

Jaynes-Cummings models with trapped electrons on liquid helium

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Jaynes-Cummings model is a typical model in quantum optics and has been realized with various physical systems [e.g. cavity quantum electrodynamics (QED), trapped ions, circuit QED, etc.] of two-level atoms interacting with quantized bosonic fields. Here, we propose an implementation of this model by using a single classical laser beam to drive an electron floating on liquid helium. Two lowest levels of the *vertical* motion of the electron act as a two-level “atom” and the quantized vibration of the electron along one of the *parallel* directions, e.g., x direction, serves as the bosonic mode. These two degrees of freedom of the trapped electron can be coupled together by using a classical laser field. If the frequencies of the applied laser fields are properly set, the desirable Jaynes-Cummings models could be effectively realized.

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Jaynes-Cummings model (JCM), describing the basic interaction of a two-level atom and a quantized electromagnetic field, is a cornerstone for the treatment of the interaction between light and matter in *quantum optics* [1]. This model can explain many quantum phenomena, such as the collapses and revivals of the atomic population inversions, squeezing of the quantized field, and the atom-cavity entanglement. Furthermore, recent experiments show that the JCMs can be implicated in quantum-state engineering and quantum information processing, e.g., generation of Fock states [2] and entangled states [3], the implementations of quantum logic gates [4], etc. Originally, JCM is physically implemented with a cavity quantum electrodynamics (QED) system (see, e.g., [5]). Certainly, there has been also interest to realize the Jaynes-Cummings Hamiltonian with other physical systems. A typical system is a cold ion trapped in a Paul trap and driven by classical laser beams [6–8]. There, the interaction between two selected internal electronic levels and the external vibrational mode of the ion can be induced. Under the so-called Lamb-Dicke (LD) limit and the well-known rotating-wave approximation, the desirable JCM (or anti-JCM) can be realized by setting the applied laser frequencies with the suitable red (or blue) sideband excitations.

Recently, Platzman and Dykman proposed that the electrons floating on liquid helium could be utilized to implement quantum computation [9,10]. In this proposal, electrons are trapped on the surface of liquid helium and controlled by a series of external electric fields, which are generated by the microelectrodes set below the liquid helium. These electrons are effectively coupled together via their Coulomb interactions. By applying microwave radiation to these electrons from the microelectrodes, their quantum states could be coherently controlled. Due to its scalability, easy manipulation, and relative long coherence time, this system has been paid much attention in recent years for quantum information processing (see, e.g., [9–13]).

In this Brief Report, we further show, theoretically that *an electron floating on liquid helium* could also be utilized to

realize the desirable JCMs. Inspired by the idea of implementing JCMs with trapped ions, we use a classical laser field to couple the vertical and parallel motional degrees of freedom of the electron on liquid helium (similar to the laser-assisted coupling between the internal and external states of trapped ions).

We consider an electron floating on the surface of liquid helium (e.g., ${}^4\text{He}$). The electron is weakly attracted by the dielectric image potential and strongly repulsed by the Pauli potential (i.e., Pauli exclusion principle), with about 1 eV potential barrier, to prevent it from penetrating into the liquid helium. As a consequence, the electron’s motion normal to the liquid-helium surface can be approximately described by a one-dimensional (1D) hydrogen with the following potential [14]:

$$V(z) = \begin{cases} -\frac{\Lambda e^2}{z}, & z > 0 \\ +\infty, & z \leq 0. \end{cases} \quad (1)$$

Where, e is electron (with mass m_e) charge, z is the distance above liquid-helium surface, and $\Lambda = (\epsilon - 1)/4(\epsilon + 1)$, with $\epsilon = 1.0568$ being dielectric constant of liquid ${}^4\text{He}$. The energy levels associated with this motion form a hydrogenlike spectrum $E_n = -\Lambda^2 e^4 m_e / 2n^2 \hbar^2 \approx -0.00065/n^2$ eV, which has been experimentally observed [15], and the corresponding wave functions can be written as [16]

$$\psi_n(z) = 2n^{-5/2} r_B^{-3/2} z \exp\left[-\frac{z}{nr_B}\right] L_{n-1}^{(1)}\left(\frac{2z}{nr_B}\right), \quad (2)$$

with the Bohr radius $r_B = \hbar^2 / (m_e e^2 \Lambda) \approx 76$ Å and Laguerre polynomials

$$L_n^{(\alpha)}(x) = \frac{e^x x^{-\alpha}}{n!} \frac{d^n}{dx^n} [e^{-x} x^{n+\alpha}]. \quad (3)$$

Beside the image potential (1), the electron is also trapped by another potential generated by the charge Q on the microelectrode, which is located at h beneath the liquid-helium surface [10]. The configuration of our model is shown in Fig. 1. For simplicity, on the helium surface, the electron is assumed to be effectively constrained to move only along the x

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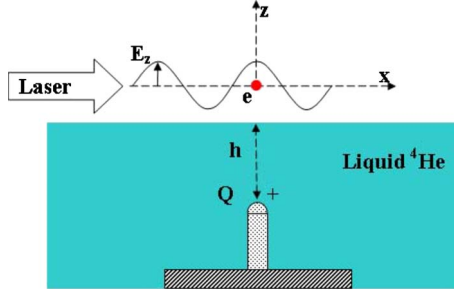


FIG. 1. (Color online) A sketch of an electron confined by a microelectrode Q submerged by the depth h beneath the helium surface and driven by a classical laser field propagating along the x direction.

axes. Therefore, under the usual condition $z, x \ll h$, the total potential of the electron can be effectively approximated as [10]

$$U(z, x) \approx -\frac{\Lambda e^2}{z} + eE_{\perp}z + \frac{1}{2}m_e\nu^2x^2, \quad (4)$$

with $E_{\perp} \approx Q/h^2$ and $\nu \approx \sqrt{eQ/(m_e h^3)} = \sqrt{eE_{\perp}/(m_e h)}$. This indicates that the motions of the trapped electron can be regarded as a 1D Stark-shifted hydrogen along the z direction and a harmonic oscillation along the x direction. Following Dykman *et al.* [10], only two lowest levels (i.e., the ground state $|g\rangle$ and first-excited state $|e\rangle$) of the 1D Stark-shifted hydrogen are considered. As a consequence, the Hamiltonian describing these two uncoupled degrees of freedom of the electron reads

$$\hat{H}_0 = \hbar\nu \left(\hat{a}^{\dagger} \hat{a} + \frac{1}{2} \right) + \frac{\hbar\omega_0}{2} \hat{\sigma}_z. \quad (5)$$

Here, \hat{a}^{\dagger} and \hat{a} are the bosonic creation and annihilation operators of the vibrational quanta (with frequency ν) of the electron's oscillation along the x direction. $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ is the Pauli operator. The transition frequency ω_0 is defined by $\omega_0 = (E_e - E_g)/\hbar$, with E_g and E_e being the corresponding energies of the lowest two levels, respectively.

In order to couple the above two uncoupled degrees of freedom of the electron, we now apply a classical laser beam $\mathcal{E}(x, t)$, propagating along the x direction, to the trapped electron (see Fig. 1). This is similar to the approach in ion trap system for coupling the external and internal degrees of freedom of the ion [7]. Suppose that the applied laser beam (of wave vector k_l , amplitude E_z , frequency ω_l , and initial phase ϕ_l) takes the form $\mathcal{E}(x, t) = E_z \hat{z} \cos(k_l x - \omega_l t + \phi_l)$, i.e., its electric field is z -direction polarization, then the Hamiltonian of the driven electron floating on the helium can be written as

$$\hat{H} = \hat{H}_0 + e z \mathcal{E}(x, t). \quad (6)$$

Certainly, $x = \sqrt{\hbar/2m_e\nu}(\hat{a} + \hat{a}^{\dagger})$ and thus the above Hamiltonian can be further written as

$$\begin{aligned} \hat{H} = & \hat{H}_0 + \hbar\tilde{\Omega}\hat{\sigma}_z (e^{i\eta(\hat{a} + \hat{a}^{\dagger}) - i\omega_l t + i\phi_l} + e^{-i\eta(\hat{a} + \hat{a}^{\dagger}) + i\omega_l t - i\phi_l}) \\ & + \hbar\Omega(\hat{\sigma}_- + \hat{\sigma}_+) (e^{i\eta(\hat{a} + \hat{a}^{\dagger}) - i\omega_l t + i\phi_l} + e^{-i\eta(\hat{a} + \hat{a}^{\dagger}) + i\omega_l t - i\phi_l}), \end{aligned} \quad (7)$$

with $\Omega = \langle g|z|e\rangle eE_z/(2\hbar)$ being the so-called carrier Rabi frequency describing the strength of coupling between the applied laser field and the electron and $\tilde{\Omega} = (\langle e|z|e\rangle - \langle g|z|g\rangle)eE_z/(4\hbar) \neq 0$ due to the broken parities of the quantum states of the above 1D hydrogen. Also, $\eta = k_l \sqrt{\hbar/2m_e\nu}$ is the so-called LD parameter, which describes the strength of coupling between the motions of z and x directions of the trapped electron. Finally, $\hat{\sigma}_- = |g\rangle\langle e|$ and $\hat{\sigma}_+ = |e\rangle\langle g|$ are the usual raising and lowering operators, respectively.

Now, we assume that the frequencies of the applied laser fields are sequentially set as $\omega_l = \omega_0 + K\nu$, with $K=0, \pm 1$ corresponding to the usual resonance ($K=0$), the first blue- ($K=1$), and red- ($K=-1$) sideband excitations [17], respectively. The LD parameters introduced above become $\eta = (\omega_0 + K\nu)\sqrt{\hbar/(2m_e\nu)}/c$ (where c is the velocity of light) and are sensitive to the frequencies ω_0 and ν , which are further relative to the applied trap field E_{\perp} and the depth h of the microelectrode set beneath the liquid-helium surface. Furthermore, by properly setting the values of E_{\perp} and h , the above LD parameters could be significantly small (i.e., $\eta \ll 1$) and the so-called LD approximation [7] would work very well. Therefore, under the usual rotating-wave approximation [18], the effective Hamiltonian of the system (in the interaction picture) reads

$$\hat{H}_{\text{eff}}^0 = \hbar\Omega e^{i\phi_l} \hat{\sigma}_+ + \text{H.c.} \quad \text{for } K=0, \quad (8)$$

$$\hat{H}_{\text{eff}}^r = i\eta\hbar\Omega e^{i\phi_l} \hat{\sigma}_+ \hat{a} + \text{H.c.} \quad \text{for } K=-1, \quad (9)$$

and

$$\hat{H}_{\text{eff}}^b = i\eta\hbar\Omega e^{i\phi_l} \hat{\sigma}_+ \hat{a}^{\dagger} + \text{H.c.} \quad \text{for } K=1, \quad (10)$$

respectively.

Obviously, Hamiltonians \hat{H}_{eff}^r and \hat{H}_{eff}^b are nothing but just those of the usual JCM and anti-JCM, respectively. Also, all the dynamical evolutions corresponding to the above effective Hamiltonians (8)–(10) are exactly solvable. For example, if the x direction's harmonic oscillator is prepared initially at the Fock state $|m\rangle$ (m is its occupation number), then we have:

(i) For $K \leq 0$,

$$\begin{cases} |m\rangle|g\rangle \rightarrow |m\rangle|g\rangle, & m < k \\ |m\rangle|g\rangle \rightarrow \cos(\Omega_{m-k,t})|m\rangle|g\rangle + i^{k-1}e^{i\phi_l} \sin(\Omega_{m-k,t})|m-k\rangle|e\rangle, & m \geq k \\ |m\rangle|e\rangle \rightarrow \cos(\Omega_{m,k,t})|m\rangle|e\rangle - (-i)^{k-1}e^{-i\phi_l} \sin(\Omega_{m,k,t})|m+k\rangle|g\rangle, \end{cases} \quad (11)$$

(ii) For $K \geq 0$,

$$\begin{cases} |m\rangle|g\rangle \rightarrow \cos(\Omega_{m,k}t)|m\rangle|g\rangle + i^{k-1}e^{i\phi_l} \sin(\Omega_{m,k}t)|m+k\rangle|e\rangle \\ |m\rangle|e\rangle \rightarrow |m\rangle|e\rangle, \quad m < k \\ |m\rangle|e\rangle \rightarrow \cos(\Omega_{m-k,k}t)|m\rangle|e\rangle - (-i)^{k-1}e^{-i\phi_l} \sin(\Omega_{m-k,k}t)|m-k\rangle|g\rangle, \quad m \geq k, \end{cases} \quad (12)$$

with $\Omega_{m,k} = \Omega \eta^k \sqrt{(m+k)!/m!}$ being the effective Rabi frequency and $k=|K|$. In principle, arbitrary quantum-state engineering, e.g., generations of nonclassical quantum states, implementations of quantum logic gates, etc. [4,7,8], could be realized by the above evolutions.

The experimental feasibility of the JCMs proposed here involves with two important factors: the value of the introduced LD parameter η and the decoherence of the electron. In fact, decoherence is always a challenge in various quantum coherence systems. Platzman and Dykman [10] showed that the main source of decoherence in the present system is the so-called riplons, i.e., the thermally excited surface waves of liquid helium [9,10]. The coherence time due to this fluctuation is estimated [9,10] to be 10^{-4} s (for the typical frequencies, a few tens of GHz), but could be increased by enhancing the frequency of the electron vibrating in plane.

For the typical parameters $E_{\perp} = 10^4$ V/m and $h = 5 \times 10^{-7}$ m [10], the transition frequency of the z direction's 1D hydrogen and the vibrational frequency of the x direction's oscillation are estimated as $\omega_0/(2\pi) \approx 180$ GHz and $\nu/(2\pi) \approx 9.4$ GHz, respectively. Consequently, the LD parameter in the above JCM is $\eta \approx 1.2 \times 10^{-4} \ll 1$. Thus, the usual LD approximation is valid. Note that the LD parameter in present system is significantly smaller than that (there $\eta \sim 0.2$) in the experimental ion trap system [7,8]. This is because the ‘‘atomic’’ frequency $\omega_0/(2\pi)$ of the trapped ion ($\sim 10^6$ GHz) is significantly *larger* than that in the present system and the vibrational frequency $\nu/(2\pi)$ ($\sim 10^{-4}$ GHz) is significantly however *smaller* than that in the present system. Note that the LD parameters could, in principle, be enlarged by decreasing the value of $\nu/(2\pi)$ (via properly adjusting E_{\perp} and h). For example, $\eta \approx 0.16$ with $\omega_0/(2\pi) \approx 118$ GHz and $\nu/(2\pi) \approx 2.1$ KHz for $E_{\perp} = 10^{-5}$ V/m and $h = 10^{-2}$ m. However, riplons-induced decoherence affects stronger for the smaller frequency of the vibrations in the plane (correspond to a large in-plane localization length).

Fortunately, although the LD parameters in present system are relatively small, the JCMs presented above still work within the typical coherence time ($\sim 10^{-4}$ s). Our numerical estimations show that the duration of a π pulse is $t = \pi/\Omega_{m,k} < 10^{-4}$ s. For example, if the amplitude of the applied laser field is set as the typical value $E_z = 10^2$ V/m [10] and the LD parameter $\eta = 1.2 \times 10^{-4}$, we have $\pi/\Omega_{m,0} \approx 9.1 \times 10^{-9}$ s and $\pi/\Omega_{m,1} < 7.6 \times 10^{-5}$ s. Note that the occupation number m does not affect the values of $t = \pi/\Omega_{m,0}$, while $t = \pi/\Omega_{m,1}$ decreases with the increase of m ($t \propto 1/\sqrt{m+1}$). Also, the above durations could be further shortened (such that the JCMs admit more π -pulse operations) by effectively increasing the amplitude E_z of the applied laser field (i.e.,

increasing the carrier Rabi frequency Ω). In principle, if the E_z increases 10 times, then the duration of a π pulse shortens 10 times. Indeed, for $E_z = 10^3$ V/m, a π pulse could be less than 7.6×10^{-6} s.

The standard JCM requires that the bosonic field should be in a pure state. However, thermal states

$$\rho_r = \sum_{m=0}^{\infty} \frac{\langle m \rangle^m}{(1 + \langle m \rangle)^{m+1}} |m\rangle\langle m|, \quad \langle m \rangle = \frac{1}{e^{\hbar\nu/(k_B T)} - 1} \quad (13)$$

are the natural states of the vibrational particles (e.g., trapped ions [8] and the electrons in the present model), which are normally in thermal equilibrium with their surroundings. Above, k_B and T are the Boltzmann constant and the temperature of the surroundings, respectively. Certainly, if the temperature of the surrounding is further lower, the probabilities that the electron in the states with smaller occupations are much larger. Suppose that the liquid helium is cooled to $T = 0.01$ K [10], which is much colder than thermal vibration of the electron with frequency $\nu/(2\pi) \approx 9.4$ GHz ≈ 0.45 K, then $\rho_r \approx |0\rangle\langle 0|$ and thus the electronic vibration is well limited to the vacuum state.

In addition, the presented JCMs could be utilized to cool the vibrational electron. Indeed, if the out-plane state of the electron is initially in $|g\rangle$, a vibrational energy $\hbar\nu$ could be reduced by the following two steps: (i) apply a $\pi/2$ pulse with duration $t = \pi/2\Omega_{m-1,1}$ to drive a transition $|m\rangle|g\rangle \rightarrow |m-1\rangle|e\rangle$; (ii) drive the transition $|e\rangle \rightarrow |g\rangle$ but forbid the transition $|g\rangle \rightarrow |e\rangle$ by using an auxiliary atomic level $|a\rangle$ and two resonant $\pi/2$ pulses to drive the transitions $|e\rangle \rightarrow |a\rangle \rightarrow |g\rangle$. For example, if the third level of the electron is selected to be the auxiliary level $|a\rangle$, we have $\omega_{ea}/\omega_{ge} \approx 0.18$ with ω_{ea} being the transition frequency between $|e\rangle$ and $|a\rangle$ and ω_{ge} between $|g\rangle$ and $|e\rangle$. After these two steps, cooling the vibrational electron by a $\hbar\nu$ is possible

$$|m\rangle|g\rangle \rightarrow |m-1\rangle|e\rangle \rightarrow |m-1\rangle|a\rangle \rightarrow |m-1\rangle|g\rangle. \quad (14)$$

These operations (their durations are typically less than 4×10^{-6} s for $E_z = 10^3$ V/m) are repeated until the vibrational state $|m\rangle$ relaxes finally to the desirable ground state $|0\rangle$. As a consequence, an arbitrary mix state $\rho = \sum_m P_m |m\rangle\langle m|$ (with P_m being the classical probability that the electron is in the vibrational state $|m\rangle$) could be cooled to the vibrational vacuum state $|0\rangle$. Note that the above method is similar to the so-called *sideband laser cooling* technique used usually in the trapped-ion system [7].

In conclusion, we have proposed a candidate for realizing the famous JCMs, *electrons on liquid helium*, by applying classical laser fields to the trapped electrons for coupling their motions along the x and z directions. We have shown that the desirable JCMs and anti-JCMs could be implemented by properly setting the frequencies of the applied laser beams to excite the first red and blue sidebands, respec-

tively. The present proposal provides a way to apply the famous JCMs in condensed matters.

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