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# Soliton-guided phase shifter and beam splitter

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We propose, analyze, and study numerically a phase shifter for light wave packets trapped by Kerr solitons in a nonlinear medium. We also study numerically a previously proposed soliton-guided nonpolarizing beam splitter.

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## I. INTRODUCTION

This paper focuses on controlling the phase of light wave packets captured by solitons, extending the work reported in [1]. We propose, analyze, and study numerically a phase shifter, and we study in some detail both phase and magnitude characteristics of the operator determined by a soliton-guided nonpolarizing beam splitter.

A soliton imprints a waveguide in a nonlinear medium through its intensity-dependent change in refractive index for both temporal and spatial optical solitons. This waveguide can be used to capture an electromagnetic wave [2–6]. It was shown in [1] that soliton collisions can be used to transfer the captured wave, or *probe*, from one soliton to another and, when the transfer is incomplete, split the wave as in a beam splitter. We next review the model.

## II. MODEL

As in [1] we adopt the formulation in [2,7–9]: two coupled wave equations. The large-signal pump, $P(z,t)$, is described by the standard cubic nonlinear Schrödinger equation; the probe signal, $u(z,t)$, which is assumed to be very much smaller than the pump, is described by a linear wave equation; and the pump determines the potential seen by the probe:

$$
\begin{aligned}
i\frac{\partial P}{\partial z} + \gamma_p |P|^2 P - \frac{\beta_{2p}}{2}\frac{\partial^2 P}{\partial t^2} &= 0 \\
i\frac{\partial u}{\partial z} + \gamma_s |P|^2 u - \frac{\beta_{2s}}{2}\frac{\partial^2 u}{\partial t^2} &= 0.
\end{aligned} \tag{1}
$$

The variable $t$ is time in the frame moving with the pump (the retarded frame), $z$ is distance in the direction of propagation, $\beta_{2\{p,s\}}$ are the group velocity dispersions of the pump and probe, $\gamma_{\{p,s\}}$ the self-phase and cross-phase modulation indexes of the pump and probe, and we assume that the interaction lengths and group velocities are such that walkoff between the probe and the pump can be ignored. We also neglect higher-order dispersion and assume a lossless medium with an instantaneous electronic response.

The probe equation in Eq. (1) is precisely the linear Schrödinger wave equation, and its solutions in general represent the propagation of an electromagnetic wave. Note, however, that $z$ plays the role that time does in the usual Schrödinger wave equation, and $t$ the role of space.

Throughout this paper we use exact one- or two-soliton ground-state solutions of the pump equation given in [10]. The standard one-soliton solution is, in their notation,

$$
P(z,t) = \frac{|k_R|}{\sqrt{\mu_p}} e^{i[k_I t + (k_R^2 - k_I^2)x]} \operatorname{sech}[k_R(t - 2k_I x) + \phi], \tag{2}
$$

where $x = -(\beta_{2p}/2)z, \phi$ is an arbitrary phase, $\mu_p = -\gamma_p/\beta_{2p} > 0$, and the free complex parameter $k = k_R + ik_I$, where $k_R$ determines the energy of the soliton and $k_I$ its velocity. Using this for the potential, letting the soliton velocity $k_I = 0$, and launching the soliton along the $z$ axis, the probe equation becomes

$$
i\frac{\partial u}{\partial z} + \gamma_s \frac{k_R^2}{\mu_p}\operatorname{sech}^2(k_R t) u - \frac{\beta_{2s}}{2}\frac{\partial^2 u}{\partial t^2} = 0. \tag{3}
$$

The usual anzatz $u(z,t) = u(t)e^{-iEz}$ separates the variables, breaking the solution of Eq. (3) into the dynamical phase factor $e^{-iEz}$ and a function $u(t)$ determined by the eigenvalue equation

$$
\frac{\beta_{2s}}{2}u'' - \left[E + \gamma_s \frac{k_R^2}{\mu_p}\operatorname{sech}^2(k_R t)\right] u = 0. \tag{4}
$$

With the transformation $\xi = \tanh(k_R t)$ [11] this reduces to the associated Legendre equation [12,13],

$$
\frac{d}{d\xi}\left[(1-\xi^2)\frac{du}{d\xi}\right] + \left[\ell(\ell+1) - \frac{m^2}{(1-\xi^2)}\right] u = 0, \tag{5}
$$

where $m^2 = 2E/(k_R^2\beta_{2s})$, and $\ell(\ell+1) = -2\gamma_s/(\mu_p\beta_{2s})$. For the purposes of this paper, we choose the simplest ground-state solution, $m = \ell = 1$, and, as in [1], determine $\beta_{2s}$ from the other parameters to make this so. The eigenvalue $E = k_R^2\beta_{2s}/2$, and $u(t) = \operatorname{sech}(k_R t)$.

## III. A PHASE SHIFTER

In this section it is convenient to refer to the captured probe wave packet simply as a *packet*, to interpret the solitons as temporal solitons, and to assume that the probe is sufficiently low in intensity that the propagation equations are valid, but sufficiently separated in wavelength and polarization to be detected in the presence of the pump. Furthermore, we arrange all collisions of pump solitons to occur with a relative phase of $\pi$, ensuring that the collisions are repulsive; and we assume that the parameters are such that on collision a packet shepherded by one soliton is transferred completely to the soliton that it hits [1].

The proposed phase shifter works as follows. Two solitons, A and B, are launched at the same, nominal, velocity in the $z$ direction: first B, then A. Initially, soliton A carries a packet. Soliton C, a third soliton, is then launched at a greater velocity.

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When C overtakes A, the packet is captured by C; and C carries the packet with it until it overtakes B, at which point the packet is transferred from C to B. The net effect, then, is that soliton C ferries the packet from A to B. The packet accumulates extra phase during the time it travels at an altered velocity.

We simulated the pump by stitching together segments of the exact two-soliton solution from [10]. This is preferable to propagating initial conditions numerically (as is done for the probe) because the solutions are exact, and the relative phase angles at collisions can be precisely controlled. The skirts of the hyperbolic secant-shaped soliton envelopes fall off exponentially and the numerical effect of this approximation is negligible.

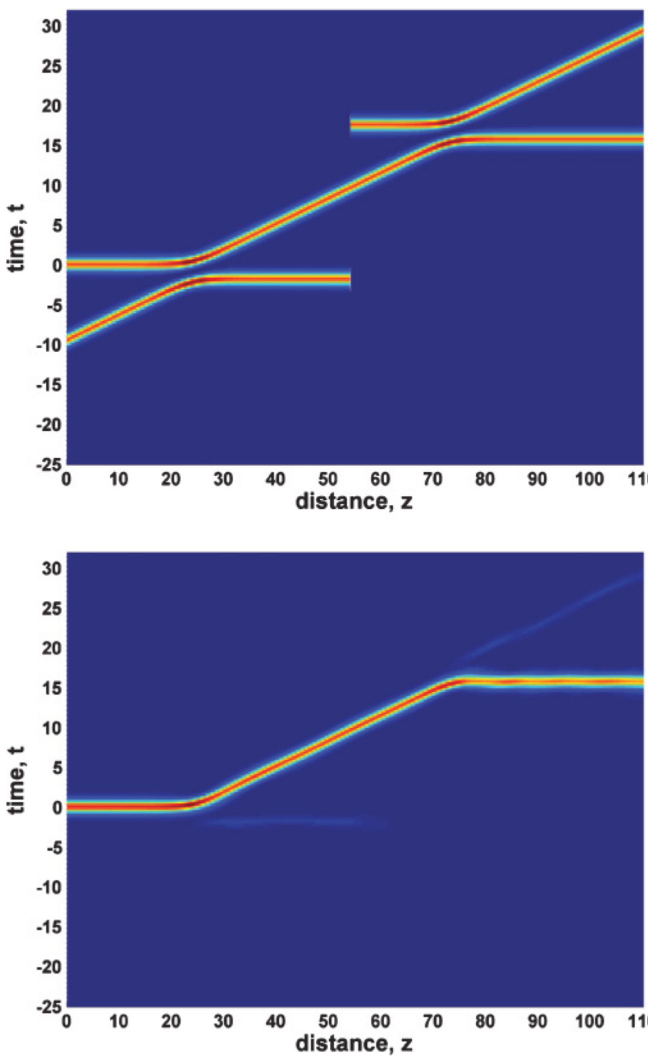



FIG. 1. (Color online) *Top*: The pump signal used in the example phase shifter, stitched together from exact two-soliton solutions. It starts at $z = 0$, $t = 0$, accelerates, and then returns to its original velocity. The soliton parameters in the notation of [10] are $k_1 = 1.5$, $k_2 = -1.5$ for the horizontal ones, $k_2 = -1.5 + 0.8i$ for the faster one, $\gamma_p = 0.02$, $\beta_{2p} = -0.4$ (so $\mu_p = 0.05$), and the amplitudes of the solitons are all 4. The relative phase at collision is arranged to be $\pi$ radians. *Bottom*: The probe signal in the phase shifter, corresponding to the pump in the figure at the top. The parameters for the probe propagation are $\gamma_s = 0.0139$, $\beta_{2s} = -0.278$, and the overall phase shift achieved is $\pi$ (see Fig. 2).

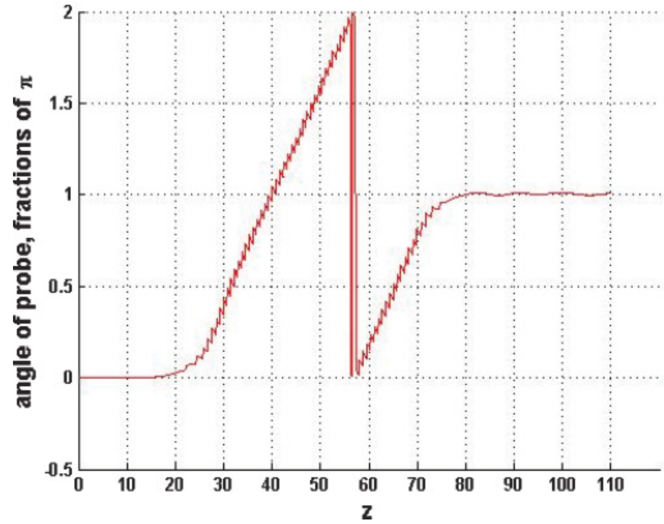



FIG. 2. (Color online) The phase of the probe as a function of the distance variable $z$. The dynamical phase has been removed.

We first give the results for an example of a phase shifter with parameters designed for the important case when the packet phase is shifted by $\pi$ radians. Figure 1 (top) shows the pump signal and the bottom of Fig. 1 shows the corresponding probe signal. The probe follows the waveguide induced by the pump well after two collisions, with small losses at the points of acceleration and deceleration and some very slight wobbling in velocity after the second collision.

The phase angle of the probe was measured numerically as a function of $z$ by scanning vertically in $t$ and using the phase at the point of peak amplitude. Figure 2 shows the the result, with the dynamical phase removed by multiplication by $e^{iEz}$ and angle reduced mod $2\pi$ to the range $[0, 2\pi]$. To verify the net phase inversion by $\pi$ radians after the second capture, we propagated an independent reference signal ahead of the probe so that the probe just meets the reference signal after its second capture. The destructive interference, shown in Fig. 3,

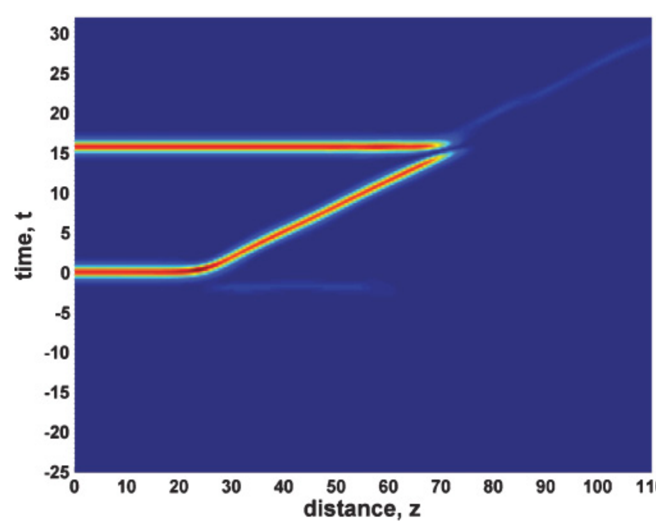



FIG. 3. (Color online) The destructive interference between a reference signal (launched at the top) and the probe shown in Fig. 1, verifying the phase shift of $\pi$ radians.

is clear. Note that this numerical experiment simply confirms the phase change and does not reflect a physically realizable cancellation.

The phase shift we might expect in the proposed system can be estimated as follows. Viewing the cross-phase modulation term in Eq. (1), $|P|^2|$, as a potential in the standard Schrödinger wave equation (with $\hbar = 1$), the effect of tailoring the pump signal is to accelerate the potential from zero to a velocity $c$, keep it there for a given (usual) time $\tau$, and then slow it back down to its original velocity. When the potential moves at constant speed $c$, an additional phase $Mc^2\tau/2$ is added to the solution of the wave equation, where $M$ is the usual mass [14]. We call this a *translational* phase, to distinguish it from the usual dynamical phase. In terms of our parameters, taking account of the scaling and interchange of time and space, the mass $M = -1/\beta_{2s}$, the velocity $c = -k_I\beta_{2p}$, and $\tau = \Delta z = \Delta t/c$, the duration of the phase shifter operation measured along the $z$ axis, the direction of propagation.

The presence of this translational phase component can be verified numerically from the phase shown in Fig. 2 in the region when the probe is moving at its greater velocity. As we see shortly, this provides only an estimate of the total accumulated translational phase induced by the phase shifter, however, because the probe does not accelerate and decelerate suddenly but changes its velocity somewhat gradually as it follows the curved induced soliton waveguide.

The accumulated translational phase of the phase shifter if the probe actually moved with exactly the extra velocity $c$ just for the time between the theoretical impact points of the guiding solitons would be

$$\Delta\phi = \frac{Mc^2\tau}{2} = \frac{Mc}{2}\Delta t = \frac{k_I\beta_{2p}}{2\beta_{2s}}\Delta t, \tag{6}$$

which provides us with an estimate of the total effective phase shift of the proposed phase shifter.

We are going to vary the cross-phase modulation index $\gamma_s$ to achieve a full range of phase shifts. Recall that $\beta_{2s}$ is determined from $\gamma_s$ and the soliton parameters $\beta_{2p}$ and $\gamma_{2p}$ to achieve the desired ground state for the probe wave, $m = \ell = 1$. Figure 4 shows the phase shift measured from the numerical propagation of the probe equation. We see that a full range of $2\pi$ is achievable by varying the cross-phase modulation index $\gamma_s$ from 0.01 to 0.03. Also shown is the estimate in Eq. (6), which is fairly close (1.7%) for $\gamma_s \approx 0.01$ but much less accurate when $\gamma_s$ approaches 0.03.

If the probe is subsequently decelerated so that its state is restored to its original trajectory, the total accumulated translational phase is not zero but *twice* the phase of the phase shifter above, since accumulated phase is proportional to $c^2$. This has been confirmed numerically. The cycle is somewhat suggestive of Berry's phase [15] but is not at all the same: the changes to the Hamiltonian are not adiabatic and, furthermore, are described by one parameter.

## IV. A BEAM SPLITTER

For certain choices of parameters the transfer of the probe wave when the guiding solitons collide is only partial, and a beam splitter results, as was illustrated in [1]. We adopt for our model the following standard two-parameter unitary transfer matrix $U(\theta,\phi)$ for a beam splitter [16]:

$$\begin{pmatrix} \cos\theta & -e^{i\phi}\sin\theta \\ e^{-i\phi}\sin\theta & \cos\theta \end{pmatrix}. \tag{7}$$

Notice that this matrix is determined by the first row alone, and that $U(\theta+\pi,\phi) = -U(\theta,\phi)$, so that when we are trying

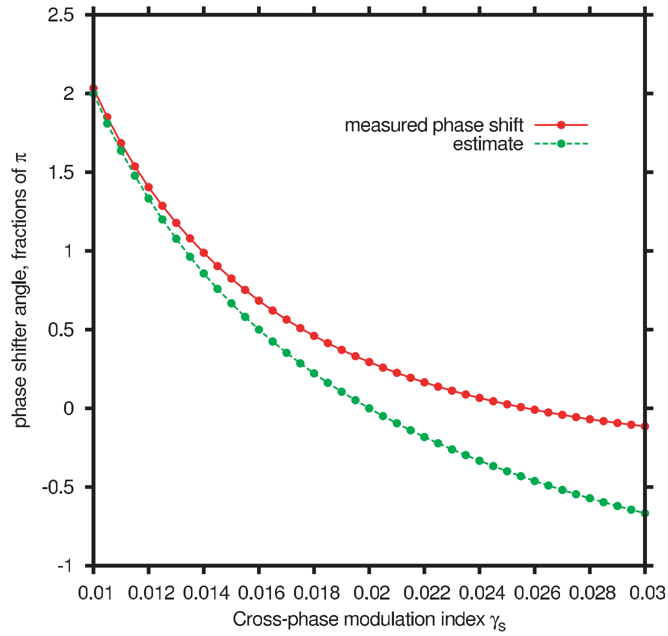



FIG. 4. (Color online) The range of achievable angles for the phase shifter. Shown is a full range of $2\pi$ for the cross-modulation index $\gamma_s = 0.01$–$0.03$. Also shown is the estimate in Eq. (6).

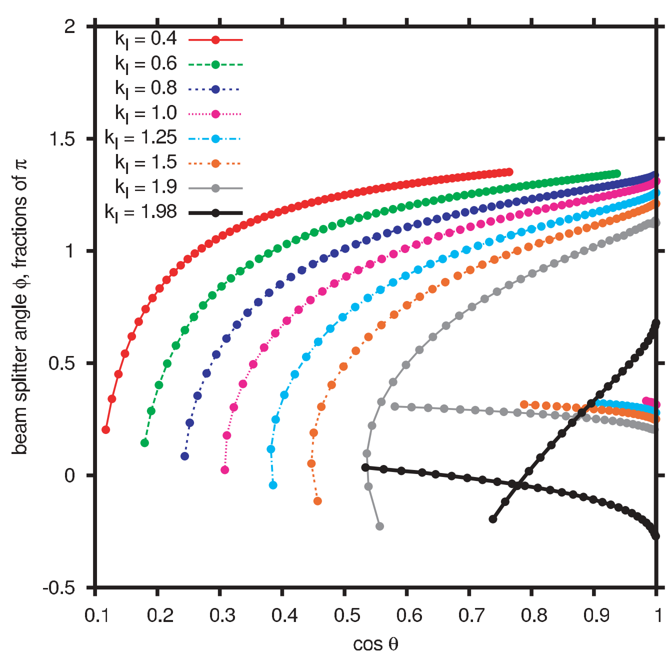
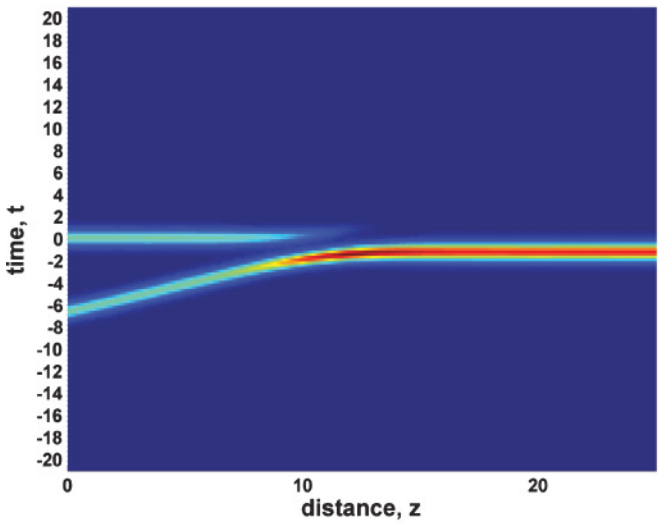



FIG. 5. (Color online) The locus of parameters of the beam splitter as $\gamma_s$ is varied from 0.017 to 0.145, for various values of the soliton velocity $k_I$. (For clarity the curve for the case $k_I = 1.98$ is shown only for $\gamma_s \geqslant 0.041$.)



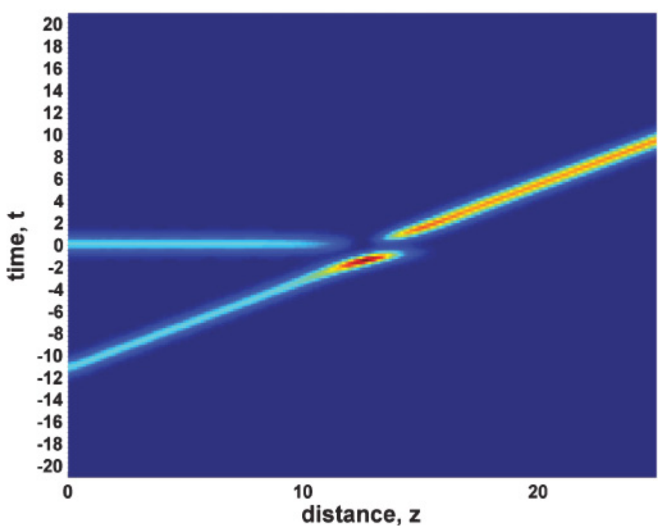




FIG. 6. (Color online) *Top*: An example of a near 50:50 beam splitter with $\phi \approx \pi$. The ideal 50:50 beam splitter has $\cos\theta = 0.7071$ and $\phi = \pi$; the parameters in this example are $\cos\theta = 0.7140$ and $\phi = 1.023\pi$, corresponding to $\gamma_s = 0.055$ and soliton velocity $k_I = 1.25$. The inputs are equal and in phase, and the signal $Y_1$ shows constructive interference. *Bottom*: The corresponding example with $\phi \approx 0$. The ideal 50:50 beam splitter has $\cos\theta = 0.7071$ and $\phi = 0$; the parameters in this example are $\cos\theta = 0.6970$ and $\phi = -0.01157\pi$, corresponding to $\gamma_s = 0.133$ and soliton velocity $k_I = 1.98$. The signal $Y_1$ now shows destructive interference.

to realize such a given unitary matrix we can assume without loss of generality that $\cos\theta > 0$ and ask that $\phi$ take on a range of values that spans $2\pi$. We next describe how the two parameters $\theta$ and $\phi$ for a wide variety of beam splitters can be chosen for the ground-state probe wave packets considered here [$m = \ell = 1$ in Eq. (5)].

Call the input packets to the beam splitter in Eq. (7) $X_1$ and $X_2$, the outputs $Y_1$ and $Y_2$, and, by an abuse of notation, use the same symbols for the corresponding complex field amplitudes. The packets $X_1$ and $Y_1$ are shepherded by the reference soliton (horizontal in the figure), and the packets $X_2$ and $Y_2$ are shepherded by the soliton that overtakes the reference soliton (lower left to upper right). We can measure $\theta$ and $\phi$ simply by measuring the complex $Y_1$ for the two input vectors $[X_1\ X_2] = [1\ 0]$ and $[0\ 1]$. Figure 5 shows the results of these measurements for a wide range of relative soliton velocities (from 0.4 to 1.98) and cross-phase modulation index $\gamma_s$ (from 0.017 to 0.145). Notice that at the right-hand axis, where $\cos\theta$ approaches 1, $\sin\theta$ passes through zero and, as a result, $\phi$ jumps by $\pi$ radians. It is remarkable that the collision of the pump solitons continues to provide an effective beam splitter for the captured probe signals throughout this wide range of parameters, and even beyond.

The large region covered by the points in Fig. 5 suggests that the design of almost any beam splitter is possible. Figure 6 shows the results when two equal-intensity, matched-phase probes are used as inputs for two contrasting beam splitter designs, both near 50:50. The first approximates the case $\phi = \pi$, and the second $\phi = 0$. The constructive and destructive interference at the outputs is clear.

## V. DISCUSSION

The schemes described in this paper for phase shifting and beam splitting wave packets are generally applicable to any medium where solitons can capture wave packets and where soliton collisions result in full or partial transfer. The problems of detecting the probe in the presence of the pump, and tailoring physical parameters to realize the virtual devices, are left for the future and are clearly heavily dependent on technological developments. In the case of optical fiber implementation, we note the remarkable developments in fabrication described in [17].

## ACKNOWLEDGMENTS

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