Interference fringes of $m = 0$ spin states under the Majorana transition caused by rapid half-rotation of a magnetic field

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The phase shift and visibility of fringes in the Ramsey atom interferometer composed of the $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ states were examined systematically for rapid half-rotation of the magnetic field. It was verified that the phase shifts by π rad in the adiabatic regime, but it does not shift from the original one in the nonadiabatic regime. These results support Robbins and Berry's claim [J. M. Robbins and M. V. Berry, J. Phys. A **27**, L435 (1994)]. The fact that the interference fringes disappear in the intermediate regime and reappear in the nonadiabatic regime can be explained by the Majorana transition caused by a rapid reverse of the magnetic field.

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I. INTRODUCTION

The transitions between the $m = 0$ sublevels of the ground and excited states are used as the "clock transition" of the primary frequency standards [1] or the "light-based beam splitter" of the Ramsey-type atom interferometer [2] because they have no first-order Zeeman frequency shift. Therefore, it is very important to properly consider the phase shift between them induced by changing the direction and strength [3] of the magnetic field.

It is known that the wave function of spinors with $m = 0$ has spatial antisymmetry for odd parity, whereas it has a spatial symmetry for even parity [4]. This parity-dependent symmetry was considered by Robbins and Berry in 1994 as follows: the wave function $|j, m = 0\rangle$ with odd j acquires π-phase shift under an adiabatic half-rotating magnetic field compared with the phase under a constant magnetic field [5]. They proposed to use an atom interferometer in order to verify this parity-dependent phase. The first verification of the π-phase shift was performed by Usami and Kozuma using a Ramsey atom interferometer, which was composed of the $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ states in the ground hyperfine states of an ensemble of cold Rb atoms that interact with microwave pulses [6]. Later, our group measured the π-phase shift between $|1,0\rangle$ and $|2,0\rangle$ states in the ground hyperfine states of sodium atoms using σ^+–σ^+ circularly polarized two-photon stimulated Raman transition [7]. We confirmed the phase shift is π rad for adiabatic half-rotations of the magnetic field and went on to show that the phase shift is 0 or π rad for any adiabatic partial rotation of the magnetic field. We explained that the phase shift arises from the negative sign of the transition amplitude between the $|1,0\rangle$ and $|2,0\rangle$ states when the wave function is rotated in the direction opposite to the original one, together with the magnetic field.

Usami and Kozuma also observed, in their experiment, that no phase shift occurred in the Ramsey fringes when the reversal was sudden and that the visibility of fringes decreased. They stated that the residual magnetic fields during reversal would have caused the mixing of the Zeeman sublevels [6]. The fact that the phase shift is zero can be easily understood if we consider that the wave function does not reverse under a nonadiabatic inversion of the magnetic field. However, the reduction of the visibility was not examined systematically and was not explained qualitatively or quantitatively. The clarification of the dependence of the phase shift and fringe visibility on the reversal speed of the magnetic field is of interest from the point of view of estimating the ultimate uncertainty of atomic clocks.

In a rapid rotation of the magnetic field, it is well known that the Majorana transitions from the $|F,0\rangle$ to $|F, \pm m_F\rangle$ states occur [8]. Such transitions reduce the population probabilities in the $m = 0$ states and break down the coherence of the superposition state so that the visibility of interference fringes decreases. Majorana derived the formula for the transition probability [8–10]; however, few quantitative results of experiments have been reported owing to the limitation of the velocity distribution of thermal atoms [11,12]. Recently, Xia *et al.* reported that the Majorana transition was clearly observed using spinor Bose-Einstein condensates and was in quantitative agreement with the Majorana formula [13]. According to their explanation, the probabilities for the Majorana transitions depend on the ratio of the Larmor rotation frequency to the rotation frequency of the magnetic field.

In this article, we first examine experimentally the phase shift and visibility of the interference fringes between the $|1,0\rangle$ and $|2,0\rangle$ states for different rotation frequencies of the magnetic field, which varies from the adiabatic to the nonadiabatic regime, in the Ramsey atom interferometer composed of two-photon stimulated Raman pulses and ensembles of cold sodium atoms. Next, the phase and visibility were examined for in case the magnetic field was reversed passing through (or close to) the zero magnetic field. Finally, the behavior of the observed visibility was compared with the calculation of the Majorana transition.

II. ATOM INTERFEROMETRY UNDER ROTATION OF MAGNETIC FIELD

As in our previous adiabatic experiment [7], we used a Ramsey atom interferometer, as shown in Fig. 1(a). The Ramsey atom interferometer was composed of the $|1,0\rangle$ and $|2,0\rangle$ states, which were coupled with two σ^+–σ^+ circularly polarized copropagating two-photon stimulated Raman pulses separated by time interval T. The Raman pulses with pulse width τ propagate in the z direction. Initially, the direction of

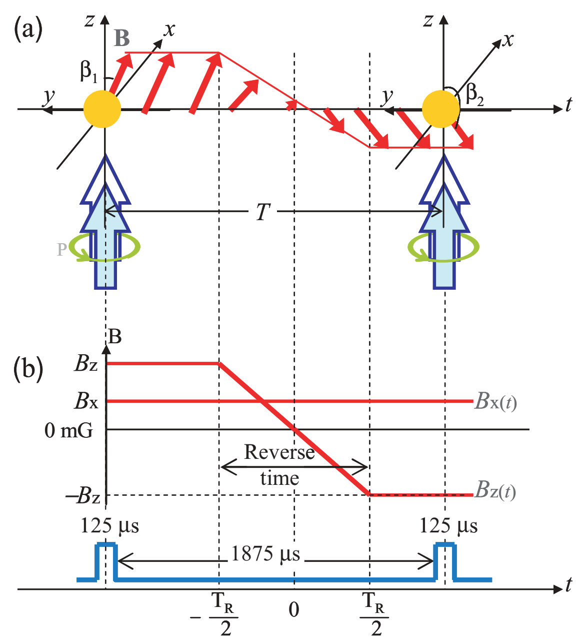

FIG. 1. (Color online) (a) Schematic of phase measurement of the Ramsey atom interferometer. Two σ^+-σ^+ polarized two-photon Raman pulses separated by T propagate in the z direction and interact with a cold ensemble of sodium atoms. The direction of the magnetic field rotates around the y axis. (b) Timing diagram of magnetic fields of $B_x(t) = B_x$ and $B_z(t)$ and two Raman pulses. $B_z(t)$ reverses from B_z to $-B_z$ with reverse time of $T_R < T$.

the magnetic field B was at an angle of β_1 with respect to the z axis, and it rotated in the x-z plane to an angle of β_2, where atoms interact with the second pulse. In the adiabatic case, the wave function rotates as the direction of the magnetic field rotates. After the interaction, the population probability of the excited state is given by [7]

$$bb^* = \frac{1}{2}\left[1 - \cos\left(\frac{\pi}{2}\cos\beta_1\right)\cos\left(\frac{\pi}{2}\cos\beta_2\right) + \sin\left(\frac{\pi}{2}\cos\beta_1\right)\sin\left(\frac{\pi}{2}\cos\beta_2\right)\cos\Phi\right]. \quad (1)$$

As the phase Φ is generally zero at resonance frequency, the phase of fringes is shifted by 0 or π rad depending on the sign of the coefficient of $\cos\Phi$, which depends on β_1 and β_2. However, in the nonadiabatic case, the wave function cannot rotate as the magnetic field rotates, so the fringes are not shifted from the initial ones.

III. EXPERIMENT

A. Apparatus

The experimental apparatus was described in detail in our previous papers [3,7]. Sodium atoms were cooled and trapped in a magneto-optical trap. After being released from the trap, all atoms were initialized to the $F = 1$ state by optical pumping. The time domain atom interferometer was composed of two $\pi/2$ Raman pulses with a pulse width of 125 μs, the separation of which was 1875 μs. After the interactions, the population probability of atoms in the $F = 2$ state was measured from the transmittance of the probe beam, which was resonant to the transition from the $F = 2$ to $F' = 3$ states. The population probability of 0.33 corresponds to a perfect transfer for this excitation, because one-third of the initial atoms were in the $F = 1$, $m_F = 0$ state. The Ramsey fringes with a cycle of 500 Hz were obtained with a visibility of 0.56 under a constant magnetic field, as shown in Fig. 2(a).

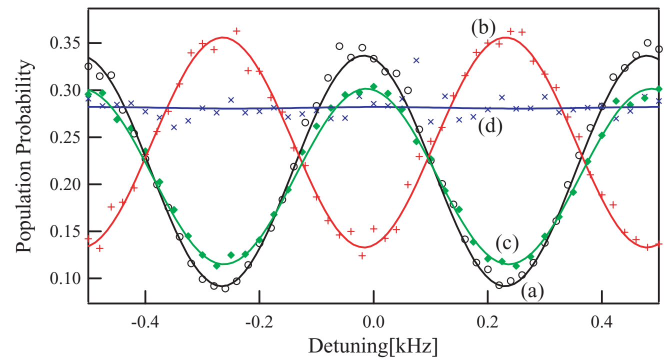

FIG. 2. (Color online) Observed Ramsey fringes with (a) constant magnetic field, (b) reverse magnetic field of $f_{\rm rot} = 2.7$ kHz (adiabatic regime), (c) $f_{\rm rot} = 2.7$ MHz (nonadiabatic regime), and (d) $f_{\rm rot} = 8.8$ kHz (intermediate region).

In the previous experiment, the adiabatic rotation of the magnetic field was produced by two mutually orthogonal pairs of Helmholtz coils, which were driven by alternating currents of about 1 kHz with a relative phase shift of $\pi/2$ rad. However, with this apparatus, it was impossible to rotate the magnetic field smoothly at a frequency greater than 2 kHz because of the inductance of the circuit. Therefore, instead of a nonadiabatic half-rotation of the magnetic field, we produced a time-dependent magnetic field in the z direction using one of the same Helmholtz coils as follows:

$$B_z(t) = \begin{cases} B_z, & t \leqslant -T_R/2 \\ -2B_z t/T_R, & -T_R/2 \leqslant t \leqslant T_R/2\,, \\ -B_z, & t \geqslant T_R/2 \end{cases} \quad (2)$$

where T_R is reverse time. The other Helmholtz coil generated a constant magnetic field $B_x(t) = B_x$ in the x direction, as shown in Fig. 1(b). At $t = 0$, namely, $B_z(0) = 0$, the Larmor frequency is $f_{\rm Lar} = g\mu_B B_x/h$, whereas the rotation frequency of the magnetic field is $f_{\rm rot} = B_z/(\pi T_R B_x)$. Then, the adiabatic regime is defined to be $f_{\rm rot} \ll f_{\rm Lar}$ and the nonadiabatic regime is $f_{\rm rot} \gg f_{\rm Lar}$; $f_{\rm rot} \sim f_{\rm Lar}$ is the intermediate region.

In order to measure B_z and B_x precisely, we used a Raman pulse with a pulse width of 250 μs. The spectrum width was measured to be 4 kHz, so that the uncertainty of the magnetic field, which was measured using the magnetic-field-sensitive transition, was usually 0.1 μT. In the present experiment, B_z was fixed at around 5.0 ± 0.1 μT, and reverse time T_R was varied from 1 μs to 1000 μs.

B. Results

At first, we set $B_x = 0.6 \pm 0.1$ μT. Under this condition, angles β_1 and β_2 are approximated to be 0 and π rad, respectively, which corresponds to a half-rotation of the magnetic field. The $f_{\rm Lar}$ is 4.2 kHz. The typical Ramsey

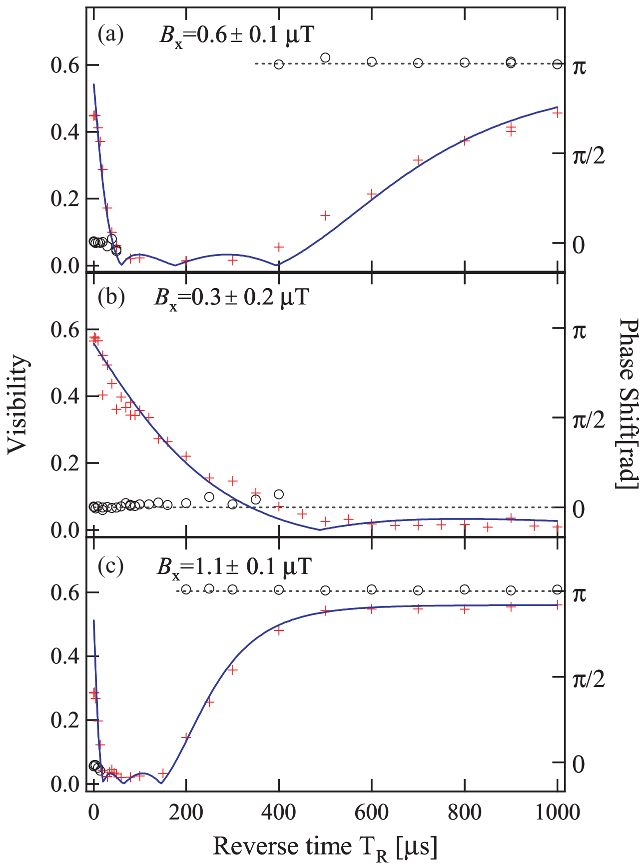

FIG. 3. (Color online) Visibility (+) and phase shift (o) of fringes as a function of reverse time T_R, together with calculated curves of visibility using Majorana transitions. $B_z = 5.0\ \mu$T. The fitted values of B_x are 0.95 for (a), 0.32 for (b), and 1.6 μT for (c).

fringes obtained under three regions are shown in Fig. 2, together with Ramsey fringes under constant magnetic field (a). The fringes in (b) were obtained at $T_R = 1$ ms, which corresponds to $f_{\rm rot} = 2.7$ kHz, namely, the adiabatic regime. The visibility of fringes is 0.47, and the phase was shifted by 3.13 ± 0.02 rad from that of (a). At $T_R = 1\ \mu$s, which corresponds to $f_{\rm rot} = 2.7$ MHz, namely, the nonadiabatic regime, the phase of fringes in (c) was shifted by 0.03 ± 0.03 rad from that in (a). This means the fringes were not shifted from the original phase. The visibility was almost the same as in (b). When $T_R = 0.3$ ms, namely, $f_{\rm rot} = 8.8$ kHz, the fringes disappeared completely, as shown in (d), although the population in the $|2,0\rangle$ state remained.

The phase shift and the visibility of fringes were summarized as a function of the reverse time T_R, as shown in Fig. 3(a). As T_R decreases from 1 ms, the visibility decreases but the phase was shifted by π. At less than 400 μs, the fringes disappeared, and we could not measure the phase shift. At less than 50 μs, fringes reappeared, and the visibility increased but the phase was not shifted. The facts that the phase shift is π in the adiabatic regime, the phase shift is zero in the nonadiabatic regime, and fringes disappear in the intermediate region confirm the result obtained by Usami and Kozuma [6]. The reason why the phase shift is zero in the nonadiabatic regime is that the wave function remains as the initial one because it does not follow the rapid rotation of the magnetic field. The fact that the fringes disappear even though the population probability is not zero means that the coherence is destroyed in the intermediate region, namely the $|2,0\rangle$ state after the reversal of the magnetic field does not interfere with that before the reversal.

Next, we changed B_x to 0.3 ± 0.2 μT, where the frequency of Zeeman splitting between the $m = 0$ and $m = 1$ states is 2.1 kHz. The splitting is considerably lower than the resonance spectrum width of 4 kHz, so that Zeeman sublevels are completely overlapped at $t = 0$. Therefore, the situation corresponds to the condition that the magnetic field reverses passing through zero magnetic field. This result is shown in Fig. 3(b), and the behaviors of shift and visibility are the same as those in the nonadiabatic regime in Fig. 3(a). At the reversal time of less than 400 μs, fringes appear and the phase shift remains zero. The visibility recovers toward 0.56 at the reverse time of less than 50 μs. The result for a sudden reversal magnetic field obtained by Usami and Kozuma will probably coincide with that at the reversal time of about 200 μs [6].

Lastly, we changed B_x to 1.1 ± 0.1 μT, where $f_{\rm Lar}$ is 7.7 kHz. The result is shown in Fig. 3(c). The result shows the behavior in the adiabatic regime in detail. At the reversal time of more than 200 μs, the phase shift is confirmed to be π.

IV. DISCUSSION

The described dependence of fringe visibilities on the reversal speed of the magnetic field was analyzed using the Majorana transition [8]. The Majorana formula gives the probability of a spin transition from a state of magnetic quantum number m to m' in a model where the magnetic field evolves as $B_y(t) = 0$ and $B_x(t) = B_x$, and $B_z(t)$ evolves as Eq. (2). For a multilevel system with total angular momentum F, the transition probability for m to m' is [8–10]

$$P^F_{m,m'} = (F+m)!(F+m')!(F-m)!(F-m')![\sin(\theta/2)]^{4F} \times \left\{ \sum_{r=0}^{2F} \frac{(-1)^r [\cot(\theta/2)]^{2r+m+m'}}{r!(r+m+m')!(F-m-r)!(F-m'-r)!} \right\}^2, \tag{3}$$

where the value of θ is given by the two-level transition

$$\sin^2(\theta/2) = P_{1/2,-1/2} = \exp\left(-\frac{f_{\rm Lar}}{f_{\rm rot}}\right) = \exp\left(-\frac{g\pi\mu_B B_x^2}{hB_z} T_R\right). \tag{4}$$

Figure 4 shows the Majorana transition probability of m to m' related to the present fringes for various reversal times under the condition of $B_x = 0.95\ \mu$T and $B_z = 5.0\ \mu$T. The transition probability from $|1,0\rangle$ to $|1,0\rangle$ decreases to zero at 175 μs, because the $|1,0\rangle$ state transitions to the $|1,\pm1\rangle$ states perfectly. The transition probability recovers to one in the nonadiabatic limit. The transition probability from $|2,0\rangle$ to $|2,0\rangle$ becomes zero at 390 μs and at 60 μs. On the other hand, the transition probabilities from $|1,\pm1\rangle$ to $|1,0\rangle$ become maximum at 175 μs.

The atoms in the $F = 1$ state excited by the first $\pi/2$ Raman pulse are in a superposition state with the $|1,0\rangle$ and

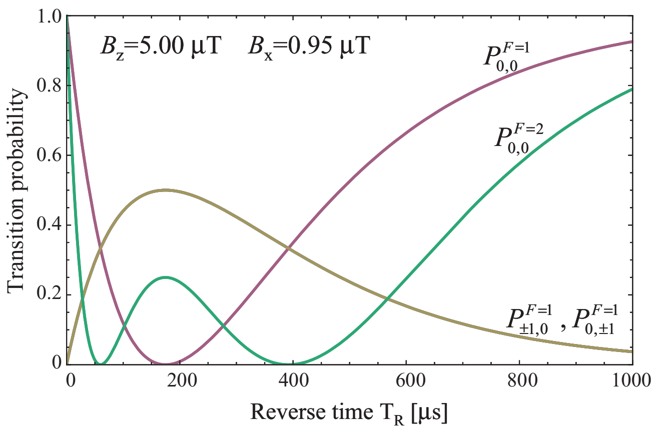

FIG. 4. (Color online) Majorana transition probabilities $P^F_{m,m'}$ from m to m' in the F state, which are related to the population probabilities in the $F = 2$, $m_F = 0$ state, for various reverse times under the conditions of $B_x = 0.95\ \mu$T and $B_z = 5.0\ \mu$T.

$|2,0\rangle$ states or in the $|1,\pm1\rangle$ states, which do not correlate with the superposition state. Because of the Majorana transition, the population probability of the $|1,0\rangle$ and $|2,0\rangle$ states in a superposition wave function varies, and the population of the noncorrelated $|1,0\rangle$ state appears from the $|1,\pm1\rangle$ states. Taking the Majorana transition probability into consideration, the transition probability of the excited state bb^* after interactions with two Raman pulses is given by

$$bb^* = \frac{1}{6}\left(\frac{P_{0,0}^{F=2} + P_{0,0}^{F=1}}{2} + \sqrt{P_{0,0}^{F=2}P_{0,0}^{F=1}}\cos\Phi + P_{1,0}^{F=1} + P_{-1,0}^{F=1}\right). \quad (5)$$

Then, the visibility is given by

$$V = \frac{2\sqrt{P_{0,0}^{F=2}P_{0,0}^{F=1}}}{P_{0,0}^{F=2} + P_{0,0}^{F=1} + 2P_{1,0}^{F=1} + 2P_{-1,0}^{F=1}}. \quad (6)$$

The calculated values of visibility were fitted to the experimental values, using B_x as a parameter. The maximum visibility was normalized to the experimental value under a constant magnetic field. The results are shown in Figs. 3(a) to 3(c) as solid curves. The coincidence between the experiment and calculation is very good. Therefore, we could explain, in terms of the Majorana transition, that the visibility of the interference fringes decreases in the intermediate region between the adiabatic and nonadiabatic regions. On the other hand, the fitted B_x values for the results in (a), (b), and (c) were 0.32, 0.95, and 1.6 μT, respectively. They were somewhat larger than the experimental values. The differences may be caused by the thermal velocity distribution of cold sodium atoms [12].

V. CONCLUSION

In summary, the phase shift and visibility of fringes in the $m = 0$ spin states were examined systematically for half-rotation of the magnetic field from the adiabatic to the nonadiabatic regime by Ramsey atom interferometry. The experimental results showed that the phase shift is π rad in the adiabatic regime or 0 rad in the nonadiabatic regime and that the visibility of fringes disappears in the intermediate region. This means that the wave function does not follow the rapid rotation of the magnetic field. The disappearance of the fringes in the intermediate regime is explained in terms of the Majorana transition caused by the rapid rotation of the magnetic field. These dependences of the phase shift and fringe visibility on the reversal speed of the quantization magnetic field should be taken into consideration when evaluating the ultimate uncertainty of an atomic clock using the transitions between the $m = 0$ sublevels.

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