Concatenated cranking representation of the Schrödinger equation and resolution to pulsed quantum operations with spin exchange

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We propose a concatenated cranking approach to resolve the dynamics for a class of time-dependent quantum systems with specific algebraic structure. By invoking a series of canonical transformations successively, concatenated representation of the Schrödinger equation is established and evolution of the system is solved in the cranking representation via discarding high-order nonadiabatic terms. The introduced method is then applied to investigate nonadiabatic dynamics and imperfection effects in pulsed gate operations of quantum dot systems regarding the existence of spin-orbit effects. The fidelity loss of the SWAP gate owing to anisotropic exchange and the fidelity retrieval of the Loss-DiVincenzo pulse sequence under nonadiabatic evolution are elaborated by virtue of the proposed method.

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I. INTRODUCTION

The study of time-dependent quantum systems is a fundamental subject of quantum mechanics and it has received increasing interests in various fields, including quantum transport [1], quantum optics [2], and quantum information processing [3]. In particular, to implement desired quantum manipulation for information processing, physical systems should undergo an evolution generated by a sequence of pulsed interactions with well-designed strength and time duration that involves coupling either in between qubits or of the qubit system with external fields. In these fabrications, accurate control of the coherence evolution requires reliable resolution to the Schrödinger equation with time-dependent Hamiltonians.

It is well known that a time-dependent quantum system can be solved rigorously if the system possesses a complete set of dynamical invariants or so-called Lewis-Riesenfeld (LR) invariants [4,5]. Such invariants could be found, e.g., by virtue of an algebraic dynamical method [5,6]. Practically, however, it happens that exact analytical expression of the LR invariants was found only for some systems of particular classes. In the case that the pulsed interaction is weakly time dependent, the evolution could be dealt with by perturbation expansion or other decomposition methods to approximate the time-ordered exponential operator [7]. Recently, a perturbation series to construct the dynamical invariant has also been proposed [8] and the method was shown able to yield an exact LR invariant for a linearly driven harmonic oscillator while resumming all order of the expansion series.

In this article we will propose a concatenated cranking approach to resolve the Schrödinger equation for a class of time-dependent quantum systems with certain algebraic structure. The main proposal of the method, which will be elucidated in detail through a typical SU(2) model, is to invoke a series of canonical transformations to the system and incorporate the nonadiabatic term successively into the resulted cranking representation. The scheme of the concatenated procedure allows a unified calculation of the time evolution operator with desired high precision to include the nonadiabatic effect for time-dependent quantum systems with a specific algebraic structure. Moreover, we apply this method to investigate nonadiabatic dynamics and imperfection effects of pulsed gate operations in quantum dot systems regarding the existence of anisotropic exchange interactions. In particular, we show that the Loss-DiVincenzo pulse sequence [9] is able to produce a faithful controlled-NOT (CNOT) operation in nonadiabatic evolution, provided that the interaction pulses have a time-reversal symmetry.

The rest of the article is organized as follows. Section II is contributed to a detailed introduction for the concatenated cranking representation of the solution of Schrödinger equation. We will employ an SU(2) model and present a schematic description of the cranking approach and relevant truncation approximation (Sec. II A). The dynamical invariants associated the series of cranking representation under truncation will be elucidated in Sec. II B. The efficiency of the method is then testified by an illustrative spin-$\frac{1}{2}$ model in a rotating magnetic field (Sec. II C). In Sec. III we shall employ the proposed method to study pulsed quantum operations on electron spins in the presence of anisotropic exchange interactions. We will calculate the fidelity loss of the SWAP gate owing to the spin-orbit effects (Sec. III A) and demonstrate that the noise resilience property of the Loss-DiVincenzo proposal against anisotropic exchange could be rigorously guaranteed in the nonadiabatic process (Sec. III B). Finally, we summarize the results in Sec. IV.

II. GENERAL DESCRIPTION OF THE METHOD

A. Concatenated cranking representation of the Schrödinger equation

Let us start from a standard SU(2) model with a linear expression of its Hamiltonian:

$$H(t) = \vec{\Omega}(t) \cdot \vec{J}, \tag{1}$$

where $\vec{J}$ denotes the angular-momentum operator with its components satisfying $[J_i, J_j] = i\varepsilon_{ijk}J_k$. The external field $\vec{\Omega}(t) = (\Omega_x(t), \Omega_y(t), \Omega_z(t))$ here assumes an analytical

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form with general time dependency. To explore the evolution of the system generated by the Schrödinger equation $i\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle$, we introduce a rotation via $|\psi(t)\rangle = G_0(t)|\psi_g^{[0]}(t)\rangle$, where $G_0(t) = e^{-i\varphi(t)J_z}e^{-i\theta(t)J_y}$ with $\theta(t) = \arccos(\Omega_z/\Omega)$ and $\varphi(t) = \arctan(\Omega_y/\Omega_x)$ accounts for a canonical transformation converting the instantaneous Hamiltonian $H(t)$ as: $G_0^\dagger H(t)G_0 = \Omega(t)J_z$. The state $|\psi_g^{[0]}(t)\rangle$ in the cranking representation defined by the transformation $G_0(t)$ satisfies a new Schrödinger equation in which the effective Hamiltonian is obtained as

$$H_g^{[1]}(t) = G_0^\dagger H(t)G_0 - iG_0^\dagger\partial_t G_0 = \Omega(t)J_z + \Delta^{[0]}(t), \tag{2}$$

where the extra term

$$\Delta^{[0]}(t) \equiv -iG_0^\dagger\partial_t G_0 = \dot\varphi\sin\theta J_x - \dot\theta J_y - \dot\varphi\cos\theta J_z \tag{3}$$

accounts for a gauge potential since $H(t)$ depends on time explicitly. As the field varies slowly and satisfies $\dot\theta(t) \ll \Omega(t)$ and $\dot\varphi(t) \ll \Omega(t)$, one derives simply the conventional adiabatic solution by discarding the term $\Delta^{[0]}(t)$. The time evolution operator is then expressed as

$$U^{[0]}(t) = G_0(t)U_g^{[0]}(t)G_0^\dagger(t_0), \tag{4}$$

where $U_g^{[0]}(t) = \exp\{-i\int_{t_0}^t \Omega(t')dt' J_z\}$.

To gain nonadiabatic corrections to the evolution, note that the term $\Delta^{[0]}(t)$ contained in $H_g^{[1]}(t)$ has a linear expression in terms of J_i, hence we can record $H_g^{[1]}(t) \equiv \vec\Omega^{[1]}(t)\cdot\vec J$. Subsequently, we introduce further a canonical transformation $G_1(t) = e^{-i\varphi_1(t)J_z}e^{-i\theta_1(t)J_y}$, in which $\theta_1(t) = \arccos(\Omega_z^{[1]}/\Omega^{[1]})$ and $\varphi_1(t) = \arctan(\Omega_y^{[1]}/\Omega_x^{[1]})$, to convert the instantaneous Hamiltonian $H_g^{[1]}(t)$ with its $\vec J$ along the z direction again. This leads to a renewed cranking representation of the Schrödinger equation in which the associated Hamiltonian takes the form of

$$H_g^{[2]}(t) \equiv \vec\Omega^{[2]}(t)\cdot\vec J = \Omega^{[1]}(t)J_z + \Delta^{[1]}(t). \tag{5}$$

At this stage, the first-order correction to the adiabatic solution is achieved by discarding the term $\Delta^{[1]}(t) \equiv -iG_1^\dagger\partial_t G_1$ and the time evolution operator herein reads

$$U^{[1]}(t) = G_0(t)G_1(t)U_g^{[1]}(t)G_1^\dagger(t_0)G_0^\dagger(t_0), \tag{6}$$

where $U_g^{[1]}(t) = \exp\{-i\int_{t_0}^t \Omega^{[1]}(t')dt' J_z\}$. The described cranking procedure can be performed repetitively and a concatenated representation of the time evolution operator of the Schrödinger equation hence is established. Specifically, the $(k+1)$th-order representation exploits a cranking transformation $G_k(t) = e^{-i\varphi_k(t)J_z}e^{-i\theta_k(t)J_y}$ which converts the orientation of the $\vec J$ of the Hamiltonian $H_g^{[k]}(t)$ along the z axis. The corresponding $H_g^{[k+1]}$ in the representation has a form of $H_g^{[k+1]}(t) = \Omega^{[k]}(t)J_z + \Delta^{[k]}(t)$ in which the gauge potential $\Delta^{[k]}(t) \equiv -iG_k^\dagger\partial_t G_k$ accounts for the $(k+1)$th-order nonadiabatic effect. The kth-order approximation to the evolution of the system is obtained by discarding $\Delta^{[k]}(t)$ and the evolution operator of the original Schrödinger equation is written as

$$U^{[k]}(t) = G_0(t)\cdots G_k(t)U_g^{[k]}(t)G_k^\dagger(t_0)\cdots G_0^\dagger(t_0), \tag{7}$$

where $U_g^{[k]}(t) = \exp\{-i\int_{t_0}^t \Omega^{[k]}(t')dt' J_z\}$.

B. Dynamical invariants associated with concatenated cranking representation

Let us consider the corresponding dynamical invariant with respect to the approximative dynamics yielded by the concatenated cranking representation. Recall that the LR invariant of a time-dependent quantum system is defined by an observable $I(t)$ that satisfies

$$i\partial_t I(t) - [H(t), I(t)] = 0. \tag{8}$$

The peculiar interest of finding such an invariant resides in that its eigenvectors, denoted as $|\phi_m(t)\rangle$, are transported diagonally during the evolution generated by the Schrödinger equation. Indeed, the basic solution of the Schrödinger equation can be derived straightforwardly by adding a total phase to $|\phi_m(t)\rangle$, i.e., $|\psi_m(t)\rangle = e^{i\Phi_m(t)}|\phi_m(t)\rangle$, where

$$\Phi_m(t) = \int_{t_0}^t \langle\phi_m(t')|i\partial_{t'} - H(t')|\phi_m(t')\rangle dt'. \tag{9}$$

Intriguingly, the above-established concatenated representation indicates a series of dynamical invariants under truncation approximation. Specifically, the first-order invariant, corresponding to an approximative evolution of Eq. (6), takes the form of

$$\begin{aligned} I^{[1]}(t) &= G_0(t)G_1(t)J_zG_1^\dagger(t)G_0^\dagger(t) \\ &= \frac{1}{\Omega^{[1]}(t)}[H(t) + \tilde\Delta^{[0]}(t)], \end{aligned} \tag{10}$$

where $\tilde\Delta^{[0]}(t) = G_0(t)\Delta^{[0]}(t)G_0^\dagger(t)$. One can recognize that the eigenvectors of $I^{[1]}(t)$, $|\phi_m^{[1]}(t)\rangle = G_0(t)G_1(t)|m\rangle$, are transported diagonally under the action of the evolution operator (6). More generally, for the kth-order approximation specified by Eq. (7), the associated dynamical invariant is derived as

$$\begin{aligned} I^{[k]}(t) &= G_0(t)\cdots G_k(t)J_zG_k^\dagger(t)\cdots G_0^\dagger(t) \\ &= \frac{1}{\Omega^{[k]}(t)}\left[H(t) + \sum_{i=0}^{k-1}\tilde\Delta^{[i]}(t)\right], \end{aligned} \tag{11}$$

where $\tilde\Delta^{[i]}(t) = G_0(t)\cdots G_i(t)\Delta^{[i]}(t)G_i^\dagger(t)\cdots G_0^\dagger(t)$. It turns out that the total phase induced to the eigenvector $|\phi_m^{[k]}(t)\rangle = G_0(t)\cdots G_k(t)|m\rangle$ can be expressed by a consistent formula

$$\begin{aligned} \Phi_m^{[k]}(t) &= \int_{t_0}^t \langle\phi_m^{[k]}(t')|i\partial_{t'} - H(t')|\phi_m^{[k]}(t')\rangle dt' \\ &= m\int_{t_0}^t \Omega^{[k]}(t')dt', \end{aligned} \tag{12}$$

in which m denotes the quantum number of J_z.

C. Spin-half model in a rotating field

To testify the efficiency of the proposed concatenated cranking approach, let us apply it to the spin-$\frac{1}{2}$ system in a rotating magnetic field with a Hamiltonian

$$H(t) = \vec B(t)\cdot\vec S, \tag{13}$$

where $\vec B(t) = B_0(\sin\theta\cos\omega t, \sin\theta\sin\omega t, \cos\theta)$. One can directly obtain, through the canonical transformation

$G_0(t) = e^{-i\omega t S_z} e^{-i\theta S_y}$ to its Schrödinger equation, the first-order cranking representation with its Hamiltonian

$$H_g^{[1]} = \vec{\Omega}^{[1]} \cdot \vec{S}, \vec{\Omega}^{[1]} = (\omega \sin\theta, 0, B_0 - \omega\cos\theta). \quad (14)$$

The effective magnetic field $\vec{\Omega}^{[1]}$ now is independent of time. Consequently, by employing further a constant rotation $G_1 = e^{-i\theta_1 S_y}$ with $\theta_1 = \arctan[\omega\sin\theta/(B_0 - \omega\cos\theta)]$, the concatenated representation terminates autonomously. The evolution of the system is then resolved exactly with the time evolution operator given as

$$U(t) = G_0(t) G_1 e^{-i\Omega^{[1]}(t-t_0)S_z} G_1^\dagger G_0^\dagger(t_0). \quad (15)$$

Accordingly, the rigorous dynamical invariant of the system is obtained

$$I(t) = G_0(t) G_1 S_z G_1^\dagger G_0^\dagger(t) = \frac{H(t) - \omega S_z}{\Omega^{[1]}}. \quad (16)$$

This leads straightforwardly to the recurrent bases of the original Schrödinger equation

$$\begin{aligned} |\phi_{1/2}(t)\rangle &= \cos\Theta|\uparrow\rangle + e^{i\omega t}\sin\Theta|\downarrow\rangle, \\ |\phi_{-1/2}(t)\rangle &= \sin\Theta|\uparrow\rangle - e^{i\omega t}\cos\Theta|\downarrow\rangle, \end{aligned} \quad (17)$$

where $\Theta = (\theta + \theta_1)/2$. From Eq. (12), the total phases induced to these bases during cyclic evolution with $T = 2\pi/\omega$ are worked out to be $\Phi_\pm = \pm\pi(\Omega/\omega + 1)$.

III. PULSED QUANTUM GATE OPERATIONS ON ELECTRON SPINS IN THE PRESENCE OF ANISOTROPIC EXCHANGE

We are now beginning to consider the pulsed quantum gate scheme in quantum dot systems based on spin exchange interactions. In the ideal situation with an isotropic exchange coupling, a SWAP gate can be generated through the evolution $U(\lambda) = \exp(-i\lambda \vec{S}_1 \cdot \vec{S}_2)$ by setting the pulse strength $\lambda = \int_{-\tau_s}^{\tau_s} J(t)dt = \pi$, where $J(t)$ is the coupling coefficient of the exchange interaction. According to Loss and DiVincenzo's proposal [9], the rigorous CNOT operation is then achieved by combining the "square-root-of-SWAP" gate $U_s \equiv U(\frac{\pi}{2})$ with single-qubit operations:

$$U_{\mathrm{CNOT}} = e^{i\frac{\pi}{2}S_{1z}} e^{-i\frac{\pi}{2}S_{2z}} U_s e^{i\pi S_{1z}} U_s. \quad (18)$$

In a realistic system, however, the presence of the spin-orbit coupling will induce additional anisotropic exchange and the resulted Hamiltonian takes the form [10–13]

$$\begin{aligned} H(t) = {} & J(t)\vec{S}_1 \cdot \vec{S}_2 + J_\beta(t)(S_{1x}S_{2y} - S_{1y}S_{2x}) \\ & + J_\gamma(t)(S_{1x}S_{2x} + S_{1y}S_{2y}), \end{aligned} \quad (19)$$

where the added two terms describe an asymmetric Dzyaloshinski-Moriya interaction and a symmetric anisotropic ingredient, respectively. As the coefficients $J_{\beta,\gamma}(t)$ cannot be tuned optionally, these anisotropic terms act as a source of noises and their influence to the gate scheme has been studied in many literatures [11–14]. Particularly, it was shown [12,13] that in the case the Hamiltonian is weakly time dependent, e.g., the pulses $J_{\beta,\gamma}(t)$ have the same form with $J(t)$ or these interaction pulses are exerted adiabatically, the noise effect from the spin-orbit coupling should cancel itself in the Loss-DiVincenzo gate sequence of Eq. (18).

In the case that the pulse of the anisotropic coupling $J_{\beta,\gamma}(t)$ is not proportional to $J(t)$, the influence from nonadiabatic effects to the gate operation requires a careful analysis [12]. We resume this issue and explore the imperfection to both quantum operations of the single SWAP gate and the Loss-DiVincenzo pulse sequence. It is noteworthy that we can demonstrate that the Loss-DiVincenzo proposal is able to produce a rigorous CNOT operation in the case of nonadiabatic evolution, provided that the interaction pulses $J(t)$ and $J_{\beta,\gamma}(t)$ have a time-reversal symmetry. We emphasize that this consequence releases the requirement for spin-based quantum computation which has not been aware of before.

A. Loss of fidelity in the SWAP gate operation

The resource of the SWAP gate alone is sufficient for universal quantum computation in an encoded space [15]. To investigate the imperfection effect to the evolution $U(\pi)$ owing to the existence of anisotropic terms, we note that the assumed interaction (19) possesses an axial symmetry hence the total spin $S_z = S_{1z} + S_{2z}$ is conserved along the evolution. The generated operation then reduces to a form $U_H(\pi) \cong (e^{-i\phi}\bar{I}_2) \oplus (e^{i\phi}U_{\mathrm{sub}})$, where $\bar{I}_2$ stands for a projector on the subspace $\{|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ and U_{sub} is an evolution operator acting on $\{|e_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)\}$ generated by

$$H_{\mathrm{sub}}(t) = \Omega_z(t)K_z + \Omega_y(t)K_y \quad (20)$$

with $\Omega_z(t) = J(t) + J_\gamma(t)$, $\Omega_y(t) = -J_\beta(t)$. The K_α ($\alpha = x, y, x$) here denote the pseudo spin operators acting on the subspace $\{|e_\pm\rangle\}$. Note that $H_{\mathrm{sub}}(t)$ contains only operators K_y and K_z, we can now employ the following cranking transformations to achieve the concatenated representation

$$G_n(t) = \begin{cases} e^{i\theta_n(t)K_x}, & n = 2k \\ e^{i\theta_n(t)K_y}, & n = 2k+1, \end{cases} \quad (21)$$

where

$$\theta_0(t) = \arctan\frac{\Omega_y}{\Omega_z}, \quad \theta_n(t) = \arctan\frac{(-1)^n\dot{\theta}_{n-1}(t)}{\Omega^{[n-1]}(t)} \quad (22)$$

and $\Omega^{[n]}(t) = [(\Omega^{[n-1]})^2 + \dot{\theta}_{n-1}^2]^{1/2}$. Correspondingly, the gauge potential in each cranking representation is obtained explicitly:

$$\Delta^{[2k]}(t) = \dot{\theta}_{2k}(t)K_x, \quad \Delta^{[2k+1]}(t) = \dot{\theta}_{2k+1}(t)K_y. \quad (23)$$

According to Eqs. (7) and (12), the occurrence of the gauge potential in the nonadiabatic evolution will result in a modified phase factor in comparing with its adiabatic counterpart. We plot the magnitude $\Omega^{[n]}(t)$ in Fig. 1, with its integral indicating the total phase factor. The pulse form of $J(t)$ and $J_\mu(t)$ ($\mu = \beta, \gamma$) is assumed as

$$J(t) = J_0\mathrm{Sech}^2(2\nu t), \quad J_\mu(t) = \mu_0\mathrm{Sech}^4(2\nu t). \quad (24)$$

The pulse $J(t)$ satisfies $\lim_{\tau\to\infty}\int_{-\tau/2}^{\tau/2} J(t)dt = J_0/\nu = \pi$ and the pulse height J_0 in our calculation is taken as $\pi \times 10^{10}\ s^{-1}$ (setting $\hbar = 1$). Besides, in the nonadiabatic evolution the recurrent bases will deviate from instantaneous eigenstates of the Hamiltonian and hence induce additional correction to the adiabatic solution. Since the anisotropic terms will not affect the states $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$, we depict in Fig. 2 the fidelity

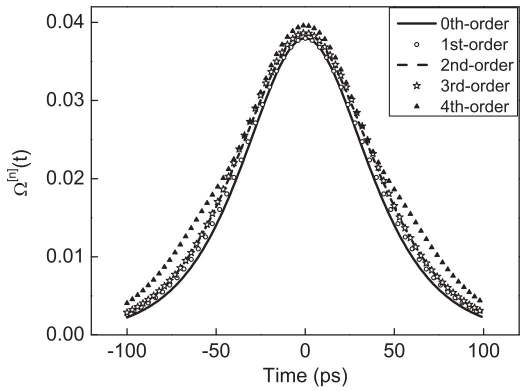

FIG. 1. Schematic of deviation of the phase factor (reflected by the integral of the curves along horizontal coordinates) in comparing with that of adiabatic evolution. The intensities of the anisotropic terms are set as $\beta_0 = 0.5J_0$ and $\gamma_0 = 0.1J_0$. The series of magnitudes $\Omega^{[n]}(t)$ in the concatenated representation of orders $n = 0, 1, 2, 3, 4$ are plotted, where $n = 0$ accounts for the adiabatic approximation.

between $U_H(\pi)$ and the ideal SWAP gate $U(\pi)$ while acting on the bases $|e_\pm\rangle$. The results reveal that as β_0/J_0 is small the approximation agrees well with the numerically exact value, while if the ratio β_0/J_0 increases high-order calculations are required to reach convergence.

B. Fidelity retrieval of the Loss-DiVincenzo pulse sequence under nonadiabatic evolution

Let us look over the noise resilience feature of the pulse sequence (18) against spin-orbit effects. Indeed, the noise cancellation process here is very similar to the refocusing technique that has been widely used in fault-tolerant quantum information processing [16]. It is seen that the single qubit operation $e^{i\pi S_{1z}}$ contained in the sequence (18) acts as a spin flip $\hat{\Pi} = e^{i\pi K_x}$ on the subspace of $\{|e_\pm\rangle\}$. In the case that the pulsed interaction is weakly time dependent, the noise cancellation of the spin-orbit effects is clearly understood since the twice evolution of $H_{\rm sub}(t)$ will cancel each other owing to the peculiarity of $H_{\rm sub}(t)\hat{\Pi} = -\hat{\Pi}H_{\rm sub}(t)$. Intriguingly, we show in the below that this noise-resilient property should hold rigorously in the case of nonadiabatic evolution, provided that the interaction pulse has a time-reversal symmetry, i.e., $H_{\rm sub}(\tau - t) = H_{\rm sub}(t)$. To prove this promising character, we write the evolution operator $U_{\rm sub}$ from time $-\tau/2$ to $\tau/2$ as

$$U_{\rm sub} = \lim_{N\to\infty} U_{\rm sub}(t_N)\cdots U_{\rm sub}(t_2)U_{\rm sub}(t_1), \tag{25}$$

where $U_{\rm sub}(t_i) = e^{-i\Delta\tau H_{\rm sub}(t_i)}$. Note that in the time interval $\Delta\tau = \tau/N$ the Hamiltonian $H_{\rm sub}(t_i)$ can be viewed as time independent, one derives $\hat{\Pi}^\dagger U_{\rm sub}(t_i)\hat{\Pi} = U_{\rm sub}^{-1}(t_i)$. Consequently, there is

$$\begin{aligned}\hat{\Pi}^\dagger U_{\rm sub}\hat{\Pi} &= \lim_{N\to\infty} U_{\rm sub}^{-1}(t_N)\cdots U_{\rm sub}^{-1}(t_2)U_{\rm sub}^{-1}(t_1)\\ &= \lim_{N\to\infty} U_{\rm sub}^{-1}(t_1)\cdots U_{\rm sub}^{-1}(t_{N-1})U_{\rm sub}^{-1}(t_N)\\ &= U_{\rm sub}^{-1},\end{aligned} \tag{26}$$

where we have used the property of time-reversal symmetry of $H_{\rm sub}(t)$. This confirms that the influence of anisotropic terms in the twice evolution inserted with $\hat{\Pi}$ will cancel each other rigorously whatever the process is adiabatic or nonadiabatic.

The noise-resilient feature described above could also be explicated in the framework of the proposed concatenated representation of the Schrödinger equation. It happens that the rigorous dynamical invariant $I(t)$ of the system, indicated by the series of $I^{[n]}(t)$ in Eq. (11) while resumming arbitrary high order, is antisymmetric under the combined transformation of the time reversal and the spin flip $\hat{\Pi}$. To prove this fact, we first note that $H(\tau - t) = H(t)$ and $H(t)$ contains only ingredients K_y and K_z, so there exists $H_{\rm sub}(\tau - t)\hat{\Pi} = -\hat{\Pi}H_{\rm sub}(t)$. Second, we verify that the rotation angles of Eq. (22) satisfy $\theta_n(-t) = (-1)^n\theta_n(t)$ and $\dot{\theta}_n(-t) = (-1)^{n+1}\dot{\theta}_n(t)$. Thus all series $G_n(t)$ specified in Eq. (21) are invariant under the combined transformation of the time reversal and spin flip $\hat{\Pi}$, whereas all terms $\Delta^{[n]}(t)$ in Eq. (23) satisfy $\Delta^{[n]}(\tau - t)\hat{\Pi} = -\hat{\Pi}\Delta^{[n]}(t)$. Consequently, all $\tilde{\Delta}^{[n]}(t)$ contained in $I^{[n]}(t)$ should satisfy the same relation with $\Delta^{[n]}(t)$. This completes our proof that $I(\tau - t)\hat{\Pi} = -\hat{\Pi}I(t)$. Therefore, one can conclude that the eigenvectors of $I(t)$ will undergo inversely in the second pulsed process and the noise cancellation of the spin-orbit effects herein is understood unambiguously in accord with the refocusing scheme.

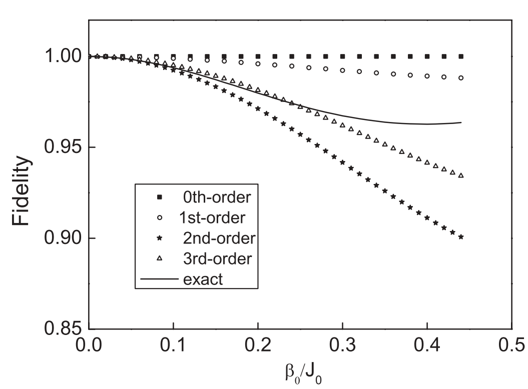

FIG. 2. Fidelity of the SWAP gate as a function of the ratio β_0/J_0 in the presence of anisotropic exchange. The interaction pulse assumes the form of Eq. (24) with truncation at $\tau/2 = 150$ ps [where $J(\pm\tau/2) \sim 10^{-4}J_0$]. We assume the pulse height $\gamma_0 = 0.1J_0$. The solid line accounts for the numerically exact value; rectangles line, dots line, stars line, and triangles line stand for results of the concatenated series $n = 0$, 1, 2 and 3, respectively.

IV. CONCLUSION

In conclusion, we have proposed a concatenated cranking approach to solve the Schrödinger equation for a class of time-dependent quantum systems with specific algebraic structure. We have resorted to an SU(2) model to characterize the truncation procedure of the method and reveal an asymptotical series to approximate the dynamical invariants of the system. In principle, this method can be extended to deal with a wide range of time-dependent physical systems which possesses a semisimple Lie algebraic structure. We mention that for the cases of high-dimensional algebraic systems, the canonical transformation employed in the successive procedure should be reconstructed so that the transformed instantaneous Hamiltonian contains only the generators of Cartan subalgebra of the semisimple Lie algebra. The Abelian property of these Cartan generators warrants that the ingredient $U_g^{[k]}$ in the evolution operator can be easily calculated irrespective of the time order of its integral. According to the theory of Lie algebras, the corresponding canonical transformation is just an element of the semisimple Lie group which can always be found [17]. We would like to stress further that the proposed approach utilizing the series of cranking transformations that approximates the dynamical evolution of the system with a desired precision differs distinctly from conventional methods based on perturbation expansion of the time-ordered exponential operator; its mathematical and physical foundations hence deserve further investigation.

For applications, we have exploited the method to study imperfection effects in pulsed quantum operations of quantum dot systems concerning the existence of the spin-orbit effects. We have investigated the fidelity loss of the SWAP gate in the case of nonadiabatic evolution where the anisotropic interaction pulses $J_{\beta,\gamma}(t)$ have different form from the isotropic $J(t)$. Furthermore, we have shown that the Loss-DiVincenzo pulse sequence has noise-resilient property against the anisotropic exchange coupling even for the case of nonadiabatic evolution, provided that the exerted interaction pulses have a time-reversal symmetry.

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