Nonequilibrium enhancement of Cooper pairing in cold fermion systems

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Nonequilibrium stimulation of superfluidity in trapped Fermi gases is discussed by analogy to the work of Eliashberg [Nonequilibrium Superconductivity, edited by D. N. Langenberg and A. I. Larkin (North-Holland, New York, 1986)] on the microwave enhancement of superconductivity. Optical excitation of the fermions balanced by heat loss due to thermal contact with a boson bath and/or evaporative cooling enables stationary nonequilibrium states to exist. Such a state manifests as a shift of the quasiparticle spectrum to higher energies and this effectively raises the pairing transition temperature. As an illustration, we calculate the effective enhancement of Cooper pairing and superfluidity in both the normal and superfluid phases for a mixture of ^{87}Rb and ^{6}Li in the limit of small departure from equilibrium. It is argued that in experiment the desirable effect is not limited to such small perturbations and the effective enhancement of the pairing temperature may be quite large.

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I. INTRODUCTION

The difficulty of observing quantum coherent phases in cold gases highlights the need to overcome low transition temperatures. In addition to Bose-Einstein condensation, there has also been recent interest in the generation of Fermi superfluids through the Bardeen-Cooper-Schrieffer (BCS) pairing mechanism [1]. This phenomenon is more difficult to observe due to prohibitively low transition temperatures [2,3] though the problem may be partially surmounted by use of Feshbach resonances [1,4,5]. Nonequilibrium effects can also be used to control and effectively cool such systems. However, this is an unexplored area of research by comparison.

Since the 1960s, it was known that superconductivity could be stimulated by radiation in microbridges [6]. In 1970, Eliashberg explained this effect as an amplification of the gap parameter by means of a stationary nonequilibrium shift in the quasiparticle spectrum to higher energies brought on by the radiation [7,8]. Over the next decade, his theory found experimental acceptance through the enhancement of critical currents and temperatures in Josephson junctions [9] and thin films [10]. At the same time, other nonequilibrium stimulation methods were developed [11] with more recent reports of enhancements of the superconducting critical temperature by up to several times its equilibrium value [12,13]. With the present interest in the application of the BCS model of superconductivity to trapped atomic Fermi gases [13–15], nonequilibrium effects represent an attractive way to magnify the quantum properties of these types of superfluids.

As in superconductors, the BCS order parameter Δ_0 for cold fermionic gases obeys a self-consistency equation [5,16,17],

$$\frac{\Delta_0}{\lambda} = \Delta_0 \sum_{\mathbf{k}} \left[\frac{1-2n_{\mathbf{k}}}{2\sqrt{\xi_{\mathbf{k}}^2+\Delta_0^2}} - \frac{1}{2\xi_{\mathbf{k}}} \right], \tag{1}$$

where $\xi_{\mathbf{k}}=\varepsilon_{\mathbf{k}}-\varepsilon_F$ is the quasiparticle dispersion centered on the Fermi energy ε_F, Δ_0 is the BCS gap, and $n_{\mathbf{k}}$ is the quasiparticle distribution function. The constant λ has the form $\lambda=-4\pi a_{\uparrow\downarrow}/m_F$, where $a_{\uparrow\downarrow}$ is the negative s-wave scattering length for collisions between hyperfine states and m_F is the mass. At equilibrium, $n_{\mathbf{k}}^{(\mathrm{FD})}$ is the Fermi-Dirac distribution function, and the only way to increase Δ_0 is either to increase the interaction strength or to lower the temperature. However, there exists a wide class of stationary nonequilibrium distributions, $n_{\mathbf{k}}$, such that Eq. (1) is still valid and has solutions with enhanced order parameters. Indeed, if a quasistationary nonequilibrium distribution is created that is different from the canonical Fermi-Dirac function, $\delta n_{\mathbf{k}}=n_{\mathbf{k}}-n_{\mathbf{k}}^{(\mathrm{FD})}$, then according to the weak-coupling BCS Eq. (1), it effectively renormalizes the pairing interaction and transition temperature as follows:

$$\frac{1}{\lambda_{\mathrm{eff}}} = \frac{1}{\lambda} + \sum_{\mathbf{k}} \frac{\delta n_{\mathbf{k}}}{E_{\mathbf{k}}} \equiv \frac{1}{\lambda} - \nu(\varepsilon_F)\chi \quad \text{and} \quad T_c^{(\mathrm{eff})} = T_c^{(0)} e^{\chi}, \tag{2}$$

where here and below $E_{\mathbf{k}}=\sqrt{\xi_{\mathbf{k}}^2+\Delta_0^2}$, $\nu(\varepsilon_F)$ is the density of states at the Fermi level, $T_c^{(0)} \sim \varepsilon_F \exp\{-1/[\nu(\varepsilon_F)\lambda]\}$ is the weak-coupling BCS transition temperature in equilibrium, and we also introduced the dimensionless parameter $\chi=-\Sigma_{\mathbf{k}}\delta n_{\mathbf{k}}/[\nu(\varepsilon_F)E_{\mathbf{k}}]$. For many nonequilibrium distributions, $\chi>0$, and this yields an effective enhancement of T_c and/or Δ. We note that even though our theory below and that of Eliashberg are limited to small deviations from equilibrium, with $|\chi|\ll 1$, this does not imply a limitation in experiment, where this parameter can be large. For such large deviations from equilibrium, the weak-coupling BCS approach and Eqs. (1) and (2) may not be quantitatively applicable, but the tendency to enhance pairing may remain. Therefore the proposed underlying mechanism may lead to substantial enhancement of fermion pairing and superfluidity. We also emphasize here that cold atom systems offer more control in creating and manipulating nonequilibrium many-body quantum states than that available in solids. Specifically, we will show that while it was impossible to drive a metal from the normal to the superconducting phase by irra-

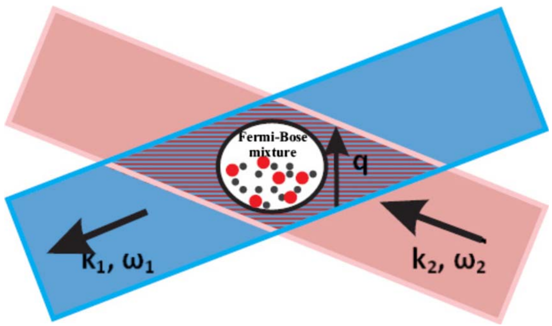

FIG. 1. (Color online) Bragg Potential: A moving lattice with wave vector $\mathbf{q}=\mathbf{k}_2-\mathbf{k}_1$ can be formed in the region of the Bose-Fermi mixture through the interference of two lasers with differing wave vectors and frequencies. By adjusting the parameters of this nonequilibrium perturbation, one can achieve states with enhanced superfluidity.

diation, it is indeed possible to drive the equivalent transition in cold gas by utilizing this additional control.

In this paper, we propose a theory of nonequilibrium stimulation of fermion pairing by considering the effect of Bragg pulses [18,19] as shown schematically in Fig. 1 on a harmonically trapped gas of fermions in the Thomas-Fermi approximation [20]. The heating induced by the external perturbation is dumped into an isothermal bath of trapped bosons via collisions, but this is not necessary in general. The pairing enhancement is calculated for a typical mixture of ^{87}Rb and ^{6}Li. It depends on the state of the gas at equilibrium. In the superfluid phase, Eliashberg's requirement that the frequency of the perturbation be less than twice the equilibrium gap ($\hbar\omega<2\Delta_0$) ensures that the pulse does not effectively heat the system by producing more quasiparticles with energies $\varepsilon\sim\Delta_0$. Though this requirement cannot be satisfied in the normal phase, where $\Delta_0=0$, the independent tunability of both the momentum and energy of the Bragg pulse allows us to protect the system from effective heating through energy conservation. This avenue, which was not available in the context of superconductors, effectively provides a means to "sharpen" the Fermi step (or even create a discontinuity at a different momentum), thereby enhancing fermion pairing.

II. MODEL

In our problem, we assume optically trapped bosons in thermal contact with fermions that occupy two hyperfine states $|\uparrow\rangle$ and $|\downarrow\rangle$. This system has a Hamiltonian of the form $\hat{\mathcal{H}}=\hat{\mathcal{H}}_0+\hat{\mathcal{H}}_I$ where the noninteracting part $\hat{\mathcal{H}}_0$ is given by

$$\hat{\mathcal{H}}_0=\int d^3\mathbf{r}\sum_p \hat{\psi}_p^\dagger(\mathbf{r})\left[-\frac{1}{2m_p}\partial^2-\mu_\alpha+V_\alpha(\mathbf{r})\right]\hat{\psi}_p(\mathbf{r}),$$

where for brevity, we introduced the subscript $p=B,F_\uparrow,F_\downarrow$ which labels bosons ($p=B$) and fermions in the two hyperfine states: "up"($p=F_\uparrow$) or "down" ($p=F_\downarrow$) and $m_{F_\uparrow}=m_{F_\downarrow}\equiv m_F$ is the fermion mass and m_B is the mass of the bosons. We assume that fermions in either hyperfine state feel the same trapping potential. Thus, $V_p(\mathbf{r})$ is given by $V_{F,B}(\mathbf{r})=\frac{1}{2}m_{F,B}\Omega_{F,B}^2r^2$ where the subscript F (B) refers to fermions (bosons). There is also an interaction Hamiltonian $\hat{\mathcal{H}}_I$ which has the form

$$\hat{\mathcal{H}}_I=\frac{1}{2}\int d^3\mathbf{r}\sum_{p_1,p_2} g_{p_1,p_2}\hat{\psi}_{p_1}^\dagger(\mathbf{r})\hat{\psi}_{p_1}(\mathbf{r})\hat{\psi}_{p_2}^\dagger(\mathbf{r})\hat{\psi}_{p_2}(\mathbf{r}),$$

with $g_{p_1p_2}$ being the strength for s-wave collisions between the particles labeled by $p_1,p_2=\{B,F_\uparrow,F_\downarrow\}$. While Pauli exclusion requires that $g_{F_\uparrow F_\uparrow}=g_{F_\downarrow F_\downarrow}=0$, an attractive coupling $g_{F_\uparrow F_\downarrow}\equiv g_{F_\downarrow F_\uparrow}<0$ will lead to BCS pairing. A nonzero interaction g_{FB} between bosons and fermions is required for thermalization between the two populations. We need put no other restrictions on $g_{p_1p_2}$, but our desired effect will be easier to observe experimentally with some other constraints. For instance, requiring that $g_{FB}<0$ will raise the BCS condensation temperature [21] while a larger $g_{BB}>0$ facilitates thermalization between bosons and fermions.

To proceed further we use the gap equation, Eq. (1), where we have $n_\mathbf{k}=n_\mathbf{k}^{(\mathrm{FD})}$ at equilibrium. In the Thomas-Fermi approximation, the transition temperature, $T_c^{(0)}$, is given by [5]

$$k_BT_c^{(0)}\simeq\frac{8\varepsilon_Fe^{\gamma-2}}{\pi}\exp\left[-\frac{\pi}{2k_F|a_{\uparrow\downarrow}|}\right], \tag{3}$$

where k_F is the Fermi wave vector and $\gamma\approx0.577\ldots$ is Euler's constant, the scattering length $a_{\uparrow\downarrow}$ is a simple combination of the coupling strengths $g_{p_1p_2}$.

A. Nonequilibrium enhancement

It is possible to create distributions that lead to larger order parameter and effective condensation temperature by weakly perturbing the trapped fermions. Specifically, we affect a Bragg pulse (Fig. 1) by illuminating the fermions with two lasers, which are both largely detuned from any fermionic transition. In what follows, we assume the lasers to be even further detuned from any bosonic transition such that we may ignore the effect of the Bragg pulse on the bosons. The interaction of the fermions with these lasers is described by the addition of a term

$$\hat{\mathcal{H}}_{\mathrm{bg}}=\int d^3\mathbf{r}\sum_{p_f=F_\uparrow,F_\downarrow}\hat{\psi}_{p_f}^\dagger(\mathbf{r})[\hbar\Omega_{bg}\cos(\mathbf{q}\cdot\mathbf{r}-\omega t)]\hat{\psi}_{p_f}(\mathbf{r})$$

to the Hamiltonian where $\mathbf{q}$ and ω represent the difference in wave vectors and frequencies between the two lasers [18]. Now, following Schmid [22] and the general argument in the introduction [see, Eq. (2)], we introduce the function, $\delta n_\mathbf{k}$, which describes departure from equilibrium. While Eliashberg assumed in [8] that the impurity concentration was high enough in metals such that momentum relaxation happened at a much faster rate than energy relaxation, we shall not make this assumption. As such, $\delta n_\mathbf{k}$ need not be isotropic although this requirement is easily included in our model. The corresponding term, $\chi/\nu(\varepsilon_F)$ from Eq. (2), is added to the right side of the gap Eq. (1) and leads to a new solution, $\Delta>\Delta_0$, for the order parameter at the same temperature (note that in the nonequilibrium situation, the notion of a temperature of the Fermi system is undefined, and here by temperature we imply the original temperature and that of the Bose bath). For $T-T_c^{(0)} \ll T_c^{(0)}$, Eq. (1) can be cast in the form of a Ginzburg-Landau equation, which, including the nonequilibrium term, becomes

$$\left(\ln\frac{T}{T_c^{(0)}} - \chi\right)\Delta + b\left(\frac{|\Delta|}{T_c^{(0)}}\right)^2\Delta = 0, \qquad (4)$$

where we assume that the coefficient in the cubic term of Eq. (4) is only weakly affected by the perturbation and use its standard BCS value [$\zeta(z)$ is the Riemann zeta function] $b=7\zeta(3)/(8\pi^2)\approx 0.107\ldots$ (see also, Ref. [23]). Equation (4) nominally leads to an exponential enhancement of the effective critical temperature: $T_c^{\rm eff}=e^{\chi}T_c^{(0)}$, if χ is positive.

B. Kinetic equation

To calculate this enhancement for the Bose-Fermi mixture, we shall balance the Boltzmann equation for $n_{\mathbf{k}}$ in the spirit of Eliashberg, including both a contribution due to collisions and that from Bragg scattering,

$$\dot{n}_{\mathbf{k}} = I_{\rm coll}[n_{\mathbf{k}}] + I_{\rm Bragg}[n_{\mathbf{k}}]. \qquad (5)$$

For small departures from equilibrium, we can linearize the collision integral in $\delta n_{\mathbf{k}}$ and use the $1/\tau$-approximation: $I_{\rm coll}[n]=-\delta n/\tau_0$, where τ_0 is the quasiparticle lifetime. In our system, this lifetime will be dominated by the inelastic collision time between bosons and fermions. We may estimate this time as in [24,25] by means of a $1/\tau_0=n\sigma v$ approximation, where n is the boson density at the center of the trap, σ is the constant low-temperature cross section for boson-fermion scattering, and v is the average relative velocity associated with the collisions between bosons and fermions.

Note that there exist other contributions to the collision integral, in particular those coming from fermion collisions. Our model assumes pointlike interactions between fermions: Such interactions can be separated into interactions in the reduced BCS channel, which involve particles with opposite momenta that eventually form Cooper pairs and other types of scattering events, which give rise to Fermi-liquid renormalizations on the high-temperature side and superconducting fluctuations on the BCS side. Note that dropping off the latter terms would lead to an integrable (Richardson) model that does not have any thermalization processes and therefore the collision integral for its quasiparticles must vanish. In thermodynamic limit this model is described by BCS mean-field theory perfectly well and so we can say that the pairing part of fermion interactions is already incorporated in our theory. Of course, including fermion-boson collision and the second type of fermion interaction processes break integrability and lead to two types of effects: First, such interactions lead to Fermi-liquid renormalizations of the effective mass and the quasiparticle Z-factor. However, these effects are not germane to the physics of interest, and we may assume that all relevant corrections are already included and treat our system as that consisting of Fermi-liquid quasiparticles. However, there is of a course a second dissipative part coming from interactions, such as those due to bosons already included into τ_0 and quasiparticle scatterings and decay processes due to non-BCS fermion-fermion collisions. Similarly to the work of Eliashberg, we will assume that the latter contribution to the collision integral is less significant than τ_0^{-1} due to the bosons. Fermi Liquid quasiparticles are exactly defined precisely on the Fermi sphere, but they have a finite lifetime due to decay processes elsewhere. By not including the lifetime $\tau_{\mathbf{k}}$ of a quasiparticle at momentum $\mathbf{k}$ in our linearization of the Boltzmann equation, we have implicitly assumed that $\tau_{\mathbf{k}} \gg \tau_0$. Because $\frac{1}{\tau_{\mathbf{k}}} \propto (\pi k_B T)^2 + (\varepsilon_{\mathbf{k}}-\varepsilon_F)^2$ in a Fermi Liquid [26], there will always be an energy region where this assumption will indeed be true for low temperatures. The most important contribution to the integral in the expression for χ comes from states for which $\varepsilon_{\mathbf{k}}$ is within $k_B T$ of ε_F. As such, if we require that $T \ll T_F$ and $\hbar\omega \ll \varepsilon_F$, then our linearization of the Boltzmann equation with respect to τ_0 will be legitimate for the calculation of an enhancement of superfluidity. Again, we stress the importance of recognizing that the aforementioned requirements are necessary only for quantitative accuracy of our model. As with Eliashberg's enhancement of superconductivity, we expect our effect to be observable far outside the constrained parameter space that is necessary for strict validity of our simple model, which provides a proof of principle for using nonequilibrium perturbations to enhance fermion pairing in cold atom systems.

With these caveats in mind, we shall tune the frequency of our Bragg pulse such that $\omega\tau_0 \gg 1$. This will ensure that any nonstationary part of the distribution function will be small [8]. Equivalently, we may think of this requirement as the statement that the Bragg pulse pumps the system out of equilibrium must faster than the system relaxes. We may note here that the aforementioned assumption that δn is small also implies that $\chi \ll 1$, thereby diminishing the desired effect, $T_c^{\rm eff}=(1+\chi)T_c^{(0)} \sim T_c^{(0)}$. Again, this approximation greatly simplifies our theoretical problem by allowing us to expand the Boltzmann equation, but it is only a mathematical convenience that represents no impediment to an experimentalist looking for striking enhancements of $T_c^{(0)}$.

Equation (5) can now be solved for δn to yield

$$\delta n_{\mathbf{k}} = \tau_0 I_{\rm Bragg}[n_{\mathbf{k}}^{\rm (FD)}](1-e^{-t/\tau_0}), \qquad (6)$$

which shows that a stationary nonequilibrium state is formed in a characteristic time τ_0. The Bragg part in Eq. (5), $I_{\rm Bragg}[n]$, now depends only on $n_{\mathbf{k}}^{\rm (FD)}$ and can be computed with Fermi's golden rule. When the wavelength of the Bragg pulse is much larger than the DeBroglie wavelength of the fermions and the reciprocal frequency of the pulse is much smaller than the relaxation time ($\lambda_F|\mathbf{q}| \ll 1$ and $\omega\tau_0 \gg 1$), Fermi's golden rule yields

$$I_{\rm Bragg}[n_{\mathbf{k}}^{\rm (FD)}] = \frac{2\pi}{\hbar}\Omega_{bg}^2\{n_{\mathbf{k}-\mathbf{q}}^{\rm (FD)}(1-n_{\mathbf{k}}^{\rm (FD)})\,\delta(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\mathbf{q}}-\hbar\omega) - n_{\mathbf{k}}^{\rm (FD)}(1-n_{\mathbf{k}+\mathbf{q}}^{\rm (FD)})\,\delta(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}-\hbar\omega)\}.$$

The determination of $I_{\rm Bragg}[n_{\mathbf{k}}^F]$ allows us to find χ and ultimately $T_c^{\rm (eff)}$. The optimal parameters depend on whether the fermionic gas is in the superfluid or normal phase at the time the Bragg pulse is applied. In the former case, the energy gap in the quasiparticle density of states protects a Bragg pulse with $\hbar\omega < 2\Delta_0$ from producing new quasiparticles that will hinder superfluidity. However, in the normal phase, the energy conservation requirement that $\epsilon_{\mathbf{k+q}}-\varepsilon_{\mathbf{k}}=\pm\hbar\omega$ and an independent control of $\mathbf{q}$ and ω allow us to engineer a pulse that ensures that only "thermal" quasiparticles with energies $\varepsilon>\varepsilon_F$ are pushed to even higher energies, while the fermions below ε_F are not affected.

Thus far, we have assumed that heat is being dissipated from the fermions into the bosons through collisions. Because Bose-Einstein condensation inhibits collisions with fermions and severely reduces thermalization between the two populations, our simple analysis depends on the bosons being at a constant temperature T greater than their Bose-Einstein condensation temperature $T_{\text{BEC}}\simeq 0.94\hbar\Omega_B N_B^{1/3}$ [27]. Condensation can be prevented at temperatures close to the BCS transition temperature by having $\Omega_B\ll\Omega_F$ and $m_B\gg m_F$. Treating the bosons classically, we expect their temperature to increase no faster than $\frac{dT}{dt}=\frac{1}{C}\Omega_{bg}\omega$, which is the energy pumping rate due to the Bragg pulse for a specific heat C. For a harmonically confined classical gas, we use a specific heat given by $C=3k_B N_B$. If t_{Bragg} is the time over which the Bragg pulse is turned on, then so long as $t_{\text{Bragg}}\frac{dT}{dt}$ is much less than the temperature of the bosons, we may consider the bosonic population to be an isothermal bath. This may be accomplished by having a large number of bosons at low density. One may be able to avoid the assumption of a bosonic population altogether when driving the transition from normal to superfluid at temperatures above $T_c^{(0)}$ by allowing energetic particles to leave the trap as in evaporative cooling. We shall show later that this is possible because the Bragg pulse may be tuned to couple only to particles with energies above a threshold energy depending on ω and $\mathbf{q}$. For concreteness however, we shall keep the bosonic population throughout the following section.

III. NUMERICAL RESULTS

As an example, we calculate the nonequilibrium enhancement of T_c for a trapped mixture of ^{87}Rb and ^{6}Li under the aforementioned assumptions with $\delta n\ll 1$. We assume a cloud of 10^5 lithium atoms and 10^7 rubidium atoms in traps of frequencies $\Omega_F=200\Omega_B=3$ kHz correspondingly. We use scattering lengths $a_{BB}=109a_0$, $a_{FF}=-2160a_0$, $a_{FB}=-100a_0$ [21] and a typical collision time $\tau_0\approx 136$ ns as estimated via the $1/\tau_0=n\sigma v$ approximation from above. With these parameters, the equilibrium BCS and BEC condensation temperatures are $T_c^{(0)}\approx 0.15\ T_F=291$ nK and $T_{\text{BEC}}\approx 23.2$ nK. The quasiparticle lifetime $\tau_{\mathbf{k}}$ for the normal fluid is estimated as $\tau_{\mathbf{k}}\approx 3\ \mu$s for $|\varepsilon_{\mathbf{k}}-\varepsilon_F|\simeq\hbar\omega$ via the methods in Refs. [26,28].

A. Superfluid at equilibrium

Let us assume that the system is initially in the superfluid phase at equilibrium with a low enough temperature $T<T_c^{(0)}$ such that $2\Delta>\hbar\omega=0.15\varepsilon_F$. Using Eq. (6), we may calculate the stationary distribution function (Fig. 2) for Bragg parameters $\Omega_{bg}=70\Omega_F$ and $|\mathbf{q}|=0.1k_F$. Some comments on the form of $\delta n_{\mathbf{k}}$ are necessary. As shown in Fig. 2(a), the first-order approximation to $n_{\mathbf{k}}$ has unphysical singularities at $E=\Delta$ and $E=\Delta+\omega$. These are due to first-order transitions to $E=\Delta+\omega$ from $E=\Delta$ where the quasiparticle density of states diverges for a superfluid at equilibrium. The exact distribution function, schematically drawn in Fig. 2(b), has no infinities. Higher orders in the expansion of the Boltzmann equation are necessary to curtail the singularities at $\varepsilon=\Delta_0+n\omega$ $(n=0,1,2\ldots)$. However, so long as these singularities are localized on energy intervals that are much smaller than ω, the approximate $\delta n_{\mathbf{k}}$ calculated from Eq. (6) will be suitable for the calculation of both the enhanced order parameter via Eqs. (1) and (2) as well as the value of χ in the Ginzburg-Landau equation [Eq. (4)] [8]. As expected from the analogy to Eliashberg's work, our singularities have energy widths of about $\Delta[2\pi\tau_0\Omega_{bg}^2/\Omega_F(6N_F)^{1/3}]$. Hence, we may consider the inequality $\frac{\Delta}{\omega}[2\pi\tau_0\Omega_{bg}^2/\Omega_F(6N_F)^{1/3}]\ll 1$ as a further requirement for the validity of our linearization of the Boltzmann equation for the calculation of χ in the superfluid case. We may also note that due to the fact that $\hbar\omega<2\Delta_0$, no new quasiparticles are excited from the lower branch by pair breaking. Hence, the quasiparticle number is conserved in this first-order approximation $(\Sigma_{\mathbf{k}}\delta n_{\mathbf{k}}=0)$. The quasiparticles are simply redistributed from the gap edge to higher energies. Substituting $\delta n_{\mathbf{k}}$ into Eq. (2), we find that at $T\simeq 0.13\ T_F$ we calculate an increase in Δ by a factor of 1/10. So long as $T<T_c^{(0)}$, the relative enhancement increases with temperature because there are more particles to redistribute and the pulse does not have enough energy to break Cooper pairs. Unlike in the normal fluid where $\Delta_0=0$ at equilibrium, the enhancement χ depends on the initial value of the gap. In Fig. 3, we plot the temperature dependence of the enhanced nonequilibrium gap with a slightly stronger pulse given by $\Omega_{bg}=110\Omega_F$, $\hbar\omega=0.15\varepsilon_F$, and $|\mathbf{q}|=0.1\ k_F$. Note that after the

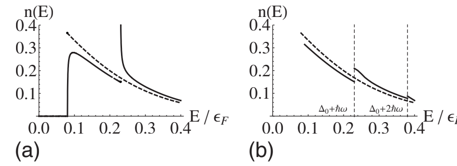

FIG. 2. (a) The first-order approximation to the quasiparticle occupation as a function of $E=\sqrt{\xi^2+\Delta_0^2}$ for parameters $\Omega_{bg}=70\ \Omega_F$, $|\mathbf{q}|=0.1k_F$, and $\hbar\omega=0.15\varepsilon_F$. The unphysical singularities at $E=\Delta_0$ and $E=\Delta_0+\hbar\omega$ are not included in the calculation of χ. See text for details. The equilibrium values for this system are $T=0.14T_F$ and $\Delta_0=0.08\varepsilon_F$. (b) The exact distribution function schematically drawn for the same parameters with Δ_0 and $\hbar\omega$ in units of ε_F. The thick dashed lines represent the occupation at equilibrium.

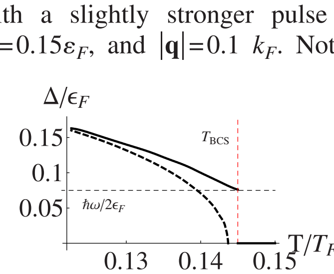

FIG. 3. (Color online) The order parameter Δ as function of the temperature which solves the nonequilibrium gap equation for parameters $\Omega_{bg}=110\Omega_F$, $|\mathbf{q}|=0.1k_F$, and $\hbar\omega=0.15\varepsilon_F$. The black dashed line is the equilibrium dependence while the red dashed line gives the nonequilibrium transition temperature $T_{\text{BCS}}>T_{\text{BCS}}^{(0)}$. We have constrained $\Delta_0>\hbar\omega/2$ to avoid pair breaking.

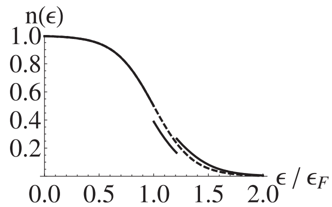

FIG. 4. The $T=0.18$ T_F normal phase occupation as a function of energy for $\Omega_{bg}=110$ Ω_F, $|\mathbf{q}|=0.1k_F$, and $\hbar\omega=0.21\varepsilon_F$. The enhanced nonequilibrium transition temperature is $T_{\mathrm{BCS}}=1.3T_{\mathrm{BCS}}^{(0)}$ The dashed line is the occupation at equilibrium.

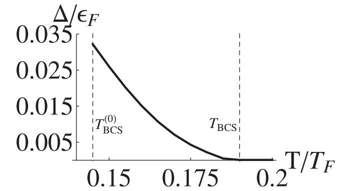

FIG. 5. The enhanced gap Δ as function of temperatures above $T_{\mathrm{BCS}}^{(0)}$ for fermions in the normal phase at equilibrium and parameters $\Omega_{bg}=110\Omega_F$, $|\mathbf{q}|=0.1k_F$, and $\hbar\omega=0.21$ ε_F

pulse is applied at equilibrium, the temperature of the bath may be increased above $T_c^{(0)}$ while maintaining a nonzero gap. This temperature dependence is exactly what would be expected from the analogous plot in Ref. [8]. The new BCS transition temperature T_c is given by the maximum of the nonequilibrium plot where the inequality $\hbar\omega<2\Delta_0$ is saturated. If the temperature is further increased, Δ discontinuously vanishes again. For these parameters we calculate a small 3% increase in the transition temperature as expected from our requirement that nonlinear effects (even gap enhancing effects) be ignored in the Boltzmann equation. The temperatures we have considered are sufficiently close to the equilibrium transition temperature such that the approximation $T_{\mathrm{BCS}}\approx T_{\mathrm{BCS}}^{(0)}(1+\chi)$ is justified. For small Δ_0 a crude order-of-magnitude estimate of χ may be given by $\chi\sim[2\pi\hbar\Omega_{bg}^2\tau_0/\varepsilon_F]n_0(\Delta_0)n_0(\Delta_0+\hbar\omega)$.

B. Normal at equilibrium

For contrast, let us now assume that we start in the normal phase at equilibrium. The initial distribution function is simply the Fermi-Dirac function for the noninteracting quasiparticles of Fermi Liquid theory. We have $\Delta_0=0$, so Eliashberg's requirement that $\hbar\omega<2\Delta_0$ cannot be satisfied. However, we can guarantee that particles are not excited from below the Fermi level by choosing ω and $\mathbf{q}$ such that the constraint $4\varepsilon_{\mathbf{q}}\varepsilon_F\leq(\hbar\omega-\varepsilon_{\mathbf{q}})^2$ is enforced. Because of this requirement, momentum and energy conservation cannot be simultaneously achieved for particles with energies less than ε_F. As such, only particles outside the Fermi sphere can undergo transitions. The lower equilibrium occupation number and higher density of states at high energies ensures that quasiparticles just outside the Fermi sphere are excited to higher energies.

Although in our system we have particle conservation just as in the superfluid case, we comment here that if these excited particles are allowed to leave the trap, then we have effectively cooled the fermions by sharpening the Fermi step, and the boson bath is unnecessary. Substituting $\delta n_{\mathbf{k}}$ into Eq. (2), we see that the depression of the population at the Fermi level shown in Fig. 4 allows for a nonzero Δ above $T_{\mathrm{BCS}}^{(0)}$. As the temperature is increased, the nonequilibrium gap enhancement is overpowered by thermal smearing of the distribution function. There are more quasiparticles at energies near ε_F where they most strongly hinder superfluidity. This contrasts with the superfluid situation wherein the enhancement increases with temperature so long as $T<T_{\mathrm{BCS}}^{(0)}$. This effect can be seen in Fig. 5, where we find an enhancement of T_{BCS} by about 30% for parameters $\Omega_{bg}=110\Omega_F$, $|\mathbf{q}|=0.1k_F$, and $\hbar\omega=0.21\varepsilon_F$. This increase is much more drastic than the enhancement in the superfluid phase because the requirements that $\delta n_{\mathbf{k}}\ll n_{\mathbf{k}}^{(\mathrm{FD})}$ and $\hbar\omega\ll\varepsilon_F$ are much less stringent than the superfluid requirements $\frac{\Delta}{\omega}\frac{2\pi\tau_0\Omega_{bg}^2}{\Omega_F(6N_F)^{1/3}}\ll1$ and $\hbar\omega\leq\Delta$. Thus, we may use a stronger pulse while still linearizing the Boltzmann equation legitimately. As such, fermionic superfluidity is expected to appear at temperatures as high as $T\approx1.3T_c^{(0)}$.

IV. SUMMARY

To conclude, we have shown that perturbing a system of trapped fermions creating a stationary quasiparticle distribution can be an effective way to stimulate fermion pairing and superfluidity. To demonstrate this, we calculate enhancements of the BCS order parameter and the transition temperature for a mixture of ^{87}Rb and ^{6}Li that is pushed out of equilibrium by a Bragg pulse. The mechanism by which fermions within the Fermi sphere are not excited differs depending on initial conditions. If the gas is a superfluid at equilibrium, these excitations are precluded by keeping $\hbar\omega<2\Delta_0$. In the normal phase, the parameters of the pulse can be chosen such that fermions below a certain energy cannot simultaneously satisfy momentum and energy conservation. Thus, they are not excited. In both cases, the enhancements that we calculate are small, but this is a consequence of our perturbative treatment rather than a physical constraint. This is evidenced by the strong effects reported from experiments on superconductors [12,13], which are based on the same underlying mechanism. Finally, we suggest that by enhancing or creating a discontinuity in the quasistationary strongly nonequilibrium distribution of fermions (not necessarily at the Fermi momentum) via the technique proposed in this paper, one may achieve effective BCS pairing at nominally very high temperatures of the bath.

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