Symmetry for the nonadiabatic transition in Floquet states

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The frequency of the Rabi oscillation driven by a periodic external field varies with the parameters of the external field, e.g., frequency and amplitude, and it becomes zero at some points of the parameters, which is called coherent destruction of tunneling. This phenomenon is understood as a degeneracy of the Floquet quasienergies as a function of the parameters. We prove that the time-reversal symmetry of the external field is a necessary condition of the degeneracy. We demonstrate the gap opening in the quasienergy spectrum in asymmetrically periodically driven systems. Moreover, an adiabatic transition of the Floquet states is demonstrated and analyzed in the analogy to the Landau-Zener transition.

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I. INTRODUCTION

The coherent dynamics of quantum states under a time-dependent external field is an important subject in establishing methods for manipulation of quantum systems. Various mechanisms have been developed in both experimental and theoretical physics, as well as in chemistry and engineering. These are also fundamental ingredients for quantum information processing [1].

In particular, driven two-level systems have been studied for a long time [2–13]. A combination of the energy gap at the avoided crossing point and the periodic external field causes a kind of Rabi oscillation. Its mechanism has also been investigated in the context of the quantum chaos. Grossmann *et al.* found that the quantum tunneling in double-well potentials was destroyed at a certain oscillating external field. They called this phenomenon coherent destruction of tunneling (CDT) [14,15]. The mechanism of CDT is understood as a quantum interference at periodic avoided level crossings [16]. Dunlap and Kenkre have found that a long time average of the wave packet neither moves nor spreads in the infinite tight binding system when the system is derived in a certain frequency. This quenching effect is called dynamical localization (DL) [3,17]. Recently, the relationship between CDT and DL was clarified by Kayanuma and Saito [18]. Various examples have been pointed out, e.g., a resonance of the magnetization in nanoscale uniaxial magnets, superfluid-insulator transition in an optical lattice [19], and the Landau-Zener interference of a charge qubit based on the Cooper-pair box [20]. So far, mainly the cases with the sinusoidal field which is symmetric in a period have been studied, although some cases with asymmetric fields have also been studied, e.g., "multiphoton transition" [21,22] and "the Zamboni effect" [23]. Moreover, recently, dependence of CDT on the symmetry of the system has been studied for tunneling in a driven double-well potential achieved by an optical light shift potential [24].

These phenomena can be interpreted as a result of the destructive interference in repeated Landau-Zener-Stückelberg transitions, and its mechanism has been studied in terms of a Floquet dynamics [25–27]. This is a universal phenomenon in effective two-level systems driven by periodic external fields. The CDT corresponds to nontrivial degeneracy of Floquet quasienergies. In general, the symmetries of the system cause conserved quantities and also degeneracy of the eigenstates. Therefore, we expect some symmetry for such degeneracy of Floquet states.

In the present work, we study conditions for the nontrivial degeneracy of the Floquet quasienergy, and we prove that the time-reversal symmetry of the external field is a necessary condition for the degeneracy. According to this mechanics, we demonstrate the gap opening in the quasienergy spectrum of the transverse Ising model driven by an asymmetrically periodically external field. Moreover, we demonstrate a sweeping velocity dependence of the nonadiabatic transitions between Floquet states at the avoided crossing point of the quasienergy levels of a Floquet operator in analogy to the Landau-Zener transition [28–30].

II. FLOQUET THEORY AND THE COHERENT DESTRUCTION OF TUNNELING

It is convenient to describe the dynamics of periodically driven systems in terms of the Floquet theorem [25]. In solid-state physics, the analogous result is known as the Bloch's theorem [31]. We shortly review the Floquet theory in this section. The time evolution in the quantum systems is described by the time-dependent Schrödinger's equation

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = \mathcal{H}(t)|\Psi(t)\rangle, \tag{1}$$

where we set $\hbar = 1$. A formal solution of this equation is

$$|\Psi(t)\rangle = \mathcal{T}\exp\left[-i\int_{t_0}^{t}\mathcal{H}(\tau)d\tau\right]|\Psi(t_0)\rangle, \tag{2}$$

where $\mathcal{T}$ denotes the time-ordering operator. A time-evolution operator is defined as

$$\mathcal{U}(t,t_0) = \mathcal{T}\exp\left[-i\int_{t_0}^{t}\mathcal{H}(\tau)\,d\tau\right]. \tag{3}$$

When the Hamiltonian of the system is periodic with a period T, that is, $\mathcal{H}(t+T) = \mathcal{H}(t)$, the time evolution for one period is given by

$$\mathcal{F} \equiv \mathcal{U}(T,0) = \mathcal{T}\exp\left[-i\int_{0}^{T}\mathcal{H}(\tau)\,d\tau\right]. \tag{4}$$

This $\mathcal{F}$ is called the Floquet operator. Eigenvalues of the Floquet operator are given in form

$$\mathcal{F}\,|\alpha\rangle = \exp[i\varepsilon_\alpha]|\alpha\rangle, \tag{5}$$

where $|\alpha\rangle$ is an eigenvector of the Floquet operator and $\exp[i\varepsilon_\alpha]$ is its eigenvalue. Here, ε_α are defined in modulo 2π and is called a quasienergy. This quasienergy is sometimes related to the nonadiabatic geometrical phase (the Aharonov-Anandan phase) [3,32,33]. Since the set of $\{|\alpha\rangle\}$ is the complete orthonormal set, we can expand any state using $\{|\alpha\rangle\}$:

$$|\Psi\rangle = \sum_\alpha |\alpha\rangle\,\langle\alpha|\,\Psi\rangle = \sum_\alpha c_\alpha|\alpha\rangle, \tag{6}$$

where $c_\alpha = \langle\alpha\,|\,\Psi\rangle$. The expectation value of a physical quantity $\mathcal{O}$ after n periods ($|\Psi_n\rangle = \mathcal{F}^n\,|\Psi_0\rangle$) is given by

$$\langle\Psi_n|\,\mathcal{O}\,|\Psi_n\rangle = \sum_{\alpha,\beta} e^{-i(\varepsilon_\alpha-\varepsilon_\beta)n} c_\beta c_\alpha^*\,\langle\alpha|\,\mathcal{O}|\beta\rangle. \tag{7}$$

If some of eigenvalues degenerate, say $\varepsilon_\alpha = \varepsilon_\beta$, then the expectation value $\langle\Psi_n|\,\mathcal{O}\,|\Psi_n\rangle$ has time (n) independent components. The CDT can be regarded as an example of this degeneracy.

III. ENERGY LEVEL STRUCTURE AND QUASIENERGY LEVEL STRUCTURE OF A SIMPLE MODEL

The CDT has been demonstrated in models which possess an avoided level crossing structure in their energy level as a function of an external field. The CDT can be interpreted as a degeneracy of the Floquet eigenstate, and it takes place not only in cases with fast frequency but also in those with slow frequency. This situation was demonstrated in the following model [26]:

$$\mathcal{H}_0 = -J\sum_{\langle i,j\rangle}\sigma_i^z\sigma_j^z - \Gamma\sum_j^L\sigma_j^x - H\,(t)\sum_j^L\sigma_j^z, \tag{8}$$

where $H(t) = h_0\cos(\omega t)$, and σ^α ($\alpha = x, y, z$) denote the Pauli matrices. In Fig. 1(a), we present the energy level structure of the model as a function of $H(t) = H_z$. Here, we adopt the parameters $J = 4$ and $\Gamma = 2$, and the system size is $L = 4$. Hereafter we fix $J = 4$ and measure the energy in this unit. We find the avoided crossing near $H_z = 0$ between the lowest level and the first excited level. In Fig. 1(b), we depict the quasienergies of this model with $\omega = 0.2$ as a function of the amplitude of the external field h_0.

Now let us study how the ground state $|G\rangle$ of the system (8) with $H(0) = h_0$ is expressed by the eigenstates of the Floquet operator:

$$|G\rangle = \sum_\alpha c_\alpha|\alpha\rangle. \tag{9}$$

In the present paper, we call $|c_\alpha|^2$ the α population of $|G\rangle$. It is noted that when $h_0 = 0$, the eigenstates of the Floquet operator coincide with those of the Hamiltonian in modulo 2π. It is found that as long as h_0 is not large ($h_0 \leqslant 0.4$), only two states give significant contribution. That is, only two populations have large values of $|c_\alpha|$. We find a ribbon structure of quasienergies of the dominant Floquet modes, which are shown by thick curves. Throughout the ribbon structure, each population is found to be almost constant. This structure was reported also by Grossmann *et al.* [15]. In the ribbon structure, the quasienergies of dominant modes degenerate at some points. A similar sequence of degeneracy was found in the energy spectra of uniaxial magnets as a function of the external transverse field, and we found a symmetry which characterizes the degeneracy [34–36]. We expect that the degeneracy of quasienergies in the present model also reflects some symmetry.

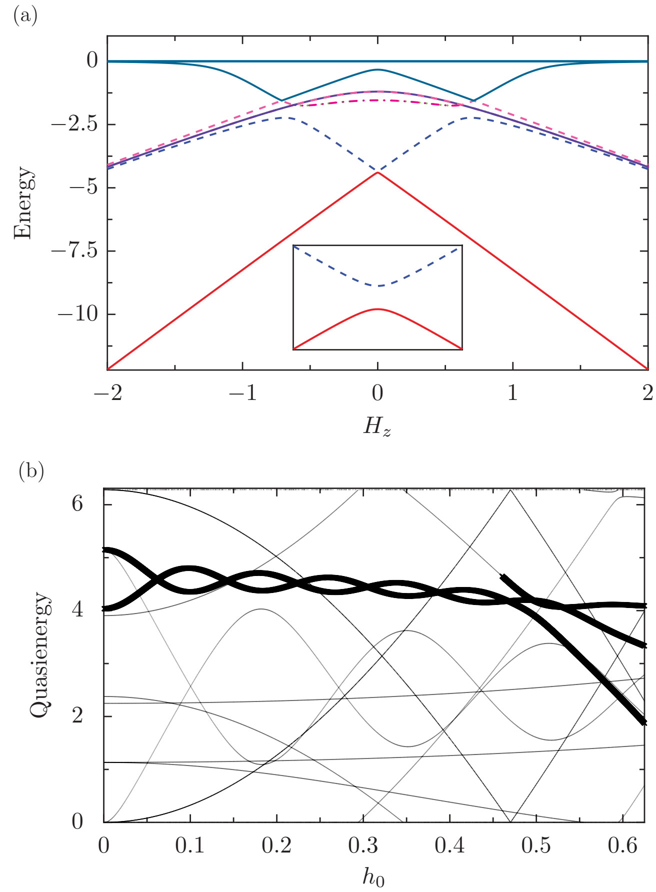

FIG. 1. (Color online) (a) Energy spectrum of the model Eq. (8) with $H(t) = H_z$. Inset: Two lowest levels near $H_z = 0$. (b) Quasienergies of dominant Floquet states ($|c_\alpha|^2 > 0.01$) plotted as thick curves; other quasienergies are plotted as thin curves.

IV. LIFT OF THE DEGENERACY OF FLOQUET QUASIENERGY

In order to resolve the symmetry for the degeneracy, we study the simplest case of a two-level system

$$\mathcal{H}\,(t) = -\Gamma\sigma_x - H\,(t)\,\sigma_z, \tag{10}$$

with

$$H(t) = \begin{cases} h_0\cos(\omega_1 t) & 0 \leqslant t < T_1, \\ -h_0\cos[\omega_2(t - T_1)] & T_1 \leqslant t < T, \end{cases} \tag{11}$$

where $T_1 = \pi/\omega_1$, $T_2 = \pi/\omega_2$, and $T_1 + T_2 = T$. This system has a period T. For the convenience of expression, we define a Hamiltonian

$$\mathcal{H}_0(t,\omega) = -\Gamma\sigma_x - h_z\cos(\omega t)\sigma_z. \tag{12}$$

The Floquet operator is expressed by

$$\mathcal{F} = U(T,0) = U(T,T_1)U(T_1,0), \tag{13}$$

where

$$U(T_1,0) = \mathcal{T}e^{-i\int_0^{T_1}\mathcal{H}_0(\tau,\omega_1)d\tau} \equiv V_1, \tag{14}$$

and

$$\begin{aligned} U(T,T_1) &= \mathcal{T}e^{-i\int_{T_1}^{T}\mathcal{H}(\tau)d\tau} \\ &= e^{-i\pi\sigma_x}\mathcal{T}e^{-i\int_0^{T_2}\mathcal{H}_0(\tau,\omega_2)d\tau}e^{i\pi\sigma_x} \\ &\equiv e^{-i\pi\sigma_x}V_2e^{i\pi\sigma_x}, \end{aligned} \tag{15}$$

where we use the unitary transformation

$$e^{-i\pi\sigma_x}X(\sigma_x,\sigma_y,\sigma_z)e^{i\pi\sigma_x} = X(\sigma_x,-\sigma_y,-\sigma_z). \tag{16}$$

Thus, the Floquet operator is given by

$$\mathcal{F} = e^{-i\pi\sigma_x}V_2e^{i\pi\sigma_x}V_1 = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}. \tag{17}$$

We use the adiabatic states as the basis vectors. $|G\rangle$ is the ground state of $\mathcal{H}_0$ at $t=0$, $|1\rangle$ is the excited state at $t=0$, $|G'\rangle$ is the ground state at $t=T_1$, $|1'\rangle$ is the excited state at $t=T_1$, $|G''\rangle$ is the ground state at $t=T$, and $|1''\rangle$ is the excited state at $t=T$. Because the dynamics is periodic, $|G''\rangle = |G\rangle$ and $|1''\rangle = |1\rangle$. We can the write the time evolution from $t=0$ to $t=T_1$ as

$$\begin{aligned} V_1|G\rangle &= t_{11}^{(1)}|G'\rangle + t_{12}^{(1)}|1'\rangle, \\ V_1|1\rangle &= t_{21}^{(1)}|G' + t_{22}^{(1)}|1'\rangle. \end{aligned} \tag{18}$$

And we can write the time evolution from $t=T_1$ to $t=T$ as

$$\begin{aligned} U(T,T_1)|G'\rangle &= t_{11}^{(2)}|G\rangle + t_{12}^{(2)}|1\rangle, \\ U(T,T_1)|1'\rangle &= t_{21}^{(2)}|G\rangle + t_{22}^{(2)}|1\rangle. \end{aligned} \tag{19}$$

We relate $\{|G\rangle,|1\rangle\}$ and $\{|G'\rangle,|1'\rangle\}$ using a unitary operator, because

$$e^{-i\pi\sigma_x}\mathcal{H}_0(0,\omega_1)e^{i\pi\sigma_x} = \mathcal{H}_0(T_1,\omega_2). \tag{20}$$

So we obtain $|G'\rangle = e^{-i\pi\sigma_x}|G\rangle$, and $|1'\rangle = e^{-i\pi\sigma_x}|1\rangle$. We represent the time evolution, using the basis $|G\rangle$, $|1\rangle$ as

$$\begin{cases} V_1|G\rangle = t_{11}^{(1)}|G'\rangle + t_{12}^{(1)}|1'\rangle \\ \qquad\;\; = t_{11}^{(1)}e^{-i\pi\sigma_x}|G\rangle + t_{12}^{(1)}e^{-i\pi\sigma_x}|1\rangle, \\ V_1|1\rangle = t_{21}^{(1)}|G'\rangle + t_{22}^{(1)}|1'\rangle \\ \qquad\;\; = t_{21}^{(1)}e^{-i\pi\sigma_x}|G\rangle + t_{22}^{(1)}e^{-i\pi\sigma_x}|1\rangle, \end{cases} \tag{21}$$

and similarly for V_2,

$$\begin{cases} V_2|G\rangle = t_{11}^{(2)}e^{i\pi\sigma_x}|G\rangle + t_{12}^{(2)}e^{i\pi\sigma_x}|1\rangle, \\ V_2|1\rangle = t_{21}^{(1)}e^{i\pi\sigma_x}|G\rangle + t_{22}^{(1)}e^{i\pi\sigma_x}|1\rangle. \end{cases} \tag{22}$$

Thus, we can write $V_1 = e^{-i\pi\sigma_x}T^{(1)}$, $V_2 = e^{i\pi\sigma_x}T^{(2)}$, and

$$\mathcal{F} = e^{-i\pi\sigma_x}V_2e^{i\pi\sigma_x}V_1 = T^{(2)}T^{(1)}. \tag{23}$$

Operators $T^{(1)}$ and $T^{(2)}$ given by $\{t_{ij}^{(1,2)}\}$ are unitary, so these must have following forms:

$$\begin{aligned} T^{(j)} &= \exp[i\alpha_j] \\ &\times \begin{pmatrix} \exp[i\theta_j]\sqrt{p_j} & \exp[i\phi_j]\sqrt{1-p_j} \\ \sqrt{1-p_j} & -\exp[i(-\theta_j+\phi_j)]\sqrt{p_j} \end{pmatrix}, \end{aligned} \tag{24}$$

where α_j, θ_j, and ϕ_j are real numbers, and $0 < p_j < 1$ for $j = 1$ and 2. The concrete form of this Floquet operator is given by

$$\begin{aligned} F_{11} &= e^{i(\theta_1+\theta_2)}\sqrt{p_1p_2} + e^{i\phi_2}\sqrt{(1-p_1)(1-p_2)}, \\ F_{12} &= e^{i(\theta_2+\phi_1)}\sqrt{p_2(1-p_1)} - e^{i(\phi_1+\phi_2-\theta_1)}\sqrt{p_1(1-p_2)}, \\ F_{21} &= e^{i\theta_1}\sqrt{p_1(1-p_2)} - e^{i(\phi_2-\theta_2)}\sqrt{p_2(1-p_1)}, \\ F_{22} &= e^{i\phi_1}\sqrt{(1-p_1)(1-p_2)} + e^{i(\phi_1+\phi_2-\theta_1-\theta_2)}\sqrt{p_1p_2}, \end{aligned} \tag{25}$$

with a phase factor $e^{i(\alpha_1+\alpha_2)}$.

We can diagonalize the unitary matrix $\mathcal{F}$ using a unitary matrix M,

$$M^\dagger\mathcal{F}M = \begin{pmatrix} \exp[i\varepsilon_1] & 0 \\ 0 & \exp[i\varepsilon_2] \end{pmatrix}. \tag{26}$$

In the case with degenerate eigenvalues, $\varepsilon_1 = \varepsilon_2$, $M^\dagger\mathcal{F}M$ must have a form $e^{i\varepsilon_1}I$. From this form, we have the following conditions:

$$\exp[i(\theta_1+\theta_2-\phi_2)] = \sqrt{\frac{p_1(1-p_2)}{p_2(1-p_1)}}, \tag{27}$$

and

$$\phi_1 = \phi_2. \tag{28}$$

The right-hand side of Eq. (27) is real and positive, thus

$$\theta_1 + \theta_2 - \phi_2 = 0 \pmod{2\pi}, \tag{29}$$

Moreover, from the fact that the absolute value of the left-hand side of Eq. (27) is unity, we have another condition for the degeneracy:

$$p_1 = p_2. \tag{30}$$

It is noted that the above logic works for the general cases in which $H(t+T) = H(t)$ and $H(T_1) = -H(0)$.

When we estimate p_j $(j = 1, 2)$ by the Landau-Zener formula, transition probabilities are given by a function of the sweeping velocity and of the energy gap at the avoided crossing point ΔE:

$$p_j = 1 - e^{-\pi(\Delta E)^2/(4\omega_j|h_zM_0|)}, \tag{31}$$

where M_0 is the magnetization at $t=0$. When we sweep the field within a finite range, the transition probability p_j is not precisely given by the Landau-Zener formula. However, it is expected that p_j is a single-valued function of v (or frequency). Therefore the condition (30) is equivalent to $\omega_1 = \omega_2$. Thus, we conclude that the time-reversal symmetry $\omega_1 = \omega_2$ is the necessary condition for the degeneracy. This means that the velocity at the avoided crossing point must be the same for the degeneracy. Therefore, for the cases with $\omega_1 \neq \omega_2$, the degeneracy is lifted and the CDT does not occur.

V. DEMONSTRATION OF THE LIFT OF THE DEGENERACY

A. Two-level system

First, we demonstrate the gap opening in an asymmetrically periodically driven model [Eq. (10)] with $\omega_1 = 0.2$ and $\omega_2 = 0.22$. Here, we redefine the range of Floquet quasienergies in two-level systems (10) with (11) from $0 \leqslant \varepsilon \leqslant 2\pi$ to $-\pi \leqslant \varepsilon \leqslant \pi$. Hereafter, we call the state of a negative Floquet quasienergy state "the first Floquet state," and the one with a positive Floquet quasienergy "the second Floquet state." In the case $\omega_1 = \omega_2$, the system has a ribbon structure with a sequence of crossings (not shown) as that in Fig. 1(b) with the bold curve. In Fig. 2(a), we depict the quasienergies as a function h_0, where only a part of one crossing is shown. Now, we study the population $|c_\alpha|^2$ for the ground state of the model at $t = 0$. Here, we find the tunneling frequency has a nonzero minimum value and the CDT does not take place any more. While in the symmetric case, the populations are almost constant (not shown); in the asymmetric case, the populations of the modes smoothly exchange as depicted in Fig. 2(b).

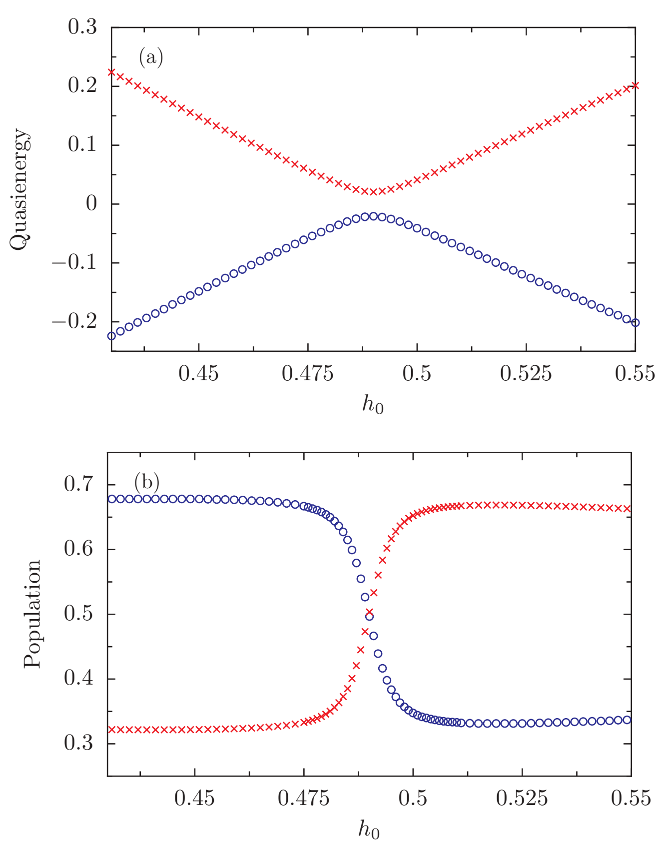

FIG. 2. (Color online) (a) Quasienergies of Floquet states near avoided crossing points. $\Gamma = 0.1$, $\omega_1 = 0.2$, $\omega_2 = 0.22$. The blue symbol ($\circ$) denotes the lower-level quasienergy (for the first Floquet state). The red symbol ($\times$) denotes the upper-level quasienergy (for the second Floquet state). (b) Populations of the ground state of the system at $t = 0$ in the states. Symbols denote data for the same levels as in (a).

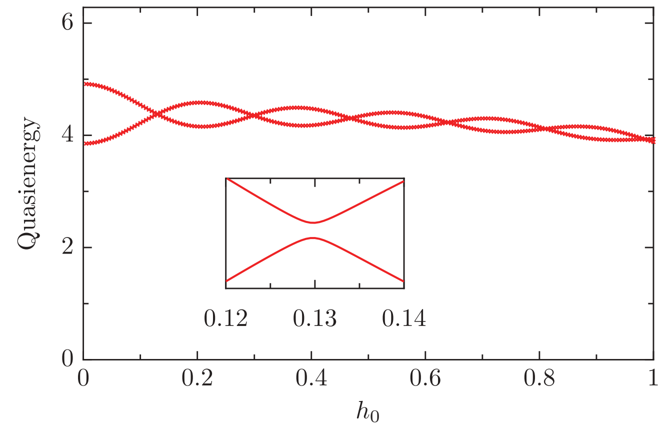

FIG. 3. (Color online) Floquet quasienergies of dominant modes. Inset: Quasienergy near a node of the ribbon structure.

B. Transverse Ising model

Next, we demonstrate the Floquet quasienergy gap opening in the transverse Ising model defined by Eq. (8) with the asymmetric alternating field (11). Here, we use the parameters $J = 4$, $\Gamma = 2$, $\omega_1 = 0.2$, and $\omega_2 = 0.22$.

We show the Floquet quasienergy of dominant modes in Fig. 3. We find ribbonlike structures of Floquet quasienergies similar to those in Fig. 1(b). But in detail, we find gap openings at the crossing points of the ribbon structure (magnified in the inset).

We also show the initial state population of the dominant modes $[|c_\alpha|^2 = |\langle \Psi(t=0)|\alpha(h_0)\rangle|^2]$ in Fig. 4, where we find a kind of ribbon structure. This ribbon structure shows exchanges at the avoided crossing points of the quasienergies, as we see in Fig. 2(b). In the symmetric case, we find the similar structure but without the exchanges. Here, it should be noted that crossings in Fig. 4 simply mean that the two modes have the same populations and do not have any specific meaning.

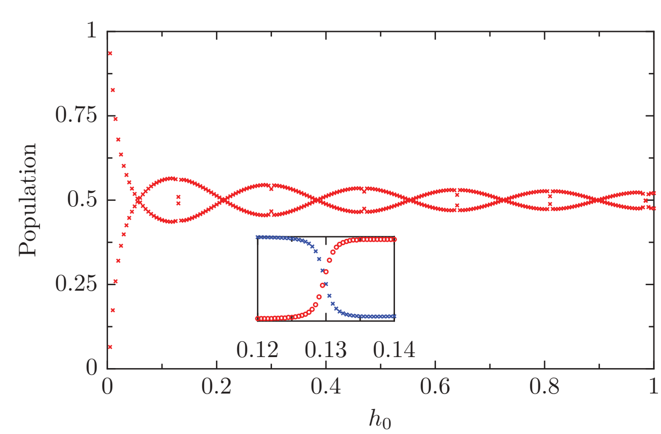

FIG. 4. (Color online) The initial state populations $\{|c_\alpha|^2\}$ of dominant Floquet modes. Inset: Dominant Floquet modes near a node of the ribbon structure of the Floquet quasienergies.

VI. ADIABATIC CHANGE OF FLOQUET STATES

Analogous to the adiabatic change of the eigenstate of the Hamiltonian, when we change h_0 the Floquet state shows a kind of adiabatic change. Such a phenomenon was pointed out as a "multiphoton transition" in laser pulse manipulation [21,22]. The amount of scattering of states at the crossing point under a field sweeping is given by a kind of Landau-Zener-mechanism. We study the transition in the two-level system (10) as a function of the sweeping velocity v of the amplitude of the external field, that is,

$$h_0 \to h_0(t) = h_0(0) + vt. \tag{32}$$

Here, we adopt v much smaller than T^{-1} in order to apply the Floquet description, e.g., $v \sim (10\,000T)^{-1}$.

We study the motion of the magnetization

$$M_{h_0(t)} = \langle\Psi(t)|\sum_j S_j^z|\Psi(t)\rangle, \tag{33}$$

by solving the Schrödinger equation (1) for the Hamiltonian (10) with the time-dependent field (11).

In Fig. 5, the sweep-velocity dependence of $M_{h_0(t)}$ is depicted. There, values of the magnetization at periodic points of time $t = nT$ (n is an integer) are plotted. The initial state was set to be the first of the Floquet eigenstates for $h_0(0) = 0.3$. Other parameters are the same as in the previous section ($\Gamma = 0.1$, $\omega_1 = 0.2$, $\omega_2 = 0.22$). In the figure, the solid curve denotes the value of the magnetization (33) of the Floquet eigenstates for the given value of h_0 which is regarded as the adiabatic Floquet state:

$$\mathcal{F}(h_0)\,|\alpha(h_0)\rangle = e^{i\varepsilon_\alpha}|\alpha(h_0)\rangle. \tag{34}$$

When v is small ($v = 0.000001$), the values of magnetization plotted by (+) are almost equal to that of the adiabatic state, and they are on the solid line for the adiabatic case (hardly seen). On the other hand, when the velocity is increased, the magnetization becomes scattered around the solid curve. In Fig. 5, the data for $v = 0.000002$ and 0.000003 are plotted by green squares and purple triangles, respectively. The amount of the scattering increases when the velocity becomes large. The scatterings indicate mixing of two Floquet states.

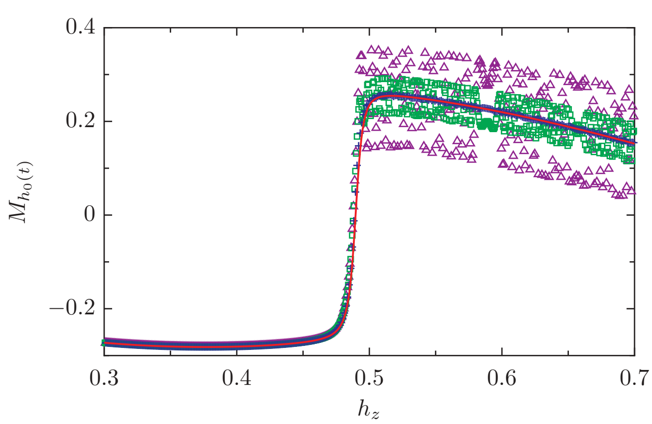

FIG. 5. (Color online) Magnetization under the sweeping, $h_0(t) = h_0 + vt$. The data are plotted at periodic times. The solid red line denotes the adiabatic case, which corresponds to $v = 0$. The blue symbols (+) denote data for $v = 0.000001$, which are almost on the line (hardly seen). The green □ and purple △ symbols denote the data $v = 0.000002$, and $v = 0.000003$, respectively. All points are plotted at $t = nT$, where n is given by multiples of 30 for $v = 0.000001$ ($n = 30m$; m is an integer), $n = 16m$ for $v = 0.000002$, and $n = 10m$ for $v = 0.000003$.

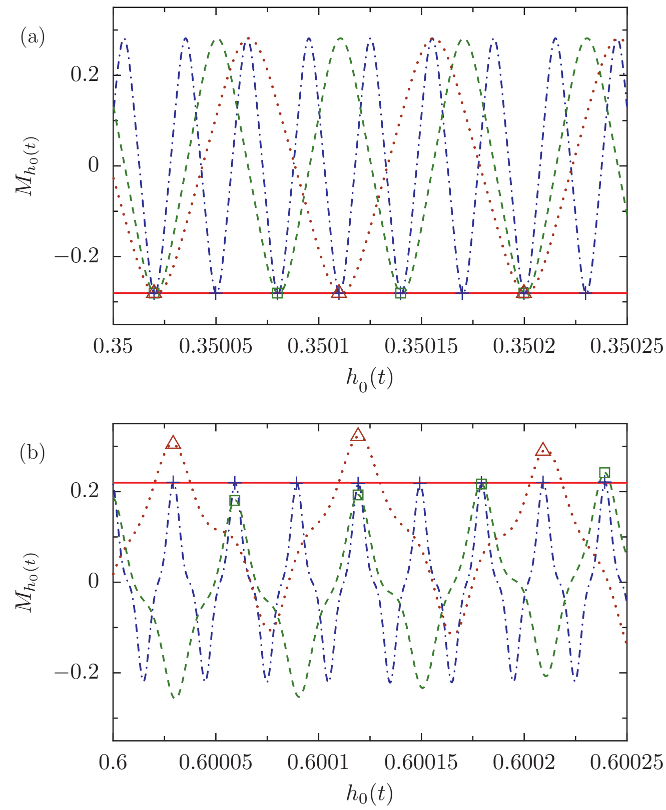

FIG. 6. (Color online) Magnetization $M_{h_0(t)}$ (33) under the sweeping (32). The solid line denotes $M_{h_0(t)}$ of the adiabatic Floquet state (34). The symbols (△) and the red dotted line denote the data for $v = 0.000001$. The symbols (□) and the green dashed line denote the data for $v = 0.000002$. The symbols (+) and the blue dash-dotted line denote the data for $v = 0.000003$. The symbols are plotted at the periodic points ($t = nT$; n is an integer). (a) Motions before the scattering; i.e., $h_0(t) < 0.5$. (b) Motions after the scattering; i.e., $h_0(t) > 0.5$.

Here, it should be noted that in Fig. 5 the values at the periodic points ($t = nT$; n is an integer) are plotted by pluses for $v = 0.000001$ with $n = 30m$, squares for $v = 0.000002$ with $n = 16m$, and triangles for $v = 0.000003$ with $n = 10m$, where $m = 1, 2, 3, \ldots$. In Fig 6, we depict the full time evolution of data in the continuum time t for various sweeping velocities v. Between the points at the periodic points, the magnetization $M_{h_0(t)}$ (33) shows a motion due to the time evolution of the system (1).

The solid line denotes the value of $M_{h_0(t)}$ for the adiabatic Floquet state (34). The dash-dotted, dashed, and dotted lines show $M_{h_0(t)}$s for $v = 0.000001$, $v = 0.000002$, and $v = 0.000003$, respectively. The values at the periodic points ($t = nT$; n is an integer) are plotted by pluses, squares, and triangles. In Fig. 6(a), the evolutions before the scattering (i.e., $h_0(t) < 0.5$) are plotted. The points are on the solid line, which means that the states evolve adiabatically.

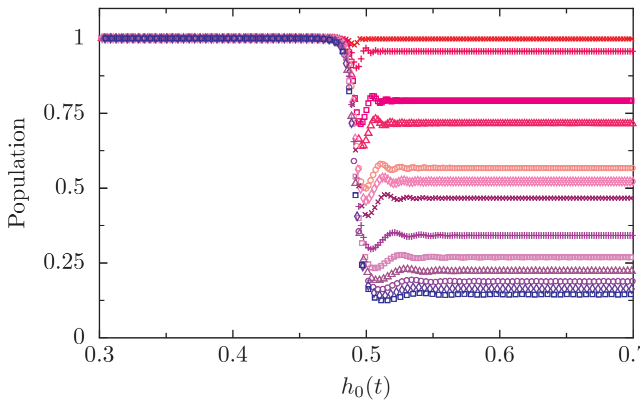

FIG. 7. (Color online) Time evolution of the population $|d_\alpha|^2$ (35) of the adiabatic Floquet state for value $h_0 = h_0(t)$. The value of v changes from 0.000002 to 0.00008 from the top to the bottom.

In Fig. 6(b), we depict the same quantities after the scattering (i.e., $h_0(t) > 0.5$). There, only the state with $v = 0.000001$ evolves almost adiabatically; i.e., the triangles stay at the solid curve. The squares and crosses show deviations from the solid curve, which indicates a nonadiabatic transition takes place at the scattering region $h_0(t) \simeq 0.5$.

Now we study amount of nonadiabatic transition between the Floquet states. We calculate the population of first Floquet states $|\alpha\rangle$ at a time t,

$$|d_\alpha|^2 = |\langle\alpha(h_0)|\ \Psi(t)\rangle|^2 . \tag{35}$$

The time evolution of the populations for various values of v are plotted in Fig. 7. The change of the population of each value of v denotes the amount of nonadiabatic change, which increases with the values of v. We study velocity dependence of the among quantitatively in the analogy of the Landau-Zener theory. That is, in Fig. 8, we plot the logarithm of the amount of scattered populations $\ln(1 - |d_\alpha|^2)$ as a function of $1/v$. We find a good linear dependence, which confirms the Landau-Zener type dependence of the form $\exp[-a/v]$ with a positive constant a.

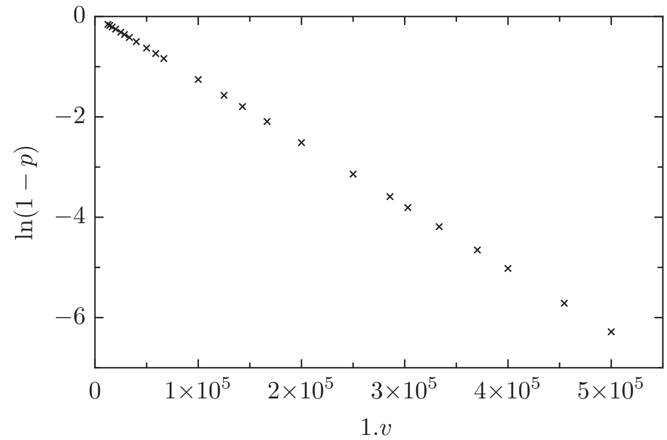

FIG. 8. Landau-Zener type dependence of the scattering of the Floquet state under the sweeping of h_0. We plot $\ln(1 - p)$ vs $1/v$, where $p = |d_\alpha|^2$ denotes the population of the adiabatic Floquet state ($|\alpha\rangle$) at the final time where $h_0 = 0.7$.

VII. SUMMARY AND DISCUSSIONS

In summary, we studied the CDT from the viewpoint of degeneracy of the Floquet quasienergies and the symmetry for that degeneracy. We studied conditions for degeneracy in models with a periodically oscillating field. We obtained explicit conditions for the two-level model (10) with (11), where we found that the CDT takes place only in the symmetric case. We demonstrated a quasienergy gap opening in asymmetric cases. We also studied the relation between the Floquet states and the ground states of the Hamiltonian at the initial value of the periodic external field, and we found a sharp change of the nature of the Floquet state at the quasicrossing point.

We demonstrated that the mechanism generally works for models with the avoided level crossing structure of the energy [e.g., the model (8)] with an asymmetric external field. There, we found that as long as the amplitude of the external field is not large, the ground state of the Hamiltonian at the initial value of the periodic external field consists of two dominant Floquet states.

We also studied a kind of adiabatic change of the Floquet states under a sweeping of the amplitude of the oscillating field and found that the velocity dependence of the population of the adiabatic change is well described by Landau-Zener type dependence.

The properties found in the present work would give basic ideas regarding the manipulation of the quantum dynamics among quantum states by alternating fields such as NMR and light irradiation.

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[1] See, e.g., M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).
[2] J. H. Shirley, Phys. Rev. **138**, B979 (1965).
[3] M. Grifoni and P. Hänggi, Phys. Rep. **304**, 229 (1998).
[4] S. Kohler, J. Lehmann, and P. Hänggi, Phys. Rep. **406**, 379 (2005).
[5] I. I. Rabi, Phys. Rev. **51**, 652 (1937).

[6] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover, New York, 1987).
[7] P. K. Aravind and J. O. Hirschfelder, J. Phys. Chem. **88**, 4788 (1984).
[8] C. E. Creffield, Phys. Rev. B **67**, 165301 (2003).
[9] K. Saito and Y. Kayanuma, Phys. Rev. B **70**, 201304(R) (2004).
[10] S. Ashhab, J. R. Johansson, A. M. Zagoskin, and F. Nori, Phys. Rev. A **75**, 063414 (2007).
[11] Ya B. Zeldovisch, Zh. Eksp. Teor. Fiz. **51**, 1492 (1966) [Sov. Phys. JETP **24**, 1006 (1967)].
[12] V. I. Ritus, Zh. Eksp. Teor. Fiz. **51**, 1544 (1966) [Sov. Phys. JETP **24**, 1041 (1967)].
[13] H. Sambe, Phys. Rev. A **7**, 2203 (1973).
[14] F. Grossmann, T. Dittrich, P. Jung, and P. Hänggi, Phys. Rev. Lett. **67**, 516 (1991).
[15] F. Grossmann, P. Jung, T. Dittrich, and P. Hänggi, Z. Phys. B **84**, 315 (1991).
[16] Y. Kayanuma, Phys. Rev. A , **50**, 843 (1994).
[17] D. H. Dunlap and V. M. Kenkre, Phys. Rev. B **34**, 3625 (1986).
[18] Y. Kayanuma and K. Saito, Phys. Rev. A **77**, 010101(R) (2008).
[19] A. Eckardt, C. Weiss, and M. Holthaus, Phys. Rev. Lett. **95**, 260404 (2005).
[20] M. Sillanpää, T. Lehtinen, A. Paila, Y. Makhlin, and P. Hakonen, Phys. Rev. Lett. **96**, 187002 (2006).
[21] K. Drese and M. Holthaus, Eur. Phys. J. D **5**, 119 (1999).
[22] S. Guérin and H. R. Jauslin, Adv. Chem. Phys. **125**, 147 (2003).
[23] G. Ramon and X. Hu, Phys. Rev. B **75**, 161301(R) (2007).
[24] E. Kierig, U. Schnorrberger, A. Schietinger, J. Tomkovic, and M. K. Oberthaler, Phys. Rev. Lett. **100**, 190405 (2008).
[25] M. G. Floquet, Ann. Ecole Norm. Suppl. **12**, 47 (1883).
[26] S. Miyashita, K. Saito, and H. De Raedt, Phys. Rev. Lett. **80**, 1525 (1998).
[27] Y. Teranishi and H. Nakamura, Phys. Rev. Lett. **81**, 2032 (1998).
[28] L. D. Landau, Phys. Z. Sowjetunion **2**, 46 (1932); C. Zener, Proc. R. Soc. London A **137**, 696 (1932); E. C. G. Stueckelberg, Helv. Phys. Acta **5**, 369 (1932); E. Majorana, Nuovo Cimento **9**, 43 (1932).
[29] S. Miyashita, J. Phys. Soc. Jpn. **65**, 2734 (1996).
[30] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, Oxford, 1976).
[31] F. Bloch, Z. Phys. **52**, 555 (1929).
[32] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).
[33] Y. Kayanuma, Phys. Rev. A **55**, R2495 (1997).
[34] A. Garg, Europhys. Lett. **22**, 205 (1993).
[35] W. Werensdorfer and R. Sessoli, Science **284**, 133 (1999).
[36] K. Hijii and S. Miyashita, Phys. Rev. B **78**, 214434 (2008).