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Decoherence suppression by quantum measurement reversal

Alexander N. Korotkov and Kyle Keane
Department of Electrical Engineering and Department of Physics & Astronomy, University of California, Riverside, California 92521, USA
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We show that qubit decoherence due to zero-temperature energy relaxation can be almost completely suppressed by using the quantum uncollapsing (measurement reversal) procedure. To protect a qubit state, a partial quantum measurement moves it toward the ground state, where it is kept during the storage period, while the second partial measurement restores the initial state. This procedure preferentially selects the cases without energy decay events. Stronger decoherence suppression requires smaller selection probability; a desired point in this trade-off can be chosen by varying the measurement strength. The experiment can be realized in a straightforward way using the superconducting phase qubit.

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Qubit decoherence can be efficiently suppressed via the quantum error correction by encoding the logical qubit in several physical qubits and performing sufficiently frequent measurement and correction operations [1]. The use of a larger Hilbert space is also needed in the idea of decoherence-free subspace [2]. Without increasing the physical Hilbert space, it is possible to suppress decoherence using the technique of dynamical decoupling based on sequences of control pulses, for example, by the "bang-bang" control [3]. However, the dynamical decoupling does not help [3,4] when the decoherence is due to processes with short correlation time scales, as, for example, for the standard (Markovian) energy relaxation. The energy relaxation can in principle be suppressed by changing properties of the qubit environment, as for suppression of spontaneous radiation from atoms [5]; however, this possibility does not seem practical for solid-state qubits. In this Rapid Communication we show that the energy relaxation in a single physical qubit can also be suppressed by using quantum uncollapsing [6,7].

The uncollapsing is a probabilistic reversal [6] of a partial quantum measurement by another measurement with an "exactly contradicting" result, so the total classical information is zeroed, thus making it possible to restore any initial quantum state. If the second measurement gives this desired result, the initial state is restored, while if the measurement result is different, the uncollapsing attempt is unsuccessful. The probability of success (selection) decreases with increasing strength of the first measurement, so that uncollapsing has zero probability for a traditional projective measurement. Perfect uncollapsing requires an ideal (quantum-efficient) detector. The quantum uncollapsing has been demonstrated experimentally [7] for a superconducting phase qubit [8], attracting some general interest [9] (a single-photon qubit has been uncollapsed in Ref. [10]).

The logic states in the phase qubit are represented by two lowest energy levels in a quantum well, separated by $\sim 25\ \mu$eV, and energy relaxation presents the major decoherence process, often nearly dominating in comparison with pure dephasing [11]. The experimental temperature of ~ 50 mK in this case corresponds to essentially the zero-temperature limit. This is exactly the regime, in which uncollapsing can be used to suppress the qubit decoherence (similar zero-temperature regime with negligible pure dephasing is realized in transmon qubits [12]).

In order to protect the qubit against zero-temperature energy relaxation, we first apply a partial quantum measurement (Fig. 1), which moves the qubit state toward the ground state in a coherent but nonunitary way (as in Ref. [13]). Then after the storage period we apply the uncollapsing procedure (for the phase qubit consisting of a π pulse, second partial measurement, and one more π pulse) which restores the initial qubit state. The protocol is very close to the existing uncollapsing experiment [7]. The procedure is probabilistic, since it selects only specific results of both measurements. (In this respect it is similar to linear optics quantum computing [14], which also relies on specific measurement results.) If an energy relaxation event happens during the storage period, then such a case will be preferentially rejected at the selection of the second measurement result. However, there is a trade-off: by increasing the strength of measurements we obtain stronger decoherence suppression, but decrease the selection probability.

To analyze the procedure quantitatively, let us assume that the initial state of the qubit in the rotating frame is $|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$. The partial measurement is performed in the standard for the phase qubit way [11,13] by allowing the state $|1\rangle$ to tunnel out of the quantum well with the probability p, while the state $|0\rangle$ cannot tunnel out. In the null-result case of no tunneling the qubit state becomes [13,15]

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle = \frac{\alpha|0\rangle + \beta\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)}}, \qquad (1)$$

and the probability of no tunneling is $P_1 = |\alpha|^2 + |\beta|^2(1-p)$. After the storage period τ the qubit state is no longer pure because of (zero-temperature) energy relaxation with the rate $\Gamma = 1/T_1$. However, it is technically easier to use the mathematical trick of "unraveling" the relaxation into "jump" and "no jump" scenarios and work with pure states. We can think that after the storage time τ the qubit jumps into the state $|0\rangle$ with the total probability $P_2^{|0\rangle} = P_1|\beta_1|^2(1-e^{-\Gamma\tau})$, while it ends up in the state

$$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle = \frac{\alpha|0\rangle + \beta\sqrt{1-p}\,e^{-\Gamma\tau/2}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)\,e^{-\Gamma\tau}}} \qquad (2)$$

with "no jump" probability $P_2^{\rm nj} = |\alpha|^2 + |\beta|^2(1-p)\,e^{-\Gamma\tau}$. Notice that we made the Bayesian-like update [16] of the qubit state $|\psi_2\rangle$ in the "no jump" scenario; such an update must be

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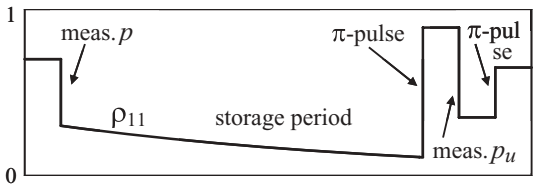

FIG. 1. Illustration of the uncollapsing sequence suppressing energy relaxation in a phase qubit: partial measurement with strength p, relatively long "storage" period, π pulse, second measurement with strength p_u, and π pulse. The line illustrates evolution of the element ρ_{11} of the qubit density matrix. We select only null-result cases for both measurements.

done even when the jump is not monitored, as can be easily checked by comparing the resulting density matrices.

After applying the π pulse the qubit state becomes either $|1\rangle$ or $\alpha_2|1\rangle + \beta_2|0\rangle$ with the same probabilities $P_2^{|0\rangle}$ and $P_2^{\rm nj}$. Then after the second (uncollapsing) measurement with strength p_u, in the no-tunneling case the qubit remains in the state $|1\rangle$ with the total probability $P_3^{|1\rangle} = P_2^{|0\rangle}(1-p_u)$, while its state becomes

$$\alpha_3|0\rangle + \beta_3|1\rangle = \frac{\beta\sqrt{1-p}\,e^{-\Gamma\tau/2}|0\rangle + \alpha\sqrt{1-p_u}\,|1\rangle}{\sqrt{|\alpha|^2(1-p_u) + |\beta|^2(1-p)\,e^{-\Gamma\tau}}} \qquad (3)$$

with probability $P_3^{\rm nj} = |\alpha|^2(1-p_u) + |\beta|^2(1-p)\,e^{-\Gamma\tau}$. Finally, the second π pulse produces either the state $|0\rangle$ with probability $P_f^{|0\rangle} = P_3^{|1\rangle}$ or the final state $|\psi_f\rangle = \beta_3|0\rangle + \alpha_3|1\rangle$ with probability $P_f^{\rm nj} = P_3^{\rm nj}$.

It is easy to see that in the "no jump" scenario the best (exact) restoration of the initial state is when $p_u = 1 - e^{-\Gamma\tau}(1-p)$, and in this case the final state is

$$|\psi_f\rangle = |\psi_{in}\rangle \text{ with probability } P_f^{\rm nj} = (1-p)\,e^{-\Gamma\tau}, \qquad (4)$$

$$|\psi_f\rangle = |0\rangle \text{ with } P_f^{|0\rangle} = |\beta|^2(1-p)^2 e^{-\Gamma\tau}(1-e^{-\Gamma\tau}). \qquad (5)$$

In the language of density matrix this means that both measurements produce null results (no tunneling) with the selection probability $P_f = P_f^{\rm nj} + P_f^{|0\rangle}$, and in such a case the final qubit state is

$$\rho_f = \left(P_f^{\rm nj}|\psi_{in}\rangle\langle\psi_{in}| + P_f^{|0\rangle}|0\rangle\langle 0|\right)\Big/\left(P_f^{\rm nj} + P_f^{|0\rangle}\right). \qquad (6)$$

An important observation is that the "good" probability $P_f^{\rm nj}$ scales as $1-p$ with the measurement strength p, while the "bad" probability $P_f^{|0\rangle}$ scales as $(1-p)^2$. Therefore, choosing p close to 1, we can make the final qubit state *arbitrarily close to the initial state*, even in the presence of a significant decoherence due to energy relaxation, $\Gamma\tau \gtrsim 1$ (though for the price of a very small selection probability, $P_f \to 0$). This is our main result.

Decoherence suppression can be explained as occurring because the storage state is close to the ground state, where the energy relaxation is naturally suppressed. For $|\psi_{in}\rangle$ close to $|1\rangle$ the explanation is a preferential selection of "no jump" cases by the second measurement.

We can characterize the performance of the procedure by calculating the fidelity of the quantum state storage and analyzing its increase with the measurement strength p. The fidelity of a quantum operation is usually defined using the χ-matrix representation [1] as $F_\chi = {\rm Tr}(\chi\chi_0)$ where χ and χ_0 characterize the actual and desired operations. In particular, this characteristic has been used in the quantum process tomography (QPT) experiments which involve selection of certain measurement results [7,10,17], even though strictly speaking it is inapplicable in this case. The reason for the inapplicability is that the QPT approach assumes a linear quantum operation, while the selection procedure involves renormalization of the density matrix, which in general makes the mapping nonlinear. Nevertheless, as discussed below, in our case the fidelity F_χ can still be defined in a "naive" way by using 4 standard initial qubit states to calculate χ (as was done in Ref. [7]), and the result practically coincides with another, more rigorous, definition. The definition which still works in the presence of selection is the average state fidelity [1] $F_{av} = \int {\rm Tr}(\rho_f U_0|\psi_{in}\rangle\langle\psi_{in}|U_0^\dagger)\,d|\psi_{in}\rangle$, where $U_0 = \mathbb{1}$ is the desired unitary operator, $\rho_f(|\psi_{in}\rangle)$ is the actual mapping [given by Eq. (6)], and the normalized integral is over all pure initial states $|\psi_{in}\rangle$ (using the Haar measure). For trace-preserving operations (without selection) $F_{av} = (F_\chi d + 1)/(d+1)$ [18], where $d = 2$ is the dimension of our Hilbert space. Therefore, it is natural to define a scaled average fidelity $F_{av}^s \equiv (3F_{av} - 1)/2$, which would coincide with F_χ in a no-selection case.

The state fidelity $F_{st} = {\rm Tr}(\rho_f|\psi_{in}\rangle\langle\psi_{in}|)$ between the desired unevolved state $|\psi_{in}\rangle$ and the actual state ρ_f given by Eq. (6) is $F_{st} = 1 - |\beta|^2 P_f^{|0\rangle}/P_f$. In order to average F_{st} over the initial state we use the integration result

$$\left\langle \frac{|\beta|^4}{A + B|\beta|^2} \right\rangle_{\rm Bl} = \frac{1}{2B} - \frac{A}{B^2} + \frac{A^2}{B^3}\ln\left(1 + \frac{B}{A}\right), \qquad (7)$$

where $\langle..\rangle_{\rm Bl}$ denotes averaging over the Bloch sphere. Using $A = 1$ and $B = (1-p)(1-e^{-\Gamma\tau})$ [see Eqs. (4)–(6), the common factor $(1-p)e^{-\Gamma\tau}$ is canceled], we thus find

$$F_{av} = \frac{1}{2} + \frac{1}{C} - \frac{\ln(1+C)}{C^2}, \quad C = (1-p)(1-e^{-\Gamma\tau}), \qquad (8)$$

and the corresponding scaled fidelity $F_{av}^s = (3F_{av} - 1)/2$. It is important to note that while the fidelity F_{av}^s increases with the measurement strength p, this happens for the price of decreasing the average selection probability $\langle P_f\rangle_{Bl} = (1-p)e^{-\Gamma\tau}(1 + C/2)$. In particular, for $p \to 1$ we have $F_{av}^s \to 1$, but $\langle P_f\rangle_{Bl} \to 0$.

In experiments the one-qubit process fidelity F_χ is usually defined by starting with four specific initial states: $|0\rangle$, $|1\rangle$, $(|0\rangle + |1\rangle)/\sqrt{2}$, and $(|0\rangle + i|1\rangle)/\sqrt{2}$, measuring the corresponding final states ρ_f, then calculating the χ matrix, and finally obtaining F_χ. Even for a nonlinear quantum operation this is a well-defined procedure (just the result may depend on the choice of the initial states), so it is meaningful to calculate F_χ defined in this (naive) way. Such defined F_χ coincides with F_χ for a linear trace-preserving operation, which would give the same final states for the four chosen initial states. Next, we use the fact [18] that the average fidelity $\tilde{F}_{av}$ for this "substitute" operation is equal to F_{st} averaged over only six initial states: $|0\rangle$, $|1\rangle$, $(|0\rangle \pm |1\rangle)/\sqrt{2}$,

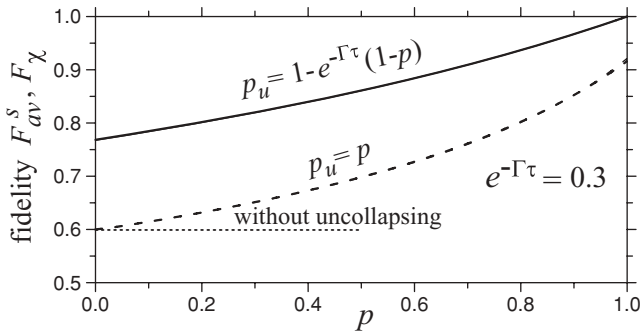

FIG. 2. Fidelity of the quantum state storage using uncollapsing as a function of the first measurement strength p for $e^{-\Gamma\tau} = 0.3$. We plot both F^s_{av} and F_χ, but they are indistinguishable by eye. Solid and dashed lines are for two choices of the second measurement strength p_u. Horizontal dotted line indicates the fidelity without uncollapsing.

and $(|0\rangle \pm i|1\rangle)/\sqrt{2}$. Since in our case F_{st} is phase insensitive, we get $\tilde{F}_{av} = [F_{st}(|0\rangle) + F_{st}(|1\rangle) + 4F_{st}(\frac{|0\rangle+|1\rangle}{\sqrt{2}})]/6$, giving

$$\tilde{F}_{av} = \frac{1}{6} + \frac{1}{6(1+C)} + \frac{4+C}{3(2+C)}. \tag{9}$$

Then the "naive" fidelity is simply $F_\chi = (3\tilde{F}_{av} - 1)/2$.

The efficiency of the energy relaxation suppression by uncollapsing is illustrated in Fig. 2 by plotting (solid lines) the scaled average fidelity F^s_{av} and the "naive" fidelity F_χ as functions of the measurement strength p for a quite significant energy relaxation: $e^{-\Gamma t} = 0.3$. Note that F^s_{av} and F_χ are practically indistinguishable (within thickness of the lines), despite different functional dependences in Eqs. (8) and (9). Also note that even for $p = 0$ the fidelities differ from the fidelity without uncollapsing ($F_\chi = 1/2 + e^{-\Gamma\tau}/4 + e^{-\Gamma\tau/2}/2 \approx 0.6$), shown by the dotted line in Fig. 2. This is because we assumed $p_u = 1 - e^{-\Gamma\tau}(1-p)$, so $p_u \neq 0$ even for $p = 0$, and the second measurement improves the fidelity. If we choose $p_u = p$ (two dashed lines, still indistinguishable by eye) as in the standard uncollapsing [6,7], then the case $p = 0$ is equivalent to the absence of any procedure. [The dashed lines are calculated in a similar way, assuming $p_u = p$ in Eq. (3).] It is interesting to notice that if we numerically maximize the fidelity F^s_{av} by optimizing over p_u, then we can get larger F^s_{av} (for the same p) than in the case $p_u = 1 - e^{-\Gamma\tau}(1-p)$; however, this will decrease the selection probability $\langle P_f\rangle_{Bl}$, and for the same $\langle P_f\rangle_{Bl}$ such optimization slightly decreases F^s_{av}.

So far we assumed that the energy relaxation happens only during the storage period, while there is no decoherence during the uncollapsing procedure (measurements and π pulses). Even though such an assumption is justified since the storage period for a quantum memory is supposed to be relatively long, let us take a step closer to reality and take into account energy relaxation during all durations illustrated by horizontal lines in Fig. 1 (except the last one, which is after the procedure is finished). The energy relaxation (still zero temperature) will be characterized by parameters $\kappa_i = \exp(-\Gamma\tau_i)$, $i = 1$–4, where τ_1 is the duration before the first measurement, $\tau_2 = \tau$ is the storage period, τ_3 is the duration between the first π pulse and second measurement, and τ_4 is between the second measurement and second π pulse (the measurements and π pulses are still assumed ideal). Using the same derivation as above and selecting only the null-result cases for both measurements, we can show that for the initial state $|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$ the final state can be unraveled as

$$|\psi_f^{\rm nj}\rangle = \frac{\alpha\sqrt{\kappa_3\kappa_4(1-p_u)}|0\rangle + \beta\sqrt{\kappa_1\kappa_2(1-p)}|1\rangle}{(P_f^{\rm nj})^{1/2}} \tag{10}$$

with the "no jump" probability $P_f^{\rm nj} = |\alpha|^2\kappa_3\kappa_4(1-p_u) + |\beta|^2\kappa_1\kappa_2(1-p)$, the state $|\psi_f\rangle = |0\rangle$ with probability $P_f^{|0\rangle} = |\alpha|^2[1-\kappa_3+\kappa_3(1-p_u)(1-\kappa_4)] + |\beta|^2[1-\kappa_1+\kappa_1(1-p)(1-\kappa_2)][1-\kappa_3+\kappa_3(1-p_u)(1-\kappa_4)]$, and also $|\psi_f\rangle = |1\rangle$ with probability $P_f^{|1\rangle} = |\beta|^2[1-\kappa_1+\kappa_1(1-p)(1-\kappa_2)]\kappa_3(1-p_u)\kappa_4]$ (all terms in these formulas have rather obvious physical meaning). Actual density matrix is then $\rho_f = (P_f^{\rm nj}|\psi_f^{\rm nj}\rangle\langle\psi_f^{\rm nj}| + P_f^{|0\rangle}|0\rangle\langle 0| + P_f^{|1\rangle}|1\rangle\langle 1|)/P_f$, where $P_f = P_f^{\rm nj} + P_f^{|0\rangle} + P_f^{|1\rangle}$ is the selection probability. It is also rather simple to take into account the additional decoherence due to the pure dephasing with rate Γ_φ. It can be shown that the only change will be the pure dephasing of the state (10) with the factor $\kappa_\varphi = \exp(-\Gamma_\varphi\sum_{i=1}^4\tau_i)$.

The state fidelity then can be calculated in a straightforward way, and the averaging over the initial state can be performed as above using the integration result (7) and similar result $\langle|\alpha|^4/(A+B|\beta|^2)\rangle_{Bl} = -(3/2B) - (A/B^2) + (1/B)(1+B/A)^2\ln(1+B/A)$. The final result for the scaled averaged fidelity F^s_{av} is analytical but rather lengthy (as well as for F_χ and $\langle P_f\rangle_{Bl}$).

Solid lines in Fig. 3 show the p dependence of the fidelities F^s_{av} and F_χ (they are still indistinguishable, being within the thickness of the line), for which we choose p_u from equation $\kappa_3\kappa_4(1-p_u) = \kappa_1\kappa_2(1-p)$ which comes from Eq. (10) and generalizes the equation $1 - p_u = e^{-\Gamma\tau}(1-p)$. For all solid lines we assume $k_2 = 0.3$. The upper line is for the ideal case $\kappa_1 = \kappa_3 = \kappa_4 = \kappa_\varphi = 1$ (so it is the same as in Fig. 2). For all other lines $\kappa_\varphi = 0.95$, while $\kappa_1 = \kappa_3 = \kappa_4 = 1$,

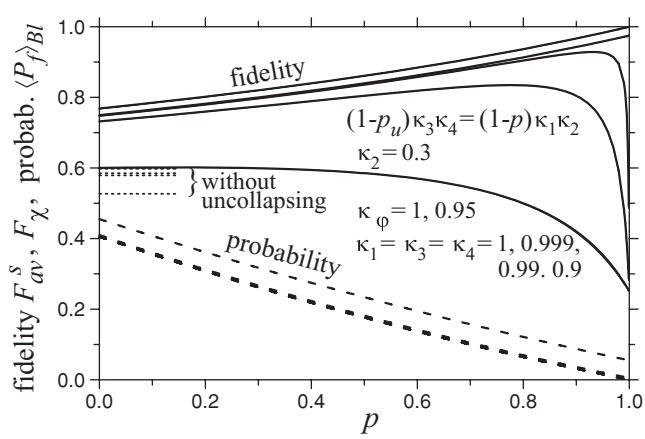

FIG. 3. Solid lines: fidelities F^s_{av} and F_χ of the state storage (still practically indistinguishable from each other), taking into account the energy relaxation and pure dephasing during all parts of the uncollapsing procedure for several sets of parameters (see text). Dashed lines: corresponding selection probabilities $\langle P_f\rangle_{Bl}$ (reverse order of curves). Dotted lines: corresponding fidelities without uncollapsing ($p = p_u = 0$).

0.999, 0.99, 0.9 (from top to bottom). Dotted lines show corresponding fidelities in the absence of uncollapsing ($p = p_u = 0$; then $F^s_{av} = F_\chi = 1/4 + \kappa_E/4 + \kappa_\varphi\sqrt{\kappa_E}/2$, where $\kappa_E = \kappa_1\kappa_2\kappa_3\kappa_4$). The dashed lines show the selection probability $\langle P_f \rangle$ of the procedure; these lines go in the opposite sequence (from bottom to top) compared to the solid and dotted lines.

As we see from Fig. 3, the uncollapsing essentially does not affect decoherence due to the pure dephasing (κ_φ), while the energy relaxation during the elements of the procedure (κ_1,κ_3,κ_4) has a less trivial effect: for small p it just reduces the fidelity, while for $p \to 1$ it causes fidelity to drop down to 0.25 (this value corresponds to complete decoherence; the fidelity decrease is mainly affected by κ_3). Note that the lowest solid line does not show a noticeable increase of the fidelity with p before it starts to decrease. This behavior is similar to the results of the uncollapsing experiment [7], in which the "storage" time between the first measurement and π pulse was not longer than other durations. Changing the experimental protocol of Ref. [7] by a relative increase of the storage time, we would expect to observe an initial increase of the fidelity with p, thus confirming that uncollapsing can suppress decoherence.

Notice that all solid lines in Fig. 3 are significantly above the standard fidelity (dotted lines, $p = p_u = 0$) for moderate measurement strength p. Significant increase of the fidelity is especially remarkable in view of the fact [4] that arbitrary Hamiltonian evolution cannot even slightly improve the fidelity in our case. So, the uncollapsing is *the only* known to us method of improving the qubit storage fidelity against energy relaxation, which does not rely on encoding a logical qubit in a larger Hilbert space, and the only method experimentally realizable today (the price though is a selection of certain measurement results). Our idea also works for entangled qubits [19].

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[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
[2] D. A. Lidar, I. L. Chuang, and K. B. Whaley, Phys. Rev. Lett. **81**, 2594 (1998).
[3] L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999).
[4] L. P. Pryadko and G. Quiroz, Phys. Rev. A **80**, 042317 (2009).
[5] S. Haroche and D. Kleppner, Phys. Today **42**, 24 (1989).
[6] A. N. Korotkov and A. N. Jordan, Phys. Rev. Lett. **97**, 166805 (2006); A. N. Jordan and A. N. Korotkov, Contemp. Phys. **51**, 125 (2010).
[7] N. Katz *et al.*, Phys. Rev. Lett. **101**, 200401 (2008).
[8] J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901 (2002).
[9] Z. Merali, Nature **454**, 8 (2008).
[10] Y. S. Kim *et al.*, Opt. Express **17**, 11978 (2009).
[11] J. M. Martinis, Quant. Info. Proc. **8**, 81 (2009).
[12] A. A. Houck, J. Koch, M. H. Devoret, S. M. Girvin, and R. J. Schoelkoph, Quant. Info. Proc. **8**, 105 (2009).
[13] N. Katz *et al.*, Science **312**, 1498 (2006).
[14] E. Knill, R. Laflamme, and G. J. Milburn, Nature **409**, 46 (2001).
[15] In real experiment [13] the partial measurement leads to an additional phase shift between states $|0\rangle$ and $|1\rangle$. We neglect it in Eq. (1) because it can be easily compensated and sometimes cancels out automatically [7].
[16] A. N. Korotkov, Phys. Rev. B **60**, 5737 (1999).
[17] J. L. O'Brien *et al.*, Phys. Rev. Lett. **93**, 080502 (2004).
[18] M. A. Nielsen, Phys. Lett. A **303**, 249 (2002).
[19] Suppose now that the qubit is entangled with other qubits, which do not decohere. Parameterizing initial state as $|\alpha 0\rangle|\psi_0\rangle + \beta|1\rangle|\psi_1\rangle$, it is easy to show that Eq. (10) changes only trivially, and the contribution to the state fidelity from the "no jump" scenario does not change at all, while several terms in the contribution from $|0\rangle$ and $|1\rangle$ will be multiplied by $\langle\psi_0|\psi_1\rangle^2$. Therefore, the results of this paper do not change qualitatively for entangled qubits, though they change quantitatively.