\phi=\arg(A_N)$ is fixed and known, $\phi=\phi_0$, and therefore can be amended.

We may quantify the performance of the scheme by the transfer fidelity $F_\psi=\langle\psi|\rho_N|\psi\rangle$, where $\rho_N\equiv{\rm Tr}_{\bar{N}}(|\Psi\rangle\langle\Psi|)=(1-|\beta|^2|A_N|^2)|0\rangle\langle0|+|\beta|^2|A_N|^2|1\rangle\langle1|+\alpha\beta^*A_N^*|0\rangle\langle1|+\alpha^*\beta A_N|1\rangle\langle0|$ is the reduced density operator for the Nth site of the chain [10]. We then have $F_\psi=|\alpha|^2+|\beta|^2(1-2|\alpha|^2)|A_N|^2+2|\alpha|^2|\beta|^2|A_N|\cos(\phi)$, while the mean transfer fidelity F, obtained by averaging F_ψ over all possible $|\psi\rangle$ and after compensating for ϕ_0, is given by [10]

$$F=\frac{1}{2}+\frac{|A_N|^2}{6}+\frac{|A_N|\cos(\phi-\phi_0)}{3}. \tag{3}$$

Thus, for the amplitude $|A_N| = 1$ but completely random phase ϕ, the fidelity is equal to the classical value of $F = 2/3$, while for $|A_N| = 0$ we have $F = 1/2$ corresponding to a random guess of the qubit state $|0\rangle$ or $|1\rangle$.

III. STATE TRANSFER PROTOCOLS

The state or excitation transfer in a spin chain described by Hamiltonian (1) [or (2)] is mediated by the nearest-neighbor couplings J_j. Clearly, in any practical realization of the spin chain there will be some upper limit for achievable coupling strength, $J_{\max} \equiv \max\{J_j\}$, determined by physical or technological constraints. On a fundamental level, this follows from the fact that the energy of the system is bounded, which, in turn, limits the speed of the state transfer, $t_{\text{out}} \gtrsim N/J_{\max}$ [11,12].

A. Noiseless spin chains

Let us first recall the key facts pertaining to an idealized spin chain with no disorder. We assume uniform on-site energies $h_j := 0\ \forall\ j \in [1, N]$, while the individual couplings J_j can be freely controlled, subject to the constraint $J_j \leqslant J_{\max}$.

(a) Perhaps conceptually the most straightforward approach to the state transfer between the two ends of the chain is to apply a sequence of SWAP operations implemented by π pulses between the pairs of neighboring sites. To that end, with all the couplings J_j set initially to zero, we switch on J_1 for time $t_1 = \pi/(2J_1)$, then J_2 for time $t_2 = \pi/(2J_2)$, etc., until the Nth site is reached. At the end of each step, the corresponding state amplitude is $A_j(t_{j-1}) = -i\sin(J_{j-1}t_{j-1})A_{j-1}(t_{j-2}) = (-i)^{j-1}$ for $j = 2, \ldots, N$. If all the couplings' strengths can be pulsed to the maximal possible $J_{\max}$, and there are $N-1$ steps, the total transfer time is $t_{\text{out}} = (N-1)\pi/(2J_{\max}) \simeq (\pi/2)(N/J_{\max})$ $(N \gg 1)$ with the final state amplitude $A_N(t_{\text{out}}) = (-i)^{N-1}$, that is, $|A_N(t_{\text{out}})| = 1$ and $\phi_0 = (-\pi/2)(N-1) \pmod{2\pi}$.

(b) We next consider a spin chain with static couplings J_j arranged in an appropriate way to facilitate the perfect state (or excitation) transfer. By "static" we mean that during the transfer the coupling strengths are fixed, but to initiate (at time t_{in}) and to terminate (at time t_{out}) the transfer process at least J_1 and J_{N-1} should be quickly switched on and off, respectively. (Alternatively the state initialization of the first site at t_{in} and state retrieval from the last site at t_{out} should be accomplished very fast, on a time scale that is short compared to $J_{1,N-1}^{-1}$.) Among the many [1]—in fact, infinitely many [13]—possible static protocols for perfect state transfer, we focus here on the one proposed in [2,14], and much earlier [15] and in a different context (that of population transfer in laser-driven multilevel atomic or molecular systems [16]), which was shown to be the optimal one [12] in terms of the transfer time. In this so-called spin-coupling protocol, the coupling constants are arranged according to $J_j = J_0\sqrt{(N-j)j}$, which makes the system formally analogous to a spin-$\mathcal{J}$ in a magnetic field.[1] This leads to the equidistant energy spectrum $\lambda_k = 2J_0k - J_0(N+1)$, with $k = 1, 2, \ldots, N$, and consequently perfectly periodic oscillations of the single excitation between the two ends of the chain, according to

$$A_j(t) = \binom{N-1}{j-1}^{1/2} [-i\sin(J_0t)]^{(j-1)}\cos(J_0t)^{(N-j)}.$$

Thus, at time $t_{\text{out}} = \pi/(2J_0)$ the amplitude of the final state is $A_N(t) = [-i\sin(J_0t_{\text{out}})]^{N-1} = (-i)^{N-1}$. Note that the strongest coupling is in the center of the chain: at $j = N/2$ for N even, $J_{N/2} = \frac{1}{2}J_0N \equiv J_{\max}$; or at $j = (N \pm 1)/2$ for N odd, $J_{(N\pm1)/2} = \frac{1}{2}J_0\sqrt{N^2-1} \simeq J_{\max}$ $(N \gg 1)$. Hence, the transfer time expressed through $J_{\max}$ is given by $t_{\text{out}} = (\pi/4)(N/J_{\max})$, which is twice as short as that for the sequential SWAP protocol.

(c) The last protocol that we consider here is the adiabatic state or excitation transfer between the two ends of the spin chain using slowly varying couplings J_j [3,17]. This is analogous to the stimulated Raman adiabatic passage (STIRAP) techniques [18] extended to multilevel atomic or molecular systems [19]. Assume that N is odd and the individual couplings J_j can be selectively and independently manipulated. In the single-excitation subspace, the Hamiltonian (1) [or (2)] has the eigenstate

$$|\Psi^{(0)}\rangle = \frac{1}{\sqrt{\mathcal{N}_0}}[J_2J_4\ldots J_{N-1}|\mathbf{1}\rangle + (-1)J_1J_4\ldots J_{N-1}|\mathbf{3}\rangle + \cdots + (-1)^{\mathcal{J}}J_1J_3\ldots J_{N-2}|\boldsymbol{N}\rangle], \quad (4)$$
$$\mathcal{J} \equiv \frac{1}{2}(N-1),$$

with eigenvalue $\lambda^{(0)} = 0$, which is conventionally called the coherent population trapping (or dark) state [18,19]. Thus the amplitude of initial state A_1 is proportional to the product of all the even-numbered couplings, while the amplitude of final state A_N is given by the product of all the odd-numbered couplings, divided by the normalization parameter $\mathcal{N}_0 = (J_2J_4\cdots J_{N-1})^2 + \cdots + (J_1J_3\ldots J_{N-2})^2$. Therefore, if all the even-numbered couplings are switched on first, the zero-energy state (4) would coincide with the initial state $|\mathbf{1}\rangle$. This is then followed by adiabatically switching on all the odd-numbered couplings, while the even-numbered couplings are switched off, which will result in state (4) rotating toward the final state $|\boldsymbol{N}\rangle$. Assuming that these two families of couplings are described by common shape functions, $J_2, J_4, \ldots, J_{N-1} = J_{\text{even}}(t)$ and $J_1, J_3, \ldots, J_{N-2} = J_{\text{odd}}(t)$, the amplitudes of the initial and the final states are given by

$$A_1(t) = \frac{[J_{\text{even}}(t)]^{\mathcal{J}}}{\sqrt{\mathcal{N}_0(t)}}, \quad A_N(t) = (-1)^{\mathcal{J}}\frac{[J_{\text{odd}}(t)]^{\mathcal{J}}}{\sqrt{\mathcal{N}_0(t)}},$$

with $\mathcal{N}_0(t) = \sum_{n=0}^{\mathcal{J}}[J_{\text{odd}}(t)]^{2n}[J_{\text{even}}(t)]^{2(\mathcal{J}-n)}$. Thus, complete state or excitation transfer between the two ends of the chain

[1] Recall from the theory of angular momentum that a spin-$\mathcal{J}$ particle in a constant magnetic field exhibits nondispersive (Larmor) precession about the field direction. With the quantization direction along an axis perpendicular to the magnetic field direction, the matrix elements for the transitions $|\mathcal{J}, m\rangle \leftrightarrow |\mathcal{J}, m+1\rangle$ between the neighboring magnetic substates $(m = -\mathcal{J}, \ldots, \mathcal{J})$ are proportional to $\sqrt{(\mathcal{J}-m)(\mathcal{J}+m+1)}$. Setting formally $N = 2\mathcal{J}+1$ and $j = \mathcal{J}+m+1$ leads to the coupling constants $J_j \propto \sqrt{(N-j)j}$, as in the text.

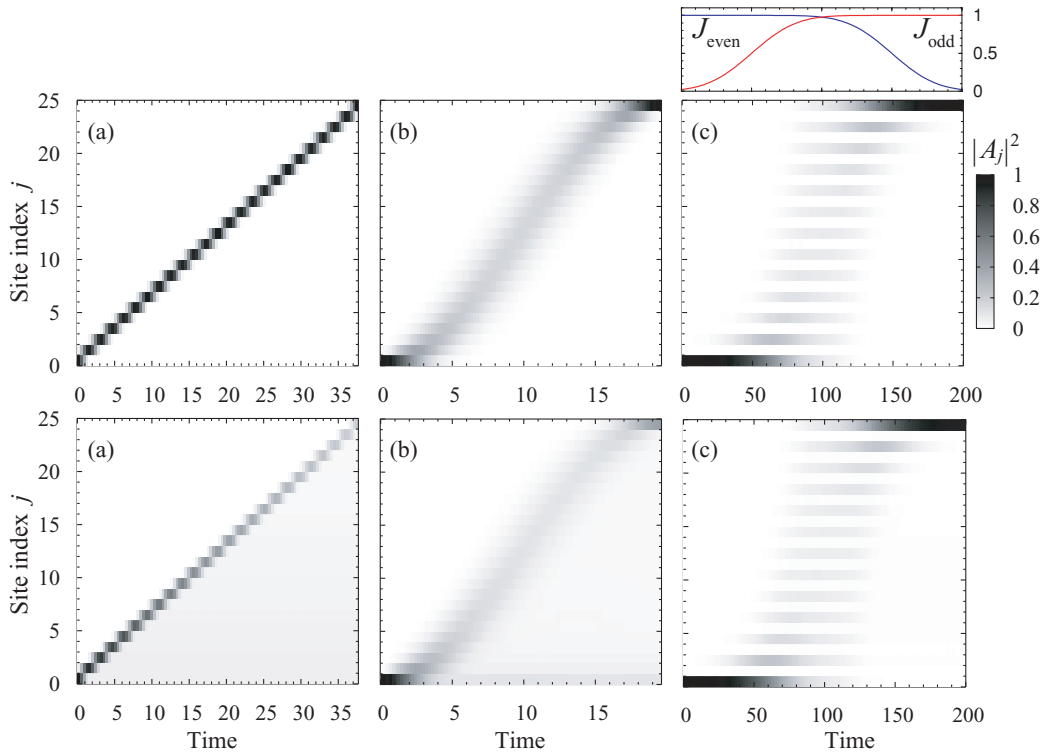

FIG. 1. (Color online) Dynamics of single-excitation transfer in a spin chain of length $N = 25$ for (a) sequential SWAP, (b) spin-coupling, and (c) adiabatic protocols. The top panels correspond to noiseless spin chains, $\sigma_h = \sigma_J = 0$, yielding complete excitation transfer $|A_N(t_{out})|^2 = 1$ for all cases (a), (b), and (c). The graph above (c) shows the time dependence of couplings J_{even} and J_{odd} normalized to J_{max} [cf. Eq. (5)]. The bottom panels illustrate the results for noisy chains with $\sigma_h = \sigma_J = 0.15 J_{max}$ averaged over 1000 independent realizations, leading to $\langle |A_N(t_{out})|^2 \rangle \simeq 0.2$, 0.42, and 0.96 for (a), (b), and (c), respectively. Time is measured in units of J_{max}^{-1} and the evolution terminates at the corresponding t_{out}.

can be achieved by applying first the J_{even} couplings and then the J_{odd} couplings, the two sets of couplings partially overlapping in time. At time t_{out}, when $J_{odd}(t_{out}) \gg J_{even}(t_{out}) \simeq 0$, the amplitude of the final state is $A_N(t_{out}) = (-1)^{\mathcal{J}}$, that is, $|A_N(t_{out})| = 1$ and $\phi_0 = (-\pi)(N-1)/2 \pmod{2\pi}$. Of course the adiabatic following of the zero-energy eigenstate (4) holds true if, during the transfer process, the nonadiabatic transitions out of $|\Psi^{(0)}\rangle$ are negligible, which requires that the rate of change of the coupling strengths be small compared to the energy separation between $|\Psi^{(0)}\rangle$ and all the other eigenstates. We can estimate the energy separation between the eigenstates in the vicinity of maximal overlap between the even and odd couplings, $J_{even} \simeq J_{odd} = J$. The energy spectrum of the chain with homogeneous coupling, $J_j = J \forall j \in [1, N-1]$, is $\lambda_k = -2J \cos[k\pi/(N+1)]$ [2,3]. The eigenstate with zero energy $\lambda^{(0)}$ is the one with $k = (N+1)/2 \equiv k_0$, and the nearest eigenstates with indices $k = k_0 \pm 1$ have energies $\lambda_{k_0 \pm 1} = \pm 2J \sin[\pi/(N+1)] \simeq \pm 2J\pi/N$ $(N \gg 1)$. With $J \lesssim J_{max}$, the excitation transfer time t_{out}, being roughly equal to the couplings' switching time, should then satisfy the condition $t_{out} \gg N/(2\pi J_{max})$.

To summarize the results for noiseless spin chains, the transfer time t_{out} for all three protocols scales with the number of sites N and the maximal intersite coupling J_{max} as (N/J_{max}). The fastest is the spin-coupling protocol with $t_{out} = (\pi/4)(N/J_{max})$. It is followed by the sequential SWAP protocol, for which $t_{out} \simeq (\pi/2)(N/J_{max})$. Finally the slowest is the adiabatic protocol $t_{out} \simeq C(N/J_{max})$, with $C \simeq 8$ being a safe estimate[2] for smooth coupling functions that we use:

$$J_{\substack{\text{odd} \\ \text{even}}}(t) = J_{max} \frac{1}{2} \left[1 \pm \text{erf}\left(\frac{t - \frac{1}{2} t_{out} \pm 2\sigma_t}{\sqrt{2}\sigma_t} \right) \right], \qquad (5)$$

with $\sigma_t = \frac{1}{8} t_{out}$. Note that the phase of the final-state amplitude for all three protocols is given by $\phi_0 = (-\pi/2)(N-1) \pmod{2\pi}$, which should be compensated for after the transfer. The single-excitation transfer for all three protocols is illustrated in the top panels of Fig. 1.

B. Disordered chains

Employing numerical simulations, we now examine the robustness of the previously described state transfer protocols

[2]We note that the smallest value of C leading to good adiabatic following is quite sensitive to the choice of the coupling functions $J_{odd}(t)$ and $J_{even}(t)$ and their overlap. (For the particular choice of the coupling functions, even for $C = 5$ we attain good adiabatic following with excitation transfer $|A_N(t_{out})|^2 \gtrsim 0.98$.) Moreover, with increasing the chain length N, $\min(C)$ slowly grows with N, because the latter determines not only the energy separation $2J\pi/N$ of the nearest eigenstate from the zero-energy eigenstate but also the number of further detuned eigenstates to which nonadiabatic transitions could occur, albeit with even lesser probabilities.

in spin chains with varying degrees of disorder. We note that for the spin-coupling scheme, related analysis has been performed in [20].

The physical origin of disorder may be twofold: (i) fabrication imperfections of the particular system realizing the spin chain, and (ii) noise of the external controls, which is assumed to vary slowly enough on the time scale of state transfer $t_{\rm out}$, as is typically the case in most experimental situations pertaining to coupled quantum dots [21], superconducting qubits [22], or atoms [23]. We shall distinguish diagonal and off-diagonal disorder. The diagonal disorder corresponds to random on-site energies, or equivalently the local magnetic fields h_j, normally distributed around $\langle h_j\rangle = 0$ with variance σ_h^2 (without loss of generality, we assume that the energies of the first and the last sites of the chain are exempt from disorder, $h_1 = h_N = 0$, since otherwise the state $|\psi\rangle$ would dephase even before and after the transfer). The off-diagonal disorder introduces randomness in the intersite coupling strengths $J_j \to J_j(1+\delta J_j)$, where δJ_j are normally distributed around $\langle \delta J_j\rangle = 0$ with variance σ_J^2. Consistent with the aforementioned description, we treat the disorder as static during each realization of the numerical experiment for the particular protocol, but completely uncorrelated between different realizations. The results presented below are obtained by averaging over many (typically 1000) independent realizations.

Figure 1 compares the single-excitation transfer for the three protocols (a), (b), and (c) in ideal and disordered spin chains of length $N = 25$. In the noiseless chain we have perfect transfer $|A_N(t_{\rm out})|^2 = 1$, while in the presence of diagonal and off-diagonal disorder characterized by standard deviations $\sigma_h = \sigma_J = 0.15 J_{\max}$, the averaged transfer probabilities are reduced to $\langle |A_N(t_{\rm out})|^2\rangle \simeq 0.2$, 0.42 and 0.96 for the cases of (a), (b) and (c), respectively. Thus, among the three transfer protocols, the sequential SWAP scheme is the most susceptible to noise, especially to the off-diagonal disorder which leads to deviations of the subsequent pulse areas from the required value of π; in this particular example, the off-diagonal disorder alone is responsible for at least 70% reduction of the transfer probability (see also the insets in Fig. 2). The spin-coupling scheme is somewhat more robust with respect to noise, with both diagonal and off-diagonal disorder comparably contributing to the reduction of the transfer probability by

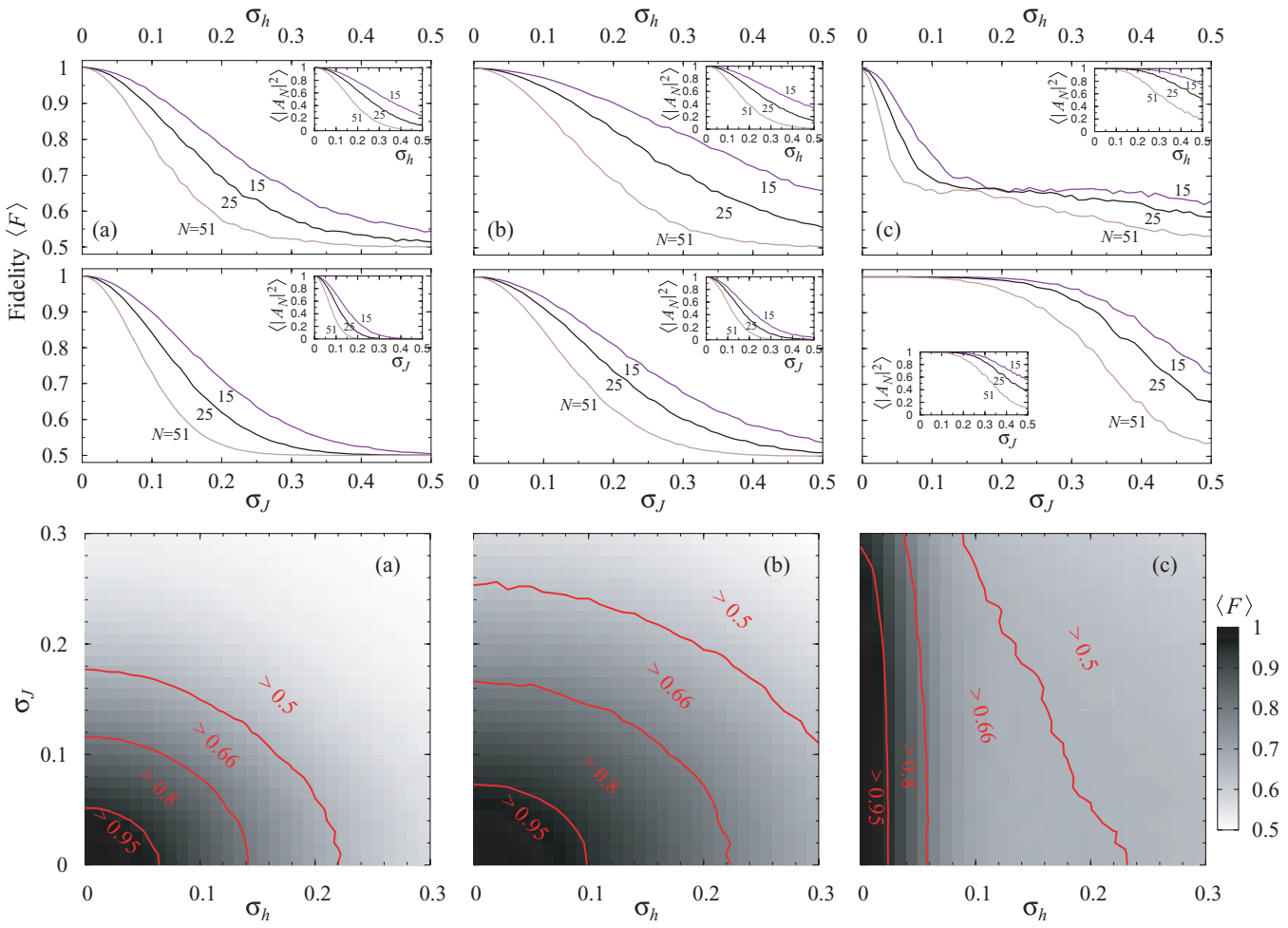

FIG. 2. (Color online) Averaged (over 1000 realizations) fidelity $\langle F\rangle$ in noisy spin chains for (a) sequential SWAP, (b) spin-coupling, and (c) adiabatic protocols. The top panel shows the dependence of $\langle F\rangle$ on the diagonal disorder σ_h with $\sigma_J = 0$ (upper plots) and on the off-diagonal disorder σ_J with $\sigma_h = 0$ (lower plots), for the chains of lengths $N = 15$, 25, 51. The inset in each graph displays the corresponding transfer probabilities $\langle |A_N(t_{\rm out})|^2\rangle$. The bottom panel shows $\langle F\rangle$ versus both σ_h with σ_J in spin chains with $N = 25$. $\sigma_{h,J}$ are measured in units of $J_{\max}$.

about 20% and 40%, respectively. Finally, the adiabatic transfer scheme is very tolerant to noise, as far as the transfer probability is concerned, but is quite slow; in fact it can tolerate even more disorder at the expense of slowing it further down (equivalent to increasing C).

The probability of excitation transfer alone is not enough to fully characterize the state transfer, since, for example, $|A_N(t_{\rm out})|^2 = 1$ but completely random phase ϕ amounts to classical information transfer only, and the resulting fidelity for quantum state transfer is merely $F = 0.66$. We therefore quantify the performance of the system subject to varying levels of noise using fidelity $\langle F\rangle$ of Eq. (3) averaged over many independent realizations of protocols (a), (b), and (c). Figure 2 summarizes the results of our numerical simulations for the chains of lengths $N = 15$, 25, and 51. Unsurprisingly, the longer the chain the lower the fidelity of the state transfer is. We find that, for the same values of diagonal σ_h and/or off-diagonal σ_J disorder, the sequential SWAP scheme yields lower fidelity than the spin-coupling scheme. Moreover, both schemes are somewhat more susceptible to the off-diagonal disorder. The behavior of the fidelity for the adiabatic transfer scheme is, however, profoundly different: it is very robust with respect to the off-diagonal disorder σ_J, but much more sensitive to the diagonal disorder σ_h: already for $\sigma_h \gtrsim 0.1 J_{\rm max}$ the fidelity $\langle F\rangle \simeq 0.66$ (but then decreases slowly with increasing σ_h). This is despite the fact that the transfer probability $\langle |A_N(t_{\rm out})|^2\rangle$ remains above 0.9 up to $\sigma_{h,J} \lesssim 0.25 J_{\rm max}$; that is, the excitation transfer is very efficient up to large values of both diagonal and off-diagonal disorder, as attested in the insets of Fig. 2(c). The adiabatic transfer protocol is so sensitive to diagonal disorder because it is slow: during the long transfer time $t_{\rm out}$ even little noise in the on-site energies σ_h accumulates to large random phase ϕ spread over $\sigma_\phi \sim \sigma_h t_{\rm out}$.

IV. CONCLUSIONS

We have critically examined the state and excitation transfer in disordered spin chains using the sequential SWAP, spin-coupling, and adiabatic transfer protocols. We have found that, depending on the character of disorder, namely, the diagonal disorder corresponding to random on-site energies (or magnetic fields) or off-diagonal disorder leading to variations in intersite couplings, either the fast spin-coupling protocol or the slow adiabatic transfer protocol is more suitable for high-fidelity transfer of quantum states between the two ends of the spin chain.

The results obtained pertain to spin chains of fixed, moderate lengths. We note some of the recent relevant studies of disordered spin chains of varying lengths which elucidated the bounds on information transfer speed [24] and distance [25] and possible ways for improvement [26,27].

Reliable quantum channels, based on, for example, spin chains, are indispensable for achieving scalable and efficient quantum information processing in solid-state systems with fixed qubit positions and finite-range interqubit interactions. Our results therefore have important implications for attaining scalability in such systems.

ACKNOWLEDGMENTS

This work was supported by the EC Marie Curie Research-Training Network EMALI (MRTN-CT-2006-035369).

[1] S. Bose, Contemp. Phys. **48**, 13 (2007); D. Burgarth, Eur. Phys. J. Spec. Top. **151**, 147 (2007).
[2] G. M. Nikolopoulos, D. Petrosyan, and P. Lambropoulos, Europhys. Lett. **65**, 297 (2004); J. Phys.: Condens. Matter **16**, 4991 (2004).
[3] D. Petrosyan and P. Lambropoulos, Opt. Commun. **264**, 419 (2006).
[4] A. Romito, R. Fazio, and C. Bruder, Phys. Rev. B **71**, 100501 (R) (2005).
[5] F. W. Strauch and C. J. Williams, Phys. Rev. B **78**, 094516 (2008).
[6] D. I. Tsomokos, M. J. Hartmann, S. F. Huelga, and M. B. Plenio, New J. Phys. **9**, 79 (2007).
[7] A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett. **90**, 100401 (2003); L.-M. Duan, E. Demler, and M. D. Lukin, *ibid.* **91**, 090402 (2003).
[8] M. Valiente and D. Petrosyan, Europhys. Lett. **83**, 30007 (2008).
[9] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, UK, 1999).
[10] S. Bose, Phys. Rev. Lett. **91**, 207901 (2003).
[11] E. H. Lieb and D. W. Robinson, Commun. Math. Phys. **28**, 251 (1972).
[12] M.-H. Yung, Phys. Rev. A **74**, 030303(R) (2006).
[13] V. Kostak, G. M. Nikolopoulos, and I. Jex, Phys. Rev. A **75**, 042319 (2007).
[14] M. Christandl, N. Datta, A. Ekert, and A. J. Landahl, Phys. Rev. Lett. **92**, 187902 (2004).
[15] R. J. Cook and B. W. Shore, Phys. Rev. A **20**, 539 (1979).
[16] B. W. Shore, *The Theory of Coherent Atomic Excitation* (Wiley, New York, 1990).
[17] A. D. Greentree, J. H. Cole, A. R. Hamilton, and L. C. L. Hollenberg, Phys. Rev. B **70**, 235317 (2004).
[18] K. Bergmann, H. Theuer, and B. W. Shore, Rev. Mod. Phys. **70**, 1003 (1998).
[19] B. W. Shore, K. Bergmann, J. Oreg, and S. Rosenwaks, Phys. Rev. A **44**, 7442 (1991).
[20] G. De Chiara, D. Rossini, S. Montangero, and R. Fazio, Phys. Rev. A **72**, 012323 (2005).
[21] W. G. van der Wiel, S. De Franceschi, J. M. Elzerman, T. Fujisawa, S. Tarucha, and L. P. Kouwenhoven, Rev. Mod. Phys. **75**, 1 (2003); F. R. Waugh, M. J. Berry, D. J. Mar, R. M. Westervelt, K. L. Campman, and A. C. Gossard, Phys. Rev. Lett. **75**, 705 (1995); L. P. Kouwenhoven, F. W. J. Hekking, B. J. van Wees, C. J. P. M. Harmans, C. E. Timmering, and C. T. Foxon, *ibid.* **65**, 361 (1990).
[22] J. Clarke and F. K. Wilhelm, Nature (London) **453**, 1031 (2008); Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).

[23] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).

[24] C. K. Burrell and T. J. Osborne, Phys. Rev. Lett. **99**, 167201 (2007); C. K. Burrell, J. Eisert, and T. J. Osborne, Phys. Rev. A **80**, 052319 (2009).

[25] J. P. Keating, N. Linden, J. C. F. Matthews, and A. Winter, Phys. Rev. A **76**, 012315 (2007).

[26] J. Allcock and N. Linden, Phys. Rev. Lett. **102**, 110501 (2009).

[27] X. Wang, A. Bayat, S. G. Schirmer, and S. Bose, Phys. Rev. A **81**, 032312 (2010).