Transfer behavior of quantum states between atoms in photonic crystal coupled cavities

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In this article, we discuss the one-excitation dynamics of a quantum system consisting of two two-level atoms each interacting with one of two coupled single-mode cavities via spontaneous emission. When the atoms and cavities are tuned into resonance, a wide variety of time-evolution behaviors can be realized by modulating the atom-cavity coupling strength g and the cavity-cavity hopping strength λ. The dynamics is solved rigorously via the eigenproblem of an ordinary coupled linear system and simple analytical solutions are derived at several extreme situations of g and λ. In the large hopping limit where $g \ll \lambda$, the behavior of the system is the linear superposition of a fast and slow periodic oscillation. The quantum state transfers from one atom to the other atom accompanied with weak excitation of the cavity mode. In the large coupling limit where $g \gg \lambda$, the time-evolution behavior of the system is characterized by the usual slowly varying carrier envelope superimposed upon a fast and violent oscillation. At a certain instant, the energy is fully transferred from the one quantum subsystem to the other. When the two interaction strengths are comparable in magnitude, the dynamics acts as a continuous pulse having irregular frequency and line shape of peaks and valleys, and the complicated time-evolution behaviors are ascribed to the violent competition between all the one-excitation quantum states. The coupled quantum system of atoms and cavities makes a good model to study cavity quantum electrodynamics with great freedoms of many-body interaction.

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I. INTRODUCTION

Since the first proposal by Purcell [1] and early works on atoms in cavities, cavity quantum electrodynamics (QED) has been actively pursued for its potential insight into the fundamental problems of light-matter interaction [2]. Recently cavity QED has been explored as a promising scheme toward realizing many important quantum information processing tasks [3–6]. One requirement of distributed quantum information processing is the coupling of the distant qubits in order to perform state transfer which is the entanglement generation or quantum gate operations between separate nodes of the systems. Atomic systems are qualified to act as qubits, as appropriate internal electronic states can coherently store information over a very long time scale. Photons are well suited to distribute information throughout the system due to high transport speed and large bandwidth. High-finesse cavities can provide good insulation against the environment and thus can hold photons over a long enough time scale before dissipation.

In accordance with a general desire to control quantum systems in an all solid platform for the sake of practical application and ease of integration, significant attention has turned toward cavity QED in photonic crystals (PCs). PCs are artificial structures with a periodic modulation in the dielectric function. They exhibit photonic band gaps (PBGs), a range of frequencies within which no propagating electromagnetic modes are allowed [7]. A point defect in PBG materials defines a PBG cavity, as any photon injected into that site cannot propagate laterally away from the defect [8,9]. In this way, PBG cavities can constitute extremely good cavities with superb photonic confinement (low loss and ultrasmall mode volume) and strong coupling for an embedded quantum system. These successes have been fueled by technological advances in microfabrication or nanofabrication in the modern era of nanoscience and nanotechnology. Therefore, a flexible design of the defect geometry is allowed in the PC platform. For example, the geometry and size of a PBG cavity can be subtly tuned to generate a very large quality factor up to several millions and simultaneously a very small modal volume down to a fraction of cubic wavelength [10–12]. Previous studies on several important quantum optics problems of atoms, such as spontaneous emission in PCs, showed that the key physical function governing the interaction of light with these quantum systems is the photon local density of states (LDOS), which is a function of atom position due to the strong inhomogeneity of Bloch-wave field in PCs [13–15]. LDOS is directly connected to the coupling strength with the quantum emitter, for example, a quantum dot (QD), which is often considered to be a two-level atom-like emitter [16,17]. In addition to the flexible control of light-atom interaction, PCs also offer a good on-chip platform to transport and communicate information of quantum states by means of different kinds of integrated optical devices.

Understanding of the properties of the single atom-cavity system can help to gain insight into many cavity QED problems such as the spectrum of one-atom lasers [18], the QD-embedded PBG cavity thresholdless laser [19,20], and giant optical nonlinearity [21]. It also forms the basis for further study of the physical effects of two coupled PBG cavities, each having a single atom in it. The coupling between two cavities can be controlled in several ways, which offers great freedom and flexibility to engineer the transport of quantum states via photonic processes. One way is to connect the two cavities indirectly by a quantum channel such as conventional optical

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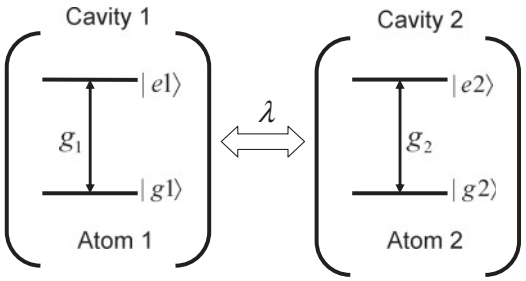

FIG. 1. Schematic diagram of a system formed by two coupled cavities, each of which contains a two-level atom. The atoms are coupled to their cavity modes with strength g_1 and g_2. Photons can hop between the cavities at rate λ.

fiber [22–24] or the PC waveguide [3,25,26] along which the photonic state propagates. Another way is to directly link the two cavities by the overlap of evanescent cavity fields [27–30], which allows photons to hop from one cavity to the other.

In this article we study cavity QED problems in the direct-coupling system, which is schematized in Fig. 1. Two defects in the PC lattice were placed at a certain distance from each other to form evanescently coupled cavities. The coupling strength can be conveniently tuned by changing the cavity-cavity distance in a nearly exponentially decaying manner [31,32]. Each cavity involves a single atom having the same resonant frequency. When atoms and cavities are tuned into resonance and dissipation is neglected, the system exhibits a coherent oscillatory energy exchange between the atom and the cavity when a single excitation is initially exerted on one of the atoms. In this system, two coupling factors are involved: one is the atom-cavity coupling strength g_j $(j = 1, 2)$, which is directly connected to the position of the embedded atom due to the inhomogeneous field distribution of the cavity defect mode. The other is the cavity-cavity hopping strength λ, whose intensity can be modulated by changing the distance between the two defects. The coupled PC cavities thus offer very many freedoms to control the interaction between atoms and photons, and photons and photons. One can expect to find different types of dynamical evolution about the atom states and cavity photon states, based on which interesting physical phenomena can be exploited and harnessed.

For simplicity, we only consider a situation where $g = g_1 = g_2$. This can be achieved by simply placing the two atoms in the same position of the two cavities. The physical dynamics can be clearly illustrated by three different cases of the hopping strength relative to the atom-cavity coupling strength. The first case is that the atom-cavity coupling strength g is much smaller than the cavity-cavity hopping strength λ ($g \ll \lambda$). The quantum state transfers from one atom to the other atom periodically at a specific frequency accompanied with weak excitation of the cavity mode. The second case is the large coupling extreme situation where $g \gg \lambda$. The time-evolution behavior of the system in this limit presents the familiar form of slowly varying carrier envelope pulses. The third case is that the detuning between g and λ is small, or in other words, g and λ are comparable in magnitude. The role of competition between the coupling strength and the hopping strength becomes important and it will result in complicated amplitude modulation behaviors.

This article is organized as follows. In Sec. II, we derive the quantum mechanical equations governing the time-evolution behavior of the quantum system and present an analytical formalism to look into the dynamics of atoms and photons. In Sec. III, we make a detailed analysis of the three cases. Both numerical calculations and analytical formalisms are used to sharpen and deepen our understanding of the quantum dynamics of the coupled atom-photon system. In Sec. IV, we summarize this article.

II. THEORETICAL MODELS

The system under consideration is depicted in Fig. 1, which is created in the PC platform. It consists of two coupled single-mode cavities with the same resonant frequency. Each cavity contains a two-level atom. The transition frequency of the atom is at resonance with the frequency of the cavity mode. In addition, the dissipation induced by photon loss and atom coupling with the environment are not taken into account. By changing the location of the atom embedded in the cavity, different coupling strengths between the atom and the cavity are obtained. Moreover, the energy transfer of photons between the two cavities is described by the parameter of cavity-cavity hopping strength, which can be efficiently modulated by the distance between the two cavities.

In the interaction picture, the Hamiltonian governing the coupled quantum system is in the form

$$H_I = i\hbar \sum_{j=1,2} [g_j(a_j^\dagger \sigma_{j-} - \sigma_{j+} a_j)] + i\hbar\lambda(a_1^\dagger a_2 - a_1 a_2^\dagger), \quad (j = 1, 2), \tag{1}$$

where $a_j^\dagger$ and a_j are creation and annihilation operators for the cavity modes, σ_{j+} and σ_{j-} $(j = 1, 2)$ are the rising and lowering operators of atom j, g_j are the coupling constants between the atom states and the cavity modes, and λ is the cavity-cavity hopping strength. These parameters are referred to as the cavity QED parameters.

The solution and discussion on the dynamics of the Hamiltonian are restricted to the subspace containing zero and one excitations. We can thus write the state vector of the system in time t as

$$\begin{aligned} |\Psi(t)\rangle &= |\psi_0(t)\rangle + |\psi_1(t)\rangle, \\ |\psi_0(t)\rangle &= |g1\rangle|g2\rangle|0\rangle_1|0\rangle_2, \\ |\psi_1(t)\rangle &= a(t)|g1\rangle|g2\rangle|1\rangle_1|0\rangle_2 \\ &\quad + b(t)|g1\rangle|g2\rangle|0\rangle_1|1\rangle_2 \\ &\quad + c(t)|e1\rangle|g2\rangle|0\rangle_1|0\rangle_2 \\ &\quad + d(t)|g1\rangle|e2\rangle|0\rangle_1|0\rangle_2, \end{aligned} \tag{2}$$

where $|gj\rangle$ and $|ej\rangle$ are the ground and excited states of atom j, and $|0\rangle_j$ and $|1\rangle_j$ are the vacuum and one-photon state of the cavity j. The zero-excitation component $|\psi_0(t)\rangle$ is invariant under the action of the Hamiltonian, so we need only consider the dynamics of the single-excitation component $|\psi_1(t)\rangle$.

We consider for the initial conditions that atom 1 is prepared in the excited state, atom 2 is in the fundamental state, and both cavities are in the vacuum state. The initial state can be

written as

$$|\psi_1(0)\rangle = |e1\rangle|g2\rangle|0\rangle_1|0\rangle_2. \tag{3}$$

The coefficients of $|\psi_1(0)\rangle$ are $a(0) = b(0) = d(0)$ and $c(0) = 1$. The time evolution of the general one-excitation state given by Eq. (3) may be calculated exactly from the Schrödinger equation.

$$\begin{aligned} \dot{a}(t) &= g_1 c(t) + \lambda b(t), \\ \dot{b}(t) &= g_2 d(t) - \lambda a(t), \\ \dot{c}(t) &= -g_1 a(t), \\ \dot{d}(t) &= -g_2 b(t). \end{aligned} \tag{4}$$

In order to solve this equation set, we assume a trial solution as $a(t) = Ae^{xt}$, $b(t) = Be^{xt}$, $c(t) = Ce^{xt}$, and $d(t) = De^{xt}$. Insertion of the trial solution into Eq. (4) leads to a set of linear equations of a 4×4 eigensystem,

$$\begin{pmatrix} 0 & \lambda & g_1 & 0 \\ -\lambda & 0 & 0 & g_2 \\ -g_1 & 0 & 0 & 0 \\ 0 & -g_2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = x \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}. \tag{5}$$

In Eq. (5) x is the eigenvalue. The solution of Eq. (5) is straightforward and yields four roots. The four eigenvalues x_n are given by

$$\begin{aligned} x_1 &= \left\{ \frac{1}{2}\left[-\left(g_1^2 + g_2^2 + \lambda^2\right) \right.\right. \\ &\quad \left.\left. + \sqrt{\left(g_1^2 + g_2^2 + \lambda^2\right)^2 - 4g_1^2 g_2^2} \right] \right\}^{1/2}, \\ x_2 &= -\left\{ \frac{1}{2}\left[-\left(g_1^2 + g_2^2 + \lambda^2\right) \right.\right. \\ &\quad \left.\left. + \sqrt{\left(g_1^2 + g_2^2 + \lambda^2\right)^2 - 4g_1^2 g_2^2} \right] \right\}^{1/2}, \\ x_3 &= \left\{ \frac{1}{2}\left[-\left(g_1^2 + g_2^2 + \lambda^2\right) \right.\right. \\ &\quad \left.\left. - \sqrt{\left(g_1^2 + g_2^2 + \lambda^2\right)^2 - 4g_1^2 g_2^2} \right] \right\}^{1/2}, \\ x_4 &= -\left\{ \frac{1}{2}\left[-\left(g_1^2 + g_2^2 + \lambda^2\right) \right.\right. \\ &\quad \left.\left. - \sqrt{\left(g_1^2 + g_2^2 + \lambda^2\right)^2 - 4g_1^2 g_2^2} \right] \right\}^{1/2}. \end{aligned} \tag{6}$$

A close look at Eq. (6) reveals that all the eigenvalues are pure imaginary numbers, which reflects the fact that the quantum system does not have dissipation. As a result, each trial function is a harmonic wave function. The general solution of the dynamics can be described by these wave functions.

The coefficient $a(t)$ for the one-excitation state $|g1\rangle|g2\rangle|1\rangle_1|0\rangle_2$ is given by

$$a(t) = \sum_{n=1}^{4} A_n e^{x_n t}, \tag{7}$$

and the coefficients of other one-excitation states can be derived from $a(t)$ by going back to Eq. (4). By using the initial condition of the four one-excitation states, we can work out the numerical value of A_n by simple calculations of the following linear equations:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ \left(x_1^2 + g_1^2\right)/\lambda x_1 & \left(x_2^2 + g_1^2\right)/\lambda x_2 & \left(x_3^2 + g_1^2\right)/\lambda x_3 & \left(x_4^2 + g_1^2\right)/\lambda x_4 \\ -g_1/x_1 & -g_1/x_2 & -g_1/x_3 & -g_1/x_4 \\ \left(x_1^2 + g_1^2 + \lambda^2\right)/\lambda g_2 & \left(x_2^2 + g_1^2 + \lambda^2\right)/\lambda g_2 & \left(x_3^2 + g_1^2 + \lambda^2\right)/\lambda g_2 & \left(x_4^2 + g_1^2 + \lambda^2\right)/\lambda g_2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} a(0) \\ b(0) \\ c(0) \\ d(0) \end{pmatrix}. \tag{8}$$

Other quantities can be readily calculated from $a(t)$ by considering Eq. (4). Finally, the amplitude of each state is given by

$$\begin{aligned} a(t) &= \sum_{n=1}^{4} A_n e^{x_n t}, \\ b(t) &= \sum_{n=1}^{4} \frac{x_n^2 + g_1^2}{\lambda x_n} A_n e^{x_n t}, \\ c(t) &= -\sum_{n=1}^{4} \frac{g_1}{x_n} A_n e^{x_n t}, \\ d(t) &= \sum_{n=1}^{4} \frac{x_n^2 + g_1^2 + \lambda^2}{\lambda g_2} A_n e^{x_n t}. \end{aligned} \tag{9}$$

The amplitudes of the four wave functions are given as

$$\begin{aligned} P_{C1}(t) &= a(t), \quad P_{C2}(t) = b(t), \\ P_{A1}(t) &= c(t), \quad P_{A2}(t) = d(t). \end{aligned} \tag{10}$$

It has been noticed that all the eigenvalues x_i $(i = 1, 2, 3, 4)$ are pure imaginary numbers. Besides we find that $\mathrm{Im}(x_1) = -\mathrm{Im}(x_2)$ and $\mathrm{Im}(x_3) = -\mathrm{Im}(x_4)$. This means that the evolution dynamics of all the four one-excitation states, which are the summation of four harmonic wave functions as in Eq. (9), is characterized by an oscillatory behavior without the amplitude decaying with time. However, as each state contributes from different channels, each of which is described by a harmonic wave function, the overall dynamics may exhibit a complex interference behavior. It turns out that the dynamics reflects the complex competition and balance of the atom-photon interaction and the photon-photon interaction, which ultimately determine the value of x_i. In general, the two-cavity system with two atoms is a complicated problem that does not allow simple analytic solution; however, by considering the condition $g = g_1 = g_2$, we can get significantly simplified and clarified insight. The condition is satisfied when the two atoms are placed at equivalent positions in the two cavities.

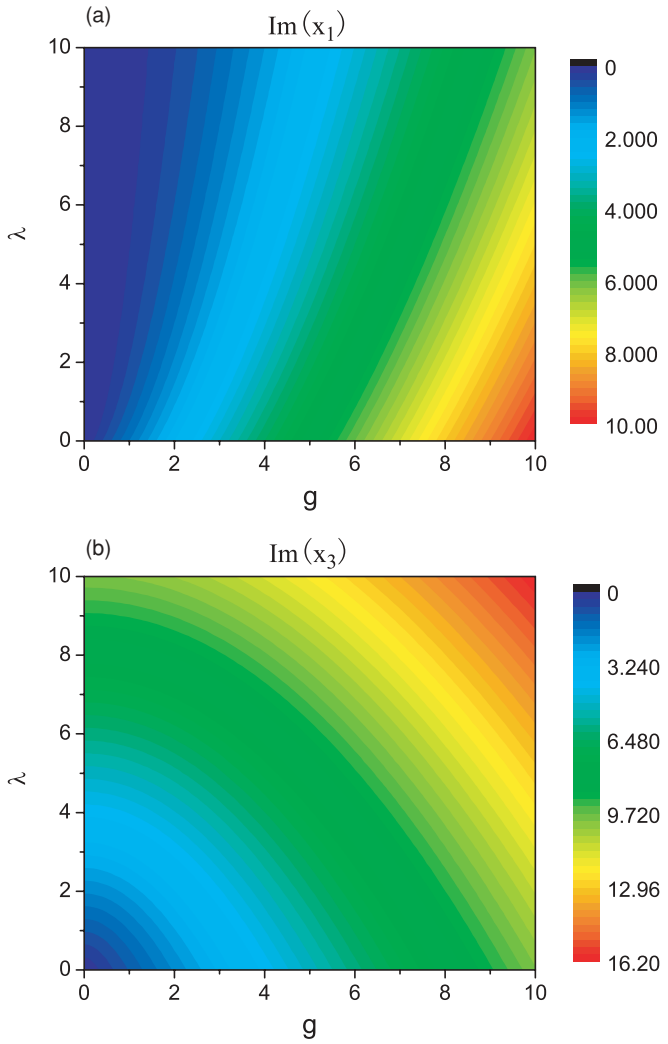

FIG. 2. (Color online) Calculated eigenvalues $\mathrm{Im}(x_1)$ and $\mathrm{Im}(x_3)$ as functions of interaction parameters of g and λ. g and λ are dimensionless.

To have some intuitive ideas about the dynamics of the quantum system, we calculate the eigenvalues $\mathrm{Im}(x_1)$ and $\mathrm{Im}(x_3)$ as functions of the two parameters g and λ from Eq. (6). The results are displayed in Fig. 2, where no approximation is made. As x_1 and x_3 represent the frequency of the oscillatory behavior of the cavity photon and the atomic state dynamics, respectively, Fig. 2 clearly shows that the oscillation frequency increases when both g and λ grow. Furthermore, $\mathrm{Im}(x_1)$ and $\mathrm{Im}(x_3)$ are subject to different influences from the two parameters. For $\mathrm{Im}(x_1)$, the influence of g is much more than that of λ in most situations, as can be clearly found by drawing a horizontal line and vertical line in Fig. 2(a). In contrast, g and λ have almost the same impact factor upon $\mathrm{Im}(x_3)$ in the entire parameter space, as can be seen from the nearly isotropic configuration of the contour plots in Fig. 2(b). Generally speaking, the dynamics of the quantum system of the coupled cavities and atoms in all situations can be visualized from the pictures in Fig. 2. However, there are several special situations where the pictures can be drastically simplified which will be addressed in much more detail in the following section. We will see the situation where the dynamics of the quantum system is determined either dominantly by the cavity-atom interaction, upon which the cavity-cavity coupling will add some fine modulations, or dominantly by the cavity-cavity coupling, upon which the atom-cavity coupling will induce some fine modifications, or cooperatively and competitively by the two parameters with none of them dominating.

III. RESULTS AND DISCUSSIONS

Since the system involves two coupling types, one is the atom-cavity coupling and the other is the cavity-cavity hopping, the detailed characterization of them in such a system still presents some open questions of significant interest, for example, the role of each coupling and the competition between them in shaping the single-excitation dynamics. The behavior of the excitation of the field mode and the atomic mode is illustrated by the following three cases. The first case is that the atom-cavity coupling strength g is far smaller than the cavity-cavity hopping strength λ expressed as $g \ll \lambda$. The second case can be expressed as $g \gg \lambda$. The third case is that g and λ have comparable magnitude, namely, $g \approx \lambda$.

A. The large hopping limit $g \ll \lambda$

In the $g \ll \lambda$ case, due to the weak coupling strength g, the system can be considered as two subsystems: one is the coupling between two atoms in which most energy is included; the other is the hopping of photons between the two cavities which acts as a pathway to transfer energy. The time evolution of the atomic states and the cavity photon states in the large hopping limit is shown in Fig. 3, which is calculated numerically from Eqs. (4)–(9). The curves represent the time evolution of the amplitude of the field modes and the atomic modes under different parameters of g and λ. Each curve in Fig. 3 includes two oscillating modes: the fast oscillation and the slow oscillation.

In Fig. 3(a), a small coupling strength $g = 0.1$ and a large hopping strength $\lambda = 1$ (all units are arbitrary) are set to satisfy the $g \ll \lambda$ limit. The behavior of the two identical atoms and the two resonant filed modes exhibits three distinct characters. First, as time goes on, the phase change of atomic mode 1 (P_{A1}) and field mode 2 (P_{C2}) undergoes a cosine oscillation. Meanwhile field mode 1 (P_{C1}) and atomic mode 2 (P_{A2}) display a sinusoidal action. There is a $\pi/2$ phase delay between atom mode 1 and atom mode 2, reflecting the transfer of energy from atom 1 to atom 2 through cavity field modes. Second, the fast oscillation period is equal to $T_f = 2\pi$, as shown more clearly in the right-hand zoomed view. The slow oscillation period is $T_s = 100 \times (2\pi)$, 100 times the fast one, as shown in the left-hand diagram. Third, the amplitudes of the slow and fast oscillations in the field mode are the same; they are both equal to 0.2. Meanwhile, the amplitude of the atomic mode fast oscillation is equal to 0.02 and that of the slow oscillation approaches unity. It illustrates that the quantum state transfer from atom 1 to atom 2 is accompanied with weak excitation of the cavity mode when the initial state has a single excitation in atom 1. The disturbance from the cavity subsystem to the atom subsystem is small.

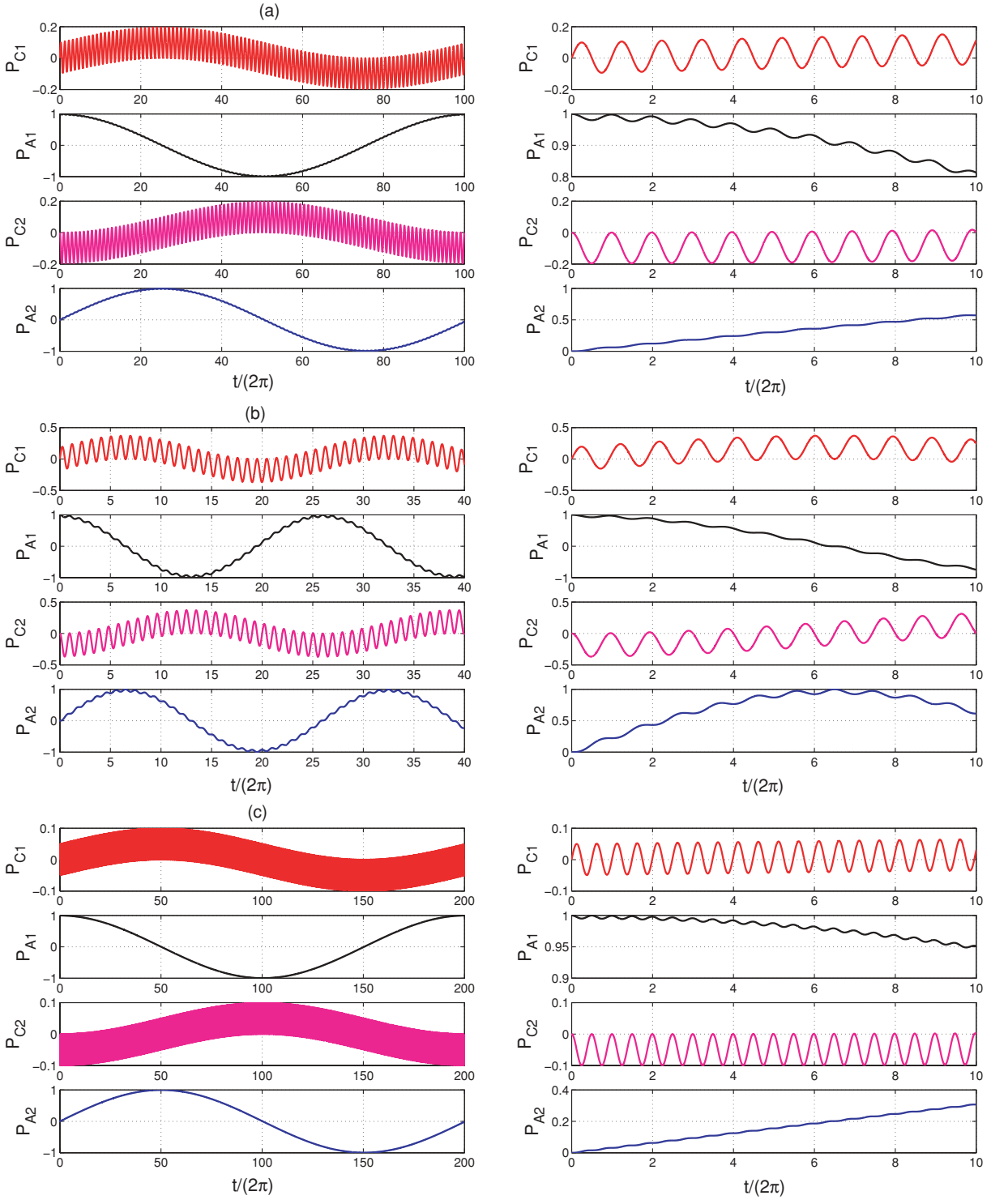

FIG. 3. (Color online) Dynamics of atomic and field excitation in the case $g \ll \lambda$. These curves give the amplitude, as a function of time, of the excitation of the field mode 1 (P_{C1}), the atomic mode 1 (P_{A1}), the field mode 2 (P_{C2}), and the atomic mode 2 (P_{A2}). The system parameters are (a) $g = 0.1$, $\lambda = 1.0$; (b) $g = 0.2$, $\lambda = 1.0$; and (c) $g = 0.1$, $\lambda = 2.0$. The panels on the right are zoomed views of the panels on the left. The time parameter t is dimensionless.

To find a more simplified and clarified description of the system dynamics behavior, we turn to Eq. (6) and try to obtain a simple description of the quantum system. Under the large hopping limit of $g_1 = g_2 = g \ll \lambda$, we obtain the following approximations based on Newton's method for the photonic state oscillation period as

$$x_1 = -x_2 \approx i\frac{g^2}{\lambda}, \qquad x_3 = -x_4 \approx i\lambda. \tag{11}$$

The corresponding amplitudes of the harmonic functions are

$$A_1 = A_3 = -A_2 = -A_4 \approx -i\frac{g}{2\lambda}.$$

The amplitudes of the four one-excitation wave functions are

$$P_{A1}(t) = \cos\left(\frac{g^2}{\lambda}t\right) + \frac{g^2}{\lambda^2}\cos(\lambda t), \tag{12}$$

$$P_{A2}(t) = \sin\left(\frac{g^2}{\lambda}t\right) + \frac{g^2}{\lambda^2}\sin(\lambda t), \tag{13}$$

$$P_{C1}(t) = \frac{g}{\lambda}\left[\sin\left(\frac{g^2}{\lambda}t\right) + \sin(\lambda t)\right], \tag{14}$$

$$P_{C2}(t) = \frac{g}{\lambda}\left[-\cos\left(\frac{g^2}{\lambda}t\right) + \cos(\lambda t)\right]. \tag{15}$$

From this equation set, the first character in Fig. 3(a) is illustrated clearly. As expected, the behavior of the system is the linear superposition of two oscillations. The terms oscillating at rate g^2/λ are considered to be slow while the terms oscillating at rate λ are considered to be fast. So the oscillating period rate between the fast and slow oscillations is g^2/λ^2. For the atomic subsystem, the transfer probability of quantum states between them is given by Eqs. (12) and (13). When an excitation is initially in atom 1, it will be fully transferred to atom 2 at times $t = (\lambda/g^2)[\pi(n + \frac{1}{2})]$ $(n = 0, 1, \ldots)$. During this process, the transfer is disturbed by the cavity subsystem leading to a fast oscillation at rate λ on the basis of the slow oscillation and with a weak amplitude of g^2/λ^2. For the cavity subsystem, the amplitudes of excitation of the field modes are given by Eqs. (14) and (15). They are also sensitive to the two parameters g and λ. The changes of the amplitudes of the two oscillations are synchronized by a factor of g/λ.

We further consider two more situations to have a better picture about the influence of the two parameters, g and λ, on the dynamics of the quantum system: the first one is $g = 0.2$ and $\lambda = 1$, and the other is $g = 0.1$ and $\lambda = 2$. The rigorous calculation results of the dynamics of the quantum system are displayed in Figs. 3(b) and 3(c), which also exhibit superposition of fast and slow oscillation characteristics. Compared with Fig. 3(a) ($g = 0.1$ and $\lambda = 1$), the fast oscillation period for the system with $g = 0.2$ and $\lambda = 1$ almost does not change ($T_f = 2\pi$), while the slow oscillation period shortens to $T_s \approx 26 \times (2\pi)$. And the fast oscillation period changes to $T_f = \pi$, whereas it extends to $T_s = 200 \times (2\pi)$ for the system with $g = 0.1$ and $\lambda = 2$. In addition, when the ratio of g/λ grows, the fast oscillation amplitude increases nearly proportionally. It can be seen that the change of the period and amplitude of the fast and slow oscillations for the quantum system shows good agreement with the approximate solution of Eqs. (12)–(15). The previous discussion clearly demonstrates that the two parameters g and λ influence the system in different ways and that the latter plays a major role in shaping and determining the dynamics of the quantum system. Simply speaking, the population will fully transfer from one atom to the other in the large hopping limit, and this transfer of quantum states between atoms placed in separate cavities is accompanied by weak population in the cavity modes. The result is consistent with the picture discussed in [22].

B. The large coupling limit $g \gg \lambda$

In contrast with the previous subsection, we now turn to consider another extreme situation where the coupling strength is much larger than the hopping strength, expressed as $g \gg \lambda$. In this large coupling limit, the system can be considered as two identical subsystems. Each one includes a Jaynes-Cummings (JC) model described by the atom-cavity interaction and the two subsystems interact through coupling between the two cavities. When $g \gg \lambda$, unlike the behavior in the large hopping limit, the excitation of each mode displays the slowly varying amplitude envelope superimposed with some small and fast oscillations. Such a feature can be seen clearly in Fig. 4, which is obtained by directly referring to Eqs. (6), (9), and (10) and performing numerical calculations. The transfer of photons in this system is clearly described in Fig. 4(a) for the parameters of $g = 10$ and $\lambda = 1$. An excitation that is initially in atomic mode 1 first passes gradually to field mode 1, then through the cavity-cavity coupling flows to another field mode 2, and finally moves from field mode 2 to atomic mode 2. At a certain instant, the amplitude of oscillation reaches it largest magnitude in one JC subsystem, which is equal to unity, and this means that the energy is fully transferred from the other JC subsystem to this one. Similar to the aforementioned large-hopping case, the transfer rate and amplitude of the quantum states can be controlled by the two parameters g and λ. This can be found from the three situations illustrated in Fig. 4.

Similar to the large hopping case, we can also obtain a simplified while more clarified picture in the current large coupling situation. Starting from Eq. (6) and applying Newton's method to the $g \gg \lambda$ condition, we find that the approximate formulas for the harmonic oscillation frequencies are given by

$$\begin{aligned} x_1 &= -x_2 \approx i(g - \lambda/2), \\ x_3 &= -x_4 \approx i(g + \lambda/2), \end{aligned} \tag{16}$$

and the amplitudes of the harmonic wave functions are

$$A_1 = A_3 = -A_2 = -A_4 \approx -\frac{i}{4}.$$

The amplitude of the four one-excitation wave functions is

$$P_{A1}(t) = \cos(gt)\cos\left(\frac{\lambda}{2}t\right), \tag{17}$$

$$P_{C1}(t) = \sin(gt)\cos\left(\frac{\lambda}{2}t\right), \tag{18}$$

$$P_{C2}(t) = -\sin(gt)\sin\left(\frac{\lambda}{2}t\right), \tag{19}$$

$$P_{A2}(t) = -\cos(gt)\sin\left(\frac{\lambda}{2}t\right). \tag{20}$$

According to Eqs. (17) to (20), perfect state transfer can be achieved both between the atom and the atom and between the atom and the field mode in the current system parameters. With this basic picture in mind, we turn back to take a closer look at the dynamics illustrated in Fig. 4 for different sets of parameters of g and λ. For the parameters of $g = 10$ and $\lambda = 1$, the dynamics displayed in Fig. 4(a) shows a periodic train of optical pulse-like oscillation patterns where a fast oscillation curve (with a period of $T_f = 2\pi/10$) is superposed

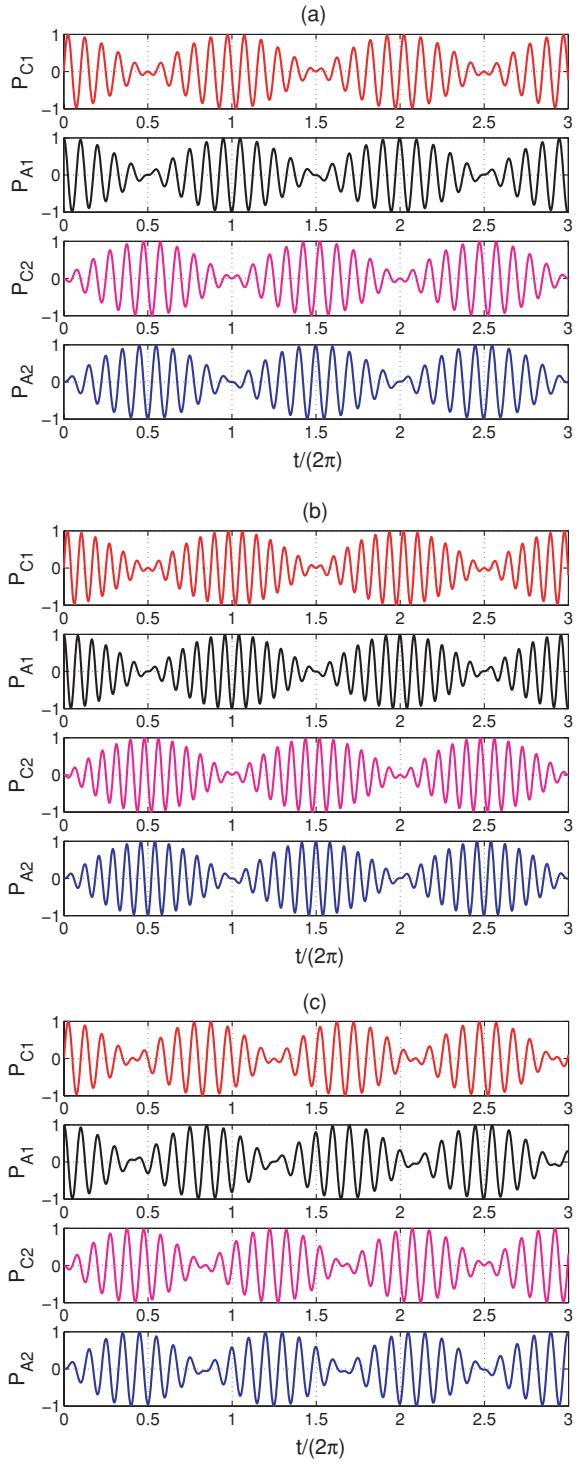

FIG. 4. (Color online) Dynamics of atomic and field excitation in the case $g \gg \lambda$. These curves give the amplitude, as a function of time, of the excitation of the field modes and the atomic modes. The system parameters are (a) $g = 10$, $\lambda = 1.0$; (b) $g = 12$, $\lambda = 1.0$; and (c) $g = 10$, $\lambda = 1.2$.

with a slowly varying oscillatory envelope (with a period of $T_s = 4\pi$). When the parameters change to $g = 12$ and $\lambda = 1$ [Fig. 4(b)], T_s does not change whereas T_f is shortened to $2\pi/12$. On the other hand, when the parameters change to $g = 10$ and $\lambda = 1.2$ [Fig. 4(c)], T_s changes to a smaller value of $T_s = 4\pi/1.2$ while T_f stays the same at $2\pi/10$. All these features of the dynamics of the quantum system conform well with the simple models depicted in Eqs. (17)–(20), which clearly show that the fast oscillation feature is entirely correlated with the large coupling strength with $T_f = 2\pi/g$ while the slowly varying envelope is merely determined by the cavity hopping strength with $T_s = 4\pi/\lambda$.

The physical picture for the quantum system is now quite clear and simple. The small hopping strength induces slow energy flow between the two cavities, while the large coupling strength leads to rapid exchange of energy between the atom and the cavity field within each cavity. As each atom-cavity subsystem only weakly interacts with its neighbor, there is enough time for the energy that is initially stored in the atom to 100% transfer to the cavity photon. Near the completion of this transfer, the photon begins to tunnel into the neighboring cavity, which also reaches 100% after some time. Finally, in the second cavity, the photon can also transfer its energy to the atom 100% after some time. As the time goes on, the inverse process starts to take action, and we see the periodic cycling dynamics of all the quantum states with respect to time. Both the numerical calculation and the analytical model show that the total cycling period is about $T = 4\pi/\lambda$. The observed oscillatory behavior manifests the fundamental characteristics of cavity QED in atom-cavity systems.

C. Small detuning between g and λ

Previously we have considered two extreme situations where one parameter (g or λ) is much larger in quantity than the other one. The dynamics of the quantum system basically involves one fast and one slow oscillatory process, in good correspondence with the nature of one strong and one weak interaction in the system. The dynamics thus looks quite simple and easy to classify the physical origin of each oscillatory process. Mathematically, one is able to greatly simplify the physical analysis and extract the accurate values of oscillation period and frequency from the simple models as described in Eqs. (12)–(15) and Eqs. (17)–(20).

Essentially it is possible to construct the aforementioned simple physical picture because the two processes do not have violent competition with each other. The reason is that the two physical processes are far apart in the time scale of interaction. For general situations where the two parameters g and λ take arbitrary values, the physics process becomes much more complicated. This is particularly true when the detuning between the coupling strength and hopping strength is small, or in other words when g is comparable to λ in magnitude. The competition between the two coupling mechanisms will lead to interesting properties in the time evolution of the dynamics of the quantum states.

We have considered the different situations. We have fixed the atom-cavity coupling strength as $g = 1$ while gradually increasing the cavity-cavity hopping strength from $\lambda = 0.8$ to $\lambda = 1.0$, $\lambda = 1.5$, and $\lambda = 2$. The aim is to see how the dynamics of the quantum system is influenced by the relative interaction strength. The calculated results of the time evolution of the four one-excitation states are displayed in Figs. 5(a)–5(d), respectively. Note that the linear quantum system has exact scalability in time scale with respect to the

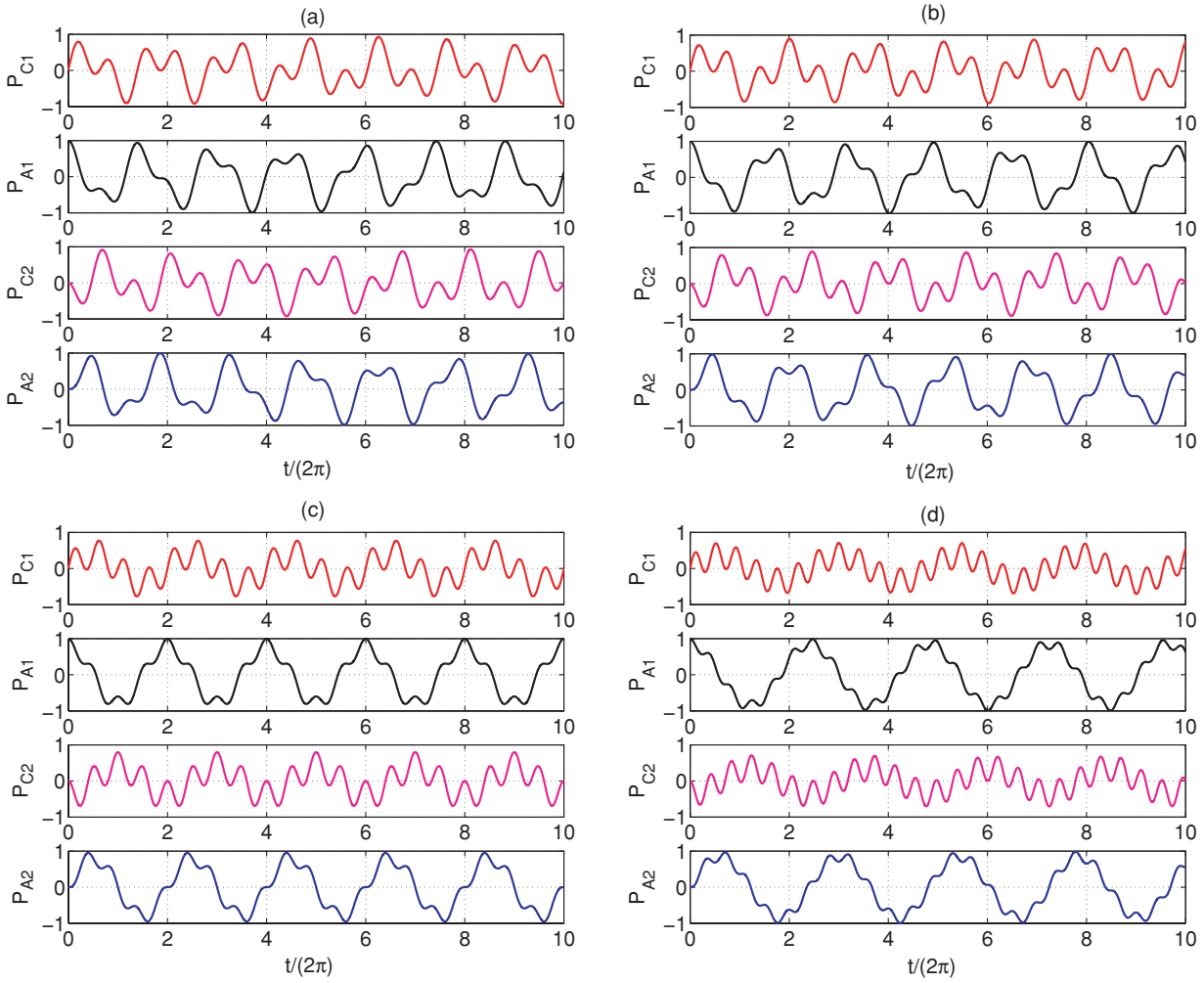

FIG. 5. (Color online) Dynamics of atomic and field excitation in the cases where the detuning between g and λ is small. These curves give the amplitude, as a function of time, of the excitation of the field modes and the atomic modes. The system parameters are $g = 1.0$, (a) $\lambda = 0.8$; (b) $\lambda = 1.0$; (c) $\lambda = 1.5$; and (d) $\lambda = 2.0$.

g and λ together as a whole, so only the relative strength has important implications. When both g and λ increase by a common factor of P, the dynamics of the quantum system does not change its form, only the time scale is scaled by a factor of $1/P$. This can be clearly seen from Eqs. (6)–(10).

The time-evolution curves in the four panels of Fig. 5 exhibit complicated structures of the variations. Unlike those shown in Figs. 3 and 4, the dynamics does not accord with a simple and regular periodic oscillation form for all values. As can be found from Eq. (9), the feature of dynamics is largely determined by the commensurability or incommensurability of the eigenvalues x_1 and x_3. The node-to-node or peak-to-peak distances and the magnitude of peaks and valleys are irregular in most cases [Figs. 5(a), 5(b), and 5(d)]. Only when the eigenvalues x_1 and x_3 are commensurable with each other, for instance $x_1 = 0.5i$ and $x_3 = 2i$ in Fig. 5(c), will regular periodic oscillations appear. However, one thing can be found here that is in common with Figs. 3 and 4. The dynamics is still composed of a faster variation form superposed with a slower variation form. In addition, there appears to be an apparent trend of change in Fig. 5. When λ increases, the frequency of the slow variation, which is not a precise word as the curves are not periodic, changes little, while the frequency of the fast variation grows gradually. The fast oscillation behavior at $\lambda = 2$ and $g = 1$ is somewhat similar to that found in Fig. 3 for the large λ situations. The complicated time evolution of the quantum states manifests the complex interaction and violent competition among the four one-excitation states that are ignited and driven by the coupled atom-cavity system. No single process dominates the time evolution and all the states contribute at the same order of magnitude and at the same time scale. The current coupled system can thus well model a strongly correlated quantum system.

IV. CONCLUSION

In summary, we have considered a quantum system that consists of two identical coupled cavities in PCs where each single-mode cavity contains a two-level atom without dissipation. We have studied systematically the one-excitation dynamics of the system via the process of spontaneous emission in the framework of cavity QED. This system makes a good model to investigate complex many-body cavity QED

problems as it offers great freedom to modulate the interaction between each atom and its cavity environment by changing the position of each atom within the PC cavity as well as the atom-atom interaction through cavity-cavity hopping by simply changing the cavity-cavity distance. As the quantum mechanical dynamics of the quantum system is correlated with abundant interaction parameters in addition to the intrinsic properties of each entity, we expect to see a wide variety of physical phenomena of cavity QED.

With appropriate numerical and analytical tools at hand, the atom-cavity coupling strength g and the cavity-cavity hopping strength λ play an important role in the behavior of the system. By changing the relative strength between them in three typical situations, abundant and diversified quantum mechanical dynamics can be clearly illustrated. The first case is the large hopping limit, namely, $g \ll \lambda$. The behavior of the system is the linear superposition of one fast and one slow oscillation. The ratio between the fast and the slow oscillation period is g^2/λ^2. For the atomic subsystem, an excitation initially in one of the atoms will be fully transferred to the other atom at certain times. During this process, the transfer is disturbed by the cavity subsystem and leads to a fast oscillation which has a weak amplitude of g^2/λ^2 and an oscillation rate of λ on the basis of the slow oscillation. For the cavity subsystem, the change of the amplitudes of the two oscillations is subject to a maximum magnitude of g/λ. The second case is opposite to the first case, namely, $g \gg \lambda$, which is the large coupling limit. The time-evolution behavior of the system is characterized by the familiar optical pulse form of slowly varying carrier envelope superimposed upon a fast and violent oscillation. At a certain instant, the energy is fully transferred from one JC subsystem to the other. The fast oscillation, whose period is $T_f = 2\pi/g$, is entirely correlated with the large coupling strength, while the slowly varying envelope is merely determined by the small cavity hopping strength with a period of $T_s = 4\pi/\lambda$. The third case is that the detuning between g and λ is small. As the two interaction strengths are comparable in magnitude, the role of competition between the coupling strength and the hopping strength becomes important. As a result, the dynamics of the quantum system acts as a continuous pulse with irregular frequency and line shape of peaks and valleys, which indicates that there is violent competition between all the one-excitation quantum states. The features manifest complex physics of a strongly correlated quantum system.

Our theory indicates that the coupled quantum system of atoms and cavities can make a good model to study cavity QED that involves great freedoms of many-body interaction because the system allows for tuning and controlling of each physical interaction parameter in a wide range. In our current work, we only consider two parameters. We expect to see more fruitful new physical phenomena emerge when more parameters are involved in the strongly correlated quantum system, such as the detuning between the atomic transition frequency and the cavity resonance frequency, the dissipation of both the cavity mode and the atomic states, and difference in the coupling strength of the two atom-cavity systems. As major progress of nanotechnology has been made in realizing PC high-Q nanocavities, we expect that these technological advances together with deeper physical insights can help to bring into reality more and more the power of controlling nanoscale light interaction with quantum systems at will.

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