Localization of spin mixing dynamics in a spin-1 Bose-Einstein condensate

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We propose to localize spin mixing dynamics in a spin-1 Bose-Einstein condensate by a temporal modulation of spin exchange interaction, which is tunable with optical Feshbach resonance. Adopting techniques from coherent control, we demonstrate the localization and freezing of spin mixing dynamics, and the suppression of the intrinsic dynamic instability and spontaneous spin domain formation in a ferromagnetically interacting condensate of ^{87}Rb atoms. This work points to a promising scheme for investigating the weak magnetic spin dipole interaction, which is usually masked by the more dominant spin exchange interaction.

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Dynamic localization is ubiquitous in nonlinear systems, both for classical dynamics as in an inverted pendulum with a rapidly modulating pivot [1] or an ion in a Paul trap [2], and for quantum dynamics like a one- or two-dimensional soliton in a Bose-Einstein condensate (BEC) when the attractive interaction strength is rapidly modulated [3–6]. It is often used to stabilize a dynamically unstable system.

Spin mixing dynamics of a spin-1 atomic condensate are dynamically unstable [7] when the spin exchange interaction is ferromagnetic, (i.e., favoring a ground state with all atomic spins aligned). When confined spatially, the unstable dynamics is known to cause formation of spin domain structures [8,9]. For many applications of spinor condensates, from quantum simulation to precision measurement [10], it is desirable that spin domain formation is suppressed. In addition, atomic spin dipolar interactions, although weak when compared to typical spin exchange interactions, induce intricate spin textures that are difficult to probe when masked by spin domain structures. Thus, the suppressing and freezing of the undesirable dynamics from spin exchange interaction is also important for investigating the effect of dipolar interaction [11,12].

Compared to conventional magnets in solid states, a spin-1 BEC has one unsurpassed advantage: its spin exchange interaction between individual atoms can be precisely tuned through optical (as well as magnetic) Feshbach resonances [13–17]. By adjusting the two s-wave scattering lengths a_0 and a_2, of two colliding spin-1 atoms via optical means, the spin exchange interaction strength, characterized by $c_2 = 4\pi\hbar^2(a_2 - a_0)/3M$ with M the mass of the atom, is tunable. Analogous to an inverted rigid pendulum with a rapidly oscillating pivot, a fast temporal modulation of the spin exchange interaction can localize the spin mixing dynamics, equivalent to a suppressing or nulling of the spin exchange interaction.

This study is devoted to a theoretical investigation of spin dynamics in a spin-1 BEC under the temporal modulation of the spin exchange interaction. As an application, we illustrate the suppression of the dynamic instability and the resulting prevention of spin domain formation in a condensate with ferromagnetic interaction. The proposed scheme to control the spin exchange interaction will potentially provide a substantial improvement to the accuracy of several envisaged magnetometer setups and to enable cleaner detections of dipolar effects.

For both spin-1 atoms ^{87}Rb and ^{23}Na, popular experimental choices, their spin-independent interaction strength, characterized by $c_0 = 4\pi\hbar^2(2a_2 + a_0)/3M$, is two to three orders of magnitude larger than $|c_2|$ [18–20]. This ensures the validity of single spatial mode approximation (SMA) [21–23] when the number of atoms is small and the magnetic field is low. The spin degrees of freedom and the spatial degrees of freedom become separated within the SMA. This allows one to focus on the most interesting spin dynamics free from density-dependent interactions.

Within the mean field framework, the spin dynamics of a spin-1 condensate under the SMA is described by [24]

$$\begin{aligned}\dot{\rho}_0 &= \frac{2c}{\hbar}\rho_0\sqrt{(1-\rho_0)^2 - m^2}\sin\theta,\\ \dot{\theta} &= \frac{2c}{\hbar}\left[(1-2\rho_0) + \frac{(1-\rho_0)(1-2\rho_0)-m^2}{\sqrt{(1-\rho_0)^2-m^2}}\cos\theta\right],\end{aligned} \tag{1}$$

where ρ_i $(i = +, 0, -)$ is the fractional population of component $|i\rangle$, $(\sum_i \rho_i = 1)$, $m = \rho_+ - \rho_-$ is the magnetization in a spin-1 Bose condensate, a conserved quantity. θ is the relative phase [24]. $\phi(\vec{r})$ is a unit normalized spatial mode function under the SMA determined from a scalar Gross-Pitaevskii equation with an s-wave scattering length of a_2. As before, the effective spin exchange interaction is given by $c(t) = c_2(t)N\int d\vec{r}|\phi(\vec{r})|^4$, albeit the time dependence, with N the total number of trapped atoms. Although the system dynamics Eq. (1) does not conserve the total spin energy,

$$\mathcal{E}(t) = c(t)\rho_0[(1-\rho_0) + \sqrt{(1-\rho_0)^2 - m^2}\cos\theta], \tag{2}$$

due to the temporal modulation, the transversal spin squared $f_\perp^2 = f_x^2 + f_y^2 = 2\mathcal{E}(t)/c(t)$ remains conserved and is determined solely by the initial condition. Because $\mathcal{E}(t)$ and $c(t)$ are modulated exactly in the same manner, replacing θ with $f_\perp^2$, Eq. (1) is further simplified to

$$(\dot{\rho}_0)^2 = \frac{4c^2}{\hbar^2}f^2(\rho_u - \rho_0)(\rho_0 - \rho_d), \tag{3}$$

where $\rho_{u,d} = f_\perp^2(1 \pm \sqrt{1-f^2})/2f^2$ with $f^2 = f_\perp^2 + m^2$ $\rho_{u(d)}$ takes the $+(-)$ sign, denoting the largest (smallest) value of ρ_0 along the orbit and satisfies $\dot{\rho}_0|_{\rho_{u,d}} = 0$. It is straightforward to find the solution,

$$\rho_0(t) = \frac{\rho_u + \rho_d}{2} + \frac{\rho_u - \rho_d}{2} \sin\left[\gamma + \int_0^t \beta(s)ds\right], \quad (4)$$

where $\beta(t) = \pm 2c(t)f/\hbar$ is the frequency of the periodic spin evolution and the $\pm$ sign denotes the forward and backward evolutions, respectively, and

$$\gamma = \text{atan}\left(\frac{\rho_0(0) - [(\rho_u + \rho_d)/2]}{\sqrt{[\rho_u - \rho_0(0)][\rho_0(0) - \rho_d]}}\right),$$

is given by the initial values of ρ_0 and θ. The above solution [Eq. (4)] is valid for an *arbitrary* temporal modulation function $c(t)$.

To control the spin dynamics, we consider several simple but practical modulations in the following. Based on these examples, we demonstrate that spin dynamics with a modulated c is very different from the free dynamics and understand how a temporal modulated $c(t)$ affects the spin dynamics.

First, we consider a sinusoidal modulation with $c(t) = d\cos(\Omega t)$. d and Ω are, respectively, the modulation amplitude and frequency. The solution Eq. (4) then becomes

$$\rho_0(t) = \frac{\rho_u + \rho_d}{2} + \frac{\rho_u - \rho_d}{2} \sin[\gamma + \eta \sin(\Omega t)], \quad (5)$$

with $\eta = \pm 2df/\hbar\Omega$. The corresponding results are illustrated in Fig. 1 for $\Omega = 0, 1/2, 1$, and 2. The case of $\Omega = 0$ is simply the free evolution without modulation with a period $T_0 = 2\pi/\beta$ determined by the initial condition [25]. For other cases, irrespective of the values for Ω, the frequency of oscillation is always Ω and the corresponding period is $2\pi/\Omega$. As shown in Fig. 1, the oscillation amplitudes show two distinctive regions: one for $\Omega \leqslant \Omega_c \equiv 2df/\pi\hbar$ (i.e., $\eta \geqslant \pi$) where the amplitude is the same with or without modulation; another for $\Omega > \Omega_c$ (i.e., $\eta < \pi$) where the amplitude decreases with modulation frequency Ω. In this latter region, it is easy to check that $A = (\rho_u - \rho_d)(1 - \cos\eta)/2$ for the case shown in Fig. 1.

Figure 2 illustrates the orbits for the corresponding spin dynamics. A full orbit is occupied if $\Omega < \Omega_c$ but only partial orbits are occupied when $\Omega > \Omega_c$. The occupied portion deceases when Ω increases. Spin dynamics for the first half-period is reversed during the second half-period evolution, irrespective of the values for $\Omega \neq 0$. This reversal is responsible for a more robust modulated dynamics against various noises as noises are not reversed and their effect can be averaged zero according to coherent control theory [26].

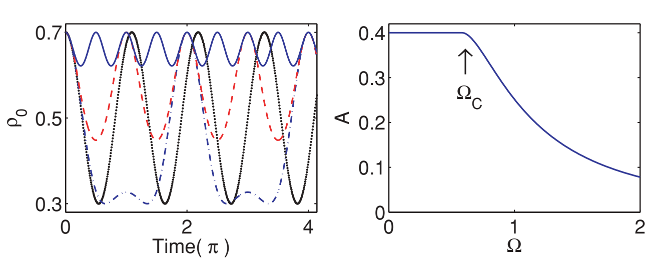

FIG. 1. (Color online) (Left) Time-dependent fractional population $\rho_0(t)$. The black dotted line refers to the free evolution without modulation, while the blue (dash-dotted), red (dashed), and blue (solid) lines are for $\Omega = 1/2$, 1, and 2, respectively. The parameters and initial conditions are $\hbar = 1, d = -1, m = 0, \rho_0(0) = 0.7, \theta(0) = 0$, and $\Omega_c \approx 0.58$. (Right) The dependence of oscillation amplitude A of ρ_0 on the modulation frequency Ω.

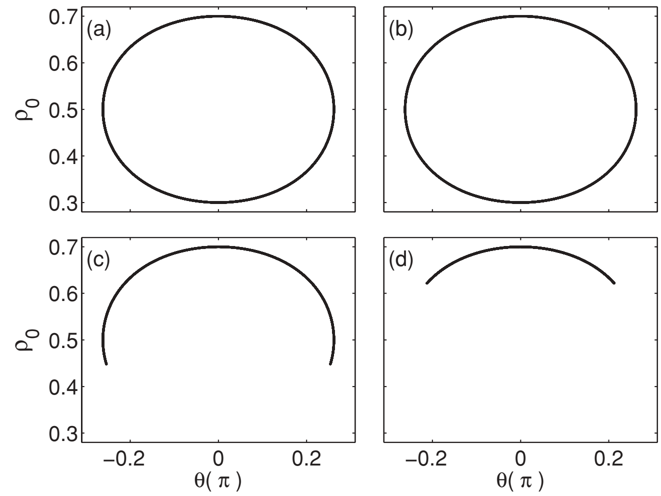

FIG. 2. (a) Orbits without modulation; orbits with modulation for (b) $\Omega = 1/2$, (c) $\Omega = 1$, and (d) $\Omega = 2$. Parameters are the same as in Fig. 1.

Next we consider a periodic square function modulation with

$$c(t) = \begin{cases} d, & n(w+\tau) \leqslant t < n(w+\tau) + w, \\ 0, & n(w+\tau) + w \leqslant t < (n+1)(w+\tau), \end{cases} \quad (6)$$

for $n = 0, 1, 2, \ldots$. The spin dynamics is halted completely if $n(w+\tau) + w \leqslant t < (n+1)(w+\tau)$ and is unmodulated if $n(w+\tau) \leqslant t < n(w+\tau) + w$. The corresponding plots for ρ_0 and θ display interesting step-like features as shown in Figs. 3(a) and 3(b).

The modulation dynamics considered above offers many interesting possibilities. A direct application is to remove a dynamical instability observed in a ferromagnetically interacting spin-1 condensate [7–9,27,28]. This instability is removed whenever the imaginary part of the eigenfrequency for the corresponding Bogoliubov excitation becomes zero in a modulation cycle. We show this instability is indeed suppressed in the following for the cosine modulation $c_2(t) = d\cos(\Omega t)$. This suppression inhibits the spontaneous formation of spin domains.

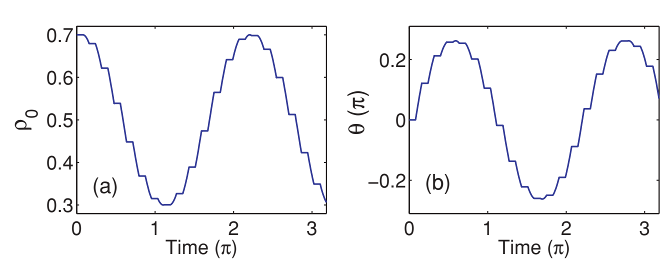

FIG. 3. (Color online) (a) Time-dependent fractional population ρ_0 and (b) time-dependent relative phase θ for the periodic square modulation Eq. (6) with $\tau = w = 1/4$ and $d = -1$.

We now consider our system of a homogeneous ^{87}Rb spin-1 condensate starting from an off-equilibrium initial state [7]. The averaged spin $\vec{f} = \langle F_x\rangle\hat{x} + \langle F_y\rangle\hat{y} + \langle F_z\rangle\hat{z}$ is conserved where $F_{x,y,z}$ are spin-1 matrices. Starting from any stationary point, the evolution of the collective excitations takes a compact form,

$$i\hbar\frac{\partial\vec{x}}{\partial t} = \mathcal{M}(t)\cdot\vec{x}, \tag{7}$$

where $\vec{x} = (\delta\Psi_+, \delta\Psi_0, \delta\Psi_-, \delta\Psi_+^*, \delta\Psi_0^*, \delta\Psi_-^*)^T$ with deviations $\delta\Psi_j$ and $\delta\Psi_j^*$ from the stationary solution [7].

The general solution to Eq. (7) is $\vec{x}(t) = U(t,0)\vec{x}(0)$ where $U(t_2,t_1) = \mathcal{T}\exp[-(i/\hbar)\int_{t_1}^{t_2} ds\mathcal{M}(s)]$ with $\mathcal{T}$ the time ordering operator. For periodic modulation, the solution during $t \in [pT,(p+1)T]$ is further simplified to $\vec{x}(t) = U(t,pT)(U_T)^p\vec{x}(0)$, where $p = 0,1,2,\ldots$ indexes the number of periods. $T = 2\pi/\Omega$ is the period of modulation, and $U_T = U(T,0) = \mathcal{T}\exp[-(i/\hbar)\int_0^T ds\mathcal{M}(s)]$ is the evolution operator for a complete period. $|\vec{x}(t)|$ will grow (decay) exponentially with p if the modulus of U_T are larger (smaller) than unity. Diagonalizing U_T and rewriting it as $U_T = V^\dagger\exp(-iTF)V$, where the Fluoquet operator F is a 6-by-6 diagonal matrix, the criteria for stable dynamics reduces to a vanishing imaginary part of F_q $(q = 1,2,\ldots,6)$. Unstable dynamics arises if the imaginary part is not zero, while stable dynamics emerges if the imaginary part of all F_q is exactly zero.

The inset of Fig. 4 shows the imaginary part of a typical spectrum for the system under a cosine modulation of c_2. We focus on the most unstable mode which in principle dominates the unstable dynamics. Compared to the modulation free results (in dashed lines), we find the most unstable mode (in double contours) is not only suppressed in amplitude but also shifted to larger wave vector. The dependence of k_- on the modulation frequency Ω is illustrated in Fig. 4. The almost independence of k_- on Ω at small values contrasts with a strong monotonic increase at large values of Ω.

The emergence of spin domains is prohibited due to the modulation. On one hand, the suppression of F implies a smaller effective spin exchange interaction thus a longer effective spin healing length ξ; on the other hand, the up-shift of k_- means shorter wave length $\lambda = 2\pi/k_-$. If the unstable mode wave length (potentially domain width) is smaller than the spin healing length, the condensate is able to heal by itself. The domain structure would never appear if ξ exceeds λ.

Because of the modulation, however, the resulting dynamics becomes completely different from the case of $c_2 = 0$ or no spin dynamics at all. The modulation does not stop spin mixing dynamics, (i.e., as we continue to observe spin waves) which is nominally disguised in experiments by the spontaneously formed spin domains [8,9]. Furthermore, we expect the modulated spin dynamics to be robust against various experimental noises because the periodic modulation effectively cancels uncorrelated noises from alternating modulation periods.

Finally, we confirm the above conclusions for a trapped spin-1 condensate with full numerical simulations. We adopt experimental parameters as in Ref. [7]: The initial conditions are as in the experiment [8], with ^{87}Rb condensates $[\rho_0(0) = 0.744,\ \theta(0) = 0,\ N = 2.0\times10^5$, and $m = 0]$, in a trap $V_{\rm ext}(\vec{r}) = (M/2)(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)$ with $\omega_x = \omega_y = (2\pi)240$ Hz and $\omega_z = (2\pi)24$ Hz. The modulation function is $c_2(t) = d\cos(\Omega t)$ with $d = c_2$, and $\Omega = \omega_z$ which is about 3 times larger than the free spin evolution frequency $2\pi/T_0$. Two cases will be considered: (1) The modulation is applied immediately (ρ_0 oscillates around ρ_u); (2) The modulation is turned on at $t = 6$ $(1/\omega_z)$ (ρ_0 oscillates around ρ_d).

The results from numerical simulations are shown in Fig. 5. With modulation, the spin dynamics is clearly localized as ρ_+

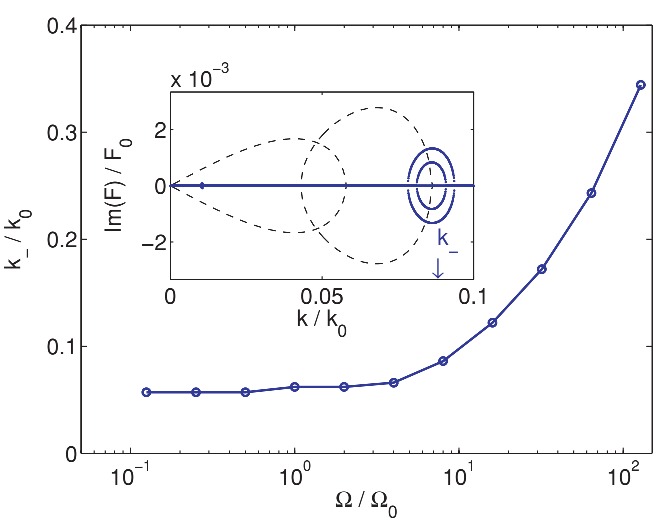

FIG. 4. (Color online) The dependence of the wave vector k_- for the most unstable mode on the modulation frequency Ω. The inset illustrates the imaginary part of a typical Bogoliubov spectrum under modulation (blue double contours). For comparison, black dashed lines denote the results without modulation.

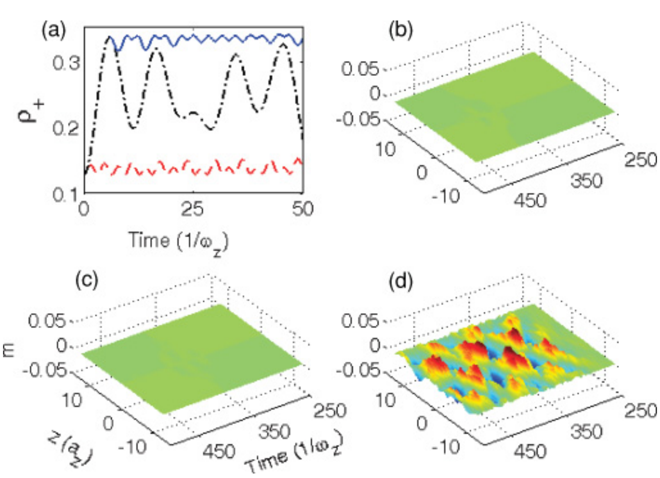

FIG. 5. (Color online) (a) Localization of the spin dynamics for a trapped spin-1 condensate in three cases: (i) modulation starts at $t = 0$ (red dashed line); (ii) no modulation (black dash-dotted line); and (iii) modulation starts at $t = 6$ (blue solid line). Spatial distribution $m(z) = \int dr2\pi r(|\Phi_+(r,z)|^2 - |\Phi_-(r,z)|^2)$ at different times for the above three cases: (b) case (i); (c) case (iii); (d) case (ii). $a_z = \sqrt{\hbar/M\omega_z}$. Trivial and flat $m(z)$ at early times $(t < 250)$ has been omitted. Dynamical instability- induced spontaneous domain formation is prohibited by the modulated spin exchange interaction.

(same for ρ_-) oscillates with a smaller amplitude around its initial value, in contrast to the large amplitude unmodulated result [Fig. 5(a)]. In addition, the unmodulated dynamics shows domain structures after $t \approx 300$, due to the intrinsic dynamical instability, while for both modulated cases, no spin domains are observed. Thus temporal modulation of spin exchange interaction does suppress the intrinsic dynamical instability and prohibit spontaneous domain formation. In our extensive numerical simulations, we find that when additional white noises are added, spin domains are found to arise quicker for the unmodulated case; yet for the two cases with modulations, almost the same behaviors are observed as if no noise were added.

Although the lifetime of the condensate at optical Feshbach resonance is reduced dramatically in ^{87}Rb gases [15], we notice that there exists at least one magic window of relative frequency, for example, between resonance β and γ in Fig. 7 of Ref. [15], where the condensate lifetime lasts several hundred milliseconds and c_2 changes sign. On the other hand, the spin domain emerges in a timescale typically shorter than 100 ms [29]. Therefore, the suppression of the domain formation in ^{87}Rb condensate is experimentally feasible.

In summary, we propose to localize the spin mixing dynamics in a spin-1 condensate by temporally modulating the spin exchange interaction. For condensed atoms, the modulation can be facilitated with the technique of optical Feshbach resonance [13,15]. We demonstrate the suppression of the intrinsic instability, thus the inhibition of spontaneous spin domain formation in a ferromagnetically interacting spin-1 Bose condensate, such as ^{87}Rb condensate in the $F = 1$ manifold. In addition, the effective freezing of spin mixing dynamics due to spin exchange interaction provides a cleaner approach to investigate magnetic spin dipolar interaction effect in a ^{87}Rb Bose condensate [11,12].

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