Optical precursors with self-induced transparency

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Optical Sommerfeld-Brillouin precursors significantly ahead of a main field of comparable amplitude have been recently observed in an opaque medium with an electromagnetically induced transparency window [Wei *et al.*, Phys. Rev. Lett. **103**, 093602 (2009)]. We theoretically analyze in this article the somewhat similar results obtained when the transparency is induced by the propagating field itself and we establish an approximate analytic expression of the time delay of the main-field arrival, which fits fairly well the result obtained by numerically solving the Maxwell-Bloch equations.

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More than one century ago, Sommerfeld examined the apparent inconsistency between the existence of superluminal group velocities and the theory of relativity. Considering an incident field switched on at time $t = 0$ (step pulse), he showed that, no matter the value of the group velocity, no field can be transmitted by a linear dispersive medium before the instant $t = L/c$, where L is the medium thickness and c the velocity of light in vacuum [1]. Subsequently, he and Brillouin studied the fast oscillatory transients appearing at $t \geqslant L/c$ in the particular case of a single-resonance Lorentz medium [2,3]. They named them "forerunners" insofar as, in proper conditions, they can distinctly precede the establishment of the steady-state field (the main field). Renamed "optical precursors," forerunners have entered classical textbooks [4,5] and continue to raise considerable interest. The theoretical results of Sommerfeld and Brillouin have been improved, even rectified (in particular, the amplitude of the precursors was strongly underestimated in their work), and different models of linear dispersive media have been considered. See [6] for a recent review.

Despite the abundant literature on precursors, there are very few articles reporting direct observation of precursors distinguishable from the main field. The difficulty of such an observation has been soundly discussed by Aaviksoo *et al.* [7], who performed in 1991 an experiment involving single-sided exponential pulses (instead of step pulses) and exploiting the dispersion originating from a narrow exciton line in AsGa [8]. For proper detuning of the optical carrier frequency ω_c from the resonance frequency ω_0, optical precursors appear as a small spike superimposed on the main pulse (see also [9,10]). The observation of precursors significantly ahead of a main field of comparable amplitude obviously requires the use of long-enough square pulses and a medium fairly transparent at the optical carrier frequency, the corresponding group delay being long compared to the duration of the precursors. As discussed in [11,12], the latter conditions are met in an opaque medium with a narrow transparency window (slow-light medium). Such an experiment has been recently conducted by Wei *et al.* [13] in an opaque cloud of cold atoms with an electromagnetically induced transparency (EIT) window. Note that, in this experiment (as in all the studies of precursors), the propagating field interacts linearly with the medium.

For comparison, we will examine here the nonlinear situation where *the medium transparency is induced by the propagating field itself* [14,15]. Figure 1 shows the result of an experiment performed in such conditions [16]. The medium is $HC^{15}N$ gas at low pressure contained in a 182-m-long oversized waveguide and the incident wave is on resonance with the molecular rotational line $J = 0$, $M = 0 \rightarrow J = 1$, $M = 0$ (wavelength $\lambda_c \approx 3.5$ mm). The gas behaves as a two-level medium [17] characterized by T_1 (T_2), the relaxation time for the population difference (the polarization); T_2^*, the Doppler time; and α, the resonant absorption coefficient at low intensity (extrapolated from the Lorentzian wings of the line). See [18] for details. The incident wave is characterized by I_0, its intensity normalized to the saturation intensity; and τ_r, its rise time. The observed step responses clearly have some similarities with those obtained in the EIT experiment [13], with a short transient preceding the establishment of a steady-state regime (main field). The quasi-Rabi oscillations [15] accompanying the latter are obviously absent in the EIT experiments but oscillations having a linear origin (postcursors) can also be observed in this case [12].

To analyze the previous results, we provisionally neglect the Doppler broadening and assimilate the guided wave to a plane wave propagating in the z direction ($0 < z < L$), with an electric field polarized in the x direction. As long as τ_r, T_1, $T_2 \gg 1/\omega_c$ and $\alpha \ll \omega_c/c$, the slowly varying envelope approximation (SVEA) [17] holds [19] and we write the E_x component of the electric field as

$$E_x(z,t) = \mathrm{Re}\,[e^{i\omega_c t}\widetilde{E}(z,t)], \quad (1)$$

where, as in all the following, t is a *local time* (real time minus z/c), and $\widetilde{E}(z,t)$ is the slowly varying field envelope. Denoting μ the dipole matrix element for the transition (chosen real), $R(z,t) = \mu\widetilde{E}(z,t)/\hbar$ the Rabi frequency, $n(z,t)$ the population difference per volume unit (n_0 its value at equilibrium), and $\widetilde{P}(z,t)$ the envelope of the electric polarization induced in the medium, it is convenient to introduce the dimensionless quantities $D = n/n_0$, $P = \frac{i\widetilde{P}}{n_0\mu}\sqrt{\frac{T_1}{T_2}}$ and $E = \mu\widetilde{E}\sqrt{T_1T_2}/\hbar = R\sqrt{T_1T_2}$, all real in the resonant case. $I = E^2$ is the intensity normalized to the saturation intensity. The Maxwell-Bloch (MB) equations governing the system evolution take then the

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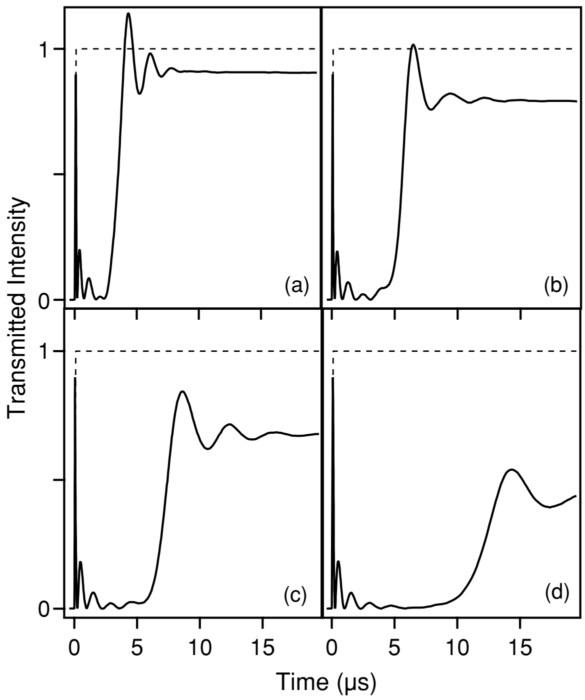

FIG. 1. Observed step response of a resonant absorbing medium. Parameters: $\alpha L \approx 200$, $T_1 \approx T_2 \approx 10\,\mu$s ($T_2/\alpha L \approx 50$ ns), $T_2^* \approx 1.3\,\mu$s, $\omega_c = 5.4 \times 10^{11}\,\mathrm{s}^{-1}$ ($\omega_c^{-1} \approx 1.8$ ps), $\tau_r = 12$ ns; $I_0 \approx$ (a) 2100, (b) 960, (c) 620, and (d) 350. In each case, the intensity is normalized to that of the step transmitted in the absence of gas (dashed line).

simple form

$$\frac{\partial E}{\partial z} = -\frac{\alpha}{2} P \tag{2}$$

$$T_2 \frac{\partial P}{\partial t} = DE - P \tag{3}$$

$$T_1 \frac{\partial D}{\partial t} = -PE + (1 - D). \tag{4}$$

We assume that the rise time τ_r of the incident intensity, while long compared to $1/\omega_c$ (as previously mentioned), is short with respect to all the other characteristic times of the system ($1/R$, T_1, T_2, and $T_2/\alpha L$). The response $E(L,t)$ of the medium (with a time resolution equal to τ_r) is then obtained by solving the MB equations with $P(z,0) = 0$, $D(z,0) = 1$, and $E(0,t) = E_0\Theta(t) = \sqrt{I_0}\Theta(t)$, where $\Theta(t)$ is the unit step function. This problem has been examined by Crisp [15] when the relaxation effects are negligible, a condition obviously not met in the experiments.

The long-term behavior of the step response ($t \gg T_1, T_2$) is obtained by solving the MB equations in steady state. Combining Eqs. (3) and (4), we find $P = E/(1 + E^2)$ and, putting this result in Eq. (2), we easily retrieve the transmission equation [20–22]

$$I(\infty) + \ln I(\infty) = I_0 + \ln I_0 - \alpha L, \tag{5}$$

where $I(t)$ is a short-hand notation of the transmitted intensity $I(L,t)$. The medium being optically thick in the linear regime ($\alpha L \gg 1$), the absorption is fully saturated [$I(\infty)/I_0 \approx 1$] only when the incident (normalized) intensity is extremely large ($I_0 \gg \alpha L$). In fact, the transmitted field (main field) will be significant (partial transparency) as soon as $I_0 - \alpha L = O(\alpha L)$. The transmission equation takes then the approximate form $I(\infty)/I_0 \approx 1 - \alpha L/I_0$ and a transmission $I(\infty)/I_0 > 1/3$ is obtained for $I_0 > 3\alpha L/2$.

Consider now the short-term behavior of the step response. By combining the integral form of Eqs. (3) and (4) and taking into account that $D(z,t) \leqslant 1$, one can establish the inequality [23]

$$1 - D(z,t) < \left| \int_0^t R(z,t')dt' \right|^2 < R_0^2 t^2, \tag{6}$$

where R_0 is the Rabi frequency associated with the incident step ($R_0^2 = \frac{I_0}{T_1 T_2}$). When $R_0^2 t^2 \ll 1$, $D(z,t) \approx 1$ and the MB equations are reduced to the couple of linear equations $\partial E/\partial z = -\alpha P/2$ and $T_2 \partial P/\partial t = E - P$. So, at least in this time domain and though $I_0 \gg 1$, the medium behaves as a linear system (small pulse-area approximation [23]). Its response $E(L,t)$ is easily retrieved from the previous couple of equations and can be written as [24,25]

$$E(L,t) = E_0\Theta(t)\left[1 - \alpha L \int_0^{t/T_2} \frac{\mathrm{J}_1(\sqrt{2\alpha L u})}{\sqrt{2\alpha L u}} \mathrm{e}^{-u} du\right]. \tag{7}$$

When $\alpha L \gg 1$, the integral can be transformed to obtain

$$E(L,t) \approx E_0\Theta(t)\mathrm{e}^{-t/T_2}\mathrm{J}_0(\sqrt{2\alpha L t/T_2}). \tag{8}$$

For $x > 1$, $\mathrm{J}_0(x) \approx \sqrt{\frac{2}{\pi x}}\cos(x - \frac{\pi}{4})$ and $E(L, t > \frac{T_2}{2\alpha L}) \approx E_+(t) + E_-(t)$, where

$$E_\pm(t) = \frac{E_0}{\sqrt{2\pi}} \mathrm{e}^{-t/T_2} \frac{\exp[\pm i(\sqrt{2\alpha L t/T_2} - \pi/4)]}{(2\alpha L t/T_2)^{1/4}}. \tag{9}$$

So the optical field is made of two components of equal amplitude and instantaneous frequency $\omega_c \pm \sqrt{\frac{\alpha L}{2tT_2}}$, which are nothing other than the Sommerfeld (E_+) and Brillouin (E_-) precursors as determined by the saddle-point method of integration [12,26,27]. The linear character of the short-term response (and thus its analysis in terms of precursors) is well supported by the experiments. As shown Fig. 1, the shape of the corresponding transient is roughly independent of the incident intensity. By numerically solving the MB equations, we find that the condition $R_0^2 t^2 \ll 1$ is much too severe and that the linear approximation satisfactorily holds up to $t = 2\pi/R_0$, the Rabi period of the incident field. It even holds later in the experiments because the transversal inhomogeneity of the field partially washes out the (nonlinear) quasi-Rabi oscillations while it does not affect the linear response (the precursors).

In the EIT experiments, the probe field interacts linearly with the medium at every time and the arrival of the main field is determined by the (slow) group velocity [12]. In the present case, this arrival is fixed by fully nonlinear phenomena, the study of which requires the resolution of the complete MB equations. We first examine the solution obtained in the rate equations approximation (REA) [17]. The equations to solve are then reduced to $\partial I/\partial z = -\alpha ID$ and $T_1 \partial D/\partial t = -D(1 + I) + 1$ [20,22], with $D(z,0) = 1$ and $I(0,t) = I_0\Theta(t)$. Eliminating D and integrating in z, we

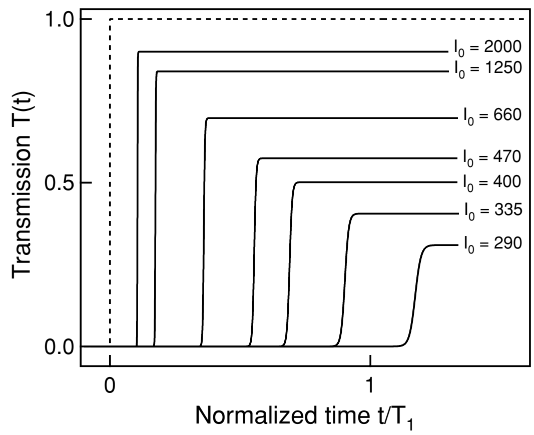

FIG. 2. Step response obtained in the rate equations approximation (REA) as a function of the normalized time t/T_1. Optical thickness $\alpha L = 200$. Each step response is labeled by the corresponding incident intensity I_0. The step response obtained for $I_0 \to \infty$ is given for reference (dashed line).

get [22]

$$T_1 d(\ln I)/dt = \ln I_0 + I_0 - \alpha L - \ln I - I, \tag{10}$$

with $I(0) = I_0 \exp(-\alpha L)$. The transmitted intensity $I(t)$ is finally given by the implicit equation

$$\frac{t}{T_1} = \int_{I_0 \exp(-\alpha L)}^{I(t)} \frac{dI'}{I'(\ln I_0 + I_0 - \alpha L - \ln I' - I')}. \tag{11}$$

The transmission $T(t) = I(t)/I_0$ monotonously increases from $\exp(-\alpha L)$ to $I(\infty)/I_0$, where $I(\infty)$ is given by Eq. (5). In the conditions considered here [$\alpha L \gg 1$, $I_0 - \alpha L = O(\alpha L)$], $T(0) \approx 0$, $T(\infty) \approx 1 - \alpha L/I_0$, and the transition between these two values is very steep (Fig. 2). The time delay of the arrival of the main intensity is conveniently defined as the time τ_d such that $I(\tau_d) = I(\infty)/2$. It is given by Eq. (11) by taking $I(\infty)/2$ as upper limit of integration. When $I_0 \gg \alpha L$ (full saturation limit), Eq. (11) can be explicitly integrated to give $T(t) \approx [1 + \exp(\alpha L - I_0 t/T_1)]^{-1}$ in agreement with the result given in [28]. The 10%–90% rise time Δt of the intensity and the time delay τ_d then read as $\Delta t \approx 4 \ln 3\,(T_1/I_0)$ and $\tau_d \approx \alpha L(T_1/I_0) \ll T_1$. When the saturation is only partial, the time delay τ_d as a function of $1/I_0$ increases much faster than $\alpha L T_1/I_0$ and values of the order of T_1 can be attained while keeping a significant transmission (Fig. 2).

The REA does not take into account the coherent effects. It eliminates in particular the quasi-Rabi oscillations accompanying the main field. One may, however, expect that the signals obtained in this way are a satisfactory approximation of the exact signals, the oscillatory parts of which would have been filtered out. To check this idea, we have compared, αL and I_0 being fixed, the step response obtained by using the REA (independent of T_2) to those obtained by numerically solving the MB equations for two different values of T_2/T_1 (Fig. 3).

The three step responses are obviously different but the time delays τ_d (as defined earlier in this article) are very close. Similar simulations made for different values of the parameters show that this result is not accidental. It appears that Eq. (11) provides the exact time delay with a precision better than 10% in all the cases of physical interest, that is, when the precursors are well developed before the arrival of the main field and the latter has a significant amplitude.

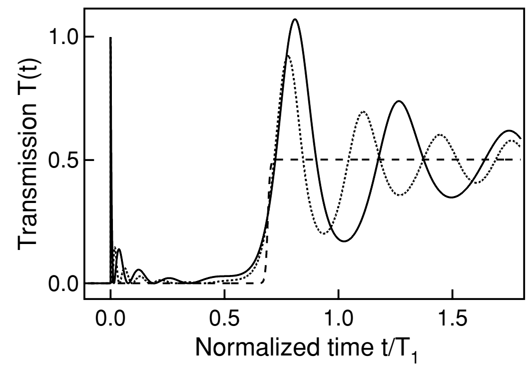

FIG. 3. Comparison of the step response obtained by the REA (dashed line) to those obtained by numerically solving the Maxwell-Bloch (MB) equations with $T_2/T_1 = 1$ (solid line) and with $T_2/T_1 = 1/2$ (dotted line). Other parameters: $\alpha L = 200$ and $I_0 = 400$, leading to $T(\infty) \approx 1/2$. Note that the pseudoperiod of the oscillations superimposed to the steady state in the MB solutions are nearly equal to the corresponding Rabi period $2\pi/R_\infty = 2\pi\sqrt{T_1 T_2/I(\infty)}$, namely $0.44T_1$ ($0.31T_1$) for $T_2/T_1 = 1$ ($T_2/T_1 = 1/2$).

We will now examine the modifications brought to the step response by some effects neglected in the previous theoretical analysis. The most important one results from the transverse inhomogeneity of the guided wave. Figure 4 shows a typical step response obtained by using a MB numerical code extended to include a transverse variation of the field [29]. As expected, the linear part of the response (precursors) is not changed (it is even slightly prolonged) but the quasi-Rabi oscillations (strongly depending on the field amplitude) are dramatically affected. Their amplitude is considerably reduced and their damping is accelerated, in agreement with the experimental result (Fig. 1). However, we remark that the time delay τ_d is not significantly larger than that obtained in the plane-wave and rate-equations approximations. Similar calculations including the Doppler broadening instead of the field inhomogeneity in the plane-wave MB numerical

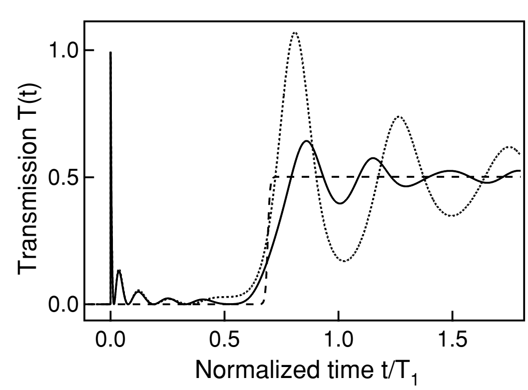

FIG. 4. Numerical solution of the MB equations taking into account the transverse distribution of the field (solid line) for $\alpha L = 200$, $I_0 = 400$, and $T_2 = T_1$. The REA (dashed line) and MB (dotted line) solutions obtained with the plane-wave model are given for reference.

code show that, even when $T_2^* = 0.13T_2$ (parameters of Fig. 1), the Doppler effect negligibly affects the precursors and slightly reduces the time delay τ_d. This can be explained by observing that the right time scale for the precursors and the nonlinear response is not T_2 but, respectively, $T_2/\alpha L \ll T_2^*$ and $1/R_0 < T_2^*$. Finally, the finite rise time of the incident step essentially affects the most rapidly varying part of the step response, namely the transient associated with the precursors and first the intensity I_1 of its first peak. When $\alpha L \gg 1$, I_1 only depends on $r = \alpha L\tau_r/T_2$ and attains the intensity I_0 of the incident wave when $r \ll 1$. This condition is approximately met in the experiment reported in [16], where $I_1 \approx 0.9I_0$ (Fig. 1). Similar results could be obtained at optical wavelength by propagating a Gaussian beam in an ensemble of laser-cooled two-level atoms. We have then $T_2^* \gg T_2$ and the Doppler effect negligibly affects the precursors and the quasi-Rabi oscillations. In other respects, T_2 (typically 50 ns) is about 200 times shorter than in the microwave experiment. For a good observation of the precursors, the rise time of the incident step should also be 200 times shorter, namely in the 50-ps range (attained with electro-optic modulators).

To summarize, we have shown that the experiments involving self-induced transparency are a good alternative to the EIT experiments in order to observe optical precursors well ahead of the main field, both having intensities comparable to that of the step-modulated incident wave. By using a plane-wave model and the REA, we have established an analytical expression for the time delay of the main-field arrival, which generalizes that previously obtained in the infinite saturation limit, and we have shown that this expression provides a good estimate of the real time delay as long as precursors and main field are well separated and of significant amplitude.

[1] A. Sommerfeld, Phys. Z. **8**, 841 (1907).
[2] A. Sommerfeld, Ann. Phys. (Leipzig) **44**, 177 (1914).
[3] L. Brillouin, Ann. Phys. (Leipzig) **44**, 204 (1914).
[4] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).
[5] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed.
[6] K. E. Oughstun, *Electromagnetic and Optical Pulse Propagation 1* (Springer, Berlin, 2007), Chap. 1.
[7] J. Aaviksoo, J. Lippman, and J. Kuhl, J. Opt. Soc. Am. B **5**, 1631 (1988).
[8] J. Aaviksoo, J. Kuhl, and K. Ploog, Phys. Rev. A **44**, R5353 (1991).
[9] H. Jeong, A. M. C. Dawes, and D. J. Gauthier, Phys. Rev. Lett. **96**, 143901 (2006).
[10] S. Du, C. Belthangady, P. Kolchin, G. Y. Yin, and S. E. Harris, Opt. Lett. **33**, 2149 (2008).
[11] H. Jeong and S. Du, Phys. Rev. A **79**, 011802(R) (2009).
[12] B. Macke and B. Ségard, Phys. Rev. A **80**, 011803(R) (2009).
[13] Dong Wei, J. F. Chen, M. M. T. Loy, G. K. L. Wong, and S. Du, Phys. Rev. Lett. **103**, 093602 (2009).
[14] S. L. McCall and E. L. Hahn, Phys. Rev. **183**, 457 (1969).
[15] M. D. Crisp, Phys. Rev. A **5**, 1365 (1972).
[16] B. Ségard, B. Macke, J. Zemmouri, and W. Sergent, Ann. Phys. (Paris), Colloque No. 1, Supplément au No. 2, Vol. **15** (1990), p. 167.
[17] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover, New York, 1987).
[18] B. Ségard, B. Macke, L. A. Lugiato, F. Prati, and M. Brambilla, Phys. Rev. A **39**, 703 (1989).
[19] These conditions are met in all the experiments having led to a direct observation of precursors.
[20] A. Selden, Br. J. Appl. Phys. **18**, 743 (1967).
[21] L. W. Hillman, R. W. Boyd, J. Krasinski, and C. R. Stroud, Opt. Commun. **45**, 416 (1983).
[22] B. Macke and B. Ségard, Phys. Rev. A **78**, 013817 (2008).
[23] M. D. Crisp, Phys. Rev. A **1**, 1604 (1970).
[24] A. Laubereau and W. Kaiser, Rev. Mod. Phys. **50**, 607 (1978).
[25] B. Ségard, J. Zemmouri, and B. Macke, Europhys. Lett. **4**, 47 (1987).
[26] W. R. LeFew, S. Venakides, and D. J. Gauthier, Phys. Rev. A **79**, 063842 (2009).
[27] This asymptotic method of integration is sometimes opposed to the SVEA. In fact, it can pertinently be used in the frame of the latter [12].
[28] P. G. Kryukov and V. S. Letokhov, Sov. Phys. Usp. **12**, 641 (1970).
[29] E. M. Pessina, B. Ségard, and B. Macke, Opt. Commun. **81**, 397 (1991).