Stability criterion for Gaussian pulse propagation through negative index materials

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We analyze the dynamics of propagation of a Gaussian light pulse through a medium having a negative index of refraction employing the recently reported projection operator technique. The governing modified nonlinear Schrödinger equation, obtained by taking into account the Drude dispersive model, is expressed in terms of the parameters of Gaussian pulse, called collective variables, such as width, amplitude, chirp, and phase. This approach yields a system of ordinary differential equations for the evolution of all the pulse parameters. We demonstrate the dependence of stability of the fixed-point solutions of these ordinary differential equations on the linear and nonlinear dispersion parameters. In addition, we validate the analytical approach numerically utilizing the method of split-step Fourier transform.

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I. INTRODUCTION

The present-day research fraternity has been witnessing a great surge of interest over the avenue of negative index materials (NIMs) due to their bizarre characteristics and soaring prospects for the potential applications for the future [1]. While the concept of this much celebrated avenue was hypothesized by Veselago much earlier in the 1960s [2], it was the experimental realization of the macroscopic demonstration of NIMs in the early 2000s which stirred the tremendous interest in this field [3–5]. These NIMs, which are still in their infancy, exhibit a variety of enthralling effects such as reversal of Snell's law, reversed Cerenkov radiation and Doppler effect, subdiffraction imaging, photon tunneling, backward wave antennas, phase combination, and electrically small resonators, and they are anticipated to fulfill a multitude of other requirements in complex environments in the imminent generation of researchers. Today's NIM literature is well alive with wide coverage of topics ranging from experimental design of suitable metamaterials to achieve negative index of refraction to their amazing applications encompassing perfect focusing, invisibility cloaking, and so on.

Theoretical curiosity on wave propagation through such NIMs is also recently emerging as another challenging topic of this area. As of now, linear propagation in NIMs has been extensively analyzed [4,6,7] with the derivation of appropriate partial differential equations and development of transfer functions [8], arrival of *ab initio* calculations of the nonlinear dielectric and magnetic properties of split-ring resonator lattice structures [9], and so on. The possibility of NIMs possessing a nonlinear electromagnetic response on the level of structural elements has further elevated the study of NIMs into the nonlinear regime.

As far as ordinary materials are considered, the nonlinear interaction of ultrashort pulses has been widely studied in the framework of the nonlinear Schrödinger equation (NLSE). The evolution of an envelope function described by the NLSE completely relies on the slowly varying envelope approximation (SVEA) over an optical cycle. In fact, quite a few of the recent studies have attempted to derive various accurate yet solvable equations that extend beyond the much celebrated SVEA. Relaxing the SVEA, Brabec and Krausz had introduced the nonlinear envelope equation describing the wave packet envelope down to pulse durations as short as one [10], which has paved the way to several other versions, including the first-order propagation equation [11], the reduced Maxwell's equation [12], and so on. Notably few practical versions of the derivation of pulse propagation equations could also be seen in the recent literature like the early projection operator approach [13,14] leading to unidirectional pulse propagation equation, the directional field approach [15], and the factorization approach [16–18]. Very recently, Kinsler had reexpressed the Maxwell's equation into a unidirectional first-order wave equation for media with both electric and magnetic responses which has much futuristic relevance to the field of NIMs [19].

Of late, exploring the feature of propagation of nonlinear pulse through NIMs has become feasible with the arrival of quite a few but accurate propagating equations governing the evolution of ultrashort pulses in NIMs. A system of coupled nonlinear Schrödinger equations for the envelopes of the propagating electric and magnetic fields in NIM have been derived by Lazarides and Tsironis [20] in which they have showed that the proposed system is equivalent to the well-known Manakov model which admits bright and dark soliton solutions. A generalized nonlinear Schrödinger equation has been introduced by Scalora *et al.* [21] which paves the way to realize a wide class of solitary waves. Out of the few other models that have been derived to describe second- and third-order nonlinear optical phenomena through NIMs [22–27], the system of negative refractive index media presented with the Drude dispersive model [26,27] possesses a vivid combination of a second-order nonlinear temporal dispersion and a higher order linear dispersion originating out of dispersive permeability leading to a modified nonlinear Schrödinger equation (MNLSE). Very recently we have analyzed the existence of small-amplitude solitons by establishing an interesting connection between the Korteweg-de Vries equation and MNLSE, through the reductive perturbation process, and we have examined the influence of nonlinear dispersion terms over the modulation instability windows [28]. Furthermore, through the study of electromagnetic wave propagation in NIMs, the role of the status of a Gaussian beam in beam self-focusing and defocusing has been identified recently [29].

At this juncture, by realizing the growing need for analytical techniques to analyze pulse propagation through NIMs, we have tried to explore the aspect of propagation of a Gaussian pulse in NIM through a novel projection operator technique (POT) [30,31]. The usage of POT in this contribution exposes a different perspective of studying the various parameters, such as amplitude, chirp, width, and so on, associated with the Gaussian pulse as it propagates through NIMs. We further utilize the analysis of fixed points to study the stability of propagation of a Gaussian pulse through NIMs.

In the following discussion, we present the description of the governing equation under consideration for NIMs in Sec. II. Then in Sec. II A, we start with a short introduction to the method of projection operator and apply it to the system of NIM to arrive at the system of ordinary differential equations (ODEs). Later on in Sec. II B, we deal with the consideration of the stability of the fixed-point solutions of the ODEs governing the evolution of pulse parameters. In Sec. III, we present the results obtained by integrating numerically the POT ODEs and by direct numerical simulation of the equation through a split-step Fourier transform method. Finally we conclude in Sec. IV.

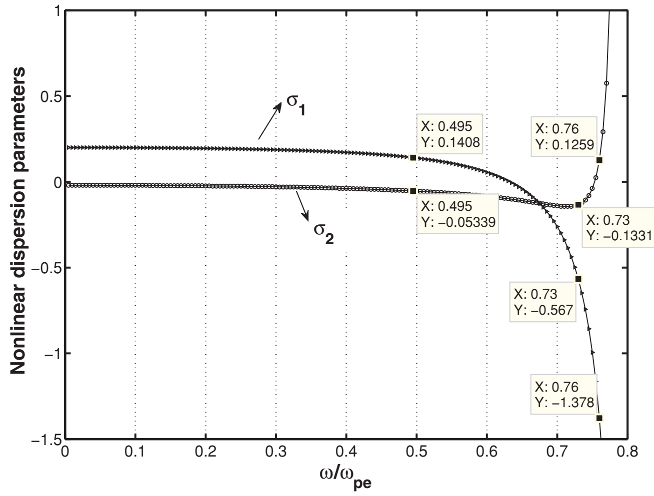

FIG. 1. Nonlinear dispersive parameters used for depicting pulse parameter dynamics through NIM at $\omega_{pe}/\omega_{pm} = 0.8$.

II. THEORETICAL FORMULATION OF THE PROBLEM

The governing equation for the propagation of an ultrashort pulse in negative index material with Kerr polarization is given by the following modified nonlinear Schrödinger equation:

$$\frac{\partial E}{\partial Z} - i\sum_{n=2}^{\infty}\frac{i^n\beta_n}{n!}\frac{\partial^n E}{\partial T^n} - \sum_{n=0}^{\infty}\frac{i^{n+1}\gamma_n}{n!}\left(1+\frac{i}{\omega_0}\frac{\partial}{\partial T}\right) \times \frac{\partial^n(|E^2|E)}{\partial T^n} - \frac{i}{2k_0}\left(\frac{\partial^2 E}{\partial Z^2} - \frac{2}{V_g}\frac{\partial^2 E}{\partial T\partial Z}\right) = 0, \quad (1)$$

where E is the amplitude of the pulse, Z and T are the propagation distance and time in co-moving reference, parameter V_g is the group velocity of the pulse, β_n is the nth-order dispersion parameter, γ_n is the nth-order nonlinear parameter, and k_0 is the wave number in the medium at the carrier frequency ω_0. Note that Eq. (1) has been derived by carrying out the approximations as in Refs. [26,27]. That is, the envelope and carrier part of the field are assumed to vary in the form of $E(Z,T)\exp[i(k_0 Z - \omega_0 T)]$, where $k_0 = n(\omega_0)\omega_0/c$, and by the introduction of co-moving variables. Furthermore, by supposing the propagation of pulses at least a few tens of wave cycles in duration with all higher order derivatives giving negligible contributions, the first-order non-SVEA corrections are approximated to $\frac{\partial^2 E}{\partial Z^2} \approx i\gamma_0\frac{\partial(|E|^2 E)}{\partial Z}$ and $\frac{\partial^2 E}{\partial T\partial Z} \approx i\gamma_0\frac{\partial(|E|^2 E)}{\partial T}$. We then introduce the normalized variables $t = T/T_0$, $z = Z/l_d$, and $U = E/E_0$ retaining the linear and nonlinear dispersion coefficients up to second order, with $l_d = T_0^2/\beta_2$, $l_{nl} = 1/\gamma_0|E_0|^2$, $l_{snl} = 1/\gamma_0|E_0|^4\varsigma$, and $l_{ss} = T_0/\gamma_0|E_0|^2 S_a$ as the dispersion, nonlinear, saturation nonlinear, and self-steepening lengths, respectively. After doing so, Eq. (1) transforms into the following MNLSE:

$$\frac{\partial U}{\partial z} = -i\frac{\alpha}{2}\frac{\partial^2 U}{\partial t^2} + \kappa\left(i|U|^2 U - \sigma_1\frac{\partial(|U|^2 U)}{\partial t} - i\sigma_2\frac{\partial^2(|U|^2 U)}{\partial t^2}\right), \quad (2)$$

where $\alpha = \mathrm{sgn}(\beta_2) = \pm 1$ represent the normal and anomalous group velocity dispersion (GVD) and nonlinear coefficient $\kappa > 0$ and $\kappa < 0$, which represent the focusing and defocusing cases, and $\sigma_1 = l_{nl}/l_{ss1}$ and $\sigma_2 = l_{nl}/l_{ss2}$ stand for the normalized first- and second-order nonlinear dispersion coefficients, respectively. To categorize the MNLSE to depict the explicit behavior of negative refraction, it is further correlated with the Drude model [26] delineating the frequency dispersion with the permittivity and permeability of the form $\epsilon(\omega) = \epsilon_0(1 - \frac{\omega_{pe}{}^2}{\omega(\omega+i\gamma_e)})$, $\mu(\omega) = \mu_0(1 - \frac{\omega_{pm}{}^2}{\omega(\omega+i\gamma_m)})$, where ω_{pe} and ω_{pm} are the electric and magnetic plasma frequencies and γ_e and γ_m are the electric and magnetic loss terms, respectively. Accordingly the nonlinear dispersion coefficients take up the form of $\sigma_1 = \frac{1}{\omega t}(1 + \frac{\omega_{pm}^2\omega_{pe}^2 - \omega_0^4}{\epsilon\mu\,\omega_0^4} - \frac{\omega_{pm}^2+\omega_0^2}{\omega_{pm}^2-\omega_0^2})$ and $\sigma_2 = \frac{1}{\omega^2 t^2}\frac{\omega_0^2}{\omega_0^2-\omega_{pm}^2} - \frac{1}{4\epsilon\mu}[1 + \frac{3\omega_{pm}^2\omega_{pe}^2}{\omega_0^4} + \frac{1}{4\epsilon^2\mu^2}(1 - \frac{\omega_{pm}^2\omega_{pe}^2}{\omega_0^4})^2]$. Note that these parameters are chosen depending on which ratio of ω_{pm}/ω_{pe} is chosen to determine the engineered size of the constituents of NIM structures, the split-ring resonators, and so on. Figure 1 shows the variation of nonlinear dispersive parameters plotted against normalized frequency at the value of $\omega_{pm}/\omega_{pe} = 0.8$.

A. Application of POT to NIMs

In a previous study, Tchofo Dinda *et al.* [30] proposed a collective variable (CV) technique for obtaining the pulse propagation dynamics through optical fiber. The CV treatment reduces the dynamics of the pulse field, which involves an infinite number of degrees of freedom, to the dynamics of a simple mechanical system having only a few degrees of freedom, each called a collective variable, which could be then associated with a relevant physical parameter of the pulse. The achievement of this approach results in the possibility of transforming the partial differential equation for the original field into a set of ordinary differential equations for the collective variables. We have tried to view and analyze the system of nonlinear negative index materials in the light of the generalized POT proposed very recently by Nakkeeran and Wai [31] for the nonlinear Schrödinger equation, which was proved to be equivalent to the bare approximation of collective variable theory proposed by Tchofo Dinda *et al.* [30] or to the Lagrangian variational method (LVM) [32] under the appropriate choice of the phase constant in the projection operator. The POT treatment starts with the introduction of new variables which could be associated with nonlinear localized modes, the amplitude, width, chirp, and phase of the pulse for which the equations of motion can be constructed. The actual propagating amplitude is decomposed in the following manner:

$$U = F[x_1, x_2, x_3 \ldots\ldots x_k, t], \tag{3}$$

where x_k are the various parameters of the propagating wave functionally depending on the propagation coordinate z. Just replacing U with any appropriate form of the functions $F[x_k, t]$ introduces extra degrees of freedom, thereby enlarging the phase space of the system. There arises the need for constraining the system of new variables to retain the system in its original phase space. Eventually the constraints imposed allow the system to be projected only in the particular direction, and POT gets its name from this particular notion.

Application of the POT to the MNLSE governing the propagation of pulses in negative index materials with Kerr-type polarization starts with the introduction of the generalized projection operator

$$P_k = e^{i\theta} F^*_{xk}, \tag{4}$$

where θ is an arbitrary phase constant. To obtain the CV's equations of motion, we project Eq. (2) in the direction of P_k. An appropriate ansatz function $F(X_k(z), t)$ is chosen with X_k being the pulse parameters and substituted for U in Eq. (2), and then the resulting equation is multiplied with the projection operator P_k, integrated then with respect to t and the real part extracted.

$$\begin{aligned}\int_{-\infty}^{\infty} \mathrm{Re}[F_z F^*_{x_k} e^{i\theta}]dt - \frac{\alpha}{2}\int_{-\infty}^{\infty} \mathrm{Im}[F_{tt} F^*_{x_k} e^{i\theta}]dt \\ + \int_{-\infty}^{\infty} \kappa |F|^2 \mathrm{Im}[F F^*_{x_k} e^{i\theta}] + \kappa\sigma_1 \mathrm{Re}[(|F|^2 F)_t F^*_{x_k} e^{i\theta}]dt \\ - \kappa\sigma_2 \int_{-\infty}^{\infty} \mathrm{Im}[(|F|^2 F)_{tt} F^*_{x_k} e^{i\theta}]dt = 0.\end{aligned} \tag{5}$$

Regarding the choice of ansatz, one should note the point that hyperbolic secant- or raised cosine-like functions result in a different set of dynamical equations from the LVM and bare approximation of CV technique. As a result, this circumstance demands investigations both from LVM and CV theory for the complete study of dynamics of NLSE. But, fortunately, it was proved that Gaussian ansatz has an inherent symmetric property between the pulse parameters which will result in the same set of the dynamical equations derived either from the LVM or from the bare approximation (BA) of the CV theory [33]. Moreover, the analytical tractability of the ansatz simplifies the study of dynamics of NLSE. Keeping in mind the stated uniqueness of the Gaussian ansatz, we have chosen $F[X_k(z), t]$ as the Gaussian function of various measurable pulse parameters, such as amplitude, pulse width, frequency, temporal position, chirp, and phase, and have fed it to the propagating amplitude E to obtain the spatial evolution of these fundamental parameters. The Gaussian function is of the form

$$\begin{aligned}F(X_k(z), t) = X_1(z)\exp\{-[t - X_2(z)]^2 X_3(z)^2 \\ + iX_4(z)[t - X_2(z)]^2 + iX_5(z)[t - X_2(z)] \\ + iX_6(z)\},\end{aligned} \tag{6}$$

where $X_1(z)$, $X_2(z)$, $X_3(z)$, $X_4(z)$, $X_5(z)$, and $X_6(z)$ represent the amplitude, temporal position, inverse width, chirp, frequency, and phase, respectively, of the propagating wave. The resultant ODEs for the various pulse parameters are termed the CV equations of motions which are as follows:

$$X_1' = \frac{1}{8} X_1 \left(\sqrt{2}\sigma_2\kappa X_1^2 + 8\alpha\right) X_4, \tag{7}$$

$$X_2' = \frac{1}{4}\left[3\sqrt{2}\kappa X_1^2(\sigma 1 - 2\sigma_2 X_5) - 4\alpha X_5\right], \tag{8}$$

$$X_3' = \frac{1}{4}\left(8\alpha + 5\sqrt{2}\sigma_2\kappa X_1^2\right) X_3 X_4, \tag{9}$$

$$\begin{aligned}X_4' = \frac{\kappa\left\{-21\sigma_2 X_3^4 + 2\left[X_5(\sigma_1 - \sigma_2 X_5) - 1\right] X_3^2\right\} X_1^2}{2\sqrt{2}} \\ \times \frac{+\kappa\sigma_2 X_4^2 X_1^2 + 4\sqrt{2}\alpha\left(X_4^2 - X_3^4\right)}{2\sqrt{2}},\end{aligned} \tag{10}$$

$$X_5' = \sqrt{2}\kappa X_1^2 X_4 (\sigma_1 - 2\sigma_2 X_5), \tag{11}$$

$$\begin{aligned}X_6' = \frac{\sqrt{2}\kappa\left\{33\sigma_2 X_3^4 + 2[X_5(\sigma_1 - 7\sigma_2 X_5) + 5]X_3^2\right\} X_1^2}{16X_3^2} \\ \times \frac{+3\sqrt{2}\kappa\sigma_2 X_4^2 X_1^2 + 8\alpha X_3^2\left(2X_3^2 - X_5^2\right)}{16X_3^2}.\end{aligned} \tag{12}$$

The numerical studies of the evolution of the pulse parameters along the distance of propagation are performed by integrating the pulse dynamical equations for the Gaussian ansatz adopting the fourth-order Runge-Kutta scheme. The projection operator method is validated by performing a direct numerical simulation of MNLSE using the split-step Fourier transform method.

B. Fixed points and their stability

Furthermore, we aim to study the existence of stable and unstable solutions in the parameter space of MNLSE. The stable fixed points of a system correspond to stable solutions of the system. Hence, we begin by arriving at the fixed points of the system by imposing the left-hand side of the CV equations to be zero (i.e., $X_i' = 0$) [34]. The stability of the fixed points can be determined by the analysis of the eigenvalues of the Jacobian matrix $M_{ij} = \partial X_i/\partial X_j$. To start with this analysis, we set $X_1 = Am + \Delta Am$, $X_3 = W_{\mathrm{inv}} + \Delta W_{\mathrm{inv}}$, $X_4 = Ch + \Delta Ch$, and $X_5 = Fr + \Delta Fr$, with Am, W_{inv}, Ch, and Fr denoting the amplitude, inverse width, chirp, and frequency, respectively, and the corresponding Δ quantities denoting the small perturbations added to them. Then we linearize the CV equations around the steady-state solutions (Am, W_{inv}, Ch, and Fr) to derive the evolution equations for the small deviations ΔAm, ΔW_{inv}, ΔCh, and ΔFr which are as follows:

$$\begin{aligned}
\Delta \mathrm{Am}' &= M_{11}\mathrm{Am} + M_{12}W_{\mathrm{inv}} + M_{13}Ch + M_{14}Fr,\\
\Delta W'_{\mathrm{inv}} &= M_{21}\mathrm{Am} + M_{22}W_{\mathrm{inv}} + M_{23}Ch + M_{24}Fr,\\
\Delta \mathrm{Ch}' &= M_{31}\mathrm{Am} + M_{32}W_{\mathrm{inv}} + M_{33}Ch + M_{34}Fr,\\
\Delta F\mathrm{r}' &= M_{41}\mathrm{Am} + M_{42}W_{\mathrm{inv}} + M_{43}Ch + M_{44}Fr,
\end{aligned} \tag{13}$$

where the matrix coefficients are given as

$$\begin{aligned}
M_{11} &= \frac{3Ch\,\kappa\,\sigma_2\,\mathrm{Am}^2}{4\sqrt{2}} + \alpha\ Ch,\\
M_{12} &= 0,\\
M_{13} &= \frac{\kappa\,\sigma_2\,\mathrm{Am}^3}{4\sqrt{2}} + \alpha\ Am,\\
M_{14} &= 0,\\
M_{21} &= \frac{5\ \mathrm{Am}\ Ch\ W_{\mathrm{inv}}\ \kappa\ \sigma_2}{\sqrt{2}},\\
M_{22} &= \frac{1}{4}\mathrm{Ch}\,(5\sqrt{2}\,\kappa\,\sigma_2\,\mathrm{Am}^2 + 8\alpha),\\
M_{23} &= \frac{1}{4}\,W_{\mathrm{inv}}(5\sqrt{2}\,\kappa\,\sigma_2\,\mathrm{Am}^2 + 8\,\alpha),\\
M_{24} &= 0,\\
M_{31} &= \frac{\mathrm{Am}\,\kappa\left[-21\,\sigma_2\,W_{\mathrm{inv}}^4 - 2(\sigma_2\,Fr^2 - \sigma_1\,Fr)W_{\mathrm{inv}}^2\right]}{\sqrt{2}}\\
&\quad\times\frac{-2\mathrm{Am}\,\kappa\,W_{\mathrm{inv}}^2 + \mathrm{Am}\,\kappa\,Ch^2\,\sigma_2}{\sqrt{2}},\\
M_{32} &= -W_{\mathrm{inv}}\big[\sqrt{2}\,\kappa\left(\sigma_2\,Fr^2 - \sigma_1\,Fr + 21\,W_{\mathrm{inv}}^2\sigma_2\right)\mathrm{Am}^2\\
&\quad + \sqrt{2}\,\kappa\,\mathrm{Am}^2 + 8\,W_{\mathrm{inv}}^2\alpha\big],\\
M_{33} &= \frac{Ch\,\kappa\,\sigma_2\,\mathrm{Am}^2}{\sqrt{2}} + 4\,Ch\,\alpha,\\
M_{34} &= \frac{\mathrm{Am}^2\,W_{\mathrm{inv}}^2\kappa\,(\sigma_1 - 2\,Fr\,\sigma_2)}{\sqrt{2}},\\
M_{41} &= 2\sqrt{2}\mathrm{Am}\ Ch\,\kappa\ \ (\sigma_1 - 2\,Fr\,\sigma_2\,),\\
M_{42} &= 0,\\
M_{43} &= \sqrt{2}\ \mathrm{Am}^2\kappa\ \ (\sigma_1 - 2\,Fr\,\sigma_2),\\
M_{44} &= -2\sqrt{2}\ \mathrm{Am}^2\ Ch\,\kappa\,\sigma_2.
\end{aligned} \tag{14}$$

Now one can get the stability criterion as the real part of at least one of the eigenvalues is positive; then the corresponding fixed point is unstable. Thus, to have a stable fixed point, and hence a stable solution, the real parts of all the eigenvalues of the matrix M_{ij} have to be negative. Now we determine the stability of solutions by calculating the characteristic polynomial of the matrix and by checking the eigenvalues analytically, which is complex because the phase space involved is four dimensional. So we substitute the realistic values for the NIM case obtained from the parameter plots of Fig. 1 and check for stable fixed points.

TABLE I. Stability criteria.

S.No.	σ_1	σ_2	α	κ	Fixed point
i.	+ve	−ve	−ve	−ve	Stable
ii.	−ve	−ve	−ve	−ve	Stable
iii.	−ve	+ve	−ve	+ve	Stable
iv.	±ve	±ve	+ve	±ve	Unstable

III. RESULTS AND DISCUSSION

As we analyze the stability of the propagation of a Gaussian pulse through NIM by explicitly utilizing the values of σ_1 and σ_2 from Fig. 1, we see that there exist three different combinations of signs of σ_1 and σ_2: (i) $\sigma_1 > 0, \sigma_2 < 0$, (ii) $\sigma_1 < 0, \sigma_2 < 0$, and $\sigma_1 < 0, \sigma_2 > 0$. The fixed points at these conditions are marked in Fig. 1. We observe that the real part of all the eigenvalues of the matrix M_{ij} tend to be negative only at the conditions described in Table I. As a result, we notice that the sign of the first-order nonlinear dispersion term σ_1 has no contribution to the stability of the fixed points. It is $\sigma_2 > (<0)$, $\kappa > 0\ (<0)$ and $\alpha > 0\ (<0)$ which determines the stability of fixed points. By analyzing the characteristic polynomial for sufficient proof of these conditions, we identify that the real part of the eigenvalues tends to be negative only when "$\alpha - \kappa\sigma_2$" tends to be negative. By interpreting in the other sense, we could say that it is the combination of second-order nonlinear dispersion with nonlinearity that balances the linear dispersion resulting in stability of fixed points leading to smoothly evolving Gaussian pulses.

For demonstration of a typical case, we choose $Am = 0.1153$, $W_{\mathrm{inv}} = 1/47.0950$, and $Ch = 0.01$ and find that for the NIM case, with $\sigma_1 = 0.1408$ and $\sigma_2 = -0.0534$ at $n = -2.2286$, there is a stable fixed point at the defocusing case ($\kappa = -1$) of anomalous dispersion ($\alpha = -1$) with all the eigenvalues of the matrix becoming negative. This stable fixed point corresponds to a smoothly evolving Gaussian pulse as depicted in Fig. 2. As we change conditions from the anomalous to normal dispersion regime for the focusing or defocusing case of nonlinearity for the same NIM parameters, we observe that the real parts of all the eigenvalues are no more negative and, thus, that the unstable fixed point results in unstable propagation which is depicted in Fig. 3. Note that we have chosen the nonlinear dispersion parameter values σ_1 and σ_2 from the NIM region of Fig. 1. While the solid line of Fig. 2(a) and Fig. 3(a) shows the variation of significant pulse parameters (amplitude, width, and chirp) with propagation distance obtained from the POT, the circled line is plotted from the direct numerical solution of MNLSE using the split-step Fourier transform method at conditions of stable and unstable fixed points, respectively. These plots show good agreement

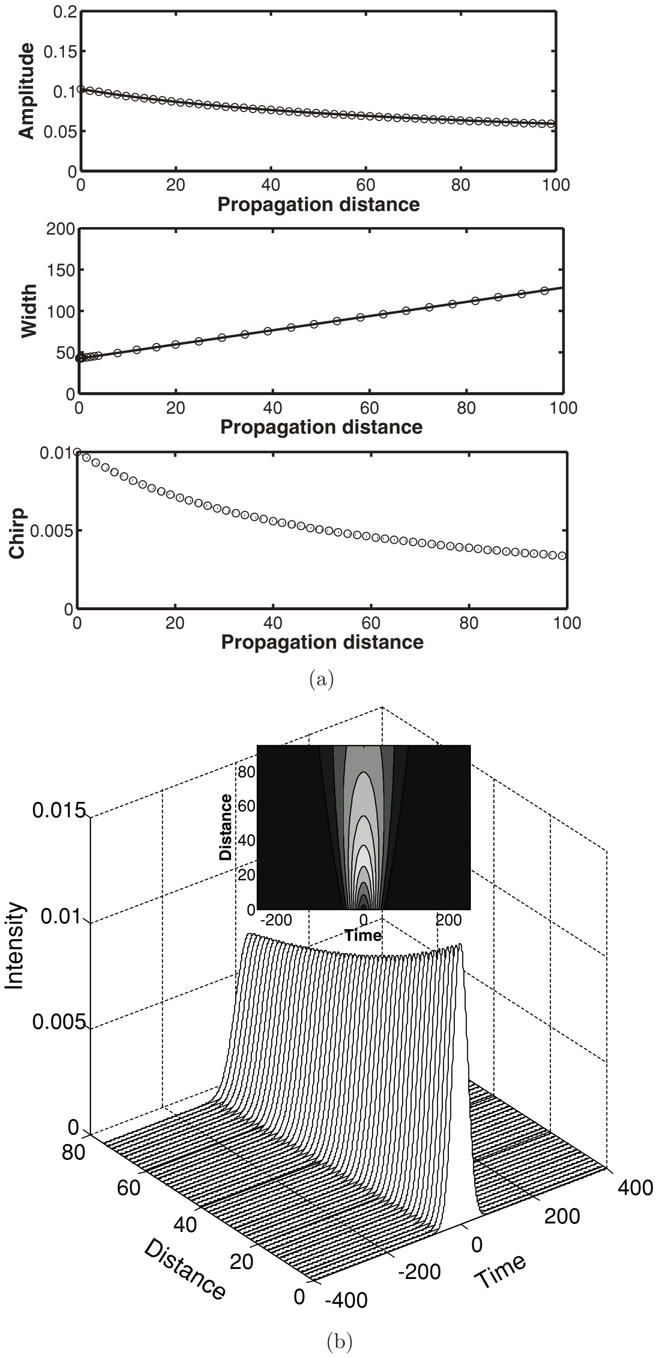

FIG. 2. Propagation of a smoothly evolving Gaussian pulse through NIM at a stable fixed point. (a) Variation of pulse parameters with respect to propagation distance with the solid line denoting that plotted by the projection operator method and the circled line corresponding to that plotted by the direct split-step Fourier transform method. (b) Intensity of a smoothly evolving Gaussian pulse propagating through NIM.

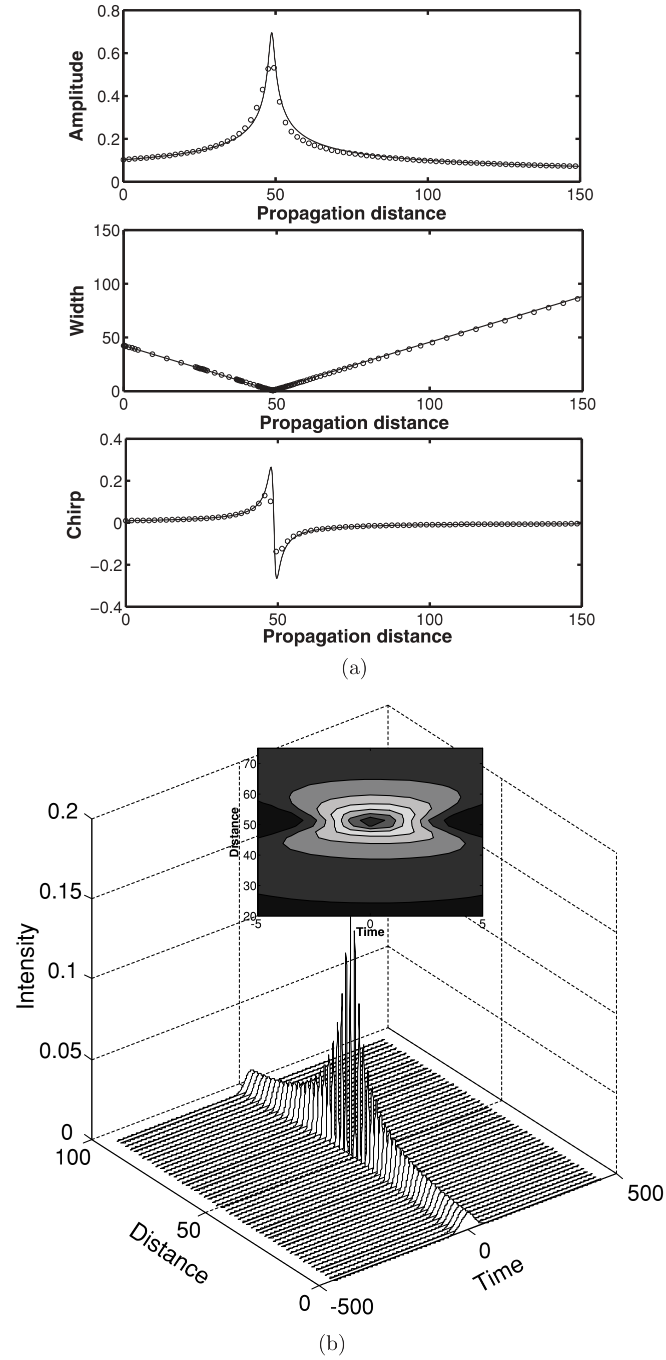

FIG. 3. Propagation of an unstable Gaussian pulse through NIM at an unstable fixed point. (a) Variation of pulse parameters with respect to propagation distance with the solid line denoting that plotted by the projection operator method and the circled line corresponding to that plotted by the direct split-step Fourier transform method. (b) Intensity of unstable Gaussian pulse propagating through NIM.

between analytical (POT) and numerical techniques. Unlike the stable propagation case, there is a dramatic variation of pulse parameters with propagation distance in the case of unstable propagation resulting from an unstable fixed point. This study of fixed points puts forth the fact which could be well interpreted as NIM facilitates the propagation of a smoothly evolving Gaussian pulse in the anomalous regime, only for positive (negative) second-order nonlinear dispersion in the focusing (defocusing) case of nonlinearity. For the sake of completeness, one can find from Ref. [16], in a positive index material, σ_2 is always positive and hence the criterion (iii) holds true. Considering a positive index material, a stable fixed point is seen at the parameter value $\sigma_1 = 0.1712$ and $\sigma_2 = 0.0531$ with $n = +0.7927$ in the focusing case of ($\kappa = 1$) of

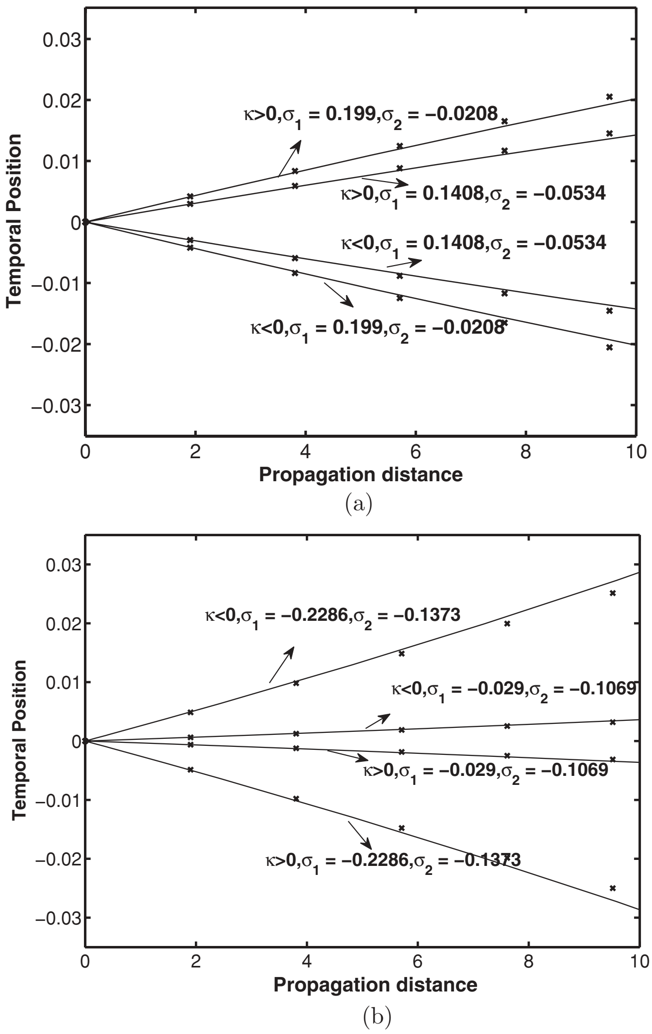

FIG. 4. Temporal position plotted as a function of propagation distance for NIM with various values of σ_1 and σ_2 at $\omega_{pm}/\omega_{pe} = 0.8$.

anomalous dispersion ($\alpha = -1$). Furthermore, while noticing the variation of temporal position with propagation distance, we observe a distinct feature. In the NIM regime, we notice that the focusing case puts forth a positive shift of the center of the pulse and that the defocusing case shifts the center of the pulse downward at positive values of σ_1. This criterion is exactly reversed at negative values of σ_1, with the focusing case bringing the downward shift and the defocusing case producing an upward shift. These two facts could be seen from Figs. 4(a) and 4(b), respectively. Here again the solid line corresponds to that obtained from POT and the starred line from direct numerical simulation.

IV. CONCLUSION

We have presented the investigation of propagation of a Gaussian pulse through negative index material using a theoretical technique of the projection operator, and we have validated the results numerically through the split-step Fourier transform method. The role of second-order nonlinear dispersion in the status of the stability of Gaussian pulse in the various categories of focusing or defocusing cases of normal or anomalous regimes is studied through fixed-point analysis, and it is identified that it is the combination of second-order nonlinear dispersion and nonlinearity that balances the linear dispersion to result in stability.

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