# Coherent-state phase concentration by quantum probabilistic amplification

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We propose a probabilistic measurement-induced amplification for coherent states. The amplification scheme uses a counterintuitive architecture: a thermal noise addition (instead of a single-photon addition) followed by a feasible multiple-photon subtraction using a realistic photon-number-resolving detector. It allows one to substantially amplify weak coherent states and simultaneously reduce their phase uncertainty, which is impossible when using a deterministic Gaussian amplifier.

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## I. INTRODUCTION

Quantum optics has an extraordinary capability to combine observations of both the wave and the particle phenomena. Information can be encoded both into the intensity and into the phase of an optical field. Although the intensity approach, involving photons, may inherently seem more "quantum" than the phase approach, neither can be described by classical physics in full [1]. In this article, it is the phase aspect of the optical field we are going to focus on. The main aim of the quantum phase information processing is to reduce noise in the system and to compensate for the loss. The classical processing methods, mostly based on measurement and repreparation, are of limited usefulness [2] because of the inherent noise present in all quantum systems. This problem is especially pronounced for optical signals with low intensities, possibly occurring as a consequence of loss. In general, we seek to enhance an unknown phase of an optical signal by deterministic or probabilistic methods, where the main benefit of probabilistic methods lies in their ability to qualitatively overcome the limits of deterministic operations.

A coherent state $|\alpha\rangle$, the approximation of a light from a stabilized laser, is a natural medium for the phase encoding of information. Coherent states are nonorthogonal and very strongly overlapping if the amplitude is small, which can easily happen after a strong attenuation. Therefore, it is highly desirable to reamplify the states in a way that improves the phase information, ideally performing the transformation $|\alpha\rangle \to |g\alpha\rangle$, where $g > 1$. One might naturally think of the displacement operation, but keep in mind we seek to amplify a coherent state with an unknown phase and therefore we lack the knowledge needed for the correct displacement. Another option is the Gaussian parametric amplification [3,4], which is phase insensitive and it can be applied to an unknown state. However, in this case the phase information of the state does actually get worse due to the fundamental quantum noise penalty [5].

The ideal amplification $|\alpha\rangle \to |g\alpha\rangle$ is nonphysical, but for small values of $|\alpha|$ it can be implemented approximatively. One approach relies on the quantum scissors paradigm, limiting the dimension of the used Hilbert state [6]. The input coherent state is split into $M$ weak copies, which can be approximated by $(|0\rangle + \alpha/M|1\rangle + \cdots)^{\otimes M}$ and probabilistically amplified to $(|0\rangle + g\alpha/M|1\rangle)^{\otimes M}$. For a small value of $|\alpha|/M$ the subsequent Gaussifying concentration yields a finite Hilbert space approximation of $|g\alpha\rangle$. However, the procedure requires multiple indistinguishable single-photon sources and high interferometric stability of the multipath interferometer. Another approach is based on a still highly sophisticated cross-Kerr nonlinearity at a single-photon level followed by homodyne detection [7]. This kind of amplification has already been suggested, in Ref. [8], to concentrate entanglement.

In this article we propose a scheme for concentration of an unknown phase of coherent states using a probabilistic highly nonlinear amplifier. Our method is based on the addition of thermal noise to the unknown coherent state, followed by a multiple-photon subtraction using a photon-number-resolving detector. This procedure probabilistically amplifies the coherent state, increasing its mean photon number and simultaneously substantially reducing the phase noise. It leads to a probabilistic concentration of phase information, which cannot be obtained by Gaussian operations alone. Remarkably, the scheme requires neither single-photon sources nor high interferometric stability—the resource for the highly nonlinear amplification is the continual thermal noise injected into the signal mode.

## II. PHASE AND AMPLIFICATION

The quality of information carried by the phase is difficult to assess, as the phase is not a quantum mechanical observable and therefore it cannot be directly and ideally measured. However, each measurement devised to obtain the phase of the state can be characterized by a real positive-semidefinite matrix $H$, which is used in computing the phase distribution $P(\theta) = \mathrm{Tr}[\rho F(\theta)]$, where $F(\theta) = (1/2\pi)\sum_{m,n=0}^{\infty} \exp[i\theta(m-n)]H_{mn}|m\rangle\langle n|$ [9], and $|m\rangle$ stands for the photon number Fock state. The actual form of the matrix $H$ depends on the process used to extract the phase information. For example, for a phase obtained by the most common heterodyne measurement, consisting of a balanced beam splitter and a pair of homodyne detectors measuring conjugate quadratures, the matrix elements are $H_{mn} = \Gamma[(n+m)/2+1]/\sqrt{n!m!}$. Ultimately, for the ideal canonical phase measurement $H_{mn} = 1$ and $F(\theta)$ is a projector on the idealized phase state $|\theta\rangle = \sum_{n=0}^{\infty} e^{i\theta n}|n\rangle$. To obtain a single parameter characterizing the quality of phase encoding, we can use the distribution $P(\theta)$ to calculate the phase variance $V = |\mu|^{-2} - 1$, where $\mu = \langle \exp(i\theta)\rangle$ and subscripts $H$ and $C$ will be used to distinguish between the heterodyne and the canonical measurements, respectively. For calculations of an arbitrary measurement we can simply use the formula $\langle \exp(i\theta)\rangle = \int_{-\pi}^{\pi} P(\theta)\exp(i\theta)d\theta = \mathrm{Tr}(\sum_{n=0}^{\infty} H_{n,n+1}|n\rangle\langle n+1|\rho)$.

The coherent states can be expressed as $|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\alpha^n/\sqrt{n!}|n\rangle$. For these states, the quality of phase encoding is fully given by the mean number of coherent photons $N = |\alpha|^2$, and the phase variances, obtained with the help of [9]

$$\mu_C = e^{-|\alpha|^2}\alpha\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!\sqrt{n+1}},$$
$$\mu_H = e^{-|\alpha|^2}\alpha\, {}_1F_1\left(\frac{3}{2};2;|\alpha|^2\right)\frac{\Gamma\left[\frac{3}{2}\right]}{\Gamma[2]}, \quad (1)$$

are both monotonically decreasing functions of the mean photon number $N$. For weak coherent states with $N < 1$, the variances can be well approximated by

$$V_C(N) \approx N^{-1} + 1 - \sqrt{2} + O(N^2),$$
$$V_H(N) \approx 4/(\pi N) + (-1 + 2/\pi) + O(N^2), \quad (2)$$

if we take only the dominating terms into account. In the following, we focus primarily on the canonical phase variance.

For coherent states the phase variance is directly related to the amplitude $|\alpha|$. The ideal noiseless amplifier, which increases the amplitude while keeping the state coherent, would be therefore a suitable amplification device, if its implementation were not so complicated. On the other hand, the deterministic phase-insensitive (Gaussian) amplifier [3,4] is experimentally quite feasible, but unfortunately it actually worsens the phase variance of the coherent state. To show this, we can use a method similar to the one used in [10] to calculate

$$\mu_C = \frac{\alpha^*}{\pi}\int_0^{\frac{1}{G}}\frac{\exp(-xGN)}{\sqrt{-\ln\frac{1-Gx}{1-(G-1)x}}}dx, \quad (3)$$

where $G = g^2$ is the linear amplification gain. We can now use (3) to obtain the phase variance and numerically verify its increase.

## III. AMPLIFICATION BY PHOTON ADDITION AND SUBTRACTION

However, there is another mechanism that can be employed for amplification and phase improvement. Consider a single-photon addition (described by $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$) followed by a single-photon subtraction (described by $a|n\rangle = \sqrt{n}|n-1\rangle$) applied to a weak coherent state (approximately, $|\alpha\rangle = |0\rangle + \alpha|1\rangle$). This corresponds to $aa^\dagger(|0\rangle + \alpha|1\rangle) \to a(|1\rangle + \sqrt{2}\alpha|2\rangle) \to |0\rangle + 2\alpha|1\rangle$. For low $N$ this reduces the phase variance roughly by a factor of 4. Note, the canonical variance actually decreases in both the creation and the annihilation process.

For a coherent state transformed in this way, $aa^\dagger|\alpha\rangle$, the total mean photon number $\langle N\rangle = N(4 + 5N + N^2)/(1 + 3N + N^2)$ increases and the canonical variance

$$\mu_C = \exp(-N)\frac{\sqrt{N}}{1+3N+N^2}\sum_{n=0}^{\infty}\frac{N^n(n+1)(n+2)}{n!\sqrt{n+1}} \quad (4)$$

is always lower than the Holevo variance from (1). For a lower $N < 1$, the canonical phase variance after the probabilistic procedure approaches the phase variance for the coherent state with $N = \langle N\rangle$. For a larger $N$ this effect tends to be less pronounced as the relative influence of single-photon operations diminishes. In this scenario it is convenient to consider a generalization, a collective $M$-photon addition followed by an $M$-photon subtraction. The phase variance is then determined by

$$\mu_C = e^{-N}\frac{\sqrt{N}}{\mathcal{N}} \times \sum_{n=0}^{\infty}\frac{N^n}{n!\sqrt{n+1}}\frac{(n+M)!}{n!}(n+1+M)(n+1)!,$$
$$\mathcal{N} = e^{-N}\sum_{n=0}^{\infty}\frac{N^n}{n!}\left(\frac{(n+M)!}{n!}\right)^2, \quad (5)$$

and it decreases as $M$ grows. Simultaneously, this also leads to an increase of the mean photon number. For sufficiently low values of $N$, the canonical variance approaches the result of the ideal noiseless amplifier and we can use the approximation

$$V_C(N) \approx \frac{1}{(M+1)^2N} + 1 - \frac{M+2}{\sqrt{2}(M+1)} + \mathrm{O}(N^2). \quad (6)$$

Comparison to the analogous formula for the noiseless amplifier (2) with $N \to g^2N$ reveals that $M + 1$ can play a role of the amplification gain $g$.

For the construction of such a probabilistic phase-insensitive amplifier, the photon addition operation is required. Furthermore, the photons have to be added coherently, perfectly interfering with the incoming coherent state. This task can be performed using a nondegenerate optical parametric amplifier with an avalanche photodiode monitoring the output idler port [11]. This approach has already been used to verify the validity of commutation relations for the annihilation operator [12], and it is therefore fully capable of demonstrating the probabilistic amplification for $M = 1$. However, the procedure is not trivial and adding and subsequently subtracting more than two photons is currently unfeasible, mainly due to low success rates.

## IV. AMPLIFICATION WITH NOISE ADDITION

Fortunately, the amplification can be made simpler. Instead of adding single photons separately, we can add a phase-insensitive thermal noise, which is characterized by its mean number of thermal photons $N_{\mathrm{th}}$. The second step is then the same as already discussed—the probabilistic subtraction of $M$ photons. Now the photon subtraction is an operation which can improve the phase properties, but the noise addition is clearly purely destructive. Why does it work then? The main point is that the photon subtraction does nothing when applied to a coherent state. However, for a mixed state the photon subtraction serves as a probabilistic filter, improving the weight of the high amplitude coherent states within the mixture. The first step of the amplification could be explained as a displacement in a random direction. This creates a phase-insensitive mixture of coherent states slightly displaced in the direction given by the initial phase. The second step, the photon subtraction, then "picks" states with the highest intensity and these states are mostly those for which the displacement had (purely by chance) the same phase as the initial signal. The state after the subtraction is still mixed, with the same mean phase as the initial coherent state, but the overall amplitude has been increased by the amplification.

Formally, the density operator of the initial coherent state after the noise addition and $M$ photon subtraction can be represented as $\rho_{\mathrm{amp}} = \sum_{n,m} \rho_{n,m} |n-M\rangle\langle m-M|$, where

$$\rho_{n,m} = \frac{1}{\mathcal{N}}\sqrt{\frac{n!}{m!}}\exp\left(-\frac{|\alpha|^2}{N_{\mathrm{th}}+1}\right)\frac{(\alpha^*)^{m-n}N_{\mathrm{th}}^n}{(N_{\mathrm{th}}+1)^{m+1}} \times \mathrm{L}_n^{m-n}\left(-\frac{|\alpha|^2}{N_{\mathrm{th}}(N_{\mathrm{th}}+1)}\right)\sqrt{\frac{n!m!}{(n-M)!(m-M)!}} \quad (7)$$

for $m \geqslant n$ and $\rho_{m,n} = \rho_{n,m}^*$ otherwise. $L_n^m(x)$ denotes the associated Laguerre polynomial. The normalization factor representing the success rate is

$$\mathcal{N} = \sum_k \exp\left(-\frac{|\alpha|^2}{N_{\mathrm{th}}+1}\right)\frac{N_{\mathrm{th}}^{k+M}}{(N_{\mathrm{th}}+1)^{k+M+1}} \times \mathrm{L}_{k+M}^0\left(-\frac{|\alpha|^2}{N_{\mathrm{th}}(N_{\mathrm{th}}+1)}\right)\frac{(k+M)!}{k!}. \quad (8)$$

It may be surprising that such an incoherent operation preserves and even improves the phase of the initial coherent state. To show that this is really the case we express the density matrix elements (7) as $\rho_{m,n} = \tilde{\rho}_{m,n}(|\alpha|)e^{i\phi(m-n)}$, where we have introduced $\phi$ as the mean phase of the initial coherent state, $\alpha = |\alpha|e^{i\phi}$. If we formally represent the amplification operation by a mapping $\mathcal{A}$ such that $\rho_{\mathrm{amp}} = \mathcal{A}[|\alpha\rangle\langle\alpha|]$, we can see that it commutes with the unitary phase shift operator $U_\theta = e^{i\theta a^\dagger a}$,

$$U_\theta \mathcal{A}[|\alpha\rangle\langle\alpha|]U_\theta^\dagger = \mathcal{A}[U_\theta|\alpha\rangle\langle\alpha|U_\theta^\dagger]. \quad (9)$$

Consequently, the mean phase of the amplified state is fully given by the phase of the initial coherent state. Also note that the amplification effects are completely covered by the density matrix given by $\tilde{\rho}_{m,n}(|\alpha|)$. In this sense, the amplification procedure is universal with respect to the phase of the initial state. To analyze the phase concentration effect we can calculate the canonical phase variance:

$$\mu_C = \frac{1}{\mathcal{N}}\sum_k \sqrt{\frac{(k+M)!}{(k+M+1)!}}\exp\left(-\frac{|\alpha|^2}{N_{TH}+1}\right) \times \frac{\alpha N_{\mathrm{th}}^{k+M}}{(N_{\mathrm{th}}+1)^{k+M+2}}\mathrm{L}_{k+M}^1\left(-\frac{|\alpha|^2}{N_{\mathrm{th}}(N_{\mathrm{th}}+1)}\right) \times \sqrt{\frac{(k+M)!(k+1+M)!}{k!(k+1)!}}. \quad (10)$$

The expression (10) can be calculated numerically and the results are presented in Fig. 1. The probabilistic amplification of the initial coherent state ($M = 0$) results in a visible reduction of the phase variance. The mean number $N_{\mathrm{th}}$ of added thermal photons was optimized to minimize the phase variance and it is saturating for larger $M$. The reduction of the phase variance saturates as well, but already for a somewhat feasible four-photon subtraction the resulting phase variance corresponds to the phase variance of a coherent state with $N = 0.36$ (as opposed to the coherent state with $N = 0.04$ before the amplification). This is equivalent to a strong amplification $|\alpha\rangle \to |g\alpha\rangle$ with gain $g = 3$. We can also look at the process from the amplification perspective and find out how the amplitude of the state increases. If we consider (without loss of generality, because the amplification is phase insensitive) the initial mean phase of $\phi = 0$, the gain of the amplification can be expressed as $\langle(a+a^\dagger)/2\rangle/\sqrt{N}$ and it is shown in Fig. 1(c) for various $M$. The addition of thermal photons is an incoherent process and the resulting state is therefore not pure. The purity after the amplification can be seen in Fig. 1(d).

The nonlinear nature of the amplification is clearly visible from a change of the contour of Wigner function (taken at full width at a half maximum) in Fig. 2. The contours are

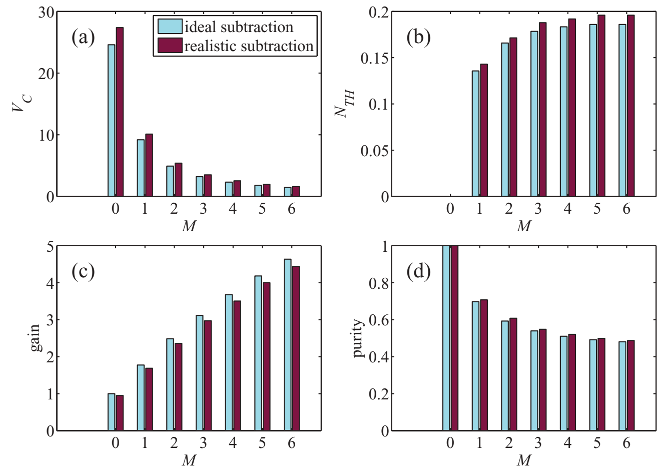


FIG. 1. (Color online) Phase concentration in a highly probabilistic amplification using thermal noise addition and photon subtraction. The separate graphs show the canonical phase variance $V_C$ (a), the optimal number of thermal photons added (b), the gain of the amplification (c), and the purity of the amplified state (d) as a function of the number of subtracted photons $M$. The different bars correspond to the ideal photon subtraction realized by the annihilation operator (left) and to the realistic photon subtraction employing a beam splitter with $T = 0.9$ and a threshold detector with efficiency $\eta = 0.4$ (right). The color coding given in panel (a) is the same for all the panels.

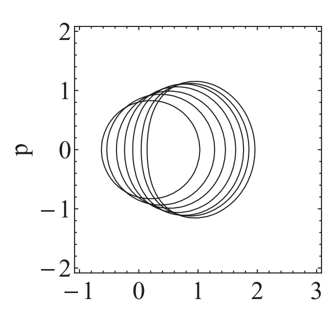


FIG. 2. Contours of Wigner functions of the states amplified by the noise addition and the beam splitter tap, similar to conditions described in Fig. 1. The contours go from left to right as the number of subtractions increases $M = 0, 1, \ldots, 6$.

plotted for an initial state with $\alpha = 0.2$ with a mean phase value of $\phi = 0$. From the contours of the group of amplified states we can see that the mean phase of the state is preserved and the state is displaced in this correct direction. At the same time the initial circular contour gains a "crescent" shape as $M$ increases. This is a difference from the ideal noiseless amplification methods [6,7], which keep the state coherent (in a suitable limit). However, although the change of shape of the Wigner function suggests greater phase uncertainty, the increase of the amplitude of the state results in a smaller phase variance.

For a physical understanding it is illustrative to consider a weak coherent state $|0\rangle + \alpha|1\rangle$ displaced by a weak thermal noise $\rho \rightarrow \rho + \epsilon_{\rm th}(a^\dagger \rho a + a\rho a^\dagger)$ and followed by a single-photon subtraction. The resulting state is $N|0\rangle\langle 0| + \epsilon_{\rm th}(|0\rangle + 2\alpha|1\rangle)(\langle 0| + 2\alpha^*\langle 1|)$ up to a normalization $\mathcal{N} = N + \epsilon_{\rm th} + 4N\epsilon_{\rm th}$. The canonical phase variance can be determined from $\mu = 2\epsilon_{\rm th}\alpha/\mathcal{N}$, and for small $N < 0.1$, the reduction approaches $V \propto \frac{1}{4N}$, approximating very well the result for the ideal amplification (2) with $g = 2$, if $\epsilon_{\rm th}$ is low enough. More generally, if the thermal noise is approximated as an addition of up to $M$ photons, the $M$-photon subtraction leads to phase variance $V \propto \frac{1}{(M+1)^2 N}$, which qualitatively matches the results for the ideal amplification with $g = M + 1$ (2), as well as the amplification by coherent addition and subtraction of $M$ photons (6).

The addition of a thermal noise can be realized by mixing the signal with a thermal state on a highly unbalanced beam splitter. The thermal state can be provided either by a thermal source, in which case the sufficient spatial and spectral overlap needs to be ensured by suitable filters, or by creating a mixture of coherent states by a proper random modulation. The benefit of the first approach lies in conceptually lower demand on resources, as there is no need for a coherent source of light. On the other hand, the second approach allows for generation of mixed states with various kinds of distributions (not just thermal), which can be used for a further optimization of the procedure.

A feasible scheme capable of approximately subtracting $M$ photons, which is required for a physical implementation of the procedure, is sketched in Fig. 3. It can be built from a linear coupling (a beam splitter with transmissivity $T$) to tap a part of the optical signal and a threshold measurement registering at least $M_0$ photons [13]. The quantum efficiency of the detector can be modeled by a virtual beam splitter with transmissivity $\eta$ inserted in front of the ideal detector. The quality of the outgoing signal depends on the transmissivity

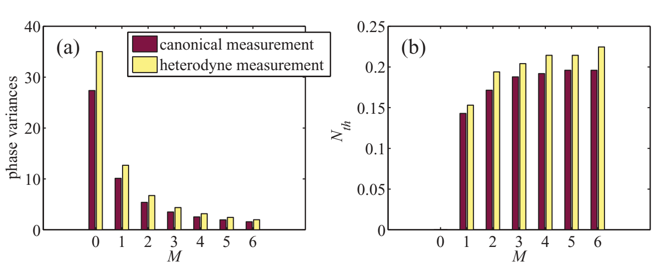


FIG. 3. (Color online) Realistic scheme for the probabilistic amplification of a coherent state.

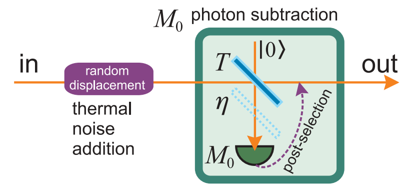


FIG. 4. (Color online) Comparison between the canonical phase measurement and the heterodyne phase measurement after the realistic phase concentration. The respective phase variances are shown in panel (a), while the optimal numbers of thermal photons are shown in panel (b).

$T$ values, $T < 1$ translates as loss, which increases the phase variance. The limited quantum efficiency of the detector only affects the success rate. However, $\eta$ that is too low may require lower $T$ to achieve sufficiently high success rates.

Generally, the amplified state can be expressed as

$$\rho'_{\rm amp} = \frac{1}{P_S}\int \Phi\left(\frac{\beta}{\sqrt{T}}\right)\mathcal{P}_\Pi\left(\frac{\beta}{\sqrt{T}}\right)|\beta\rangle\langle\beta|\frac{d^2\beta}{T}, \qquad (11)$$

where $\mathcal{P}_\Pi(\beta) = \langle\sqrt{\eta(1-T)}\beta|\Pi|\sqrt{\eta(1-T)}\beta\rangle$ and $\Pi$ denotes the positive-detection positive operator-valued measure (POVM) element, which in the case of the threshold detector looks like $\Pi = 1 - \sum_{k=0}^{M_0-1}|k\rangle\langle k|$. The initial coherent state with the addition of thermal noise is represented by $\Phi(\beta) = \exp(-|\beta-\alpha|^2/N_{\rm th})/\pi N_{\rm th}$. The normalization factor $P_S$ gives the probability of the success: $P_S = \int \Phi(\beta/\sqrt{T})\mathcal{P}_\Pi(\beta/\sqrt{T})d^2\beta/T$.

The density operator (11) fully describes the realistically amplified coherent state and its numerical evaluation is straightforward. The results are shown in Fig. 1 and we can see that, although they are quantitatively worse than those for the ideal subtraction, they follow the same qualitative pattern. The realistic multiphoton subtraction, even with the low quantum efficiency $\eta = 0.4$, is therefore a sufficient replacement for the ideal subtraction. Finally, we can check the difference between the canonical and the heterodyne phase measurements. The comparison in Fig. 4 shows a good qualitative agreement and justifies the use of the canonical measurement for the previous analysis.

## V. SUMMARY

We have proposed a probabilistic amplifier for coherent states. The amplifier setup, based on a thermal noise addition (instead of a single-photon addition) followed by a feasible multiphoton subtraction allows one to substantially reduce the phase variance of a coherent state. Note that the distribution of the random-noise-like modulation of the signal could be optimized to achieve better performance. There are also several possible applications open for future consideration. In quantum key distribution, the amplifier could be conceivably used in situations when the loss in the quantum channel prevents the secure key generation. At the same time, the amplifier is of no use to the eavesdropper because of its probabilistic nature—any gain in the rare event when the amplification succeeds is lost in the noise produced when it does not. As another possible application one could consider a probabilistic cloning of coherent states. Finally, the amplification itself need not be restricted to traveling wave quantum optics. All the necessary components, the thermal field, the coherent field, and the single-photon subtraction are also available in cavity QED [14] and this direction is open for future investigation.

*Note added.* Recently, a publication appeared that proposed another method for a noiseless amplification of coherent states based on a multiple-photon addition [15].


## ACKNOWLEDGMENTS

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