Laser oscillation and light entanglement via dressed-state phase-dependent electromagnetically induced transparency

Jingyan Li and Xiangming Hu*
Department of Physics, Huazhong Normal University, Wuhan 430079, People's Republic of China
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We show that it is possible to achieve laser oscillation and two-mode entanglement by using dressed-state phase-dependent electromagnetically induced transparency (EIT) in a double Λ system. Under certain conditions, two beams of bichromatic fields induce the depopulation of a coherent superposition state of the two excited states and the quantum beat of a pair of cavity fields. While one of two beams of bichromatic fields dresses the atoms, the other and the pair of cavity fields are in phase-controlled EIT interaction with the dressed atoms. On the basis of this, the pair of cavity fields not only operates well above threshold and exhibit subshot noise but also is in an entangled state. This mechanism suggests an efficient way to achieve bright entangled source of light.

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I. INTRODUCTION

Electromagnetically induced transparency (EIT) is one of the most important coherent phenomena. The underlying mechanism of EIT is the coherent population trapping (CPT) [1]. The essence of EIT and CPT is a creation of a coherent superposition of long-lived states [1–8]. This superposition state is usually called a dark state. In the exact EIT and CPT situations, the atoms are prepared in the dark state and are no longer excited by radiation. EIT is the basis for lasing without population inversion [9–14]. Atoms in the lower-lasing state do not absorb light and a small quantity of the atoms in the upper lasing state via a weak additional excitation contributes to the gain. The reduced absorption or excitation is also the mechanism for the suppression of spontaneous emission and the generation of squeezed light. This gives the possibilities of generating subshot light from a laser without inversion [15–20]. In some situations, the EIT configuration is created when ac-Stark splitting is included [21]. In this case, one has dressed-state EIT. At the same time, the population transfer between split levels is intrinsically existent, as in dressed-state lasers, and thus no additional pumping is required for the light amplification. This gives laser oscillation and output light with subshot noise [21]. Also, it is possible to achieve both large self-phase modulation and vanishing absorption [22], which lay a basis for frequency conversion and squeezed-light generation [23].

The dark state is created for any laser phases and intensities as in a two colored driven three-level Λ system. The only necessary condition is two-photon resonance of the atom-field interaction. As long as the applied laser fields do not change or change adiabatically, the dark state is also prepared via optical pumping in the continuous-wave regime or via adiabatic following in the pulsed regime, independent of the laser phases. This, however, is not the case when the laser fields interact with atoms in a closed loop of transitions. More often than not, one faces with a system with a closed loop of transitions. In particular, Kosachiov *et al.* [24] showed that the relative phase of the transitions plays a crucial role in determining both dynamics and the steady state of the atoms in closed-loop systems. In particular, the dark state is existent only for specific values of the phase, even if the multiphoton resonance condition is satisfied. Korsunsky *et al.* [25,26] described and observed the phase-dependent EIT. Fleischhaker and Evers [27] discussed the pulse propagation in a phase-dependent EIT medium. The phase-dependent EIT and CPT turn out to be the basis for controlling a type of the atom-field interactions. One can control the properties of medium and optical fields by the input field phases without changing neither frequencies nor input intensities. This not only gives an additional degree of freedom in the coherent optical control, but also allows one to manipulate the light-atom interactions in a more sophisticated manner than in conventional EIT.

Here we present a scheme, showing that the dressed-state phase-dependent EIT is an efficient way for obtaining bright entangled source of light. We use two pairs of bichromatic fields to couple the atoms in a double Λ configuration. On multiphoton resonances in a closed loop (phase dependent), a superposition state of the two excited states has vanishing population (coherent depopulation, i.e., the counterpart of CPT [1]), which leads to the creation of quantum beat of a pair of cavity fields. When one bichromatic field induces the Stark splitting, the other bichromatic field and the pair of cavity fields are in the phase-dependent EIT interaction with dressed atoms. By this mechanism, the two generated fields run well above threshold and display subshot noise and are entangled with each other.

So far, there have been several schemes that have been proposed for obtaining light entanglement via atomic coherent effects. Examples include those that are based on correlated spontaneous emission [28,29], atomic reservoir engineering with two-step four-wave mixing [30], dispersive atom-field interaction [31], optical bistability with EIT, and cavity dispersion [32]. Our scheme fundamentally differs from those in that it is based on the coherent effects of multiphoton resonances, which are established in a closed loop

*Author to whom correspondence should be addressed; xmhu@phy.ccnu.edu.cn

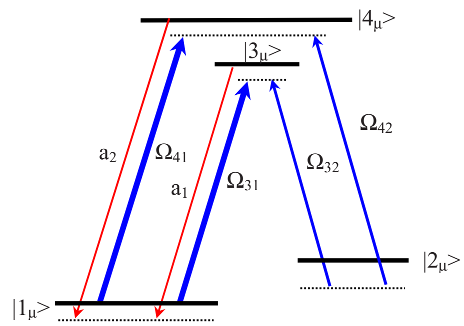

FIG. 1. (Color online) Level diagram for the double-Λ system. Four external fields are applied to atomic transitions $|k^{\mu}\rangle - |l^{\mu}\rangle$ with complex Rabi frequencies Ω_{kl} ($k=3,4;l=1,2$). a_l ($l=1,2$) are the annihilation operators for the cavity fields.

of double Λ configuration. The characteristic features are as follows. (i) Quantum beat is created due to coherent depopulation effect. (ii) Phase-dependent dressed-state EIT is formed when the ac-Stark splitting is included. (iii) Light amplification of one collective mode is achieved by combining the dressed-state EIT and the intrinsic incoherent transfer of population. (iv) Both the quantum noise squeezing and quantum entanglement are achieved via intrinsic feedback when the laser runs well above threshold. To our knowledge, such phase-controlled coherent effects of multiphoton resonances on light amplification and quantum correlations have not been reported yet. Our main purpose of the present paper is to show that the multiphoton resonances are efficient in the coherent control of laser oscillation and light entanglement.

This paper is organized as follows. In Sec. II, we present the model and derive the master equation for the atom-field system. In Sec. III, we give the steady-state intensities and quantum correlations. Realistic considerations are presented in Sec. IV and the conclusion is given in Sec. V.

II. MODEL AND MASTER EQUATION

An ensemble of N atoms in double-Λ configuration (Fig. 1) is placed in a two-mode cavity. Four external coherent fields of frequencies ω_{kl} are applied to the four transitions $|k^{\mu}\rangle - |l^{\mu}\rangle$ ($k=3,4;l=1,2$) of the μth atom in a closed loop with complex Rabi frequencies $\Omega_{kl}\exp(-i\phi_{kl})$ ($\Omega_{kl}>0$) and wave vectors $\mathbf{k}_{kl}$, respectively. Two cavity modes $a_{1,2}$ of frequencies $\omega_{1,2}$ and wave vectors $\mathbf{k}_{1,2}$ are generated from the transitions $|1^{\mu}\rangle - |3^{\mu},4^{\mu}\rangle$, respectively. The transitions $|1^{\mu}\rangle - |2^{\mu}\rangle$ and $|3^{\mu}\rangle - |4^{\mu}\rangle$ are dipole forbidden. The master equation for the atom-field density operator is derived in the dipole wave approximation and in the interaction picture as [33]

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}_a\rho + \mathcal{L}_f\rho, \tag{1}$$

where $H=H_0+V$, the Hamiltonian H_0 characterizes the interaction of the applied external fields with the atoms, and V denotes the interaction of the cavity fields with atoms

$$H_0 = \sum_{k=3,4}\sum_{\mu=1}^{N}\frac{\hbar}{2}\{\Omega_{k1}\sigma_{k1}^{\mu}\exp[i(\tilde{\mathbf{k}}_{k1}\cdot\mathbf{r}^{\mu} - \Delta_{k1}t - \phi_{k1})] + \Omega_{k2}\sigma_{k2}^{\mu}\exp[i(\mathbf{k}_{k2}\cdot\mathbf{r}^{\mu} - \Delta_{k2}t - \phi_{k2})]\} + \text{H.c.}, \tag{2}$$

$$V = \sum_{\mu=1}^{N}\hbar\{g_1^{\mu}a_1\sigma_{31}^{\mu}\exp[i(\tilde{\mathbf{k}}_{31}\cdot\mathbf{r}^{\mu} - \delta_1 t)] + g_2^{\mu}a_2\sigma_{41}^{\mu}\exp[i(\tilde{\mathbf{k}}_{41}\cdot\mathbf{r}^{\mu} - \delta_2 t)]\} + \text{H.c.} \tag{3}$$

Here, H.c. represents Hermitian conjugates. $\sigma_{kl}^{\mu}=|k^{\mu}\rangle\langle l^{\mu}|$ ($k,l=1-4$) are the atomic projection operators ($k=l$) and the flip operators ($k\neq l$) for μth atom. $a_l, a_l^{\dagger}$ ($l=1,2$) are annihilation and creation operators and $g_{1,2}^{\mu}$ describe the couplings of the μth atom to the cavity modes 1 and 2. $\Delta_{kl}=\omega_{kl}-\bar{\omega}_{kl}$ are detunings of driving field frequencies from atomic resonance frequencies, where $\bar{\omega}_{kl}$ are the atomic resonance frequencies. $\delta_1=\omega_1-\bar{\omega}_{31}$ and $\delta_2=\omega_2-\bar{\omega}_{41}$ are the detunings of the cavity frequencies from atomic resonance frequencies. The sum of wave vectors $\tilde{\mathbf{k}}_{31}=\mathbf{k}_{31}+\mathbf{k}_1$ ($\tilde{\mathbf{k}}_{41}=\mathbf{k}_{41}+\mathbf{k}_2$) originates from the simultaneous couplings of two fields to the $|1^{\mu}\rangle - |3^{\mu}\rangle$ ($|1^{\mu}\rangle - |4^{\mu}\rangle$) transition. $\mathcal{L}_a\rho$ and $\mathcal{L}_f\rho$ represent the atomic and field damping terms, respectively,

$$\mathcal{L}_a\rho = \sum_{k=3,4;l=1,2}\gamma_{kl}\mathcal{L}_{lk}\rho,$$

$$\mathcal{L}_{lk}\rho = \sum_{\mu=1}^{N}\frac{1}{2}(2\sigma_{lk}^{\mu}\rho\sigma_{kl}^{\mu} - \rho\sigma_{kl}^{\mu}\sigma_{lk}^{\mu} - \sigma_{kl}^{\mu}\sigma_{lk}^{\mu}\rho), \tag{4}$$

$$\mathcal{L}_f\rho = \sum_{l=1,2}\kappa_l\mathcal{L}_{a_l}\rho, \quad \mathcal{L}_{a_l}\rho = \frac{1}{2}(2a_l\rho a_l^{\dagger} - \rho a_l^{\dagger}a_l - a_l^{\dagger}a_l\rho), \tag{5}$$

where γ_{kl} are the atomic decay rates from $|k^{\mu}\rangle$ to $|l^{\mu}\rangle$ and κ_l ($l=1,2$) are the decay rates of the cavity fields.

A. Quantum beat due to coherent depopulation

For clarity, we replace $|2\rangle$, $|3\rangle$, and $|4\rangle$ by $\exp[i(\mathbf{k}_{32}-\tilde{\mathbf{k}}_{31})\cdot\mathbf{r}^{\mu}]|2\rangle$, $\exp(i\tilde{\mathbf{k}}_{31}\cdot\mathbf{r}^{\mu})|3\rangle$, and $\exp[i(\tilde{\mathbf{k}}_{31}-\mathbf{k}_{32}+\mathbf{k}_{42})\cdot\mathbf{r}^{\mu}]|4\rangle$, respectively, and then rewrite Hamiltonians (2) and (3) as

$$H_0 = \sum_{\mu=1}^{N}\frac{\hbar}{2}\{\Omega_{31}\sigma_{31}^{\mu}\exp[-i(\Delta_{31}t+\phi_{31})] + \Omega_{32}\sigma_{32}^{\mu}\exp[-i(\Delta_{32}t+\phi_{32})] + \Omega_{41}\sigma_{41}^{\mu}\exp[-i(\Delta\mathbf{k}\cdot\mathbf{r}^{\mu}+\Delta_{41}t+\phi_{41})] + \Omega_{42}\sigma_{42}^{\mu}\exp[-i(\Delta_{42}t+\phi_{42})]\} + \text{H.c.}, \tag{6}$$

$$V = \sum_{\mu=1}^{N}\hbar\{g_1^{\mu}a_1\sigma_{31}^{\mu}\exp(-i\delta_1 t) + g_2^{\mu}a_2\sigma_{41}^{\mu}\times\exp[-i(\Delta\mathbf{k}\cdot\mathbf{r}^{\mu}+\delta_2 t)]\} + \text{H.c.}, \tag{7}$$

where we have defined $\Delta\mathbf{k}=(\mathbf{k}_{31}-\mathbf{k}_{32})-(\mathbf{k}_{41}-\mathbf{k}_{42})+(\mathbf{k}_1-\mathbf{k}_2)$. It should be noted that the above substitutions do not change the forms of the damping terms (4) and (5). For simplicity, we consider the case of the wave vector matching $\Delta\mathbf{k}=0$. Since four applied external fields interact with the atoms in a loop of double-Λ configuration, even in the absence of the cavity fields, the system dynamics is dependent on the harmonic factor [34] $e^{in\Delta t}$, $n=0,\pm 1,\pm 2,\ldots$, with the detuning $\Delta=\Delta_{31}-\Delta_{32}+\Delta_{42}-\Delta_{41}$. This is very complicated. To simplify the treatment and clarify the physics, we assume that $\Delta=0$. In particular, we focus on the case

$$\text{(i)}\quad \Delta_{31}=\Delta_{41},\Delta_{32}=\Delta_{42}, \tag{8}$$

under which we have two-photon resonances $|3^\mu\rangle-|1^\mu\rangle-|4^\mu\rangle$ and $|3^\mu\rangle-|2^\mu\rangle-|4^\mu\rangle$ and the four-photon resonance $|1^\mu\rangle-|3^\mu\rangle-|2^\mu\rangle-|4^\mu\rangle-|1^\mu\rangle$. Note that even when the condition (i) and the following conditions (ii) and (iii) are satisfied, the four applied external fields are not necessarily in an EIT interaction configuration unless the detuning relations $\Delta_{31}=\Delta_{32}$ and $\Delta_{41}=\Delta_{42}$ are satisfied. Instead, the present scheme works in the cases of $\Delta_{31}\neq\Delta_{32}$ and $\Delta_{41}\neq\Delta_{42}$.

The collective phase of these fields is important for the interaction of the cavity fields with atoms. We assume that the driving fields satisfy the condition $\frac{\Omega_{31}\exp(-i\phi_{31})}{\Omega_{41}\exp(-i\phi_{41})}=\frac{\Omega_{32}\exp(-i\phi_{32})}{\Omega_{42}\exp(-i\phi_{42})}$, i.e.,

$$\text{(ii)}\quad \frac{\Omega_{31}}{\Omega_{41}}=\frac{\Omega_{32}}{\Omega_{42}} \tag{9}$$

and

$$\text{(iii)}\quad (\phi_{31}-\phi_{32})-(\phi_{41}-\phi_{42})=2m\pi,\quad (m=0,\pm 1,\pm 2,\ldots). \tag{10}$$

Under the conditions (i)–(iii), the interaction Hamiltonian between the atoms and the applied driving fields is written as

$$H_0=\sum_{\mu=1}^{N}\frac{\hbar}{2}[\Omega_1\sigma_{c1}^\mu\exp(-i\Delta_{31}t)+\Omega_2\sigma_{c2}^\mu\exp(-i\Delta_{32}t)]+\text{H.c.}, \tag{11}$$

where $\sigma_{cl}^\mu=|c^\mu\rangle\langle l^\mu|$, $\Omega_l=(\Omega_{3l}^2+\Omega_{4l}^2)^{1/2}$, and $l=1,2$. The state $|c^\mu\rangle$ is a coherent superposition of the higher two excited states $|3^\mu\rangle$ and $|4^\mu\rangle$. There is the other coherent superposition state $|d^\mu\rangle$ orthogonal to $|c^\mu\rangle$. These two superposition states are

$$|c^\mu\rangle=\cos\theta\exp(i\phi_{31})|3^\mu\rangle+\sin\theta\exp(i\phi_{41})|4^\mu\rangle,$$

$$|d^\mu\rangle=-\sin\theta\exp(-i\phi_{41})|3^\mu\rangle+\cos\theta\exp(-i\phi_{31})|4^\mu\rangle, \tag{12}$$

with $\tan\theta=\frac{\Omega_{41}}{\Omega_{31}}$. It is seen from Hamiltonian (11) that only the superposition state $|c^\mu\rangle$ is coupled to the applied fields while the coherent superposition state $|d^\mu\rangle$ is decoupled. This means that $|d^\mu\rangle$ is not excited. We have vanishing population (coherent depopulation) in the state $|d^\mu\rangle$,

$$\langle\sigma_{dd}^\mu\rangle=\sin^2\theta\langle\sigma_{33}^\mu\rangle+\cos^2\theta\langle\sigma_{44}^\mu\rangle-\sin(2\theta)\text{Re}\{\exp[i(\phi_{31}-\phi_{41})]\langle\sigma_{34}^\mu\rangle\}=0, \tag{13}$$

which reflects destructive interference when $\langle\sigma_{33}^\mu\rangle\neq 0$ and $\langle\sigma_{44}^\mu\rangle\neq 0$. It is the very case for the above detuning conditions. So long as the conditions (i)–(iii) are satisfied, the present four-level system is reduced to a three-level one ($|1^\mu\rangle,|2^\mu\rangle,|c^\mu\rangle$).

For the cavity fields, we assume that $\delta_1=\delta_2$, $g_1^\mu=g_2^\mu=g^\mu$ for simplicity. Hamiltonian (7) is rewritten in the equivalent form

$$V=\sum_{\mu=1}^{N}\hbar g^\mu(A\sigma_{c1}^\mu+B\sigma_{d1}^\mu)\exp(-i\delta_1 t)+\text{H.c.}, \tag{14}$$

where we have introduced a pair of collective modes

$$A=a_1\cos\theta\exp(-i\phi_{31})+a_2\sin\theta\exp(-i\phi_{41}),$$

$$B=-a_1\sin\theta\exp(i\phi_{41})+a_2\cos\theta\exp(i\phi_{31}). \tag{15}$$

It is seen from Hamiltonian (14) that the collective modes A (the sum mode) and B (the difference mode) are coupled to the $|1^\mu\rangle-|c^\mu\rangle$ and $|1^\mu\rangle-|d^\mu\rangle$ transitions, respectively. Because the state $|d^\mu\rangle$ is empty, the mode B undergoes only absorption and there is no gain mechanism for it. This fact determines that the mode B always stays in its vacuum state. This also determines that the original cavity modes are in a quantum beat [35–42]. The reasons are as follows. Since the mode B is in vacuum state, we have $\langle B\rangle=\langle B^\dagger B\rangle=0$, from which we have the phase locking and the amplitude locking

$$\phi_1-\phi_2=\phi_{31}-\phi_{41},\quad \frac{r_1}{r_2}=\cot\theta, \tag{16}$$

where $\langle a_l\rangle=r_l\exp(i\phi_l)$ $(l=1,2;r_l>0)$ have been used. For the phase locking, we have the beat signal between the two original modes $a_{1,2}$,

$$\text{Re}\langle a_1^\dagger a_2\rangle=\frac{1}{2}\sin(2\theta)\cos(\omega_{43}t+\phi_{41}-\phi_{31})\langle A^\dagger A\rangle. \tag{17}$$

Using the Hermitian operators [39] B_φ and B_r corresponding to the imaginary and real parts of the relative mode B, together with an additional phase factor

$$B_\varphi=\frac{\sin\theta}{i\sqrt{2}}(a_1e^{-i\phi_{31}}-a_1^\dagger e^{i\phi_{31}})-\frac{\cos\theta}{i\sqrt{2}}(a_2e^{-i\phi_{41}}-a_2^\dagger e^{i\phi_{41}})=-\sqrt{2}\,\text{Im}(Be^{-i(\phi_{31}+\phi_{41})}), \tag{18}$$

$$B_r=\frac{\sin\theta}{\sqrt{2}}(a_1e^{-i\phi_{31}}+a_1^\dagger e^{i\phi_{31}})-\frac{\cos\theta}{\sqrt{2}}(a_2e^{-i\phi_{41}}+a_2^\dagger e^{i\phi_{41}})=-\sqrt{2}\,\text{Re}(Be^{-i(\phi_{31}+\phi_{41})}), \tag{19}$$

we obtain the variances of B_φ and B_r as

$$\langle(\Delta B_{\varphi,r})^2\rangle=\frac{1}{2}, \tag{20}$$

which are at their vacuum-noise levels. For the particular choice $\sin\theta=\cos\theta=\frac{1}{\sqrt{2}}$, the present case is exactly the same

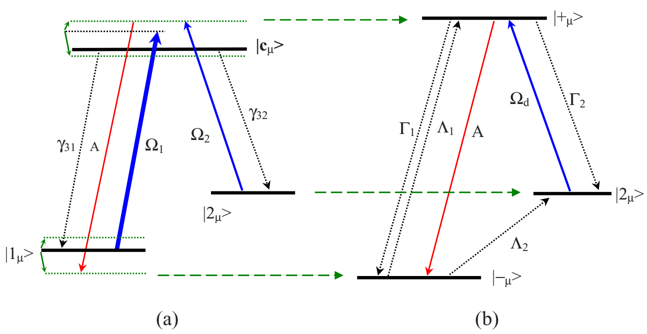

FIG. 2. (Color online) (a) An equivalent three-level system. One effective driving field (Rabi frequency Ω_1) and the collective mode A are coupled to the $|1^\mu\rangle-|c^\mu\rangle$, while the other effective driving field (Rabi frequency Ω_2) is coupled to the $|2^\mu\rangle-|c^\mu\rangle$ transition. γ_{3l} ($\gamma_{3l}=\gamma_{4l}, l=1,2$) are the decay rates. (b) Dressed state EIT configuration. In a picture dressed by the effective field Ω_1 (i.e., Ω_{31},Ω_{41}), the effective control field Ω_2 (i.e., Ω_{32},Ω_{42}) is tuned resonant with the $|2^\mu\rangle-|+^\mu\rangle$ transition and the collective mode A is resonant with the $|-^\mu\rangle-|+^\mu\rangle$ transition. Γ_l,Λ_l ($l=1,2$) are the rates for incoherent population transfer.

as in correlated spontaneous emission lasers [39]. For the quantum beat, we can separate the mode B and reduce Hamiltonian (14) for the interaction of the cavity fields with atoms to

$$V=\sum_{\mu=1}^{N}\hbar(g^\mu A\sigma_{c1}^\mu e^{-i\delta_1 t}+g^{\mu*}A^\dagger\sigma_{1c}^\mu e^{i\delta_1 t}). \tag{21}$$

The total Hamiltonian of the system is described by the sum of Eqs. (11) and (21). The present system is reduced to an equivalent three-level system, as shown in Fig. 2(a). The effective driving field Ω_1 and the collective cavity mode A are both coupled to the $|1^\mu\rangle-|c^\mu\rangle$ transition while the effective driving field Ω_2 is coupled to the $|2^\mu\rangle-|c^\mu\rangle$ transition.

B. Dressed state EIT

As has been pointed out, the four applied fields are not in the EIT interaction with the atoms unless $\Delta_{31}=\Delta_{32}$, $\Delta_{41}=\Delta_{42}$ [24–27]. For arbitrary detunings Δ_{31} and δ_1, it is necessary to use harmonic expansion [43] $\exp[in(\Delta_{31}-\delta_1)t]$, $n=0,\pm1,\pm2,\ldots$. However, it is greatly simplified for the case where the coherent driving field Ω_1 is strong, $\Omega=(\Delta_{31}^2+\Omega_1^2)^{1/2}\gg(\Omega_2,|g_l^\mu\langle a_l\rangle|,\gamma_{kl}),(k=3,4;l=1,2)$. It causes the large Stark splitting of each of levels $|1^\mu\rangle$ and $|c^\mu\rangle$ into $|-^\mu\rangle$ and $|+^\mu\rangle$, as shown in Fig. 2(a). The effective driving field Ω_2 is tuned to a sideband resonance, i.e., it is resonant with the transition $|2^\mu\rangle-|+^\mu\rangle$. This is why Ω_1 and Ω_2 are not in the EIT interaction. At the same time, the cavity fields are tuned resonant with the high-frequency transitions between split levels, which is similar to the case of two-level dressed-state lasers [44–47]. Now, the effective driving field Ω_2 and the collective mode A are both in the resonant interaction with dressed atoms. Thus the EIT interaction configuration is created in terms of dressed states.

In particular, the dressed states can be obtained by diagonalizing the interaction Hamiltonian associated with the driving field Ω_1 [48] in an appropriate frame rotating with $\exp(i\Delta_{31}t\Sigma_{\mu=1}^N\sigma_{cc}^\mu)$. After doing so, we write the dressed states in terms of the bare states as

$$|+^\mu\rangle=\sin\phi|1^\mu\rangle+\cos\phi|c^\mu\rangle,$$

$$|-^\mu\rangle=\cos\phi|1^\mu\rangle-\sin\phi|c^\mu\rangle, \tag{22}$$

where $\tan(2\phi)=-\frac{1}{\delta}$, $\delta=\frac{\Delta_{31}}{\Omega_1}$, and $0<2\phi<\pi$. The corresponding eigenvalues are $\lambda_\pm=\frac{1}{2}(-\Delta_{31}\pm\Omega)$. We tune the effective control field Ω_2 (i.e., Ω_{32},Ω_{42}) such ($\Delta_{32}=\Delta_{31}+\lambda_+$) that it is resonant with the $|2^\mu\rangle-|+^\mu\rangle$ transition and tune the cavity fields such ($\delta_1=\Delta_{31}+\Omega$) that they are resonant with Rabi sideband transition $|+^\mu\rangle-|-^\mu\rangle$. We make a rotating transform in terms of dressed states and adopt the secular approximation (neglecting the fast oscillating terms such as $e^{i\Omega t}$) as for the dressed-state lasers [44,45]. Then, the interaction Hamiltonian for the total system is then written as

$$H=\sum_{\mu=1}^{N}\hbar(\Omega_d\sigma_{+2}^\mu+g_A^\mu A\sigma_{+-}^\mu)+\text{H.c.}, \tag{23}$$

with $\sigma_{kl}^\mu=|k^\mu\rangle\langle l^\mu|$ $(k,l=\pm,2)$, $\Omega_d=\frac{1}{2}\Omega_2\cos\phi$, and $g_A^\mu=g^\mu\cos^2\phi$. After the above all transformations, we can rewrite the master Eq. (1) in terms of atomic dressed states and the collective field modes. Since the difference mode B remains in the vacuum state, the density operator for it can be separated from that of the total system. The master equation for the density $\tilde{\rho}$ of the sum mode A and the dressed atoms is derived as

$$\frac{\partial}{\partial t}\tilde{\rho}=-\frac{i}{\hbar}[H,\tilde{\rho}]+\mathcal{L}_a\tilde{\rho}+\mathcal{L}_A\tilde{\rho}. \tag{24}$$

Here, the atomic damping terms $\mathcal{L}_a\tilde{\rho}$ have the form

$$\mathcal{L}_a\tilde{\rho}=\Lambda_1\mathcal{L}_{+-}\tilde{\rho}+\Gamma_1\mathcal{L}_{-+}\tilde{\rho}+\Gamma_2\mathcal{L}_{2+}\tilde{\rho}+\Lambda_2\mathcal{L}_{2-}\tilde{\rho}+\Gamma_p\mathcal{L}_p\tilde{\rho}, \tag{25}$$

with the phase damping

$$\mathcal{L}_p\tilde{\rho}=\sum_{\mu=1}^{N}\frac{1}{4}(2\sigma_p^\mu\tilde{\rho}\sigma_p^\mu-\tilde{\rho}\sigma_p^\mu\sigma_p^\mu-\sigma_p^\mu\sigma_p^\mu\tilde{\rho}), \tag{26}$$

where $\sigma_p^\mu=\sigma_{++}^\mu-\sigma_{--}^\mu$. For the atomic damping, we have taken $\gamma_{31}=\gamma_{41}$, $\gamma_{32}=\gamma_{42}$ for simplicity. $\mathcal{L}_{lk}\tilde{\rho}$ has the same form as $\mathcal{L}_{lk}\rho$ in Eq. (4). For the damping of cavity fields, we have assumed that $\kappa_1=\kappa_2=\kappa$. The damping term $\mathcal{L}_A\tilde{\rho}$ for the collective mode A has the same form as $\mathcal{L}_{a_l}\rho$ in Eq. (5) except for the substitutions of A for a_l. The parameters in Eq. (25) are $\Gamma_1=\gamma_{31}\cos^4\phi$, $\Lambda_1=\gamma_{31}\sin^4\phi$, $\Gamma_2=\gamma_{32}\cos^2\phi$, $\Lambda_2=\gamma_{32}\sin^2\phi$, and $\Gamma_p=\frac{1}{2}\gamma_{31}\sin^2(2\phi)$. The coherent and incoherent atomic transitions described by Eq. (24) are indicated in Fig. 2(b). So far, three features should be emphasized.

(i) Dressed state EIT. It is seen from Hamiltonian (23) that the cavity mode A is on resonance with the $|+^\mu\rangle-|-^\mu\rangle$ transition and the effective driving field Ω_2 is on resonance with the $|2^\mu\rangle-|+^\mu\rangle$ transition. These two effective fields are in a standard EIT interaction configuration together with the incoherent pathways $\Lambda_{1,2}$. The coherent transition $|2^\mu\rangle-|+^\mu\rangle$

plays three roles in the manipulation of the cavity field A (or $a_{1,2}$). First, it is necessary for light amplification. Once $\Omega_d=0$, we have $\langle\sigma_{++}^{\mu}\rangle=\langle\sigma_{--}^{\mu}\rangle=0$ and $\langle\sigma_{22}^{\mu}\rangle=1$. In this case, it is impossible to have laser gain. Second, the coherent transition $|2^{\mu}\rangle-|+^{\mu}\rangle$ greatly suppresses the absorption from the state $|-^{\mu}\rangle$. A small population in state $|+^{\mu}\rangle$ will lead to the laser gain [9–14]. Third, the coherent transition is a necessary channel for recycling the electron via the loop $|2^{\mu}\rangle\to|+^{\mu}\rangle\to|-^{\mu}\rangle\to|2^{\mu}\rangle$. The intrinsic feedback is possible only in the presence of $|2^{\mu}\rangle\to|+^{\mu}\rangle$ [16–20,49–53].

(ii) Phase dependence. Under conditions (8)–(10), the coherent superposition state $|d^{\mu}\rangle$ and the collective mode B are both decoupled from the dynamics. Instead, the coherent superposition state $|c^{\mu}\rangle$ as a single state and the collective mode A as a single mode mediate into interaction. This determines that the difference phase between $a_{1,2}$ is equal to that of the driving fields Ω_{31} and Ω_{41} (or Ω_{32} and Ω_{42}).

(iii) Intrinsic incoherent transfer of population. From the master equation (24), we see two intrinsic, incoherent pathways $|-^{\mu}\rangle\to|+^{\mu}\rangle$ and $|-^{\mu}\rangle\to|2^{\mu}\rangle$, through which population is transferred from state $|-^{\mu}\rangle$ to $|+^{\mu}\rangle$ and $|2^{\mu}\rangle$. We can imagine that once the two incoherent pathways are absent, it is impossible to have laser gain. It has been shown that the $|-^{\mu}\rangle\to|+^{\mu}\rangle$ pathway provides the dressed-state inversion for the two-level dressed-state one- and two-photon lasers [44–47] or necessary excited-state population for a three-level inversionless laser [16]. On the other hand, the $|-^{\mu}\rangle\to|+^{\mu}\rangle$ is the necessary channel for the gain and noise reduction in a Raman laser [17]. The present scheme combines the benefits of these two types of systems. This provides necessary population recycling for the laser gain. The present system can operate with $\langle\sigma_{++}^{\mu}\rangle>\langle\sigma_{--}^{\mu}\rangle$ and it can also run without inversion $\langle\sigma_{++}^{\mu}\rangle<\langle\sigma_{--}^{\mu}\rangle$ in the presence of the dressed-state EIT. These two intrinsic incoherent channels are crucial for quantum correlations. Through the two incoherent pathways together with the coherent transitions

$$|2^{\mu}\rangle\xrightarrow{\Omega_d}|+^{\mu}\rangle \quad \text{and} \quad |+^{\mu}\rangle\xrightarrow{A}|-^{\mu}\rangle,$$

the electron is recycled. Such a recycling forms a deep intrinsic feedback [49–53] when the system is operated well above threshold. As a result, the sum mode A has subshot noise, which corresponds to two-mode continuous variable entanglement as shown below.

III. STEADY-STATE INTENSITIES AND QUANTUM CORRELATIONS

In this section, following the standard techniques [17] and using the relations between the original modes $a_{1,2}$ and combination modes A,B, we discuss the two-mode oscillations and quantum correlations of the original modes $a_{1,2}$. We derive the Langevin equations from the master equation by means of the generalized P representation of Drummond and Gardiner [54]. The atomic variables are described by collective operators $\sigma_{kl}=\frac{1}{N}\sum_{\mu=1}^{N}\sigma_{kl}^{\mu}$, $k,l=\pm,2$. Since the individual atoms are uncorrelated and the atomic dipole moments are randomly orientated, this leads to $\sigma_{ij}\sigma_{kl}=\frac{1}{N}\delta_{jk}\sigma_{il}$ and $\sum_{\mu,\nu=1}^{N}\frac{g^{\mu}g^{\nu *}}{N^2}=\frac{g^2}{N}$, where $g^{\mu}=ge^{i\psi^{\mu}}$ has been used. The master equation (24) can be expanded to obtain a partial differential equation containing derivatives of infinite order in the atomic number. Adopting standard scaling, we can neglect derivatives of higher than second order in the limit of many atoms [16,51]. We choose the normal ordering $A^{\dagger}$, σ_{+2}, σ_{+-}, σ_{2-}, σ_{++}, σ_{22}, σ_{--}, σ_{-2}, σ_{-+}, σ_{2+}, A and define the correspondence between the c numbers and operators as $\alpha\leftrightarrow A$ ($\alpha^{*}\leftrightarrow A^{\dagger}$), $v_1\leftrightarrow\sigma_{2+}$ ($v_1^{\dagger}\leftrightarrow\sigma_{+2}$), $v_2\leftrightarrow\sigma_{-+}$ ($v_2^{\dagger}\leftrightarrow\sigma_{+-}$), $v_3\leftrightarrow\sigma_{-2}$ ($v_3^{\dagger}\leftrightarrow\sigma_{2-}$), $z_l\leftrightarrow\sigma_{ll}$. The set of equations for the c numbers are derived as

$$\dot{\alpha}=-\frac{1}{2}\kappa\alpha-ig_A N v_2+F_{\alpha}, \tag{27}$$

$$\dot{v}_1=-\gamma_1 v_1+i\Omega_d(z_+-z_2)+ig_A\alpha v_3^{\dagger}+F_{v_1}, \tag{28}$$

$$\dot{v}_2=-\gamma_2 v_2+ig_A\alpha(z_+-z_-)-i\Omega_d v_3+F_{v_2}, \tag{29}$$

$$\dot{v}_3=-\gamma_3 v_3+ig_A\alpha v_1^{\dagger}-i\Omega_d v_2+F_{v_3}, \tag{30}$$

$$\dot{z}_-=-(\Lambda_1+\Lambda_2)z_-+\Gamma_1 z_++ig_A(\alpha v_2^{\dagger}-\alpha^{*}v_2)+F_{z_-}, \tag{31}$$

$$\dot{z}_2=\Lambda_2 z_-+\Gamma_2 z_++i\Omega_d(v_1^{\dagger}-v_1)+F_{z_2}. \tag{32}$$

where $g_A=g\cos^2\phi$. Populations follow the closure relation $z_-+z_++z_2=1$. The parameters in Eqs. (27)–(32) are $\gamma_1=\frac{1}{2}(\Gamma_1+\Gamma_2)+\frac{1}{4}\Gamma_p$, $\gamma_2=\frac{1}{2}(\Gamma_1+\Gamma_2+\Lambda_1+\Lambda_2)+\Gamma_p$, and $\gamma_3=\frac{1}{2}(\Lambda_1+\Lambda_2)+\frac{1}{4}\Gamma_p$. $F_x(t)$ are the noise forces. The noise correlations can be easily obtained from generalized Einstein relations [33].

We assume that the atomic variables change much more rapidly than the cavity fields ($\gamma_{31},\gamma_{32}\gg\kappa$). Then we can eliminate adiabatically the atomic variables. From Eqs. (27)–(32), we obtain the linear gain for the laser intensity $\langle I\rangle=g^2\langle A^{\dagger}A\rangle$ as

$$G=\frac{C\cos^4\phi(q_3+q_2\gamma_3)\Omega_d^2\kappa\gamma_{31}}{q_1\Omega_d^4+(q_1\gamma_2\gamma_3+q_3\gamma_1)\Omega_d^2+q_3\gamma_1\gamma_2\gamma_3}, \tag{33}$$

where $C=\frac{2g^2N}{\kappa\gamma_{31}}$ has been defined as the cooperativity parameter and the other parameters are $q_1=\Gamma_1+2\Lambda_1+2\Lambda_2$, $q_2=\Lambda_1+\Lambda_2-\Gamma_1$, and $q_3=\Lambda_1\Gamma_2+\Gamma_1\Lambda_2+\Lambda_2\Gamma_2$. It is not hard to find the linear gain $G>\kappa$, which means the system operates above threshold. Neglecting the noise and setting derivatives to zero in the evolution Eqs. (27)–(32), we obtain the stable intensity $\langle I\rangle$ for sum mode A. The mode B is below threshold and is in vacuum state. Using the mode transform relations (15), we obtain for two original modes $a_{1,2}$ the respective intensities $\langle I_1\rangle=g^2\langle a_1^{\dagger}a_1\rangle=\langle I\rangle\cos^2\theta$ and $\langle I_2\rangle=g^2\langle a_2^{\dagger}a_2\rangle=\langle I\rangle\sin^2\theta$.

In general, the entanglement properties of the two-mode cavity field $a_{1,2}$ can be expressed by two Einstein-Podolsky-Rosen (EPR)-like operators

$$u=x_1+x_2, v=p_1-p_2, \tag{34}$$

where

$$x_l=\frac{1}{\sqrt{2}}(a_l e^{-i\phi_l}+a_l^{\dagger}e^{i\phi_l}),$$

$$p_l = \frac{1}{i\sqrt{2}}(a_l e^{-i\phi_l} - a_l^\dagger e^{i\phi_l}), l = 1,2. \tag{35}$$

According to the criterion proposed by Duan *et al.* [55], an entangled state is created if the sum of the quantum fluctuations of operators u and v satisfies the inequality

$$M = \langle(\Delta u)^2\rangle + \langle(\Delta v)^2\rangle < 2. \tag{36}$$

From now on, we focus on the case of $\frac{\Omega_{31}}{\Omega_{41}} = \frac{\Omega_{32}}{\Omega_{42}} = 1$. Then, the operators u and v are expressed by the combination modes A and B as

$$u = Ae^{-i(\phi_1-\phi_{31})} + A^\dagger e^{i(\phi_1-\phi_{31})},$$

$$v = i(Be^{-i(\phi_1+\phi_{41})} - B^\dagger e^{i(\phi_1+\phi_{41})}), \tag{37}$$

which are associated with the amplitude quadrature of mode A and the phase quadrature of mode B. Since the relative mode B is in the vacuum state, the variance of quadrature v is in the vacuum-noise level [35–40], $\langle(\Delta v)^2\rangle = 1$. As is well known, if the laser operates far above threshold, the fluctuations in the amplitude are negligibly small compared to the amplitude itself. The variance of the operator u is related to the Mandel factor Q by the relation

$$\langle(\Delta u)^2\rangle = 1 + Q, \tag{38}$$

where $Q = \frac{\langle:(\Delta A^\dagger A)^2:\rangle}{\langle A^\dagger A\rangle}$. Here, 1 stands for the shot noise level and the Mandel Q is the normally ordered normalized variance of the sum mode intensity and measures the derivations from Poissonian statistics. Using the Q factor, we obtain the normally ordered part of the output fluctuation spectrum

$$S(\omega) = 2\int_0^\infty d\tau \cos(\omega\tau)\frac{\langle:i(t+\tau),i(t):\rangle}{\langle i(t)\rangle} = Q\left(\frac{2\kappa}{\lambda}\right)\frac{\lambda^2}{\omega^2+\lambda^2}, \tag{39}$$

where $i(t) = \kappa A^\dagger(t)A(t)$ corresponds to the output photon flux (intensity) operator and λ is proportional to the differential gain. It is known that $S(\omega) = 0$ corresponds to shot noise, $-1 < S(\omega) < 0$ to sub-Poissonian photon statistics, and $S(\omega) = -1$ to Fock state. The output spectrum for the variance sum of EPR-like operators is obtained as

$$M(\omega) = 2 + S(\omega), \tag{40}$$

which indicates that if sub-Poissonian statistics is present, the two cavity fields are entangled, i.e., $M(\omega) < 2$. Correspondingly, for the respective modes, the normally ordered parts of the output fluctuation spectra are expressed as $S_l(\omega) = \frac{1}{2}S(\omega)$. Quadrature squeezing in the respective modes occurs when sub-Poissonian statistics in the sum mode is existent.

In the numerical calculation, Rabi frequencies, detunings, and decay rates are scaled in units of γ_{32}. The intensities $\langle I_1\rangle = \langle I_2\rangle$ are in units of γ_{32}^2. In Fig. 3, we plot the zero-frequency output spectra (a) and the respective intensities of the two cavity modes (b) as functions of normalized detuning $\delta = \frac{\Delta_{31}}{\Omega_1}$ for different Rabi frequencies ($\Omega_{32} = \Omega_{42}$), $\Omega_{32} = 1.0$ (dotted line), $\Omega_{32} = 2.0$ (dashed line), and $\Omega_{32} = 3.0$ (solid line). The other parameters are chosen as $C = 500$, $\gamma_{31} = 4.0$. It is seen from Fig. 3(a) that for a wide range of parameters, the entanglement criterion is satisfied ($M < 2$) and the respective intensities are large. Both the variances and the intensities are strongly dependent on the Rabi frequency Ω_{32}. As Ω_{32} is small, the laser intensities are also relatively small. However, the range of δ for entanglement is relatively wide. As Ω_{32} increases, the laser intensities rise. However, the range of δ for entanglement becomes relatively narrow. It should also be noted that Ω_{32} cannot take too large value. This is because, as is seen from Eq. (33), this will give rise to strong saturation so that the linear gain is less than the cavity loss. As a consequence, there will be no laser output. In particular, when $\Omega_{32} = 3.0$, the zero-frequency output spectrum takes the minimal value $M(0) \approx 1.45$ at $\delta = 1.42$ and the intensities have the largest values $\langle I_1\rangle = \langle I_2\rangle \approx 179$ in units of γ_{32}^2. At the same time, we have the output spectra $S(0) \approx -0.55$ and $S_l(0) \approx -0.275$, which corresponds to squeezing of 55% in the sum mode A and squeezing of 27.5% in the sum mode $a_{1,2}$. It shows that the two cavity modes, each of which has subshot noise, are in an entangled state. In Fig. 4, the zero-frequency output spectra and the intensities are plotted for different cooperativity parameters $C = 200$ (dotted line), $C = 500$ (dashed line), and $C = 800$ (solid line). The other parameters are chosen as $\Omega_{32} = 2.0$ and $\gamma_{31} = 4.0$. Generally, increasing the cooperativity parameter will lead to an increase in the laser intensities and a widening of δ for entanglement. In particular, for the cooperativity parameter $C = 800$, the output spectrum has the minimal value $M(0) \approx 1.52$ and the intensities have the largest values $\langle I_1\rangle = \langle I_2\rangle \approx 166$ in units of γ_{32}^2. Correspondingly, we have the output spectra $S(0) \approx -0.48$ and $S_l(0) \approx -0.24$, which corresponds to

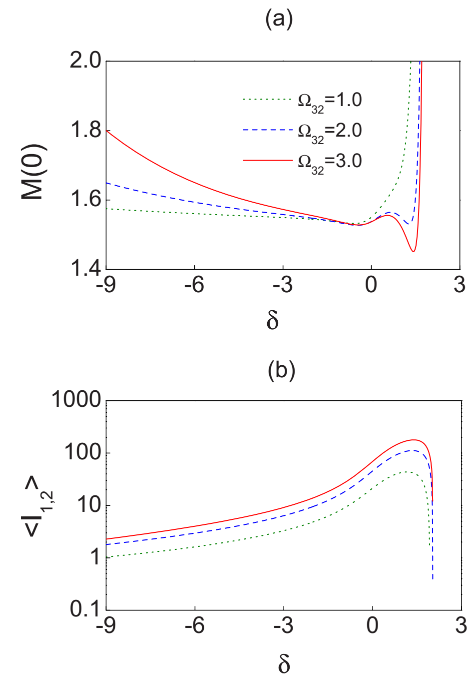

FIG. 3. (Color online) (a) The zero-frequency output spectrum $M(0)$ and (b) the steady intensities $\langle I_1\rangle = \langle I_2\rangle$ vs the normalized detuning δ for different Rabi frequencies $\Omega_{32} = 1.0$ (dotted lines), $\Omega_{32} = 2.0$ (dashed lines), and $\Omega_{32} = 3.0$ (solid lines). The other parameters are chosen as $C = 500$ and $\gamma_{31} = 4.0$.

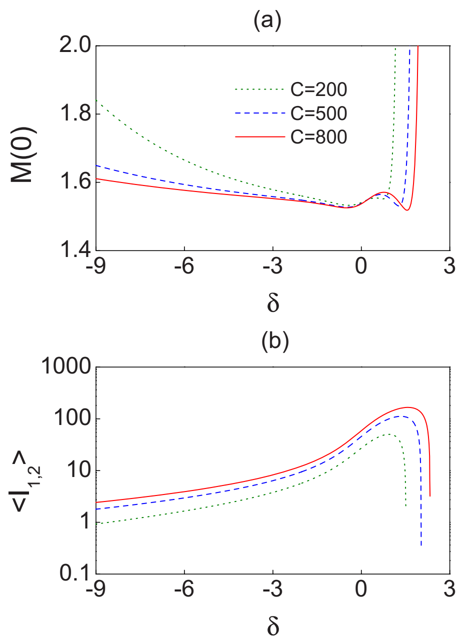

FIG. 4. (Color online) (a) The zero-frequency output spectrum $M(0)$ and (b) the steady intensities $\langle I_1\rangle=\langle I_2\rangle$ vs the normalized detuning δ for different cooperativity parameters $C=200$ (dotted lines), $C=500$ (dashed lines), and $C=800$ (solid lines). The other parameters are chosen as $\Omega_{32}=2.0$ and $\gamma_{31}=4.0$.

squeezing of 48% for the sum mode A and squeezing of 24% for the respective modes $a_{1,2}$. In a word, for a large range of parameters, two cavity fields that operate well-above threshold and display sub-Poissonian statistics are in an entangled state.

As for the mechanism of the entanglement, the following three factors are important. (i) *Quantum beat*. This is created due to coherent depopulation of the coherent superposition state $|d^\mu\rangle$. This depopulation leads to the consequence that the difference mode B is decoupled and is kept in the vacuum state. This brings us to a three-level system, where both the effective driving field Ω_1 and the sum mode A are coupled to the $|1^\mu\rangle-|c^\mu\rangle$ transition and the effective driving field Ω_2 is coupled to the $|2^\mu\rangle-|c^\mu\rangle$ transition. (ii) *Dressed state EIT*. This is formed due to the dressing by the pair of the applied fields Ω_{31} and Ω_{41}, equivalently, by Ω_1. In terms of the split levels, the sum mode A is tuned resonant with the high Rabi frequency transition $|-^\mu\rangle-|+^\mu\rangle$ and the effective driving field Ω_2 is tuned resonant with $|2^\mu\rangle-|+^\mu\rangle$. The double resonances are in an EIT configuration. This greatly suppresses the absorption and is favorable for the light amplification. Two intrinsic incoherent channels are sufficient for the population transfer for the laser gain. (iii) *Intrinsic feedback*. In the dressed picture, we see the electron recyclings

$$|-^\mu\rangle \xrightarrow{\Lambda_1} |+^\mu\rangle \xrightarrow{\Gamma_2} |2^\mu\rangle \xrightarrow{\Omega_d} |+^\mu\rangle \xrightarrow{A} |-^\mu\rangle$$

and

$$|-^\mu\rangle \xrightarrow{\Lambda_2} |2^\mu\rangle \xrightarrow{\Omega_d} |+^\mu\rangle \xrightarrow{A} |-^\mu\rangle.$$

When the laser fields are relatively strong, one has a deep feedback, which regularizes the laser electrons. This is the very mechanism for quantum noise reduction as shown previously [16–21,44–48]. Finally, the noise squeezing in the sum mode A and the vacuum state of the difference mode B correspond to the two-mode continuous variable entanglement.

IV. REALISTIC CONSIDERATIONS

So far, we have presented our scheme by considering the ideal case. For the experimental realization, some realistic factors should be taken into account. In fact, a great number of the level structures are suitable for the present scheme, as used recently in the mixing experiments based on EIT, such as alkali metal atoms [56–59], atomic hydrogen [60–63], and atomic ^{208}Pb [64].

First, in order to avoid the Doppler broadening, we can employ an ensemble of cold atoms. For the media of cold atoms, the Doppler broadening is substantially smaller than the natural linewidths and quantum coherent effects can be well preserved, as in experiments involving EIT [7,65–67] and lasing without population inversion [68].

Second, the nonresonant hyperfine interactions of applied fields with adjacent transitions should be taken into account, especially when these fields are off resonant with the atomic transitions under consideration. The nonresonant interactions will give rise to a shift of the resonance center by the order of the atomic decay rates. This was demonstrated experimentally in Ref. [7] and was analyzed theoretically in Ref. [69]. As an example, we consider the ^{87}Rb atom for the present scheme. We use the states $|1\rangle=|5S_{1/2},F=1\rangle$, $|2\rangle=|5S_{1/2},F=2\rangle$, $|3\rangle=|5P_{1/2},F'=2\rangle$, and $|4\rangle=|5P_{3/2},F''=2\rangle$. The transitions from the ground states to the two excited states are well separated from each other by the D_1 line (794.8 nm) and the D_2 line (780.0 nm). The ground states are separated from each other by 6.8347 GHz. We note that the excited state $|3\rangle$ has an adjacent level $|5P_{1/2},F'=1\rangle$. The transitions $|1,2\rangle\rightarrow|3\rangle$ undergo ac Stark shifts that are caused by nonresonant couplings of states $|5P_{1/2},F'=1\rangle$ and $|1,2\rangle$. As a result, the centers of the $|1,2\rangle\rightarrow|3\rangle$ resonances are shifted. Similarly, the transition $|1\rangle\rightarrow|4\rangle$ experiences a shift caused by the nonresonant transition between the adjacent lower state $|5P_{3/2},F''=1\rangle$ and the ground state $|1\rangle$; the transition $|2\rangle\rightarrow|4\rangle$ has a shift due to the nonresonant couplings of the two adjacent states $|5P_{3/2},F''=1\rangle$ and $|5P_{3/2},F''=3\rangle$ to the ground state $|2\rangle$. In comparison, the influence of the coupling of the adjacent lower state $|5P_{3/2},F''=1\rangle$ and $|2\rangle$ is negligibly small [70]. By $\delta\varpi_{kl}$, we denote the shifts of various transitions $|k\rangle\rightarrow|l\rangle$ $(k=3,4;l=1,2)$ under consideration and then we have the effective detunings $\Delta'_{kl}=\Delta_{kl}-\delta\varpi_{kl}$, $\delta'_1=\delta_1-\delta\varpi_{31}$, and $\delta'_2=\delta_2-\delta\varpi_{41}$. In order to satisfy the two-photon resonance conditions, we can tune the applied fields such that $\Delta'_{3l}=\Delta'_{4l}$, $l=1,2$. This also guarantees the condition for the four-photon resonance $\Delta'=\Delta'_{31}-\Delta'_{32}+\Delta'_{42}-\Delta'_{41}=0$. At the same time, we tune the cavity fields such $\delta'_{1,2}=\Delta'_{31}+\Omega$ that they are resonant with Rabi sidebands.

Essentially, the nonresonant couplings to the adjacent levels belong to the dispersive interaction, hence population

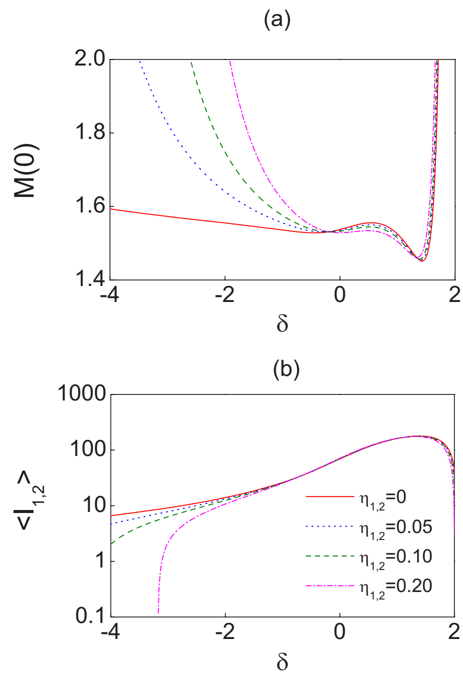

FIG. 5. (Color online) (a) The zero-frequency output spectrum $M(0)$ and (b) the steady intensities $\langle I_1\rangle=\langle I_2\rangle$ vs the normalized detuning δ for the rates $\eta_{1,2}=0$ (solid lines), $\eta_{1,2}=0.05$ (dotted lines), $\eta_{1,2}=0.10$ (dashed lines), and $\eta_{1,2}=0.20$ (dotted-dashed lines). The other parameters are chosen as $\Omega_{32}=3.0$, $C=500$, and $\gamma_{31}=4.0$.

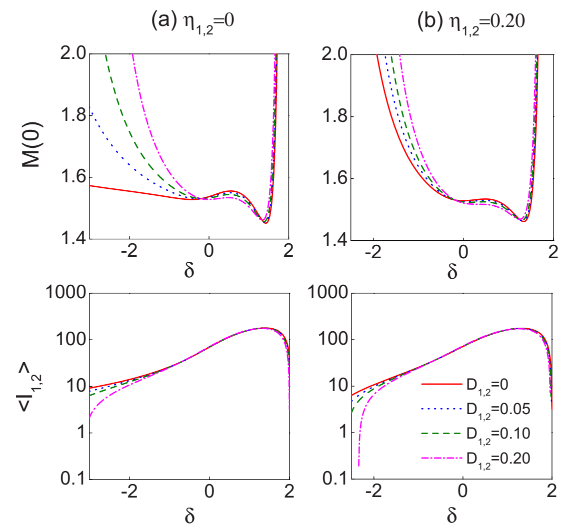

FIG. 6. (Color online) The zero-frequency output spectrum $M(0)$ (upper row) and the steady intensities $\langle I_1\rangle=\langle I_2\rangle$ (lower row) vs the normalized detuning δ for (a) $\eta_{1,2}=0$ and (b) $\eta_{1,2}=0.20$ and linewidths $D_{1,2}=0$ (solid lines), $D_{1,2}=0.05$ (dotted lines), $D_{1,2}=0.10$ (dashed lines), and $D_{1,2}=0.20$ (dotted-dashed lines). The other parameters are the same as in Fig. 5.

transfer to the adjacent levels is negligible. This only gives rise to extra phase damping, i.e., we have additional relaxations for the atomic flip operators σ_{kl} $(k,l=\pm,2;k\neq l)$. The extra phase damping can be simulated by [33]

$$\mathcal{L}'_a\tilde{\rho}=\eta_1\mathcal{L}_{22}\tilde{\rho}+\eta_2\mathcal{L}_{--}\tilde{\rho}, \tag{41}$$

where $\mathcal{L}_{kk}\tilde{\rho}$ have the same form as $\mathcal{L}_{lk}\tilde{\rho}$ in Eq. (4) except for the substitution of σ^{μ}_{kk} for σ^{μ}_{lk}. After adding this term to $\mathcal{L}_a\tilde{\rho}$ in Eq. (25), the decay rates in the set of Eqs. (27)–(32) are changed by adding $\frac{\eta_1}{2}$ to γ_1, adding $\frac{\eta_2}{2}$ and γ_2, and adding $\frac{1}{2}(\eta_1+\eta_2)$ to γ_3. Usually, the additional damping rates are much smaller than the atomic decay rates. In Fig. 5, we plot the spectra and intensities of the two cavity modes for different rates $\eta_{1,2}=0$ (solid lines), $\eta_{1,2}=0.05$ (dotted lines), $\eta_{1,2}=0.10$ (dashed lines), and $\eta_{1,2}=0.20$ (dotted-dashed lines). The other parameters are chosen as $\Omega_{32}=3.0$, $C=500$, and $\gamma_{31}=4.0$. This figure clearly shows that the spectra do not significantly change for a wide range of laser intensities. This corresponds to a regime of $-1.0\lesssim\delta\lesssim 1.5$, where the driving fields are in the resonant or near-resonant interactions with the atoms. In this regime, one has bright beams of squeezed and entangled light.

Third, in order to preserve the coherent depopulation in the presence of finite linewidths of applied fields, one should use the two pairs of correlated beams. Although the laser linewidths are usually much smaller than the atomic decay rates, they have significant influence on the quantum coherent effects. However, when a pair of fluctuating beams is highly correlated with each other, they have a defined phase relation, i.e., the diffusion coefficient of the relative phase vanishes. Such correlations can be realized by using electro-optical modulators, as in the experiments on phase-dependent EIT [25]. This has also in principle been demonstrated in the correlated spontaneous emission lasers [35–41]. In what follows, we describe the influence of the linewidths in details. The Rabi frequencies have fluctuating phases $\phi_{kl}(t)$ $(k=3,4;l=1,2)$, which are characterized by the random force $\frac{d}{dt}\phi_{kl}=\chi_{kl}(t)$ with zero averages and the white-noise correlations

$$\langle\chi_{kl}(t)\rangle=0, \tag{42}$$

$$\langle\chi_{kl}(t)\chi_{k'l'}(t')\rangle=D_{klk'l'}\delta(t-t'), \tag{43}$$

where D_{klkl} are the linewidths of the applied fields Ω_{kl} and $D_{klk'l'}\leq\frac{1}{2}(D_{klkl}+D_{k'l'k'l'})$ are the cross-correlated linewidths $(k\neq k'$ or $l\neq l')$. In a three-level CPT system, the coherence between the two ground states can be preserved by using the cross correlations [71]. For the present scheme, when we establish $D_{3l4l}=D_{3l3l}=D_{4l4l}=D_l$ $(l=1,2)$, the coherent depopulation is no longer spoiled by the laser linewidths.

Although the coherent depopulation effect is preserved due to the cross correlations, the fluctuating phases give rise to the extra phase damping term [33,71,72]

$$\mathcal{L}''_a\tilde{\rho}=D_1\mathcal{L}_{22}\tilde{\rho}+D_2\mathcal{L}_{--}\tilde{\rho}, \tag{44}$$

which is added to $\mathcal{L}_a\tilde{\rho}$ in Eq. (25). Due to the additional term, the decay rates in the set of Eqs. (27)–(32) are changed by adding $\frac{D_1}{2}$ to γ_1, adding $\frac{D_2}{2}$ to γ_2, and adding $\frac{1}{2}(D_1+D_2)$ to γ_3. In our numerical calculations, the linewidths are scaled in units of γ_{32}. Usually, the laser linewidth is much smaller than the atomic decay rates. In Fig. 6, we plot the spectra (upper row) and intensities (lower row) of the two cavity modes for (a) $\eta_{1,2}=0$ and (b) $\eta_{1,2}=0.20$ and linewidths $D_{1,2}=0$ (solid lines), $D_{1,2}=0.05$ (dotted lines), $D_{1,2}=0.10$ (dashed lines), and $D_{1,2}=0.20$ (dotted-dashed lines). The other parameters are chosen the same as in Fig. 5. What is shown in Fig. 6(a) is the effect of the linewidths in the absence of the nonresonant interactions and the combined effects of both the nonresonant interactions and the laser linewidths are given in Fig. 6(b). It is seen that, even when the two negative factors are included, the correlations are kept without significant change for relatively large values of laser intensities. This just occurs in the regime, $-1.0 \lesssim \delta \lesssim 1.5$, where the driving fields are resonantly or near-resonantly coupled to the atoms. When the laser intensities are small, the influence of the two limited factors becomes serious. This appears in the far off-resonance region. It is obvious that resonant or near-resonant coherent driving is advantageous for the generation of bright sources of squeezed and entangled lights.

V. CONCLUSION

In conclusion, we have shown the coherent effects of multiphoton resonances on laser oscillation and quantum correlations in a double Λ system. When four applied fields induce the coherent depopulation, i.e., vanishing population of a coherent superposition state of the two excited states (the counterpart of CPT), a pair of cavity fields is driven into the quantum beat. By using two of four coherent fields to dress the coupled levels (i.e., creates the dressed states), we have the phase-dependent EIT interaction of the other two applied fields and the pair of cavity fields with the dressed atoms. By this mechanism, the pair of cavity fields runs well above threshold and has subshot noise and is in an entangled state. The dressed-state phase-dependent EIT provides an efficient mechanism for obtaining entangled light from a laser device.

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