Compression of photon wave packets in the soft-x-ray region

S. Fujiki,[1] K. Tsuchiya,[2] J. Ishikawa,[1] H. Okuma,[1] T. Miyahara,[1,*] H. Sasaki,[2] T. Shioya,[2] T. Obina,[2] and S. Yamamoto[2]
[1]*Department of Physics, Tokyo Metropolitan University, 1-1 Minamiohsawa, Hachioji, Tokyo 192-0397, Japan*
[2]*Photon Factory, High Energy Accelerator Research Organization, 1-1 Oho, Tsukuba 305-0801, Japan*
(Received 19 July 2008; published 2 December 2009)

Using a tapered undulator and a double grating system we have compressed soft-x-ray photon wave packets into 37% of the original length. To confirm the shortening of the length of wave packets, we employed a special analog method to measure two-photon correlation *as it is* without elongating the wave-packet length, while the previous two-photon correlation measurements had adopted the method to largely elongate the wave-packet length up to several hundred fs using a monochromator. The present method gives a new prospect in that the compressed wave-packet length is extremely short and of the order of 1 fs, which enables various soft-x-ray spectroscopic experiments to detect in the time domain fast elementary excitations in a solid.

DOI: 10.1103/PhysRevA.80.063804 PACS number(s): 42.65.Re, 07.60.−j, 42.50.−p, 41.50.+h

I. INTRODUCTION

Demands for utilization of short-pulse soft or hard x rays have been increasing for the recent decade in the fields of solid state physics and molecular or atomic physics. It is because we sometimes need to observe various phenomena in the time domain rather than the energy domain. In the time domain we can have opportunities to obtain information on the phases of dynamical motion excited in a material, such as sinelike or cosinelike oscillation with damping, while in the energy domain we just get information of the spectra. Basically a short pulse plays two roles in a material. One is to define the initial time of an excitation that causes slower oscillations than the time duration of the input pulse, which is like what a hammer does on a macroscopic body. The other is to excite the microscopic material system by giving it the energy corresponding to the frequency of the wave packet. Of course the energy has some ambiguity because of the Fourier limit of the short time duration of the pulse.

As for synchrotron soft x rays the shortest ever attained pulse duration is of the order of 100 fs. Utilizing these short pulses researchers have done experiments to observe low energy elementary excitations, such as phonons and magnons in a solid that have the time scale of something around several ps. But recently some researchers pay more attention to investigate quicker motions caused by electronic correlations that have the energy scale of 0.1–1 eV. This situation motivated the authors to develop a method to shorten soft-x-ray pulses produced by a synchrotron without substantial loss of the intensity.

We have first noticed that there are two aspects of utilization of short x-ray pulses depending on whether one uses a short bunch length of the ensemble of photon wave packets or just uses a short wave-packet length of a single photon. The former case usually appears in some coincidence measurements with high coincidence rates. The bunch length in an electron storage ring as a synchrotron radiation source could be shortened with the bunch-slicing technique [1,2] down to 100 fs, but the intensity would be largely reduced down to around 0.1% because the typical electron bunch length in a storage ring is of the order of several tens ps from which only a tiny part corresponding to 100 fs is sliced out.

The latter case is utilized when the coincidence rate is much smaller. Under this situation researchers would pay more attention to define the initiation time of an event and the possible subsequent coherent motion induced. Under this circumstance the bunch length in a storage ring does not have to be as short as fs, while the pulse duration of each photon is of crucial importance. In this study we therefore have concentrated our effort on shortening an individual photon wave packet rather than shortening the bunch length.

The ideal best way to shorten the wave-packet length is transformation of the light in the ω-t phase space because it is ideally lossless. Transformation of the light in x-x' phase space is well known and widely used to spatially focus synchrotron radiation. The principle is just based on combination of the two transformations corresponding to straight motion and focusing (or defocusing) action. In the similar way a wave-packet length can be compressed using the transformations in the ω-t phase space as shown in Figs. 1(a) and 1(b). The broken-lined ellipses in Fig. 1(a) show chirped pulses where the frequency depends on time, while the ellipse with solid line corresponds to a well-behaved Gaussian wave packet. The figure shows how a chirped pulse is transformed into a well-behaved Gaussian wave packet when a dispersive

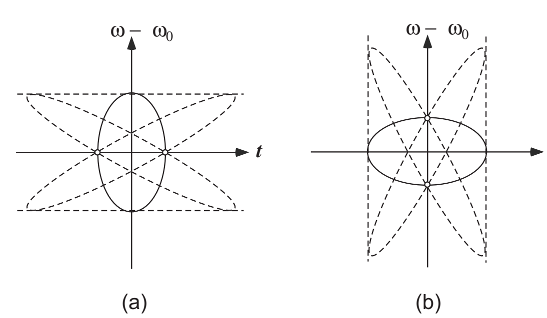

FIG. 1. (a) Transformation of the beam in the ω-t phase space using a double grating system, where the optical path depends on the wavelength. (b) Transformation of the beam by upchirping and downchirping.

*Present address: Department of Mathematical and Physical Sciences, Japan Women's University, Tokyo 112-8681, Japan.

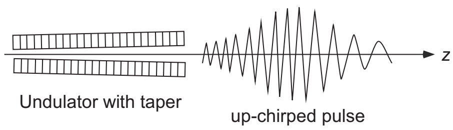

FIG. 2. Undulator with tapered gap. Upchirped pulses are radiated through the undulator.

element causes a change in the path length depending on the frequency of light. Usually two dispersive elements are used to perform this transformation because the directions of the incident and exit beams should remain constant. In fact a double prism system is often used in the visible light region, where two prisms are located to cause positive and dispersion angles, respectively. This transformation is lossless in principle though actually there is a loss caused by imperfect reflectance of the optical elements such as diffraction gratings.

However, creation of a chirped pulse from a well-behaved Gaussian wave packet is not a trivial task in the soft-x-ray region, while in the visible region combination of an intense laser and a nonlinear material can create such chirped pulses. As is shown in Fig. 1(b), a well-behaved Gaussian wave packet of visible light drawn in a solid-lined ellipse could be transformed into the ellipses with broken lines relatively easily using an appropriate nonlinear material. The trick is that an intense radiation could modify the refractive index with time response slower the inverse of the input frequency. This induces a time-dependent increase (or decrease) of the refractive index and then creates a chirped pulse.

The problem here is that an optical element to chirp a soft-x-ray pulse is not available and synchrotron radiation is not intense enough to cause such nonlinear effects. We then employed a method to chirp the pulse by modifying the undulator gap to produce tapers in the two magnet arrays. In the present experimental framework the pulses were upchirped with the taper in which the gap is wider in the down stream of the undulator. It should be noted here that as shown in Fig. 2 the weaker magnetic field at the downstream end of the undulator actually contributes to the tail of the photon wave packet which reaches the beamline optical elements later. We chose upchirped pulses because, as is shown in Fig. 3, the double grating system adopted gives longer path lengths for longer wavelengths and thus works as a compressor of the wave packets. We believe that this is a setup to obtain compressed soft-x-ray wave packets.

Of course there is the lower limit of the compression that corresponds to the Fourier transform of a chirped pulse. In fact this limit is associated to possible deterioration of the first-order coherence when the light goes through the double grating system. The difference among optical paths could increase the beam size which then could increase the vertical light beam emittance. However if the increased emittance is still within the diffraction limit, the vertical emittance of the beam does not increase substantially. Strictly speaking the fact that the lower limit of compression is restricted by the Fourier limit is identical to the restriction on the first-order coherence by the diffraction limit.

Therefore the present principle of operation is summarized as follows. First we reduce the monochromaticity of the wave packet by chirping without changing the length of the wave packets, and second we compress the wave packets down to the Fourier limit utilizing the wavelength-dependent paths in the double grating system.

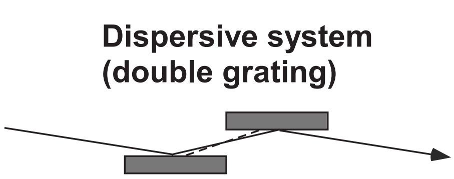

FIG. 3. Difference between optical paths for longer and shorter wavelengths in the double grating system.

II. PRINCIPLE OF THE EXPERIMENT

Because the above-mentioned shortening of pulses must be verified experimentally we employed the two-photon correlation measurement to confirm the shortening based on the fact that our synchrotron radiation is chaotic or thermal radiation. In principle two-photon correlation is related to the second-order coherence characterizing the correlation between two intensities of light. Therefore it represents the statistical property of photons included. When the statistics obeys the Bose-Einstein statistics, as is the case for thermal or chaotic radiation, the photons tend to gather together if they are in the same electromagnetic mode, enhancing the correlation. Then because the identity of the mode depends on the first-order coherence, two-photon correlation for chaotic radiation also depends on the first-order coherence. In the other extreme situation when the light is totally coherent and the statistics obey the Poisonian statistics, as is the case for lasers, two-photon correlation always has a constant value. In both cases, however, the two-photon correlation linearly depends on the wave-packet length when the average number of photons included in a unit time is kept constant.

The first measurement of two-photon correlation was performed by Hanbury-Brown and Twiss (HT) [3–6] using an analog method to measure the diameter of visible stars emitting radio waves. There was no systematic time structure of the light (or radio waves) coming from the stars, namely, the light was regarded as a stationary light source. They detected two radio wave signals using two antennas. The fluctuations of two signals were multiplied using a correlator that employed a sophisticated vacuum tube circuit. To eliminate the background noise they elaborated a kind of double lock-in modulation method. The frequency limit was about 30 MHz limited by operation of the vacuum tubes used. They also performed a similar experiment using a mercury lamp as a line-spectral light source.

An apparent advantage of the today's technology is that we can use much faster correlators with time response of several GHz. However, as will be described later, the detection efficiency is basically restricted by the ratio between the time duration of the light pulse and the response time of the correlator used. Because HT used the highly monochromatic mercury line spectrum with wavelength of 435.835 nm, then they were able to detect two-photon correlation even with

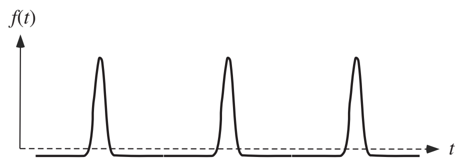

FIG. 4. Schematic view of $f(t)$. It is so defined that the integration of the function by time corresponding to the circulating period of electron bunches in the storage ring vanishes.

poor frequency response of the correlator. In the present study, however, we had to detect two-photon correlation for pulses with much shorter time duration, of the order of 1 fs, meaning that it is by no means a simple task.

Another difficulty of the present study is that the synchrotron radiation has a time structure characterized by distribution of bunches around the orbit of the electron storage ring. In fact regarding synchrotron radiation Shuryak proposed observation of the synchrotron radiation [7] and Ikonen gave a detailed discussion on the possibility of detecting two-photon correlation in the x-ray region [8]. In order to understand the nonstationary property of synchrotron radiation we first generalize the situation of two-photon correlation to the case in which the light source has a specific time structure, $N[1+f(t)]$, where N is the average number of photons of a light source. This requires the equation

$$\int f(t)dt = 0. \tag{1}$$

Obviously $f(t)$ has the periodicity of revolution frequency of the stored beam in the storage ring and schematically shown in Fig. 4. The case of the measurements by HT corresponds to the case, $f(t) \equiv 0$, leading to a stationary light source.

Now because we have two light sources, the average intensity of the two signals is

$$\langle N_1[1+f_1(t)]\rangle = \langle N_1\rangle \quad \text{and} \quad \langle N_2[1+f_2(t)]\rangle = \langle N_2\rangle. \tag{2}$$

Then the correlation of the two signals is expressed as

$$\begin{aligned}\tilde{N}_{12} &= \langle\{N_1[1+f_1(t)]-\langle N_1\rangle\}\{N_2[1+f_2(t)]-\langle N_2\rangle\}\rangle \\ &= (\langle N_1N_2\rangle - \langle N_1\rangle\langle N_2\rangle)(1+f_1+f_2+f_1f_2) + \langle N_1\rangle\langle N_2\rangle f_1f_2.\end{aligned} \tag{3}$$

HT actually measured the quantity

$$N_{12} = \langle(N_1-\langle N_1\rangle)(N_2-\langle N_2\rangle)\rangle = \langle N_1N_2\rangle - \langle N_1\rangle\langle N_2\rangle, \tag{4}$$

for which HT presented an important relation

$$N_{12} = \langle N_1\rangle\langle N_2\rangle|\gamma_{12}|^2\frac{\tau}{T}, \tag{5}$$

where τ is the wave-packet length of the photons expressed in terms of time (pulse duration) and T is the response time of the detectors and is assumed to be much longer than τ. $|\gamma_{12}|^2$ is the first-order coherence function between the two sources, taking a value between 1 and 2 for chaotic radiation with the Bose-Einstein distribution. $|\gamma_{12}|^2$ is equal to 2 only when the two chaotic light sources are in exactly the same mode of the electromagnetic field, meaning that the first-order coherence between the two sources is perfect, while for a light source with the Poisonian distribution such as lasers, the value remains to be unity. When two modes are totally different, the value is 1 even for chaotic radiation. The situation could be easily understood considering a basic aspect of statistical quantum mechanics as was described by Feynman. When two photons are exactly in the same mode we cannot distinguish these photons. In the Bose-Einstein distribution, the chance for a detector to detect two photons at the same time is doubled compared to the case when two photons are in different modes.

In this study, as was the case for the former studies, the degree of the first-order coherence has been changed as a parameter, giving rise to modification of $|\gamma_{12}|^2$. Then what we have to do is changing this value from 1 to 2 and this gives the final result of the two-photon coincidence rate as will be shown in Fig. 8. This way of measurement works in principle because synchrotron radiation is chaotic light at least in soft-x-ray or hard x-ray region. Of course there are some special cases where one observes synchrotron radiation with the wavelength comparable or longer than the bunch length. Under this situation synchrotron radiation could be similar to a laser, and the present principle of measurement does not work. In principle the present method also would work even when the light includes both chaotic and totally coherent parts at the same time if the ratio between the two is kept constant.

The linear dependence on τ in Eq. (5) is interpreted with the following consideration. Two-photon coincidence takes place when two-photon wave packets overlap. When the average number of wave packets (and therefore the number of photons) is constant, the probability of the overlap is obviously proportional to the wave-packet length or pulse duration.

The major difficulty of measuring two-photon correlation is that the ratio τ/T in Eq. (5) is extremely small for natural light sources because the wave-packet lengths are usually much shorter than those of lasers. In addition when correlation between light sources with time structure has to be measured, the second term of Eq. (3) is just harmful and should be removed using some technique.

One method to remove the annoying term is to modulate τ and use a lock-in amplifier to extract the true correlation, which is described in Refs. [9,10]. This method cannot be used in the present study because we need to know τ itself. Another method is to extract false correlation, the second term of Eq. (3), detecting the false correlation taking place between different bunches. This method was employed by Takayama *et al.* [11] and we used a similar method with slight modification. These two methods will be described later in more detail.

We understood we were facing such a difficult problem that we needed to measure two-photon correlation *as it is* without using a monochromator because what we wanted to measure was the change in the wave-packet length itself and therefore we are not allowed to change the length of wave packets, while in the past experiments the wave packets were elongated to some extent with monochromator. In other words as it is means that the pulses are not elongated.

Gluskin *et al.* [12–14], Yang *et al.* [15], and Feng *et al.* [16] measured two-photon correlation of x rays emitted at

Advanced Light Source, Illinois, USA, while Kunimine *et al.* [17] measured two-photon correlation in hard x-ray region using the TRISTAN main ring, where an ultrahigh resolution four-crystal monochromator was used to elongate the wave-packet length up to 10 ps. They employed a digital counting method to get the correlation. In the soft-x-ray region Tai *et al.* performed a two-photon correlation using the analog modulation of the monochromaticity of the incident beam to remove the huge amount of false correlation [9]. In their experiment an analog technique was used including a sharp notch filter to eliminate the effect of rf, 500 MHz, and the wave-packet length *elongated* by a grating monochromator was supposed to be about 20 fs, which is the nominal average value because the slit width was modulated. This value is much shorter than the hard-x-ray case because of poorer resolution of the monochromator chosen to keep the linearity of the modulation. In the hard x-ray region the digital counting method employed by Yabashi *et al.* [18] gave a beautiful result making use of very high brightness of Spring-8.

On the other hand a digital counting technique was used also in the soft-x-ray region by Tai *et al.* [9], where a sophisticated delay-time modulation technique was introduced for the first time to remove the false correlation. The pulse length elongated by a grating monochromator was estimated to be around 300 fs.

In this paper we show that two-photon correlation was successfully measured. The experimental results were obtained using the beamline BL16B of the Photon Factory. In this study we were able to confirm the compression of photon wave packets in the soft-x-ray region. The development of a technique to measure two-photon correlation as it is also presented.

III. EXPERIMENT

We have first noticed that there is no essential difference between an analog and a digital method in terms of statistical errors when the intensity of incident light and the time response of the detectors are identical, except for the case that time resolution of the time-to-amplitude converter (TAC) used in digital measurements is worse than that of detectors. The sampling theorem in information theory suggests that what is restricted by the frequency response of the sampling is identical to what is restricted by the frequency response of an analog detection. In fact what matters are the bandwidth of the detection system (or the inverse of response time), the intensity of the incident light, and the wave-packet length. Here it is obvious that in previous research the product of the intensity of light and the wave-packet length is basically constant when an ideal monochromator with no loss is used. Of course practically any monochromator has a loss caused by imperfect reflectance of optical elements used.

However the analog detection system employed in this work has a problem of slow response time by the following reason. In the present analog measurements the detectors have to endure a potentially very high photon flux, which means that conventional multichannel plates cannot be used because of saturation, though a multichannel plate could have a response time of the order of a few 10 ps. Then we used a pair of photodetectors designed by Hamamatsu Photonics Ltd. that were supposed to have a rise time of about 2 ns. We confirmed this response time by observing the bunch structure with 500 MHz repetition corresponding to the rf of the 2.5 GeV electron storage ring. Obviously the time response of 2 ns is substantially worse than the typical resolution of 20 ps of the standard TAC system. This is apparently a serious problem considering T in Eq. (5).

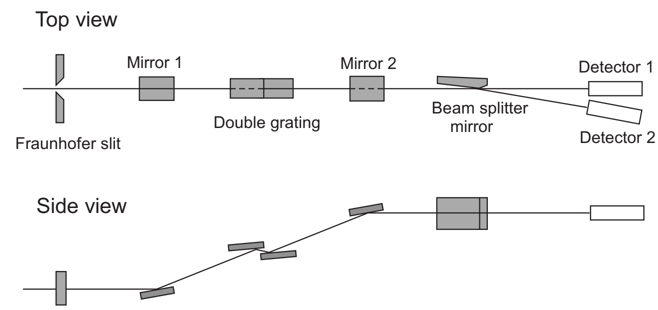

FIG. 5. Illustration of the experimental setup. The Fraunhofer slit diffracts the incident beam and changes the first-order coherence. The double grating system functions to compress the wave-packet length. The edge of beam splitter mirror is located in the middle of the beam with finite width and divides the beam into two beams with almost same intensity.

Meanwhile we knew that in the previous experiment performed by Takayama *et al.* [11] the input photon flux was reduced down to 5% of the original photon flux to avoid saturation of the TAC with single channel analyzer. Thus we learned that two-photon correlation was measurable in the present signal condition of the detectors used because we could capture extremely high photon flux without monochromatizing the beam. This is a great advantage to overcome the problem of slow response of the detectors.

The experimental setup is illustrated in Fig. 5. We used the beamline monochromator under the zeroth-order condition, which means that there is no change in the wave-packet length through the monochromator. The incident beam goes through a Fraunhofer slit to modify the first-order spatial coherence. We knew that the synchrotron radiation is chaotic radiation because all the studies [9–18] were based on the chaotic nature of spontaneous synchrotron radiation. Under this condition the two-photon coherence shows a peak when the first-order coherence increases to become almost 100%. The above Fraunhofer slit works as changing the spatial coherence.

A double grating system was employed to compress the third harmonic, 114 eV, of the undulator radiation. It should be noted here that the direction of the dispersion is vertical and the vertical emittance of the light is almost diffraction limited because of the small coupling (about 2%) between the vertical and horizontal betatron oscillations of the stored beam, while the diffraction by the Fraunhofer slit is horizontal. Accordingly we do not have to worry about some change in the vertical emittance that could happen through the double grating system.

The gap between the two gratings with the groove density of 1200 lines/mm was chosen to be 0.9 mm to optimize the compression for the 114 eV light. We chose the third har-

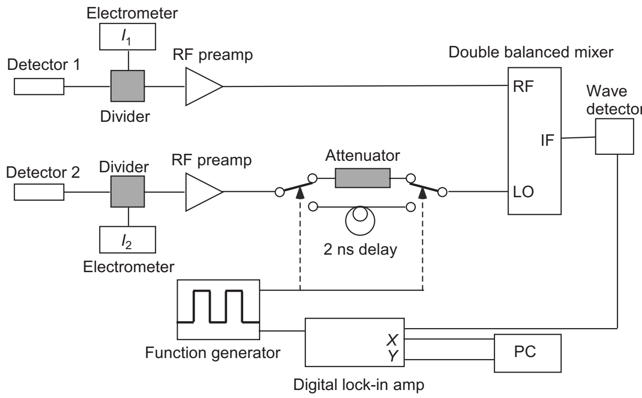

FIG. 6. Schematic diagram of the detection electronics. The power divider extracts the dc components I_1 and I_2 of the signals that are used to normalize the output signal. One of the signals is fed to a switching circuit, where delay time is modulated with the frequency of 11 Hz. The double balanced mixer multiplies the two signals and the wave detector extracts the amplitude of the high frequency oscillation. The lock-in amplifier outputs the two signals: the in-phase signal X and the out-of-phase signal Y.

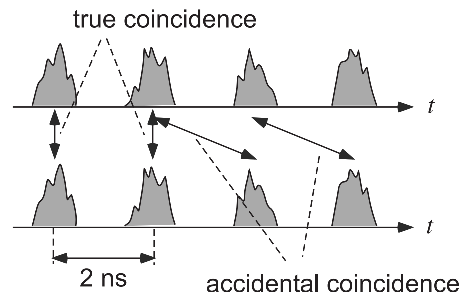

FIG. 7. Illustration of the true coincidence and the false coincidence. The detection system subtracts the coincidence signal between two different bunches from the coincidence signal in the same bunch.

monic rather than the first harmonic because a higher harmonic is more drastically influenced by the effect of tapering the undulator gap. In fact because the number of the period of the undulator is 25 the third harmonic has an energy resolution of about 1/75. On the other hand the average gap width was chosen to be 51 mm and the gap at the downstream end of the undulator was chosen to be wider by 520 μm than the gap at the upstream end considering the nonlinearity of the dependence of the photon energy on the gap width. This taper creates an additional reduction of the monochromaticity down to roughly 1/25. Accordingly it was expected that the monochromaticity of the third harmonic would be degraded down to about 1/3 due to tapering the gap width, which means that the wave-packet length can be compressed also down to 1/3 after going through the double grating system.

An additional trick was incorporated in the grating system. The two gratings were set so that the zeroth-order light reflected by the first grating is blocked by the side of the second grating and the first harmonic cannot go through the gratings because the diffraction condition is not satisfied. The fifth harmonic of the undulator radiation is greatly reduced because the incident angle to the first grating was set to be 80°. With this trick only the third harmonic can be mainly detected.

At the downstream of the grating system was installed a beam splitter mirror by which the half of the beam is reflected to go to detector 2 while the remaining part goes straight to detector 1. As shown in Fig. 6 the output analog signals were fed into power dividers to monitor the average currents, I_1 and I_2, and the corresponding rf signals were amplified by two rf preamplifiers. One signal from the preamplifier was fed to a double balanced mixer with 3 GHz bandwidth, while the other was lead to a solid state switch to select a path with high precision variable attenuator or a path with 2 ns delay relative to the first path. The modulation frequency of solid state switch was chosen to be 11 Hz.

By this modulation two different situations can be made. When there is no delay the double balanced mixer multiplies two signals coming from the same bunch but when there is 2 ns delay the double balanced mixer multiplies two signals coming from two adjacent bunches. The former case includes true correlation signals while the latter case includes only false correlation signals as shown in Fig. 7. Finally the modulated signal was detected by a digital lock-in amplifier with subtraction between the two quantities. This is the basic principle of operation to extract true correlation signals removing the huge background of false correlation signals [11].

The output signal from the wave detector just after the balanced mixer can be expressed as follows:

$$S(D) \propto \langle I_1\rangle\langle I_2\rangle\left(\alpha\frac{\tau_c}{T}G(D)+\tilde{A}\right), \tag{6}$$

where I_1 and I_2 are the intensities (currents) of the two beams, α is a constant to characterize the efficiency of the detection system, τ_c is the wave-packet length in time, T is the time resolution of the detector, and $\tilde{A}$ is the very large background mainly consisting of false correlation signals. $G(D)$ is related to the first-order coherence and expressed as

$$G(D) = \left[\frac{D^2+24\Sigma^2}{(1+\Sigma^2/\sigma_c^2)D^2+24\Sigma^2}\right] \times \cos^{-1}\left[\frac{D^2}{(1+2\sigma_c^2/\Sigma^2)D^2+48\sigma_c^2}\right], \tag{7}$$

where D is the width of the Fraunhofer slit, Σ is the beam size on the slit, and σ_c is the coherent photon size on the slit and is given using the emittance of the stored beam ε as [11,19,20]

$$\sigma_c = \left(\frac{\lambda}{4\pi}\right)\frac{\Sigma}{\sqrt{\varepsilon^2-(\lambda/4\pi)^2}}. \tag{8}$$

Then the two outputs from the lock-in amplifier are

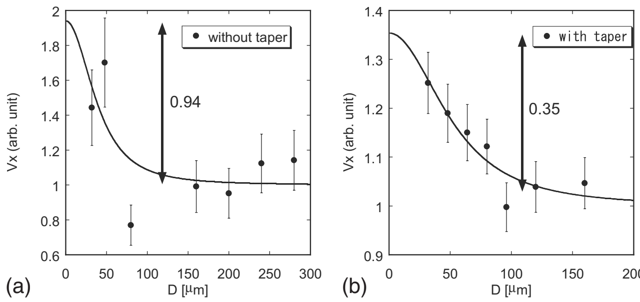

FIG. 8. (a) Two-photon correlation obtained for the undulator without taper. The abscissa shows the width of the Fraunhofer slit. The ordinate shows the quantity $F(D)$. (b) Two-photon correlation for the undulator with taper. Comparison of the two results gives the compression ratio of $37\% \pm 6\%$.

$$(X,Y) \propto I_1 I_2 \left\{ \int [\alpha \tau_c G(D) + \widetilde{A}] \sin \omega t dt, \int [\alpha \tau_c G(D) + \widetilde{A}] \cos \omega t dt \right\}, \qquad (9)$$

where the value of τ_c can be changed by compression of wave packets. The relation between $G(D)$ and the ordinary first-order coherence is expressed as

$$G(D) = \frac{\pi}{2} |\gamma(D)|^2 \sqrt{\frac{D^2 + 24\Sigma^2}{(1 + \Sigma^2/\sigma_c^2)D^2 + 24\Sigma^2}}. \qquad (10)$$

In this study the maximum photon flux captured by each photodetector was estimated to be around 3×10^{12} photons/s, which may cause true two-photon correlations with roughly 50 counts in 1 s, considering the interbunch spacing of 2 ns, the bunch length of 100 ps, and the wave-packet length of 0.8 fs estimated from the monochromaticity of the undulator radiation. Based on Eq. (6) described above and with the time resolution of the detectors, 2 ns, the frequency of true correlations detected is still in the measurable range even if the flux detected is reduced when the slit width is narrowed to increase the first-order coherence.

It is not easy to exactly cancel out the background $\widetilde{A}$ even by using a lock-in amplifier. However using a 500 MHz signal generator we carefully adjusted the high precision variable attenuator so that the two output signals in the switching circuit balance with the accuracy of 10^{-5}. After this process we finely readjusted the attenuator using real signals from synchrotron radiation. This readjustment was found to be very tiny and of the order of 10^{-6} but in principle is required because nonuniform bunch distribution could give a slightly different background for the case of 2 ns delay. It should be noted, however, that though we surely have residual nonvanishing background that could be different among different measurements it does not depend on D and accordingly it does not affect the rate of true two-photon correlation that does depend on D.

We accumulated the data for about 8 h for each value of D, which is required to obtain good statistics. The taper was added to the undulator after 1 week of measurements and the measurement with taper was performed.

IV. RESULTS AND DISCUSSIONS

Figures 8(a) and 8(b) show the final results obtained for the two cases without and with taper, respectively. The abscissa shows the width, D, of the Fraunhofer slit and the ordinate represents the values obtained after normalization by the two intensities, I_1 and I_2, and is proportional to the following quantity:

$$F(D) = \alpha \frac{\tau_c}{T} G(d) + B, \qquad (11)$$

where B is the residual background which was not perfectly eliminated in the present measurements. Between the two cases, with taper and without taper, the value of B was found to be different probably because of different operating conditions of the storage ring.

Therefore the hump above the background, which is caused by better first-order coherence with smaller width of the Fraunhofer slit, is directly proportional to the wave-packet length τ_c. From the results we conclude that correlation peak height estimated ignoring the background is substantially larger for the case without taper than the case with taper and that the wave-packet length was compressed down to $37\% \pm 6\%$ with tapering the undulator gap. As is seen from the figure the background was not canceled 100%, but it was remarkably small, suggesting that the present experimental setup is basically appropriate to measure two-photon correlation as it is. As is mentioned in Sec. III the wave-packet length estimated from the number of period of the undulator is around 0.8 fs, which is much shorter than those in the previous studies on two-photon correlation measurements for synchrotron light.

One thing to be noticed would be that the case without taper shows a worse signal-to-noise (S/N) ratio. We believe that this could be caused by accidental difference between the operation conditions of the electron storage ring in two different weeks. A longitudinal instability of stored bunches is not usually harmful to ordinary experiments using synchrotron radiation because the average bunch length does not change even under this instability, but it could influence the present photon correlation measurements.

As a matter of fact the above instability modifies the peak height and width of a pulse in Fig. 4 with the area for every pulse being kept constant. The modulation takes place in such a way that if a bunch has a larger width than the average then the adjacent bunch has more chance to have also a larger width. Along the orbit of the storage ring, however, there is a part with slightly longer bunches as well as a part with slightly shorter bunches. It is like a wave of modulated bunch length along the orbit. The problem here is that because of the $f_1 f_2$ term appearing in the first term in Eq. (3) the two-photon coincidence rate increases when a bunch is shorter and vice versa, which might give some more background noise to the experimental results.

This kind of instability could be accidentally occurring during the measurement with undulator without taper. Fortunately this modulation is a slow process with a frequency from several to hundred kHz and therefore it does not influence the average value of the long-term coincidence rate. In other words this kind of instability just degrades the S/N ratio through fluctuation of $\widetilde{A}$ in Eq. (8) but does not affect the ratio between the true and false correlations. Therefore it is concluded that what has been measured is basically the change in τ_c.

Finally we discuss something about relationship between the first-order and second-order coherences. Basically the two quantities are independent and expressed by different formulas. Quantum mechanically the first-order coherence contains one creation operator and one annihilation operator of a photon, while the second-order coherence contains two creation and annihilation operators. However when the photons are in the same mode, we have, using the commutation relations of bosons,

$$\langle a^{+}a^{+}aa\rangle = \langle a^{+}aa^{+}a\rangle - \langle a^{+}a\rangle. \tag{12}$$

Therefore information on the second-order coherence includes that of the first-order coherence. In addition there might be some argument on estimation of the first-order coherence. Lin *et al.* pointed out a non-Gaussian behavior of coherence function measured for an undulator radiation [21]. Paterson *et al.* discussed the coherence of soft-x-ray radiation influenced by the effect of speckle due to beamline optics [22]. The present author think that these phenomena could be described in the framework of the first-order coherence based on the idea presented by Takayama *et al.* who discussed spatial coherence of undulator radiation beyond the van Cittert–Zernike theorem [23].

ACKNOWLEDGMENTS

This work was performed under Scientific Proposal No. 2006G238 of the Photon Factory, KEK and was financially supported by KEK under the R & D project with Contract No. 2007-14. The authors are grateful to staff members of the Photon Factory for their continuous support and encouraging this work.

[1] A. A. Zholents and M. S. Zolotorev, Phys. Rev. Lett. **76**, 912 (1996).
[2] R. W. Schoenlein, S. Chattopadhyay, H. H. W. Chong, T. E. Glover, P. A. Heimann, C. V. Shank, A. A. Zholents, and M. S. Zolotorev, Science **287**, 2237 (2000).
[3] R. Hanbury Brown and R. Q. Twiss, Nature (London) **177**, 27 (1956).
[4] R. Hanbury Brown and R. Q. Twiss, Nature (London) **178**, 1046 (1956).
[5] R. Hanbury Brown and R. Q. Twiss, Proc. R. Soc. London, Ser. A **242**, 300 (1957).
[6] R. Hanbury Brown, *The Intensity Interferometer* (Taylor & Francis, London, 1974).
[7] E. V. Shuryak, Sov. Phys. JETP **40**, 30 (1975).
[8] E. Ikonen, Phys. Rev. Lett. **68**, 2759 (1992).
[9] R. Z. Tai, Y. Takayama, N. Takaya, T. Miyahara, S. Yamamoto, H. Sugiyama, J. Urakawa, H. Hayano, and M. Ando, Phys. Rev. A **60**, 3262 (1999).
[10] R. Z. Tai, Y. Takayama, N. Takaya, T. Miyahara, S. Yamamoto, H. Sugiyama, J. Urakawa, H. Hayano, and M. Ando, Rev. Sci. Instrum. **71**, 1256 (2000).
[11] Y. Takayama, H. Shiozawa, T. Yoshida, C. Lee, K. Obu, H. Otsubo, T. Miyahara, S. Yamamoto, and M. Ando, J. Synchrotron Radiat. **10**, 303 (2003).
[12] E. Gluskin, I. McNulty, and P. J. Viccaro, Nucl. Instrum. Methods Phys. Res. A **319**, 213 (1992).
[13] E. Gluskin, E. E. Alp, I. McNulty, W. Sturhahn, and J. Sutter, J. Synchrotron Radiat. **6**, 1065 (1999).
[14] E. Gluskin, I. McNulty, I. Yang, K. J. Randall, Z. Xu, and E. D. Johnson, Nucl. Instrum. Methods Phys. Res. A **347**, 177 (1994).
[15] L. Yang, I. McNulty, and E. Gluskin, Rev. Sci. Instrum. **66**, 2281 (1995).
[16] Y. P. Feng, I. McNulty, Z. Xu, and E. Gluskin, LS Note 258, 1997.
[17] Y. Kunimune, Y. Yoda, K. Izumi, M. Yabashi, X.-W. Zhang, T. Harami, M. Ando, and S. Kikuta, J. Synchrotron Radiat. **4**, 199 (1997).
[18] M. Yabashi, K. Tamasaku, and T. Ishikawa, Phys. Rev. Lett. **87**, 140801 (2001).
[19] Y. Takayama, T. Hatano, T. Miyahara, and W. Okamoto, J. Synchrotron Radiat. **5**, 1187 (1998).
[20] Y. Takayama, R. Z. Tai, T. Hatano, T. Miyahara, W. Okamoto, and Y. Kagoshima, J. Synchrotron Radiat. **5**, 456 (1998).
[21] J. J. A. Lin, D. Paterson, A. G. Peele, P. J. McMahon, C. T. Chantler, and K. A. Nugent, Phys. Rev. Lett. **90**, 074801 (2003).
[22] D. Paterson, B. E. Allman, P. J. McMahon, J. Lin, N. Moldovan, K. A. Nugent, I. McNulty, C. T. Chamtler, C. C. Retsch, T. H. K. Irving, and D. C. Mancini, Opt. Commun. **195**, 79 (2001).
[23] Y. Takayama, N. Takaya, T. Miyahara, S. Kamada, W. Okamoto, T. Hatano, R. Z. Tai, and Y. Kagoshima, Nucl. Instrum. Methods Phys. Res. A **441**, 565 (2000).