Steady-state and dynamical Anderson localization of counterpropagating beams in two-dimensional photonic lattices

Dragana M. Jović[1,2] and Milivoj R. Belić[2]
[1]Institute of Physics, P. O. Box 57, 11001 Belgrade, Serbia
[2]Texas A&M University at Qatar, P. O. Box 23874, Doha, Qatar
(Received 17 September 2009; published 12 February 2010)

We demonstrate Anderson localization of mutually incoherent counterpropagating beams in an optically induced two-dimensional photonic lattice. The effect is displayed in a system of two broad probe beams propagating head-on through a fixed disordered photonic lattice recorded in a photorefractive crystal. In addition to the steady-state localization, we also observe the dynamical localization; that is, the localization of time-changing beams. As compared to the localization of single beams, in which there exist no dynamical effects, the localization of counterpropagating beams is more pronounced and prone to instabilities.

DOI: 10.1103/PhysRevA.81.023813 PACS number(s): 42.65.Tg, 42.65.Sf

I. INTRODUCTION

At the half-centennial celebration [1] of the remarkable discovery of electronic localization in disordered crystals by P. W. Anderson [2], it is perhaps fitting to report another instance of localization in an unrelated physical system. Nowadays, the phenomenon of Anderson localization (AL) is one of the basic concepts in solid-state physics. Originally introduced to explain localization of electronic wave functions as they propagate through disordered crystals, it has attracted growing interest during the next few decades [3,4] for its usefulness in elucidating the phase transition from conductors to insulators.

Quite early it was realized that, as a wave phenomenon, AL can be extended to include light [5–7] and interpreted as an interference effect of counterpropagating (CP) beamlets along different possible paths in multiple scattering. Following recent experimental observation of AL in a disordered two-dimensional (2D) optically induced photonic lattice (PL) [8], AL of matter waves in a 1D disordered Bose-Einstein condensate (BEC) [9], and Anderson-like localization of light in a 1D PL with randomly distributed defects [10], the transverse localization of light as it propagates in disordered photonic crystals has become a hot topic of research.

In this article we report the steady state (SS) and the dynamical transverse AL of broad CP beams in optically induced PLs. We consider a system of two broad probe beams counterpropagating head-on in a fixed PL. In a numerical study we demonstrate AL of the probe beams by adding a varying random disorder to the lattice. It is found that the localization of CP beams is more pronounced than that of single beams and prone to dynamical instabilities, which are absent in the single beam AL.

The article is divided into four sections. Section I presents introduction, Sec. II introduces the model, Sec. III discusses results, and Sec. IV concludes the article.

II. THE MODEL

To study the effect of AL in CP geometry, a time-dependent model is adopted, describing the nonlinear (NL) propagation of mutually incoherent CP beams in optically induced PLs in photorefractive (PR) crystals [11,12]. Although the experiment [8] is done with a single propagating beam, we extend the analysis to the CP geometry to obtain more general results. Our results can be extended to other systems, such as lattices and beams in BECs [9,13], and other periodic NL systems in optical, atomic, and condensed matter physics.

We utilize the well-known model [11,12] for the NL propagation of CP beams in an optically induced lattice in the paraxial approximation:

$$i\partial_z F = -\Delta F + \Gamma E F, \quad -i\partial_z B = -\Delta B + \Gamma E B, \quad (1)$$

where z is the propagation coordinate, Δ is the transverse Laplacian, and Γ is the beam coupling constant. F and B are the forward- and the backward-propagating probe beams, E represents the space charge field (SCF) built in the crystal, normalized to the applied external field. Equations (1) are written in the dimensionless form. The generation of SCF in PR crystals is described by a relaxation-type equation:

$$\tau\partial_t E + E = -\frac{I + I_g}{1 + I + I_g}, \quad (2)$$

where τ is the relaxation time of the crystal, $I = |F|^2 + |B|^2$ is the total beam intensity, and I_g stands for the lattice intensity, both measured in units of the dark or background intensity. A slow change in SCF causes a change in the index of refraction, which modifies the SS propagation of the fast optical beams. Note that we consider a local isotropic saturable PR model for the interaction of incoherent incident beams.

When the propagation in disordered PLs is considered, Eq. (2) is modified, to include the randomized lattice intensity distribution I_r:

$$\tau\partial_t E + E = -\frac{I + I_r}{1 + I + I_r}. \quad (3)$$

Randomization is achieved by adding certain percentage of the random field to I_g. Since the lattice is uniform in the z direction, randomness is confined to the transverse plane. We use lattices with triangular arrangement of beams. As probe beams, identical hyper-Gaussian beams are launched head-on from the opposite faces, in the center of the lattice.

The propagation Eqs. (1) are solved numerically, concurrently with the temporal Eq. (3) for SCF, in the manner described in Ref. [11,12] and for typical values of parameters

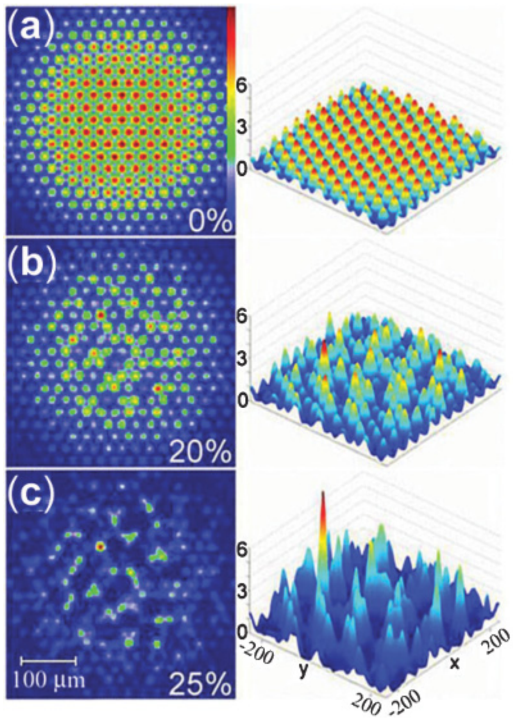

FIG. 1. (Color online) Steady-state Anderson localization. Intensity distributions of the forward probe beam are shown at its exit face for (a) zero disorder; (b) 20% disorder; (c) 25% disorder. Parameters: $\Gamma = 11$, $L = 2.5L_D = 10$ mm, FWHM of the probe beams 300 μm, input intensity $|F_0|^2 = |B_L|^2 = 1$, lattice intensity $I_g = 3$, and lattice spacing 25 μm.

used in experiments. Input peak intensities of the forward- and backward-propagating beams are set to 1/3 of the lattice beam peak intensity. When dynamical AL is considered, the input beam intensities are increased to twice the lattice beam intensity. The lattice beam spacing is 25 μm. The input FWHM of the probe beams equals $\omega_0 = 300$ μm. The coupling constant between the CP beams is set to $\Gamma = 11$. The propagation length is fixed to $L = 10$ mm. When the steady state is reached, we speak of the SS AL; when the fields keep changing, we speak of the dynamical AL.

III. RESULTS

First, we consider the case without disorder. Hyper-Gaussian CP beams focus on propagation and are captured by the lattice sites, forming a filamented triangular pattern similar to the lattice itself (Fig. 1, the first row). No transverse localization of the probe beams is observed, of course, but the transverse mutual and self-focusing of each beam as a whole are there. For the chosen values of parameters, no temporal instabilities of the beams occur—the SS is reached fast.

To observe the transverse beam localization, a random noise is added to the lattice. By increasing disorder, we discover the effect of AL (Fig. 1, the remaining rows). It is clear that the number of prominent peaks, as well as the width of the probe beam, is greatly reduced with the increase in disorder. We find that the transverse AL in SS is enhanced in the CP geometry, as compared to the single beam AL, for the same set of parameters.

For quantitative description of the phenomenon, we utilize the standard quantities: the inverse participation ratio

$$P = \int I^2(x, y, L)dxdy \Bigg/ \left[\int I(x, y, L)dxdy\right]^2, \quad (4)$$

the localization depth ξ, and the effective beam width $\Lambda = P^{-1/2}$ [8]. It should be noted that Λ is defined at the exit face for each of the beams and is *not* equal to the FWHM of input beams. The localization depth is defined as $\xi = l\exp(k_\perp l/2)$, where l is the mean free path related to the refractive index fluctuations and $k_\perp = 2\pi/\omega_0$ is the transverse wave number. In our case l is of the order of the lattice constant, while $k_\perp$ is inversely proportional to the initial beam width.

Since AL is essentially a statistical phenomenon, many realizations of the disorder are needed to measure ensemble averages for the quantities of interest. We establish different disorder realizations by starting each simulation with different random number generators. It turned out that even though different realizations lead to different transverse distributions of the probe beams (cf. insets in Fig. 2), the measured values of P and Λ stay close to each other (Fig. 2). Still, the relative fluctuation $\Delta P/P \approx 30\%$ at the 25% disorder is relatively large.

It is evident from Fig. 2 that, starting from some input values, the inverse participation ratio of any of the beams gets larger and the effective beam width gets smaller, as the amount of disorder is increased. These tendencies are similar to the experimental findings in Ref. [8], except for the shape of the functional dependencies: while in Ref. [8] they are convex (concave) for P (Λ), in our case they are the opposite. This difference is the consequence of considering AL of CP beams, instead of the single beams.

Figure 3 presents AL of the forward probe beam as a function of the propagation distance. Such dependencies are characteristic for the presentation of the single beam localization. At this point it is worthwhile to point to the differences between the single beam AL and the AL of CP beams. In the single beam AL, to reach the localization regime, the beam's propagation distance must be long enough, so that

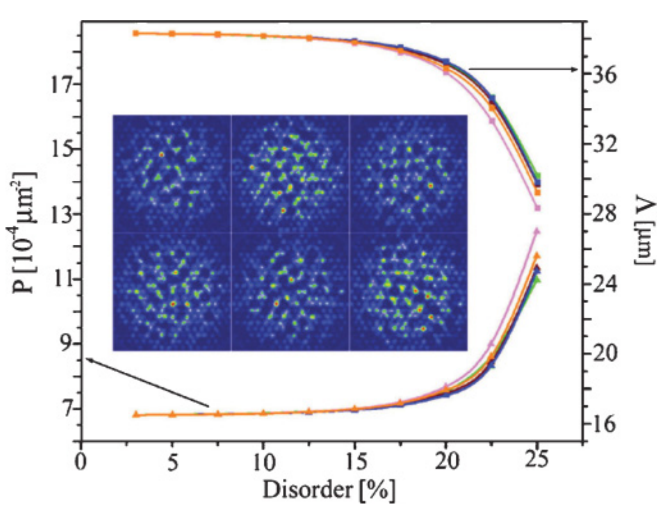

FIG. 2. (Color online) P and Λ as functions of the disorder level. Inset depicts different realizations for the same disorder of 25%.

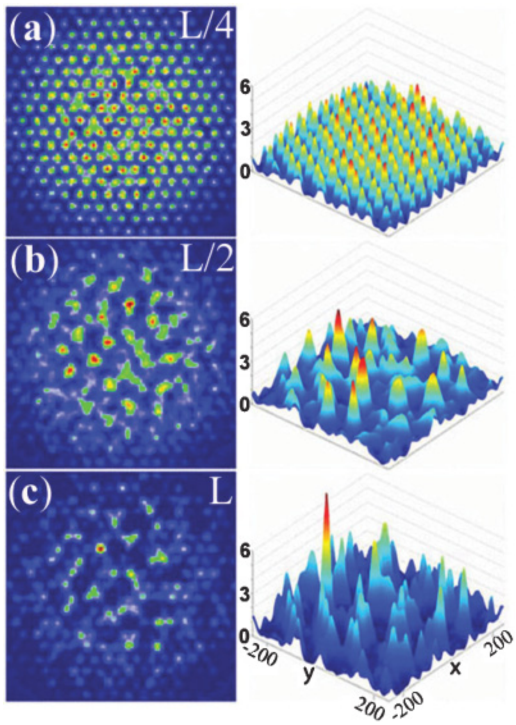

FIG. 3. (Color online) Anderson localization at different propagation distances and for 25% disorder, (a) at $L/4$; (b) $L/2$; and (c) at the exit face L. The setup and parameters are as in Fig. 1.

the localization length ξ becomes comparable to the effective beam width. As it propagates, the beam initially undergoes diffusive broadening, until the appropriate propagation distance is reached; then the localization takes place and the beam focuses, acquiring exponentially decaying tails [8]. This clear picture becomes more complex in the CP case.

In the CP localization, the initial diffusive broadening is practically absent (Fig. 4), this being the consequence of mutual and self-focusing of the beams, which suppress the broadening. The beams enter with an effective width of about 42 μm and continue to focus. No diffusive expansion is

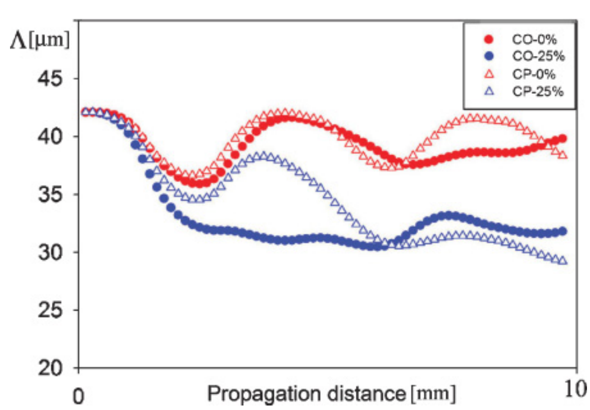

FIG. 4. (Color online) Comparison between the localization of copropagating and counterpropagating beams. Λ is shown vs. the propagation distance for the forward-propagating beam. Red (light gray) curves are for 0 disorder, blue (dark gray) curves are for 25% disorder. Oscillations in the beam widths are the consequence of focusing instabilities.

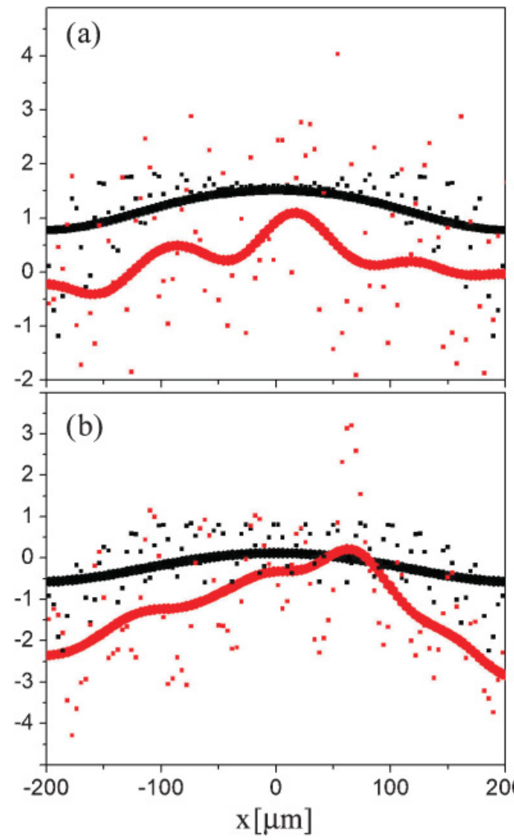

FIG. 5. (Color online) Comparison of the single beam and CP beams localization for 0 and 25% disorder. The ordinate axis is on the logarithmic scale. (a) Single beam profile after 10 mm propagation (black curve and dots, 0% disorder; red (gray) curve and dots, 25% disorder). Curves represent spline fits through the dots; (b) Same as (a) but for the forward beam of the CP pair.

seen—the beams reinforce each other on propagation and form tightly focused filamented structures. Even in the case of no disorder—no localization—there is no broadening of the beams in the NL regime we consider. The oscillations seen in Λ are the consequence of the NL mutual and self-focusing instability of CP beams. The self-focusing oscillation of the single beam, as well as its localization, is less prominent. For the parameters used, already at the level of 25% disorder a strong localization regime is reached; the diffusion broadening is negligible, but the exponential tails are there (Fig. 5). The transverse wave number equals only $k_\perp \approx 0.021\ \mu\text{m}^{-1}$, and the localization depth is estimated at $\xi \approx 32\ \mu$m, which is of the order of the effective beam width.

However, the most prominent difference between the single beam AL and the AL of CP beams is the appearance of dynamical localization in the latter case. Such localization is not possible in the single beam AL. In principle, CP beams are prone to spatiotemporal (ST) instabilities on increasing the coupling constant, the propagation distance, or the initial beam intensities. One should make a distinction between the spatial modulational instabilities, which are manifested in the beam filamentation, and the ST instabilities, which involve focusing and filamentation in both space and time. While the literature on spatial instabilities in Kerr and PR media is abundant [14–19], there is scarcely any on the ST

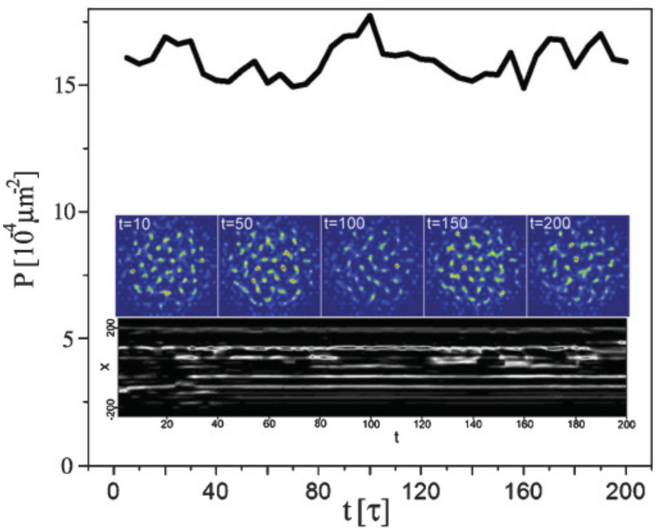

FIG. 6. (Color online) Dynamical Anderson localization. (Top) P as a function of time. (Middle) Transverse intensity distributions at different moments. (Bottom) The x-axis section through the lattice, as a function of time. The parameters are: $|F_0|^2 = |B_L|^2 = 2$, $I_g = 1$. Other parameters are as in Fig. 1.

instabilities of CP beams in the presence of optically induced lattices [12,20]. In short, such instabilities manifest themselves in many ways: from sudden transverse jumps of the CP beams as a whole, to spontaneous pattern formation, and to the continuous transverse motion of beam filaments [21]. It is the latter form of instabilities that is readily observed in the AL of CP beams.

Under dynamical localization we understand AL of CP beams in which all the quantities associated with the localization keep changing in time. As they propagate, the beams undergo all the changes characteristic of AL, but they never reach steady state. An example is shown in Fig. 6. The dynamical AL is reached there just by increasing the input beam intensities relative to the lattice intensity and keeping all other parameters as in Fig. 1(c). The time-dependent quantities keep fluctuating about the time-independent SS averages. For the example at hand, one can infer from the graph that $P_{ave} \approx 16 \times 10^{-4}\ \mu\text{m}^{-2}$, which translates into $\Lambda_{ave} \approx 25\ \mu$m.

IV. CONCLUSION

In conclusion, we report on the transverse AL of mutually incoherent CP beams in optically induced 2D photonic lattices. This is accomplished in a system of two CP broad probe beams, propagating head-on through the fixed triangular photonic lattice in a saturable PR medium. In a numerical study, we demonstrate the transverse AL of the probe beams by adding a varying random disorder to the lattice. It is found that steady state AL is more pronounced in the CP geometry than in the standard single beam geometry. Also, due to strong mutual and self-focusing tendency of CP beams, no initial diffusive broadening of beams is observed in the regime we consider.

Interestingly, we also observe the dynamical AL; that is, AL of the time-changing CP beams. Dynamical situation is introduced by increasing the input fields relative to the lattice peak input intensity, whereby an unstable regime of beam propagation is entered. The CP beams no longer reach steady state but keep changing in time. The parameters describing localization, such as P, Λ, ξ, etc., become time dependent; however, the time dependence is not dramatic. The time-dependent quantities keep fluctuating about the time-independent steady-state averages.

ACKNOWLEDGMENTS

Work performed at the Institute of Physics, Belgrade University, is supported by the Ministry of Science and Technological Development of the Republic of Serbia, under the project OI 141031. Work performed at the Texas A&M University at Qatar is supported by the project NPRP 25-6-7-2 with the Qatar National Research Foundation.

[1] See feature articles in the August issue of *Physics Today*: A. Lagendijk, B. van Tiggelen, and D. S. Wiersma, Phys. Today **62**, 24 (2009); A. Aspect and M. Inguscio, *ibid.* **62**, 30 (2009).
[2] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
[3] B. Kramer and A. MacKinnon, Rep. Prog. Phys. **56**, 1469 (1993).
[4] T. Brandes and S. Kettemann (eds.), *Anderson Localization and Its Ramifications* (Springer, Berlin, 2004).
[5] S. John, Phys. Rev. Lett. **53**, 2169 (1984).
[6] D. S. Wiersma, P. Bartolini, Ad Lagendijk, and R. Righini, Nature **390**, 671 (1997).
[7] M. Störzer, P. Gross, C. M. Aegerter, and G. Maret, Phys. Rev. Lett. **96**, 063904 (2006).
[8] T. Schwartz, G. Bartal, S. Fishman, and M. Segev, Nature **446**, 52 (2007).
[9] J. Billy, V. Josse, Z. Zuo, A. Bernard, B. Hambrecht, P. Lugan, D. Clément, L. Sanchez-Palencia, P. Bouyer, and A. Aspect, Nature **453**, 891 (2008).
[10] A. Kanshu, C. E. Rüter, D. Kip, and V. M. Shandarov, Appl. Phys. B **95**, 537 (2009)
[11] M. Belić, M. Petrović, D. Jović, A. Strinić, D. Arsenović, K. Motzek, F. Kaiser, Ph. Jander, C. Denz, M. Tlidi, and Paul Mandel, Opt. Express **12**, 708 (2004).
[12] M. Belić, D. Jović, S. Prvanović, D. Arsenović, and M. Petrović, Opt. Express **14**, 794 (2006).
[13] G. Roati, C. D'Errico, L. Fallani, M. Fattori, C. Fort, M. Zaccanti, G. Modugno, M. Modugno, and M. Inguscio, Nature **453**, 895 (2008).
[14] W. J. Firth and C. Paré, Opt. Lett. **13**, 1096 (1988).
[15] W. J. Firth, A. Fitzgerald, and C. Paré, J. Opt. Soc. Am. B **7**, 1087 (1990).
[16] G. G. Luther and C. J. McKinstrie, J. Opt. Soc. Am. B **9**, 1047 (1992).

[17] R. Chang, W. J. Firth, R. Indik, J. V. Moloney, and E. M. Wright, Opt. Commun. **88**, 167 (1992).
[18] J. B. Geddes, R. A. Indik, J. V. Moloney, and W. J. Firth, Phys. Rev. A **50**, 3471 (1994).
[19] O. Sandfuchs, F. Kaiser, and M. R. Belić, Phys. Rev. A **64**, 063809 (2001).
[20] S. Koke *et al.*, Opt. Express **15**, 6279 (2007).
[21] For a recent review on counterpropagating optical beams and solitons, see M. S. Petrović, M. R. Belić, C. Denz, and Yu. S. Kivshar, to appear in Laser & Photonics Reviews (2010) doi:10.1002/lpor.200900053, http://www.arXiv.org, Preprint [physics.optics] arXiv:0910.4700.