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Dissipation-induced Tonks-Girardeau gas of polaritons

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A scheme for the generation of a Tonks-Girardeau (TG) gas of polaritons with purely dissipative interaction is described. We put forward a master equation approach for the description of stationary light in atomic four-level media and show that, under suitable conditions, two-particle decays are the dominant photon loss mechanism. These dissipative two-photon losses increase the interaction strength by at least one order of magnitude as compared to dispersive two-photon processes and can drive the polaritons into the TG regime. Characteristic correlations of the TG gas, including quantities that distinguish it from free fermions, can be measured via standard quantum optical techniques. Our scheme thus allows one to feasibly generate highly correlated photon states, which can be of considerable use in quantum-information processing and precision measurements.

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Quantum mechanics categorizes particles into fermions or bosons. In three dimensions, only these two categories are possible, whereas more exotic anyons can exist in two dimensions [1]. In one dimension, the particle statistics cannot be considered without taking interparticle interactions into account [2]. A prominent example are bosons that interact via strong repulsive forces in a one-dimensional setting and can enter a Tonks-Girardeau (TG) gas regime [3], where they behave with respect to many observables as if they were fermions. A TG gas can be described as the strong interaction limit of the Lieb-Liniger model [4].

Strong correlations in many-particle systems, such as in the TG gas, give rise to interesting and partly not yet well understood physics. A substantial research effort is currently devoted to these systems, and progress in cooling and trapping of atoms and ions has led to very precise experimental studies of many-body systems. Eventually, this progress enabled the observation of a TG gas of atoms in an optical lattice [5]. Later, an experiment [6] with cold molecules showed that not only elastic interactions but even two-particle losses alone are able to create a TG gas. Here inelastic contact interactions effectively result in a repulsion between particles such that they never occupy the same position. Consequently, the dissipation-induced repulsion eventually inhibits the dissipation of particles. This counterintuitive result can be regarded as a manifestation of the quantum Zeno effect [6].

A classical analog where absorption leads to repulsion is given by a light wave in a medium with refractive index n_1 propagating perpendicular to the interface with a medium with refractive index n_2 [6]. The light wave will be completely reflected if $|n_2| \to \infty$, even if the medium n_2 is highly absorptive and hence n_2 imaginary.

In contrast to atoms, photons do only interact in the presence of nonlinear media, and usually these interactions are very weak. Their robustness against decoherence makes photons ideal carriers for quantum information, but the realization of strongly correlated systems or photon gates calls for strong photon-photon interactions. Recently, it has been shown that effective many-body systems of photons and polaritons can be generated via light matter interactions [7–10], and this concept is currently receiving increasing attention [11–14]. Promising experimental setups for entering the strongly correlated regime in order to access its rich physics are arrays of coupled microcavities doped with emitters [15] or optical fibers that couple to atoms [11,16,17]. In all these setups, the major challenge for realizing strong correlations is to make the polariton-polariton interactions significantly stronger than photon losses which are inevitably present in every experiment.

Here we present an effective many-body system of polaritons where the ubiquitous but usually undesired dissipative processes become the essential ingredient for the creation of strong many-particle correlations. This paradigm shift allows us to relax some conditions on the model parameters such that the achievable nonlinearities in our approach are at least an order of magnitude larger than their conservative counterparts [7,11,14]. In particular, we show that the dissipative nonlinearities in our system give rise to a TG gas of polaritons. For this regime, fermionic (e.g., Friedel oscillations) as well as nonlocal (e.g., the single-particle density matrix) correlations of the TG gas can be measured via standard quantum optical techniques.

We consider photons guided in an optical fiber that interact with nearby atoms [11,16], where stationary light [18] is created via electromagnetically induced transparency (EIT) [19]. As compared to coupled microcavities, the fiber approach is appealing because of the low photon loss of the fiber and because the longitudinal trapping of light is done optically, thus avoiding the need to build many mutually resonant cavities. We thus focus on this setup here. However, our mechanism for building up correlations works equally well in cavity arrays, and the dissipative nonlinearities we discuss here are always stronger than their conservative counterparts independent of the geometry of the experimental device.

We start with a more detailed description of our one-dimensional model shown in Fig. 1. Each of the N atoms interacts with control and probe fields denoted by $\Omega_\pm$ and $\hat{E}_\pm$, respectively. The control fields of frequency ω_c are treated classically, and Ω_+ (Ω_-) labels the Rabi frequency of the control field propagating in the positive (negative) z direction. In addition, we assume that the control fields are spatially homogeneous but may depend on time. The probe fields $\hat{E}_+$ and $\hat{E}_-$ are quantum fields that propagate in the positive and negative z direction, respectively. They are defined as $\hat{E}_\pm(z) = \sum_K a_{\pm K} e^{\pm i K z}$, where $a_{\pm K}$ are photon annihilation

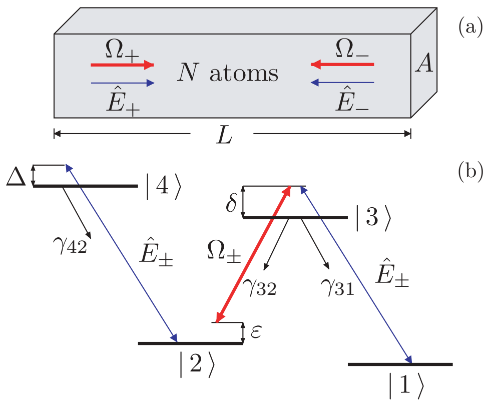

FIG. 1. (Color online) (a) Considered setup of N atoms confined to an interaction volume of length L and transverse area A. $\Omega_\pm$ are the Rabi frequencies of the classical control fields, and $\hat{E}_\pm$ are the quantum probe fields. (b) Atomic level scheme. γ_{ij} is the full decay rate on the $|i\rangle \leftrightarrow |j\rangle$ transition, δ and Δ label the detuning of the probe fields with states $|3\rangle$ and $|4\rangle$, respectively, and ε is the two-photon detuning.

operators. The wave numbers K are positive and of the order of the wave number k_c of the control field.

We model the time evolution of the atoms and the quantized probe fields by a master equation [20] for their density operator ϱ, $\dot{\varrho} = -\frac{i}{\hbar}[H,\varrho] + \mathcal{L}_\gamma\varrho$, where $\mathcal{L}_\gamma\varrho$ describes spontaneous emission from states $|3\rangle$ and $|4\rangle$, and the full decay rate on the transition $|i\rangle \leftrightarrow |j\rangle$ is denoted by γ_{ij} (see Fig. 1). In a rotating frame that removes the time dependence of the classical laser fields, the system Hamiltonian H reads $H = H_0 + H_\Lambda + H_{\text{NL}}$, where

$$H_0 = -\hbar\sum_K(\omega_p - \omega_K)(a_K^\dagger a_K + a_{-K}^\dagger a_{-K}) - \hbar\sum_{\mu=1}^N\left[\varepsilon A_{22}^{(\mu)} + \delta A_{33}^{(\mu)} + (\Delta+\varepsilon)A_{44}^{(\mu)}\right] \quad (1)$$

describes the free time evolution of the atoms and the probe fields. $A_{ii}^\mu = |i_\mu\rangle\langle i_\mu|$ is a projection operator onto state $|i_\mu\rangle$ of atom μ, the energy of level $|i\rangle$ is $\hbar\omega_i$ (we set $\omega_1 = 0$), and transition frequencies are denoted by $\omega_{ij} = \omega_i - \omega_j$. We denote the central frequency of the probe pulse by ω_p. The detuning of the probe field with respect to the transitions $|3\rangle \leftrightarrow |1\rangle$ and $|4\rangle \leftrightarrow |2\rangle$ is labeled by $\delta = \omega_p - \omega_{31}$ and $\Delta = \omega_p - \omega_{42}$, respectively, and $\varepsilon = (\omega_p - \omega_c) - \omega_2$ is the two-photon detuning. The interaction between the atoms and the probe and control fields is described by $H_\Lambda + H_{\text{NL}}$, with

$$H_\Lambda = -\hbar\sum_{\mu=1}^N\left\{S_{32}^{(\mu)}[\Omega_+(t)e^{ik_cz_\mu} + \Omega_-(t)e^{-ik_cz_\mu}] + g_1 S_{31}^{(\mu)}[\hat{E}_+(z_\mu) + \hat{E}_-(z_\mu)]\right\} + \text{H.c.}, \quad (2)$$

$$H_{\text{NL}} = -\hbar g_2\sum_{\mu=1}^N S_{42}^{(\mu)}[\hat{E}_+(z_\mu) + \hat{E}_-(z_\mu)] + \text{H.c.} \quad (3)$$

Transition operators of atom μ at position z_μ are defined as $S_{ij}^{(\mu)} = |i_\mu\rangle\langle j_\mu|$ $(i \neq j)$, and g_1 and g_2 are the single-photon Rabi frequencies on the $|3\rangle \leftrightarrow |1\rangle$ and $|4\rangle \leftrightarrow |2\rangle$ transitions, respectively. In the following, we assume that the Rabi frequencies of the control fields are identical (and real), and set $\Omega_+ = \Omega_- = \Omega_c$. With this choice, the interaction of the probe and control fields with the Λ subsystem formed by states $|1\rangle$, $|2\rangle$, and $|3\rangle$ allows us to store the probe field inside the medium [18]. On the other hand, the coupling of the probe fields to the $|4\rangle \leftrightarrow |2\rangle$ transition creates an effective photon-photon interaction [21].

Next we outline the approach we developed to reduce the master equation $\dot{\varrho} = -\frac{i}{\hbar}[H,\varrho] + \mathcal{L}_\gamma\varrho$ for the atoms and quantized probe fields into a master equation solely for dark-state polaritons [22], formed by collective excitations of photons and atoms. We represent the quantum state of dark-state polaritons by a density matrix ϱ_{D} comprised of dark states $|\alpha\rangle = \prod_{k=1}^{N_\alpha}(1/\sqrt{n_k!})(\psi_k^\dagger)^{n_k}|0\rangle$ that satisfy $H_\Lambda|\alpha\rangle = 0$, and the vacuum state $|0\rangle = |\{0\}_{\text{phot}}; 1_1,\ldots,1_N\rangle$ is the state where all photon modes of the probe fields are empty and all atoms are in state $|1\rangle$. The operators ψ_k are defined as [23]

$$\psi_k = A_k\cos\theta - X_{12}^k\sin\theta, \quad (4)$$

where $\sin\theta = \sqrt{N}g_1/\Omega_0$, $\cos\theta = \sqrt{2}\Omega_c/\Omega_0$, and $\Omega_0 = \sqrt{Ng_1^2 + 2\Omega_c^2}$. The operator $A_k = (a_{k_c+k} + a_{-k_c+k})/\sqrt{2}$ is a superposition of two counterpropagating probe field modes, and X_{12}^k describes the spin coherence, $X_{12}^k = \frac{1}{\sqrt{N}}\sum_{\mu=1}^N S_{12}^{(\mu)}e^{-ikz_\mu}$. Note that the wave number k can be positive or negative, and for all relevant k we have $|k| \ll k_c$. We assume that initially all atoms are in state $|1\rangle$ and that the total number of photons is much smaller than the number of atoms N. In this case, the dynamics induced by the Hamiltonian H is confined to a subspace $\mathcal{H}_{\text{FE}}$ of the total state space where $\langle\psi|\sum_{\mu=1}^N A_{11}^{(\mu)}|\psi\rangle \approx N$ for all $|\psi\rangle \in \mathcal{H}_{\text{FE}}$. It follows that the operators ψ_k obey bosonic commutation relations in $\mathcal{H}_{\text{FE}}$, $[\psi_k, \psi_p^\dagger] = \delta_{kp}$, where we neglected corrections of order $1/N$.

The dark-state polaritons are eigenstates of H_Λ, but the remaining parts H_0 and H_{NL} of the system Hamiltonian give rise to a nontrivial time evolution of ϱ_{D}. Fortunately, this dynamics can be studied entirely in terms of bosonic quasi-particle excitations if the system dynamics is restricted to the subspace $\mathcal{H}_{\text{FE}}$. In particular, the free time evolution H_0 introduces a coupling of dark-state polaritons to bright polaritons, $\phi_k = A_k\sin\theta + X_{12}^k\cos\theta$, and photons, $D_k = (a_{k_c+k} - a_{-k_c+k})/\sqrt{2}$. These excitations are in turn coupled to the excited state $|3\rangle$. Furthermore, H_{NL} introduces a direct coupling of dark-state polaritons ψ_k to the excited state $|4\rangle$ via a two-particle process [24]. Excitations in the states $|3\rangle$ and $|4\rangle$ are created by $P_{k,+}^\dagger$, $P_{k,-}^\dagger$ and $U_{k,+}^\dagger$, $U_{k,-}^\dagger$, respectively, where $P_{k,\pm}^\dagger = \sum_{\mu=1}^N[S_{31}^{(\mu)}e^{i(k_c+k)z_\mu} \pm S_{31}^{(\mu)}e^{-i(k_c-k)z_\mu}]/\sqrt{2N}$, $U_{k,\pm}^\dagger = \sum_{\mu=1}^N[S_{41}^{(\mu)}e^{i(k_c+k)z_\mu} \pm S_{41}^{(\mu)}e^{-i(k_c-k)z_\mu}]/\sqrt{2N}$. Spontaneous emission from states $|3\rangle$ ($|4\rangle$) results in decays of excitations created by $P_{k,\pm}^\dagger$ ($U_{k,\pm}^\dagger$).

We employ projection operator techniques [20] to derive a master equation for ϱ_{D} which is obtained from ϱ by a partial trace over all excitations except for the dark-state polaritons ψ_k. We restrict our analysis to the so-called slow-light regime, where $\sin^2\theta \approx 1$ and $\cos^2\theta \ll 1$. In this case, the coupling of dark-state polaritons to excitations in states $|3\rangle$ and $|4\rangle$ is much slower than the decay of the relevant correlation functions $\langle\phi_k\phi_p^\dagger\rangle(\tau)$, $\langle D_k D_p^\dagger\rangle(\tau)$, and $\langle U_{k,+}U_{p,+}^\dagger\rangle(\tau)$, which happens on a time scale given by the lifetimes of the exited states $|3\rangle$ and $|4\rangle$. This existence of two different time scales allows us to derive a master equation in the Born-Markov approximation if $4g_2^2\cos^2\theta N_{\rm ph} \ll \gamma_{42}^2, \cos^2\theta c^2 k_{\max}^2/\Omega_0^2 \ll 1$, $\cos^2\theta\Delta\omega^2/\Omega_0^2 \ll 1$, and $\Omega_0 \gg \gamma_{ij}, |\delta|$. Here c is the speed of light, $\Delta\omega = \omega_p - \omega_c$ is the frequency difference between the probe and control fields, and $N_{\rm ph}$ is the number of photons in the pulse. We describe the polariton pulse by the field operator $\psi(z) = (1/\sqrt{L})\sum_k e^{ikz}\psi_k$ which obeys the commutation relations $[\psi(z), \psi^\dagger(z')] = \delta(z - z')$. The maximal wave number contributing to ψ is $k_{\max}$. For a small two-photon detuning $\varepsilon = -\cos^2\theta\Delta\omega$, we obtain

$$\hbar\dot{\varrho}_{\rm D} = -iH_{\rm eff}\varrho_{\rm D} + i\varrho_{\rm D}H_{\rm eff}^\dagger + \mathcal{I}\varrho_{\rm D} + \mathcal{L}_1\varrho_{\rm D} + \mathcal{L}_2\varrho_{\rm D}, \quad (5)$$

where $H_{\rm eff}$ is a non-Hermitian Hamiltonian,

$$H_{\rm eff} = \frac{\hbar^2}{2m_{\rm eff}}\int_0^L dz\,\partial_z\psi^\dagger\,\partial_z\psi + \frac{\tilde{g}}{2}\int_0^L dz\psi^{\dagger 2}\psi^2, \quad (6)$$

$m_{\rm eff} = -\hbar\Omega_0^2/(2\delta c^2\cos^2\theta)$ is the effective mass of the polaritons, $\tilde{g} = 2\hbar L g_2^2\cos^2\theta/(\Delta - \cos^2\theta\Delta\omega + i\gamma_{42}/2)$ is the complex coupling constant, and

$$\mathcal{I}\varrho_{\rm D} = -{\rm Im}(\tilde{g})\int_0^L dz\psi^2\varrho_{\rm D}\psi^{\dagger 2}, \quad (7)$$

$$\mathcal{L}_1\varrho_{\rm D} = -\frac{\hbar\Gamma\Delta\omega^2\mathcal{D}[\psi]}{2\Omega_0^2/\cos^2\theta}, \quad \mathcal{L}_2\varrho_{\rm D} = -\frac{\hbar\Gamma c^2\mathcal{D}[\partial_z\psi]}{2\Omega_0^2/\cos^2\theta}. \quad (8)$$

Here $\mathcal{D}[\hat{X}] = \int_0^L dz(\hat{X}^\dagger\hat{X}\varrho_{\rm D} + \varrho_{\rm D}\hat{X}^\dagger\hat{X} - 2\hat{X}\varrho_{\rm D}\hat{X}^\dagger)$ is a dissipator in Lindblad form [20] for an operator $\hat{X}$, and $\Gamma = \gamma_{31} + \gamma_{32}$ is the full decay rate of state $|3\rangle$. For optical fibers, photon losses due to leakage are very low and can be neglected. If they need to be taken into account, an additional decay term with the same structure as $\mathcal{L}_2\varrho_{\rm D}$ but with a decay rate $\kappa\cos^2\theta$ appears (κ is the bare photon leakage rate). To confirm the accuracy of our results, we compared the predictions of the master equation (5) for the Λ subsystem ($\tilde{g} = 0$) with the results of a full numerical integration of Maxwell-Bloch equations for classical fields and found excellent agreement.

Next we derive the essential results of this letter from the master equation (5) that describes a one-dimensional system of interacting bosons. The first contribution to $H_{\rm eff}$ in Eq. (6), $(\hbar^2/2m_{\rm eff})\int_0^L dz\partial_z\psi^\dagger\partial_z\psi$, represents a kinetic-energy term with quadratic dispersion relation for the polaritons. The term proportional to $\tilde{g}$ in Eq. (6) and $\mathcal{I}\varrho_{\rm D}$ in Eq. (7) account for the elastic and inelastic two-particle interactions that originate from the coupling of dark-state polaritons to the excited state $|4\rangle$. More precisely, the real part of $\tilde{g}$ gives rise to a Hermitian contribution to $H_{\rm eff}$ that accounts for elastic two-particle collisions. On the other hand, the imaginary part of $\tilde{g}$ together with $\mathcal{I}\varrho_{\rm D}$ gives rise to a two-particle loss term that can be written in Lindblad form as ${\rm Im}(\tilde{g}/2)\mathcal{D}[\psi^2]$.

The contributions $\mathcal{L}_1\varrho_{\rm D}$ and $\mathcal{L}_2\varrho_{\rm D}$ describe single-polariton losses that can be omitted under the following conditions. Since $\mathcal{L}_1\varrho_{\rm D}$ is proportional to $\Delta\omega^2$, single-particle losses are minimized by minimizing $|\Delta\omega|$. Note that this fact has not been pointed out so far. From now on, we assume that $\Delta\omega^2$ is small enough such that $\mathcal{L}_2\varrho_{\rm D}$ represents the dominant single-particle losses. This is reasonable if $|\Delta\omega|$ is at most of the order of GHz and implies $|\varepsilon| \ll |\gamma_{24}|$. The term $\mathcal{L}_2\varrho_{\rm D}$ is negligible if two conditions are met. First, the dynamics induced by the kinetic-energy term proportional to $m_{\rm eff}$ in Eq. (6) must be fast as compared to the inverse decay rate of polaritons introduced by $\mathcal{L}_2\varrho_{\rm D}$. This can be achieved if we set $|\delta| \gg \Gamma$. Second, losses due to $\mathcal{L}_2\varrho_{\rm D}$ must be negligible, which imposes a limit on the maximal evolution time $t_{\max} \ll 2\Omega_0^2/(\Gamma c^2 k_{\max}^2\cos^2\theta)$. This implies that $t_{\max}$ can be of the order of $1/(\cos^2\theta\Gamma) \gg 1/\Gamma$.

Under these conditions, the master equation (5) reduces to $\hbar\dot{\varrho}_{\rm D} = -iH_{\rm eff}\varrho_{\rm D} + i\varrho_{\rm D}H_{\rm eff}^\dagger + \mathcal{I}\varrho_{\rm D}$ and can be identified with the generalized Lieb-Liniger model [25] for a one-dimensional system of bosons with mass $m_{\rm eff}$ and complex interaction parameter $\tilde{g}$. All features of the Lieb-Liniger model [4,25] are characterized by a single, dimensionless parameter $G = m_{\rm eff}\tilde{g}/(\hbar^2 N_{\rm ph}/L)$, where $N_{\rm ph}$ is the number of photons in the pulse. The absolute value of G is

$$|G| = \frac{g_1^2 g_2^2 L^2 N}{c^2|\delta|\sqrt{\Delta^2 + \frac{\gamma_{42}^2}{4}}N_{\rm ph}} = \frac{(1/16)\Gamma\gamma_{42}(d_{\rm opt})^2}{|\delta|\sqrt{\Delta^2 + \frac{\gamma_{42}^2}{4}}NN_{\rm ph}}, \quad (9)$$

where $d_{\rm opt} = 4Ng_1^2L/(c\Gamma) = 4Ng_2^2L/(c\gamma_{42})$ is the optical depth on the probe field transitions. Note that the parameters g_1^2L and g_2^2L are independent of the length of the system, since $g_1, g_2 \sim 1/\sqrt{AL}$. It follows that the parameter G and the optical depth depend only on the transverse area A of the interaction volume, but not on the length L of the cell. The absolute value of G characterizes the effective interaction strength between the particles. In the strongly correlated regime $|G| \gg 1$, the interaction between the particles creates a Tonks-Girardeau gas, where polaritons behave like impenetrable hard-core particles that never occupy the same position. Formally, this result can be derived via the pair-correlation function $g^{(2)}(z, z') = \langle\psi^\dagger(z)\psi^\dagger(z')\psi(z)\psi(z')\rangle/[\langle\hat{n}(z)\rangle\langle\hat{n}(z')\rangle]$ with $\hat{n}(z) = \psi^\dagger(z)\psi(z)$. For the ground state of the generalized Lieb-Liniger model in the strongly correlated regime, $g^{(2)}(z, z) = (1 - 1/N_{\rm ph}^2)4\pi^2/(3|G|^2)$ is close to zero and vanishes in the limit $|G| \to \infty$ [25]. Moreover, this ground state is the same [25] as in the original model with repulsive interaction for $|G| \to \infty$. It follows that $g^{(2)}(z, z')$ for $z \neq z'$ exhibits Friedel oscillations [26] that indicate a crystallization of photons in the fiber.

The parameter $|G|$ is maximal if the interaction between the polaritons is purely dissipative ($\Delta = 0$). Since the realization of a regime where the two-particle interactions are dominated by elastic processes requires $\Delta \gg \gamma_{42}/2$, the conservative nonlinearities are at least an order of magnitude smaller than the dissipative counterparts for $\Delta = 0$. It follows that purely dissipative interactions between the polaritons we discuss here are most effective for the generation of correlations. Furthermore, we point out that $|G| \propto N/A^2$ and thus increases linearly with the number of atoms N. In contrast to cavity QED systems [14], the condition $g_2 \gg \gamma_{42}$ is thus not required to obtain large values of $|G|$.

An analysis of dissipation-induced correlations ($\Delta = 0$) requires at least two photons. Assuming $|\delta|/\Gamma = 10$ such that the single-particle loss term $\mathcal{L}_1\varrho_{\rm D}$ in Eq. (5) is negligible and $N_{\rm ph} = 2$, Eq. (9) shows that $|G|$ is larger than unity for $(d_{\text{opt}})^2/N > 160$. A recent experiment [16] with laser-cooled atoms loaded into a hollow fiber reports a value of $(d_{\text{opt}})^2/N \approx 0.3$ for $N = 3000$ atoms. Since $(d_{\text{opt}})^2/N \propto N$, the required value of $(d_{\text{opt}})^2/N > 160$ could be achieved with $N > 1.6 \times 10^6$ atoms if the transverse area A of the fiber core is kept fixed. For the experimental parameters [16] of $L = 3$ cm and $A = 11.3\ \mu\text{m}^2$, the value of $N > 1.6 \times 10^6$ atoms corresponds to a density of $\rho > 4.7 \times 10^{18}$ atoms/m^3. This value is comparable to both the typical density of laser-cooled atoms and the density of hot atoms used in a stationary light experiment [18].

The observation of the dissipation-induced TG gas regime requires that the system can be prepared in low-energy states. One possibility is the procedure described in [11] which relies on an adiabatic state transfer realized by a time-dependent detuning Δ. A second possibility does not require any tuning of the two-particle losses. The master equation (5) implies that losses due to inelastic two-particle interactions are related to the pair-correlation function $g^{(2)}(z,z)$ via $\partial_t\langle\hat{n}(z)\rangle = (2/\hbar)\text{Im}(\tilde{g})g^{(2)}(z,z)\langle\hat{n}(z)\rangle^2$. It follows that uncorrelated states with $g^{(2)}(z,z) \approx 1$ decay much faster than those where $g^{(2)}(z,z) \approx 0$. Therefore, a regime where $g^{(2)}(z,z) < 1$ should be entered on a time scale $\hbar/[2\text{Im}(\tilde{g})N_{\text{ph}}/L]$, which is much shorter than the allowed evolution time t_{max} for $k_{\text{max}} \leqslant 2\pi N_{\text{ph}}/L$ and $|G| > 1$. Since the ground state of the generalized Lieb-Liniger model decays at the smallest rate [25], two-particle losses themselves are then able to drive the system into states close to the ground state. A more rigorous investigation of this point would require a numerical integration of equation (5), which is beyond the scope of this work.

For measurements, we note that the polariton pulse can be released from the fiber without distortion if one control field is adiabatically switched off [11,18]. It follows that spatial correlations $\langle\psi^\dagger(z)\psi(z')\rangle$ and $\langle\psi^\dagger(z)\psi^\dagger(z')\psi(z)\psi(z')\rangle$ of the trapped pulse are mapped into first- and second-order correlations in time of the output light, respectively. Since the latter can be detected via standard quantum optical techniques, the Friedel oscillations of $g^{(2)}(z,z')$, the correlations $\langle\psi^\dagger(z)\psi(z')\rangle$, and the characteristic momentum distribution of the TG gas can be measured with high precision.

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