# Comment on "Full quantum reconstruction of vortex states"

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It is pointed out that the *bona fide* Wigner distribution recently introduced [Phys. Rev. A **78**, 060101(R) (2008)] is in fact the rotational Wigner function earlier derived and thoroughly studied [Phys. Rev. A **49**, 3255 (1994); **71**, 069901(E) (2005)], so the latter remains indeed the natural phase-space representation for the rotation-angle and angular-momentum pair.

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In a recent paper, Rigas *et al.* [1] propose a *bona fide* Wigner function as a proper phase-space representation for the eigenstates of the orbital-angular-momentum operator, after saying that "although some interesting attempts have been published, they seem of difficult application for the problem at hand," referring to the works by Mukunda [2] and Bizarro [3]. However, by making a simple, straightforward comparison of Eqs. (5) and (8) of Rigas *et al.* [1] with Eqs. (3.25) and (3.26) of Bizarro [3], one can immediately conclude that such a *bona fide* Wigner function is no other than the so-called rotational Wigner function: the two are one and the same distribution, hence this Comment.

In fact, recalling the expression

$$\hat{\varrho} = |\Psi\rangle\langle\Psi|$$

for the density matrix of a generic quantum state $|\Psi\rangle$, Eqs. (5) and (8) of Rigas *et al.* [1] combine to yield

$$W(\ell,\phi) = \frac{1}{2\pi}\sum_{\ell'\in Z} e^{-2i\ell'\phi}\langle\ell-\ell'|\Psi\rangle\langle\Psi|\ell+\ell'\rangle + \frac{1}{2\pi^2}\sum_{\ell',\ell''\in Z}\frac{(-1)^{\ell''-\ell}}{\ell''-\ell+1/2} \times e^{-i(2\ell'+1)\phi}\langle\ell''-\ell'|\Psi\rangle\langle\Psi|\ell''+\ell'+1\rangle,$$

which can be easily cast into the form

$$W(\ell,\phi) = \frac{1}{2}w(\ell,\phi) + \frac{1}{2}\sum_{\ell''\in Z}\frac{(-1)^{\ell''-\ell}}{(\ell''-\ell+1/2)\pi}w(\ell''+1/2,\phi),$$

where

$$w(\ell+\mu/2,\phi) = \frac{1}{\pi}\sum_{\ell'\in Z} e^{-i(2\ell'+\mu)\phi} \times\langle\ell-\ell'|\Psi\rangle\langle\Psi|\ell+\ell'+\mu\rangle,$$

$\mu$ being either 0 or 1. Now, apart from some minor cosmetic changes, which amount essentially to changing $\ell$'s into $m$'s, $\phi$'s into $\theta$'s, and $\Psi$'s into $\psi$'s, and allowing for an explicit time dependence, the last two expressions are precisely Eqs. (3.25) and (3.26) of Bizarro [3].

So, the rotational Wigner function, rigourously derived from a set of uniqueness conditions and thoroughly studied by Bizarro [3], is indeed the natural quasiprobability distribution for the pair rotation angle and angular momentum, being thus the appropriate tool for carrying out the quantum tomographic reconstruction of vortex states as proposed by Rigas *et al.* [1]. Therefore, it is not true that "the construction of a proper Wigner function for [vortex states] is still an open question," as stated by Rigas *et al.* [1]: such a problem has been solved for more than a decade.

Incidentally, notice that the rotational Wigner function has been successfully employed by Piovella *et al.* [4] to obtain a quantum model for the free electron laser in terms of a Wigner distribution for the electron beam. It is worth mentioning still that the same function has been independently derived in signal processing by Morris and Wu [5] as a Wigner distribution suitable for time-frequency analysis of discrete-time signals, and that it has been applied as such by Bizarro and co-workers [6] to process data from nuclear-fusion experiments.

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[1] I. Rigas, L. L. Sánchez-Soto, A. B. Klimov, J. Řeháček, and Z. Hradil, Phys. Rev. A **78**, 060101(R) (2008).
[2] N. Mukunda, Am. J. Phys. **47**, 182 (1979).
[3] J. P. Bizarro, Phys. Rev. A **49**, 3255 (1994); **71**, 069901(E) (2005). *Note:* J. P. S. Bizarro appears as J. P. Bizarro in some earlier papers.
[4] N. Piovella, M. M. Cola, L. Volpe, R. Gaiba, A. Schiavi, and R. Bonifacio, Opt. Commun. **274**, 347 (2007); N. Piovella, M. M. Cola, L. Volpe, A. Schiavi, and R. Bonifacio, Phys. Rev. Lett. **100**, 044801 (2008).
[5] J. M. Morris and D. Wu, IEEE Trans. Signal Process. **44**, 1355 (1996).
[6] A. C. Figueiredo and J. P. S. Bizarro, IEEE Trans. Plasma Sci. **30**, 54 (2002); J. P. S. Bizarro, A. C. A. Figueiredo, and JET EFDA Contributors, Nucl. Fusion **46**, 645 (2006).

 