Collective oscillations of ultracold matter

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We consider the collective oscillations of an ultracold atomic cloud by using a wave kinetic description which retains phonon recoil effects. We derive an exact quantum-wave dispersion relation for hybrid plasma-acoustic oscillations and the associated atomic Landau damping. The quasilinear kinetic theory is also extended into the quantum regime, leading to a Boltzmann type of equation for the atoms in a collective force field. Comparison with our previous quasiclassical results is considered. Diffusion in velocity space due to density fluctuations is discussed in detail. This will us allow to establish an additional temperature limit, different from the usual Doppler limit, below which the laser cooling process is prevented by the density fluctuations. This work could also be useful to describe low-frequency Doppler instabilities, as well as collective phonon laser processes.

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I. INTRODUCTION

In recent years, there has been an increasing interest on the physics of very low temperature atomic gas, confined in a magneto-optical trap and cooled down to the micro-Kelvin temperature range [1,2]. One of the most interesting and surprising properties of this ultracold neutral gas is that it manifests itself as a "non-neutral plasma" due to the existence of an effective electric charge associated with the neutral atoms [3]. As a consequence, neutral atoms repel each other, as if they had the same electric charge. In the absence of magnetic confinement, this effective charge will lead to the occurrence of Coulomb explosion, as observed by [4]. It will also lead to the possible occurrence of many collective processes, such as those recently described by us [5]. In particular, new modes of the acoustic type, but with a plasma frequency cutoff, which we have called hybrid plasma-acoustic modes, were shown to exist, as well as a series of internal Tonks-Dattner resonances. They cannot be confused with similar modes and resonances which can also be found in real cold plasmas, as recently reviewed by Killian *et al.* [6], because we are not dealing with an ionized medium, but with a gas of neutral atoms.

Here we return to the problem of collective modes in a neutral ultracold gas by extending our previous results into the quantum domain. This can be done by using a quantum-wave kinetic equation where the atom recoil effects are retained. Such an approach has been considered before [7,8]. But here, instead of looking for recoil due to photon emission and absorption associated with the laser cooling process, we focus on phonon recoil effects.

We apply the wave kinetic equation to study the collective oscillations in the ultracold gas and obtain the exact quantum-wave dispersion relation and the atomic Landau damping. The quasilinear kinetic theory is also discussed in the quantum regime, leading to a Boltzmann type of equation for the atoms in a collective force field. This extends our previous quasiclassical work [5] into the quantum domain and clarifies the energy and momentum exchange of the atoms with the collective fields. A diffusion process in the atomic velocity space is also considered, and it will be shown that it leads to a new temperature limit, different from the usual Doppler limit [8], but related here to the spectrum of density fluctuations of the gas.

II. BASIC EQUATIONS

It is well known that the wave equation describing the evolution of an atom inside the ultracold gas can be transformed into a wave kinetic equation of the Wigner-Moyal type [9,10], by introducing the correlation function $K(\mathbf{r},\mathbf{s},t) = \langle \mathbf{r}, +\mathbf{s}/2|\mathbf{r}, -\mathbf{s}/2\rangle$, where $\mathbf{r}$ is the center of mass the atom and $|\mathbf{r}, t\rangle$ is the atomic state vector describing its translational motion with respect to the laboratory frame. Taking its Fourier transformation, we arrive at the atomic Wigner function

$$W(\mathbf{r}, \mathbf{q}, t) = \int K(\mathbf{r}, \mathbf{s}, t) \exp(-i\mathbf{q}\cdot\mathbf{s})\, d\mathbf{s}. \tag{1}$$

We can then follow the usual Moyal procedure which will allow us to establish an evolution equation for the quantity W, in the well-known form [11]

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) W = \frac{-i}{\hbar}\int V_0(\mathbf{k}')[W^{(-)} - W^{(+)}]e^{i\mathbf{k}'\cdot\mathbf{r}}\frac{d\mathbf{k}'}{(2\pi)^3}, \tag{2}$$

where $V_0(\mathbf{k}')$ are the Fourier components of an applied external potential $V_0(\mathbf{r})$. We want to focus on the atom density perturbations that can occur in an atomic gas cloud and generalize our previous quasiclassical treatment [5] to the quantum domain. In order to do this, we need to specify the external potential, in the following practical way, as

$$V_0(\mathbf{r}) = V_B(\mathbf{r}) + V_{\mathrm{eff}}(\mathbf{r}), \tag{3}$$

where $V_B(\mathbf{r})$ is the static confining potential of the magneto-optical trap and the effective potential $V_{\mathrm{eff}}(\mathbf{r})$ describes the collective influence of the nearby atoms. It is well known that $V_B(\mathbf{r})$ can approximately be described by a three-dimensional parabolic potential [12]. On the other hand, the effective potential is determined by the local atomic density $n(\mathbf{r})$, as [5]

$$\nabla^2 V_{\mathrm{eff}} = -Qn \equiv -Q\int W(\mathbf{r}, \mathbf{v}, t)\, d\mathbf{v} \tag{4}$$

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with

$$Q = (\sigma_R - \sigma_L)\sigma_L I_0/c, \tag{5}$$

where σ_R and σ_L are the radiation and the laser absorption cross sections as defined by [4] and others, and I_0 is the laser intensity. In Eq. (4), we have normalized the Wigner function such that

$$\int d\mathbf{r} \int d\mathbf{v}\, W(\mathbf{r}, \mathbf{v}, t) = \int n(\mathbf{r}, t)\, d\mathbf{r} = N, \tag{6}$$

where N is the total number of confined atoms. In this sense, Eq. (2) will describe the evolution of N identical atoms, assumed as independent, except for the mean-field potential $V_{\rm eff}$ which creates an effective collective force associated with the exchange of photons between nearby atoms. It is well known that such a force results from the balance between a negative shadow force term [13] and a positive repulsive force term [3], as stated by the two terms with opposite sign appearing in the definition of the quantity Q.

III. LINEAR DISPERSION RELATION

We now consider the linear evolution of atom density perturbations around some equilibrium value defined by W_0, as determined by the confining potential V_B and the equilibrium collective potential. We therefore assume perturbations of the atom quasiprobablity distribution W and of the collective potential $V_{\rm eff}$ of the form

$$\tilde{W}, \tilde{V}_{\rm eff} \propto \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t). \tag{7}$$

Perturbative analysis of Eqs. (4) and (2) then lead to the following two expressions:

$$\tilde{V}_{\rm eff} = \frac{Q}{k^2}\int \tilde{W}\, d\mathbf{v} \tag{8}$$

and

$$\tilde{W} = \tilde{V}_{\rm eff}\frac{[W_0^{(-)} - W_0^{(+)}]}{\hbar(\omega - \mathbf{k}\cdot\mathbf{v})} \tag{9}$$

with

$$W_0^{(\pm)} = W_0(\mathbf{v}_\pm) \equiv W_0(\mathbf{v} \pm \hbar\mathbf{k}/2M). \tag{10}$$

From these equations, we can easily get the dispersion relation for density perturbations with wave vector $\mathbf{k}$ and frequency ω in the ultracold gas, as

$$1 - \frac{Q}{\hbar k^2}\int \frac{[W_0^{(-)} - W_0^{(+)}]}{(\omega - \mathbf{k}\cdot\mathbf{v})} d\mathbf{v} = 0. \tag{11}$$

Before discussing the properties of the dispersion relation in this exact form, it is useful to consider its quasiclassical limit, where the momentum carried by an emitted or absorbed phonon $\hbar\mathbf{k}$ can be considered negligible with respect to the atomic translational momentum $\hbar\mathbf{q} = \hbar M\mathbf{v}$. In this case, we can use the approximate expressions

$$W^{(\pm)} \simeq W(\mathbf{r}, \mathbf{q}, t) \pm \frac{\mathbf{k}}{2}\cdot\frac{\partial W}{\partial \mathbf{q}} + \frac{\partial}{\partial \mathbf{q}}\cdot\frac{\mathbf{kk}}{4}\cdot\frac{\partial}{\partial \mathbf{q}} W \pm \cdots. \tag{12}$$

From this we get

$$[W^{(-)} - W^{(+)}] = -\mathbf{k}\cdot\frac{\partial W}{\partial \mathbf{q}}. \tag{13}$$

In the quasiclassical limit, the dispersion relation (11) reduces to

$$1 + \frac{Q}{Mk^2}\int \frac{\mathbf{k}\cdot\partial W_0/\partial\mathbf{v}}{(\omega - \mathbf{k}\cdot\mathbf{v})} d\mathbf{v} = 0, \tag{14}$$

which coincides with our previous result [5].

Going back to the exact dispersion relation (11), we can now consider the important case of a monoenergetic atomic beam, as determined by the equilibrium Wigner function

$$W_0(\mathbf{v}) = n_0\delta(\mathbf{v} - \mathbf{v}_0), \tag{15}$$

which describes ultracold atoms in the $T \to 0$ limit, moving with velocity $\mathbf{v}_0$ with respect to the laboratory frame. The result is

$$1 - \frac{Qn_0}{Mk^2}\left[\frac{1}{(\omega - \mathbf{k}\cdot\mathbf{v}_-)} - \frac{1}{(\omega - \mathbf{k}\cdot\mathbf{v}_+)}\right] = 0, \tag{16}$$

where $\mathbf{v}_\pm$ are defined by Eq. (10). Noting that $(\mathbf{v}_+ - \mathbf{v}_-) = \hbar\mathbf{k}/M$, and introducing the effective plasma frequency $\omega_p = \sqrt{Qn_0/M}$, we arrive at the following result:

$$1 - \frac{\omega_p^2}{(\omega - \mathbf{k}\cdot\mathbf{v}_+)(\omega - \mathbf{k}\cdot\mathbf{v}_-)} = 0. \tag{17}$$

In the classical limit, we can use $\mathbf{v}_\pm \simeq \mathbf{v}_0$, and we recover another expression of our previous work [5]

$$\omega_p^2 = (\omega - \mathbf{k}\cdot\mathbf{v}_0)^2, \tag{18}$$

which describes oscillations of the atomic gas at the plasma frequency in its proper frame. In the particular case where the atoms are at rest, Eq. (17) leads to

$$\omega^2 = \omega_p^2 + \frac{\hbar^2}{4}\frac{k^4}{M^2}, \tag{19}$$

which describes these same collective oscillations but with a quantum dispersion term.

Let us now consider temperature corrections. We return to the general dispersion relation (11) and assume an arbitrary equilibrium distribution $W_0(\mathbf{v})$. It is useful to write this equation in the form

$$1 + \chi(\omega, \mathbf{k}) = 0, \tag{20}$$

where the atomic susceptibility is defined by

$$\chi(\omega, \mathbf{k}) = -\frac{Q}{\hbar k^2}\int \frac{[W_0^{(-)} - W_0^{(+)}]}{(\omega - \mathbf{k}\cdot\mathbf{v})} d\mathbf{v}. \tag{21}$$

It is now useful to introduce the parallel and perpendicular velocity components, such that

$$\mathbf{v} = u\frac{\mathbf{k}}{k} + \mathbf{v}_\perp. \tag{22}$$

We clearly see from Eq. (21) that there is a resonant velocity, $u_0 = \omega/k$, such that the atom moves in phase with the wave propagation. We can also write the atomic susceptibility in terms of the parallel velocities as

$$\chi(\omega, \mathbf{k}) = -\frac{Q}{\hbar k^3}\int G(u)\left[\frac{1}{u - u_0^{(-)}} - \frac{1}{u - u_0^{(+)}}\right] du, \tag{23}$$

where we have introduce the parallel quasidistribution $G(u) = \int W_0(u, \mathbf{v}_\perp) d\mathbf{v}_\perp$ and used the quantities $u_0^{(\pm)} = u_0 \pm \hbar k/2M$.

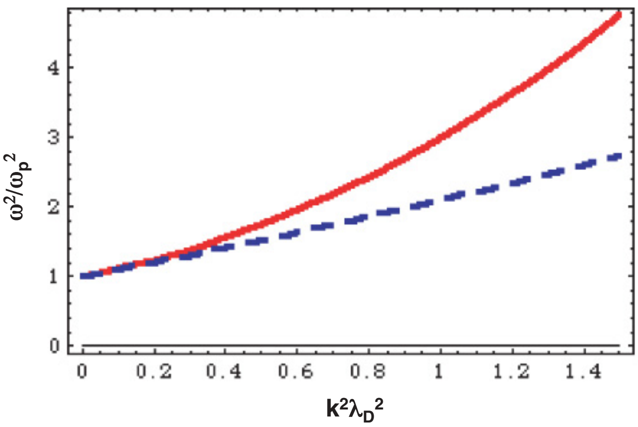

FIG. 1. (Color online) Dispersion relation, representing the dimensionless variables, frequency (ω/ω_P) versus wave number ($k\lambda_D$). The red plain curve was obtained for the dimensionless quantum parameter $H \equiv \hbar^2/4M^2u_s^2\lambda_D^2 = 1$. The blue dashed line was obtained for $H = 0.1$ and represents the quasiclassical regime.

We should notice that the integral in this expression can be divided into its principal part and its resonant contribution. This leads to a complex susceptibility of the form

$$\chi(\omega, \mathbf{k}) = \chi_r(\omega, \mathbf{k}) + i\,\chi_i(\omega, \mathbf{k}). \tag{24}$$

We can easily solve the principal part of the integral using the plausible assumption that the root-mean-square velocity deviation of the atoms in the cloud is much smaller than the phase velocity of the wave perturbation. This means that the main contribution to the integral comes from regions where $u \ll u_0$. Assuming an even function $G(u) = G(-u)$, and noting that

$$n_0 = \int G(u)du, \quad u_s^2 = \frac{1}{n_0}\int G(u)u^2du, \tag{25}$$

where u_s can be called the sound speed, we arrive at the following result for the real part of the dispersion relation (Fig. 1):

$$1 - \frac{Qn_0}{\hbar k^2}\left[\frac{\mathbf{k}\cdot(\mathbf{v}_- - \mathbf{V}_+)}{(\omega - \mathbf{k}\cdot\mathbf{v}_-)(\omega - \mathbf{k}\cdot\mathbf{v}_+)}\right]\left(1 + \frac{k^2}{\omega^2}u_s^2\right) = 0. \tag{26}$$

This leads to

$$\omega^2 = \omega_p^2\left(1 + \frac{k^2}{\omega^2}u_s^2\right) + \frac{\hbar^2}{4}\frac{k^4}{M^2}, \tag{27}$$

which for $\omega^2 \simeq \omega_p^2$ allows us to write

$$\omega^2 = \omega_p^2\left(1 + k^2\lambda_D^2\right) + \frac{\hbar^2}{4}\frac{k^4}{M^2}, \tag{28}$$

where the Debye length $\lambda_D = u_s/\omega_p$ is the characteristic scale length for the collective interactions. This dispersion relation generalizes our previous result on hybrid (or plasma-acoustic) modes [5] by including quantum dispersion. Apart from its cutoff frequency ω_p, it also strongly resembles the dispersion relation of Bogolioubov oscillations in a Bose-Einstein condensate [14]. Neglecting the cutoff and quantum dispersion terms, we would get the dispersion relation for the usual acoustic waves in the gas, $\omega^2 = k^2u_s^2$.

Let us now turn to the imaginary part of the atomic susceptibility. By solving the resonant contribution to the integral (23), we get

$$\chi_i(\omega, \mathbf{k}) = \frac{\pi Q}{\hbar k^3}[G^{(-)}(u_0) - G^{(+)}(u_0)]. \tag{29}$$

This imaginary part implies the existence of a complex mode frequency $\omega = \omega_r + i\gamma$, where γ is the damping (or growth) rate, for a given wave vector $\mathbf{k}$. For $|\gamma| \ll \omega_r$, it is known that

$$\gamma = -\frac{\chi_i(\omega_r, \mathbf{k})}{(\partial\chi_r/\partial\omega)_r}, \tag{30}$$

where the derivative is taken at $\omega = \omega_r$. We then get the expression for the atomic Landau damping of the hybrid plasma-acoustic modes, as determined by

$$\gamma = -\frac{\pi Q}{\hbar k^3}\frac{[G(u_0 - \hbar k/2M) - G(u_0 + \hbar k/2M)]}{(\partial\chi_r/\partial\omega)_r}. \tag{31}$$

This result retains the exact atom recoil due to the emission or absorption of an hybrid phonon. The resulting damping rate is due to the difference in population for translational states distant by an amount of momentum $\hbar\mathbf{k}$. For an inversion of population, we get an instability, $\gamma > 0$, and the collective oscillations of the ultracold gas can start to grow from out of noise. This instability could lead to the phonon laser effect with the coherent emission of hybrid plasma-acoustic phonons. In that respect this process would generalize the recently demonstrated process of a phonon laser for a single trapped ion [15,16] to the case where real phonons are emitted in a gaseous medium.

It is now useful to take the quasiclassical limit where this momentum increment can be considered negligible and where we can approximately write

$$G^{(-)} - G^{(+)} \simeq -\frac{\hbar k}{M}\left(\frac{\partial G}{\partial u}\right)_{\omega/k}. \tag{32}$$

By taking the derivative in the denominator of (31) as $\sim 1/\omega$, we can then recover the expression derived in our previous work [5]

$$\gamma = \frac{\pi}{\omega}\frac{Q}{Mk^2}\left(\frac{\partial G}{\partial u}\right)_{\omega/k}. \tag{33}$$

We can then see that the quasiclassical limit of atomic Landau damping (or growth) rate is determined by the derivative of the parallel distribution $G(u)$, a result that is also known for collective oscillations in classical plasma physics.

IV. QUASILINEAR DIFFUSION

In the previous calculations we have assumed that a given equilibrium function $W_0(\mathbf{r}, \mathbf{v})$ remains constant along the process of atom density oscillations, and wave propagation, with damping and growth. This is certainly valid on the time scale of the wave period $1/\omega_r$ but is no longer valid for a much larger time scale (larger than $1/\gamma$), because of the energy exchange between the kinetic energy of the atoms and the resonant oscillation modes.

In order to establish the long time evolution for W_0, we can go back to the exact wave kinetic equation (2) and retain the long time contribution of the nonlinear terms due to the existence of fast time oscillations of the medium. The slowly varying part of the wave kinetic equation can then be written as

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) W_0(\mathbf{v}) = \frac{1}{i\hbar}\int \tilde{V}_{\rm eff}(-\mathbf{k})[\tilde{W}(\mathbf{v}_-,\mathbf{k}) + \tilde{W}(\mathbf{v}_+,\mathbf{k})]\frac{d\mathbf{k}}{(2\pi)^3}. \quad (34)$$

Now, using Eq. (9) which relates the potential perturbations $\tilde{V}_{\rm eff}$ to the perturbations of the distribution function $\tilde{W}$, we obtain

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r},}\right) W_0(\mathbf{v}) = \frac{Q^2}{\hbar^2 k^4}\int \frac{|\tilde{n}(\mathbf{k})|^2}{i(\omega-\mathbf{k}\cdot\mathbf{v})}[W_0(\mathbf{v}-\hbar\mathbf{k}/M) - W_0(\mathbf{v}+\hbar\mathbf{k}/M)]\frac{d\mathbf{k}}{(2\pi)^3}, \quad (35)$$

where we have used the density fluctuations $\tilde{n}(\mathbf{k}) = k^2\tilde{V}_{\rm eff}(\mathbf{k})/Q$. This is a kinetic equation of the Boltzmann type, associated with the inelastic collisions between the atoms and the hybrid phonons of the density fluctuation spectrum. Emission and absorption of one phonon will make the atom velocity to jump between $\mathbf{v}$ and $\mathbf{v}\pm\hbar\mathbf{k}/M$, as it should be expected.

Now, let us consider the quasiclassical limit of this equation, when the atomic recoil becomes negligible, In this limit, the population difference in Eq. (35) is replaced by a derivative, and we can write

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) W_0(\mathbf{v}) = \frac{1}{i\hbar}\int \tilde{V}_{\rm eff}(-\mathbf{k})\delta\tilde{W}(\mathbf{k})\frac{d\mathbf{k}}{(2\pi)^3}, \quad (36)$$

where

$$\delta\tilde{W}(\mathbf{k}) = -\frac{\hbar\mathbf{k}}{M}\cdot\frac{\partial}{\partial\mathbf{v}}\tilde{W}(\mathbf{v},\mathbf{k}). \quad (37)$$

In the same limit, we also have

$$\tilde{W}(\mathbf{v},\mathbf{k}) = -\frac{\tilde{V}_{\rm eff}}{M}\frac{\mathbf{k}\cdot\partial W_0/\partial\mathbf{v}}{(\omega-\mathbf{k}\cdot\mathbf{v})} \quad (38)$$

from where we get

$$\delta\tilde{W}(\mathbf{k}) = \tilde{V}_{\rm eff}(\mathbf{k})\frac{\hbar\mathbf{k}}{M^2}\cdot\frac{\partial}{\partial\mathbf{v}}\frac{\mathbf{k}}{(\omega-\mathbf{k}\cdot\mathbf{v})}\cdot\frac{\partial}{\partial\mathbf{v}}W_0. \quad (39)$$

Replacing this in Eq. (36), we finally obtain

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}} - \frac{\partial}{\partial\mathbf{v}}\cdot\mathbf{D}(\mathbf{v})\cdot\frac{\partial}{\partial\mathbf{v}}\right]W_0(\mathbf{v}) = 0 \quad (40)$$

with the diffusion tensor in the velocity space determined by

$$\mathbf{D} = \frac{Q^2}{M^2k^4}\int\frac{|\tilde{n}(\mathbf{k})|^2}{i(\omega-\mathbf{k}\cdot\mathbf{v})}\mathbf{k}\mathbf{k}\frac{d\mathbf{k}}{(2\pi)^3}. \quad (41)$$

We can now explore the spectral symmetries, well known from the quasilinear theory [17], by noting that $|\tilde{n}(\mathbf{k})|^2 = |\tilde{n}(-\mathbf{k})|^2$. Here we are assuming that, for each mode of the density fluctuation spectrum, there is a complex frequency $\omega \equiv \omega(\mathbf{k}) = \omega_{\mathbf{k}} + i\gamma_{\mathbf{k}}$, where $\omega_{\mathbf{k}}$ is the real part of the mode frequency and $\gamma_{\mathbf{k}}$ is the corresponding Landau damping. We also notice that $\omega_{-\mathbf{k}} = \omega_{\mathbf{k}}$. This allows us to rearrange the terms inside the integral of Eq. (42), leading to the following new expression for the diffusion tensor:

$$\mathbf{D} = \frac{Q^2}{M^2k^4}\int|\tilde{n}(\mathbf{k})|^2\frac{\gamma_{\mathbf{k}}\mathbf{k}\mathbf{k}}{(\omega-\mathbf{k}\cdot\mathbf{v})^2+\gamma_{\mathbf{k}}^2}\frac{d\mathbf{k}}{(2\pi)^3}. \quad (42)$$

This new expression is physically more satisfactory because it clearly states that diffusion is a real process. Of particular interest is the case where Landau damping is a very small quantity, and where we can use the limit

$$\lim_{\gamma_{\mathbf{k}}\to 0}\frac{\gamma_{\mathbf{k}}}{(\omega-\mathbf{k}\cdot\mathbf{v})^2+\gamma_{\mathbf{k}}^2} = \pi\delta(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v}). \quad (43)$$

We can then rewrite the diffusion tensor in a much simpler form

$$\mathbf{D} = \pi\frac{\omega_p^2}{n_0k^4}\int|\tilde{n}(\mathbf{k})|^2\,\mathbf{k}\mathbf{k}\,\delta(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v})\frac{d\mathbf{k}}{(2\pi)^3}. \quad (44)$$

This new expression for $\mathbf{D}$ states that diffusion in velocity space is due to a succession of wave modes which are at a given time in exact resonance with the atoms moving with velocity $\mathbf{v}$, as shown by the Dirac delta function. The stronger the wave component energy, the faster diffusion occurs. The existence of the density fluctuation spectrum will then introduce a temperature limit for the laser cooling process.

In order to estimate this temperature limit, we have to describe the competing influence of the density fluctuations and the laser cooling force. Including the well-known expression for this force, we can write the quasiclassical kinetic equation in the form

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}\right)W_0 = \frac{\partial}{\partial\mathbf{v}}\cdot\left[\mathbf{A}+\mathbf{D}\cdot\frac{\partial}{\partial\mathbf{v}}\right]W_0, \quad (45)$$

where $\mathbf{A}$ is the friction coefficient associated with the cooling force, as given by [18,19]

$$\mathbf{A} = \beta\mathbf{v}, \quad \beta = 8\hbar k^2\Gamma\frac{\Delta|\Omega_R|^2}{M(4\Delta^2+\Gamma^2)^2}, \quad (46)$$

where Ω_R is the Rabi frequency, Γ the spontaneous decay time, and Δ the frequency detuning between the radiative transition frequency and the cooling laser frequency. This expression is valid for $|\omega_R|^2 \ll \Gamma^2/2$. Now, assuming spherical symmetry in velocity space, a steady-state solution for this equation can be derived as

$$W_0(\mathbf{v}) = W_{00}\exp\left[-\frac{MV^2}{2T_{\rm eff}}\right], \quad (47)$$

where W_{00} is a constant and the effective temperature $T_{\rm eff}$ is determined by

$$T_{\rm eff} \simeq \frac{\pi^2c^2}{\hbar\omega^2}\frac{\omega_p^2}{n_0k^2}\int|\tilde{n}(\mathbf{k})|^2\frac{d\mathbf{k}}{(2\pi)^2}. \quad (48)$$

This quantity establishes a new temperature limit for the laser cooling process, which is conceptually different and eventually larger than the well-known Doppler limit associated with spontaneous emission.

V. CONCLUSIONS

In this work we have applied the wave kinetic description to the discussion of atomic density fluctuations in a confined ultracold gas and have extended our previous results [5] into the quantum regime. A general dispersion relation for the hybrid plasma-acoustic modes which includes a quantum dispersive term was derived, and an exact quantum expression for the atomic Landau damping of these modes was established. Finally, a Boltzmann type of equation was derived for the long time evolution of the atom distribution, leading in the quasiclassical limit to a diffusion coefficient in velocity space. In this limit, our previous results were recovered. A new temperature limit for laser cooling, different from the usual Doppler limit, and due to the existence of a density fluctuation spectrum, was also established.

This approach is adequate for describing atom recoil, due to the emission and absorption of phonons, which are the elementary excitations of the collective field. These phonons are the quasiparticles of the hybrid plasma-acoustic wave modes, and the understanding of its quantum dispersive properties was one of the main purposes of this work. In particular, we have demonstrated that a negative Landau damping can eventually occur, leading to the coherent emission of hybrid phonons. This could be explored to extend the range of single-particle phonon lasers, recently discussed in the literature [15,16], into the collective regime where real density oscillations of a background gas can coherently be excited. This particular problem will be the object of a separate publication [20].

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