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Improved far-field optical trap for cold atoms (molecules) with phase-modulated circular aperture diffraction

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We propose a scheme to trap cold atoms (or molecules) by using an improved red- or blue-detuned far-field optical trap, which is formed by an optical system composed of a binary phase plate and a circular aperture illuminated by a plane light wave. We calculate the relative intensity distribution of the far-field trap and study its dependence on the phase φ of the binary phase plate. Our study shows that the binary phase plate can be used to improve some geometric and optical parameters of the far-field optical trap, and realize the evolution of the far-field optical trap from a red-detuned trap to a blue-detuned one. In particular, when the phase φ of the binary phase plate is changed from 0 to $-\pi$, the maximum intensity of the far-field trap and its well depth will be enhanced by ~4 times, and the corresponding trapping volume will be increased by ~8 times, with respect to a trapping beam obtained from the diffraction by a circular aperture.

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Usually, optical dipole traps for cold atoms or molecules can be mainly classified into two kinds [1]: one is the red-detuned optical dipole trap, and the other is the blue-detuned optical dipole trap [2]. These two kinds of optical traps have their individual advantages and disadvantages [3] and have been widely applied in the fields of atomic, molecular, and optical (AOM) physics [1–3], and so on. However, these review articles [1–3] did not include a kind of special optical trap formed from the diffraction light by a circular aperture: near- or far-field optical trap with a red or blue detuning [4–9], which also have some important and interesting applications in AOM.

As early as 1994 and 1995, Letokhov's group proposed some simple schemes to trap or focus cold atoms by using the near-field diffraction light from a round aperture [4–6], which is the simplest trapping or focusing scheme in the optical dipole traps or atomic lens by only using a single unfocused laser beam. Particularly, this method is of benefit to form one-dimensional (1D) or two-dimensional (2D) array of near-field optical nanotraps by only using a complex diffraction mask composed of 1D or 2D array of circular apertures, while don't need to simultaneously use 1D or 2D array of microlenses (e.g., Fresnel lens). However, until recently, this near- or far-field optical trap scheme or atom nano-optics has been interested and studied again by a few groups [7–9] such as Gillen *et al.* in 2006 who used the Hertz vector diffraction theory to investigate the feasibility of this near-field optical trap and performed the corresponding optically experimental demonstration [7]. In 2008, Bandi *et al.* proposed an array of near-field atom microtraps to perform site-selective manipulation of cold atoms by using near-field Fresnel diffraction light from an array of microsized circular apertures [8]. In addition, our group (2007) used the scalar-wave diffraction theory to calculate the far-field diffracted intensity distribution, and studied the dependence of the geometric and optical parameters of this far-field optical trap on the parameters of our optical system, and discussed its applications in all-optical integrated atom (molecule) optics, atom (molecule) chip and 1D or 2D optical lattice [9]. Here it should be pointed out that usually the trap configuration of the used optical fields with subwavelength scale is called the "near-field trap" or "nanotrap" while the trap one of the optical fields with a scale of larger than the laser wavelength (i.e., a scale of larger than 1 μm) is called the "far-field trap" or "microtrap."

To improve the intensity utilization efficiency of this far-field optical trap and enhance its well depth, in this Brief Report, we studied a scheme (see Fig. 1) to form a red- or blue-detuned far-field optical trap for cold atoms (molecules) by using an optical system composed of a circular aperture and a binary phase plate illuminated by a plane light wave, and obtain some interesting results. Figure 1(a) shows the phase distribution of the binary phase plate, where the phase plate with a radius a is embedded in the central circular aperture of the thin screen. The circular binary phase plate is composed of two concentric phase rings, their phase values are 0 and φ respectively, and they have the same areas (that is, the radius of the 0 phase ring is $\frac{\sqrt{2}}{2}a$) and the change range of the phase φ is from $-\pi$ to π.

The formation mechanism of our improved far-field opti-

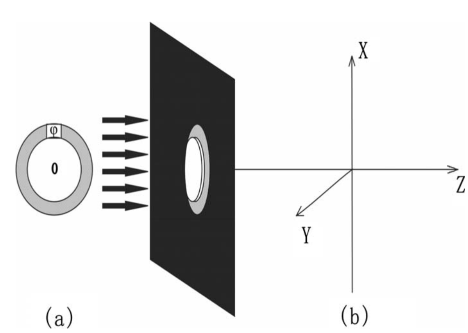

FIG. 1. (a) A binary phase plate with two phase zone $(0,\varphi)$. (b) Scheme to generate a far-field atom trap by using a binary phase plate and an infinite opaque plate with a central circular aperture.

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cal trap can be explained as follows: as shown in Fig. 1(b), when a collimated incident laser beam passes through a binary phase plate, the light wave transmitted from the phase plate will be divided into two parts with a different phase (i.e., with a phase difference of φ), and the corresponding radial and axial intensity distributions of the transmitted diffraction light field will be changed due to the spatial phase modulation of the binary phase plate for the incident light field. In particular, when the phase difference between the two phase rings are $-\pi$ and π, respectively, the constructive or destructive interference will be happened, and the red- or blue-detuned optical trap with the maximum improved well depth will be formed in the far-field diffraction light, while the phase difference between the two phase rings is equal to 0, the far-field optical trap has no any improvement, which is the case of our previous far-field optical trap [9]. In our calculation, as an example, a 50 W arbitrary-polarized yttrium aluminum garnet (YAG) laser for a red-detuned far-field optical trap (or a 50 W frequency-doubled YAG laser for a blue-detuned far-field trap) and a waist of $w_0 = 200\ \mu$m is used, and the radius of the circular binary phase plate is $a = 20\ \mu$m, and they satisfy the condition: $w_0 \gg a$ for obtaining a good focusing and an ideal near- or far-field diffraction.

Our previous study showed [9] that when the distance between the diffraction screen and the axial position is larger than $3a = 60\ \mu$m, the intensity distribution calculated from Fresnel diffraction formula is consistent with that from Rayleigh-Sommerfeld diffraction. If the second intensity maximum region ordered (numbered) from the infinite place (i.e., the right second intensity maximum region on the z axis) is chosen as our far-field optical trap, we can use Fresnel diffraction formula to calculate the radial and axial intensity distribution of the far-field optical trap. So from Fresnel diffraction theory, the far-field distribution of the circular aperture diffraction light can be calculated by [10]

$$E(r,z) = \frac{1}{i\lambda z}\exp\left[ik\left(z + \frac{r^2}{2z}\right)\right] \times \left[\int_0^{(\sqrt{2}/2)a} + \exp(-i\varphi)\int_{(\sqrt{2}/2)a}^{a}\right]E_0 \times \exp\left[ik\left(\frac{r_0^2}{2z}\right)\right]\int_0^{2\pi}\exp\left(\frac{-ik}{z}rr_0\cos\theta\right)r_0 dr_0 d\theta, \tag{1}$$

where E_0 is the complex amplitude of the incident plane wave (the incident intensity is $I_0 = |E_0|^2 \approx 2\times 10^8\ \mathrm{W/m^2}$ when $P = 50$ W), and λ is the laser wavelength. The corresponding intensity distribution of the far-field trap is given by

$$I(r,z) = |E(r,z)|^2. \tag{2}$$

From Eqs. (1) and (2), we calculate the diffracted-field relative axial and radial intensity I/I_0 distributions as the modulated phase φ of the phase plate is changed from $\varphi = 0$ to $-\pi$, and the corresponding results are shown in Fig. 2. It is clear from Fig. 2(a) that when the modulated phase $\varphi = 0$

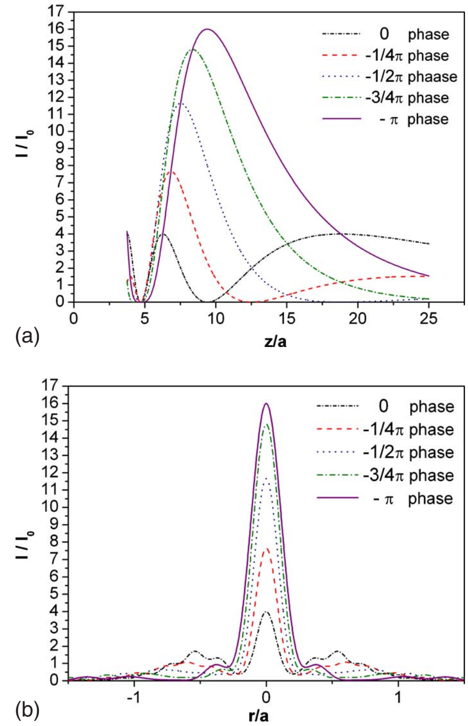

FIG. 2. (Color online) Dependence of the relative (a) axial and (b) radial intensity distributions of the far-field diffraction light on the phase φ for $a = 20\ \mu$m, $P = 50$ W, $\lambda = 1.064\ \mu$m, $\varphi = 0$ (the short dashed line), $-1/4\pi$ (the dashed line), $-1/2\pi$ (the dotted line), $-3/4\pi$ (the dashed-dotted line), and $-\pi$ (the solid line).

(i.e., the case of conventional far-field optical trap [9]), the relative axial intensity of the second diffraction maximum ordered from the infinite place [i.e., the final (or right) second diffraction maximum as shown in Fig. 2(a)] is approximately equal to four times of the incident light intensity. Although there are a few regions of the localized high and low intensity distributions in the far-field diffraction light, the right second diffraction maximum region is our interest and a desirable red-detuned far-field optical trap for cold atoms (molecules). Also, we find from Fig. 2(a) that with the reduction in the phase φ from $\varphi = 0$ to $-\pi$, the relative axial intensity of the right second diffraction maximum will be increased from about 4–16, and the relative intensity of the right first diffraction maximum will be reduced from about 4 to zero, while the position of the trapping center (i.e., the axial position of the right second diffraction maximum) will be moved toward the right along the z axis. In particular, when $\varphi = -\pi$, the relative axial intensity (i.e., the well depth) of the red-detuned far-field optical trap will be enhanced by 4 times relative to the case of $\varphi = 0$. This shows that with the reduction of the phase φ from 0 to $-\pi$, an enhanced or improved far-field optical trap with a red detuning will be formed.

From Fig. 2(b), we can see that with the reduction of the phase φ from $\varphi = 0$ to $-\pi$, the relative radial intensity of the red-detuned far-field optical trap and its width will be increased, while all high-order diffraction maxima will be greatly decreased, which is of very benefit to form a 1D or 2D array of far-field optical microtraps for cold atoms or molecules by using a 1D or 2D array of microsized circular apertures. In particular, when the phase $\varphi=-\pi$, the ratio of the second-order diffraction maximum to the zero-order diffraction one will be reduced to 0.06 from 0.33 as compared with the case of $\varphi=0$, and the other minor maximums will be more low. Also, we can find from Fig. 2 that when $\varphi=-\pi$, we obtain the maximum intensity $I_{max}=6.37\ \text{GW/m}^2$ and a neat far-field optical trap.

It is clear from Fig. 2 that the phase φ of the binary phase plate must be carefully adjusted to optimize the radial intensity distribution of the zeroth-order diffraction light, and we should choose a binary $-\pi$ phase plate to improve the intensity distribution of the red-detuned far-field optical trap. Our calculation also shows that when the phase φ is changed from 0 to $-\pi$, the maximum intensity and trapping volume of the improved far-field optical trap will be enhanced by about four and eight times, respectively. In our calculation, the trapping volume is defined by $\Delta V_{1/2}=\frac{4}{3}\pi\Delta x_{1/2}\Delta y_{1/2}\Delta z_{1/2}$, and calculated by using $\Delta x_{1/2}=\Delta y_{1/2}\approx 0.1298a$ and $\Delta z_{1/2} \approx 0.8089(2a/\lambda)\Delta x_{1/2}$ for the red-detuned trap, or by using $\Delta x_{1/2}=\Delta y_{1/2}\approx 0.1489a$, $\Delta z_{1/2}\approx 0.1293(2a/\lambda)\Delta x_{1/2}$ for the blue-detuned one when $a \gg \lambda$; here $\Delta x_{1/2}$, $\Delta y_{1/2}$, and $\Delta z_{1/2}$ are the geometric sizes of the far-field optical trap in the x, y, and z directions when the maximum intensities are reduced to its half value.

We also calculate the relative axial and radial intensity distributions at each far-field trap center as the modulated phase φ is changed from 0 to π, and the corresponding results are shown in Fig. 3. We can find from Fig. 3(a) that with the increase of the phase φ from $\varphi=0$ to π, the relative axial intensity of the right second diffraction maximum will be decreased to zero, while the relative axial intensity of the right first diffraction maximum will be increased from about 4 to 16, and the axial position of the right first diffraction minimum will moved toward the left along the z axis. This shows that with the increase of the phase φ from 0 to π, an enhanced or improved far-field localized hollow optical trap with a blue detuning will be formed. In particular, when $\varphi=\pi$, a blue-detuned far-field optical trap with an enhanced well depth of ~4 times will be formed.

Similarly, we can see from Fig. 3(b) that with the increase of the phase φ from $\varphi=0$ to π, the relative radial intensity of the principal maximum (i.e., the zero-order diffraction maximum) will be decreased to zero, while the relative radial intensity of the first-order diffraction ring will be increased from about 1.5 to 3.5, which can be used to form a blue-detuned far-field optical trap. At the same time, other high-order diffraction maximums will be increased by about two times. In this case, it is not of benefit to form a 1D or 2D array of far-field hollow optical microtraps for cold atoms (or molecules) by using a 1D or 2D array of microsized circular apertures.

Since the blue-detuned optical trap has some unique physical properties [2,3], such as efficient intensity-gradient cooling, small trapping volume, tight confinement, low photon scattering rate, and long trap lifetime [11] and so on, it has many important applications [1–3] in the fields of laser cooling [12,13] and trapping, cold atomic physics and atom optics, cold molecular physics and molecule optics, etc. From Fig. 3, we can also see that with the increase of the phase φ from $\varphi=0$ to π, an optimal blue-detuned far-field trap with an enhanced well depth and trapping volume of ~4 times will be formed. Because our blue-detuned far-field-diffracted optical trap has very high intensity gradient [~$(2.5-8.1)\times 10^{14}\ \text{W/m}^3$], it can also be used to cool the trapped atoms by its efficient intensity-gradient cooling [12,13].

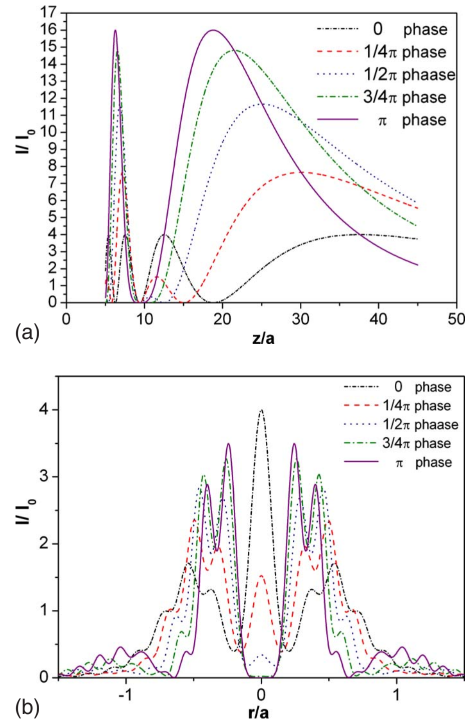

FIG. 3. (Color online) Dependence of the relative (a) radial and (b) radial intensity distributions of the far-field diffraction light on the phase φ for $a=20\ \mu\text{m}$, $P=50$ W, $\lambda=0.532\ \mu\text{m}$, $\varphi=0$ (the short dashed line), $-1/4\pi$ (the dashed line), $-1/2\pi$ (the dotted line), $-3/4\pi$ (the dashed-dotted line), and $-\pi$ (the solid line).

It is well known that the resolution of current micro- or nanofabrication techniques can reach a few ten nm even several nm [14–17], this shows that the diameter of our phase-modulated circular aperture can be equal to or smaller than 100 nm, so our scheme can also be used to form an improved near-field atom nanotrap [7] even a 2D optical lattice with a lattice constant of a few hundred nm [8]. It is clear that the lattice constant of our optical lattice can cover a wide range from a few hundred nm to several hundred μm, so it should have some important applications in quantum atom optics, nonlinear atom optics, quantum manipulating and controlling, quantum computing and information processing [18], and so on [19]. In addition, since our proposed trap scheme with $-\pi$ phase-modulated circular aperture can be further miniaturized, integrated and fabricated on the surface of the substrate (i.e., atom chip), our improved far-field optical trap has some important applications in the fields of all-optical integrated atom optics and atom chip [20]. Also, some applications mentioned above are suitable for cold molecules.

In conclusion, we have presented a scheme to improve the far-field optical trap by using a single plane wave to illuminate an optical system composed of a binary phase plate and a circular aperture, and calculated the corresponding intensity distribution and studied the dependence of the relative intensity of the far-field trap on the phase φ of the binary phase plate. Our study shows that our binary phase plate can be used to improve the geometric and optical parameters of the far-field optical trap, and realize the evolution of the far-field optical trap from a red-detuned trap to a blue-detuned one. In particular, when the phase φ of the binary phase plate is changed from 0 to $-\pi$, the maximum intensity of the far-field trap and its well depth will be increased by $\sim$4 times, and the corresponding trapping volume will be increased by $\sim$8 times, and the maximum intensity in all high-order diffraction rings around our far-field trap will be reduced by $\sim$5.5 times. While the phase φ of the phase plate is changed from 0 to π, a far-field Gaussian-like optical trap will be evolved as a blue-detuned far-field trap with an enhance trapping intensity and volume of $\sim$4 times. This shows that our negative π phase-modulation scheme is very beneficial to form 1D or 2D array of the red-detuned far-field microtraps (or near-field nanotraps) by using a 1D or 2D array of $-\pi$ phase-modulated circular apertures. So such an improved far-field optical trap cannot only be used to prepare an optical lattice with cold atoms or molecules, but also has some important applications in the fields of AOM [1–3], laser cooling [12,13] and trapping, all-optical integrated atom (or molecule) optics and its chip [20], quantum computing and information processing [18], and so on [19].

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