Non-Markovian analysis of the phase-damped Jaynes-Cummings model in the presence of a classical homogeneous gravitational field

M. Mohammadi*

Department of Physics, Islamic Azad University–Shahreza Branch, Shahreza, Isfahan 86145-31, Iran

(Received 21 January 2009; published 17 November 2009)

In this paper, the non-Markovian dissipative dynamics of the phase-damped Jaynes-Cummings model in the presence of a classical homogeneous gravitational field will be analyzed. The model consists of a moving two-level atom simultaneously exposed to the gravitational field and a single-mode traveling radiation field in the presence of a non-Markovian phase damping mechanism. First, the non-Markovian master equation for the reduced density operator of the system in terms of a Hamiltonian describing the atom-field interaction in the presence of a homogeneous gravitational field will be presented. Then, the superoperator technique will be generalized and an exact solution of the non-Markovian master equation will be obtained. Assuming that initially the radiation field is prepared in a Glauber coherent state and the two-level atom is in the excited state, the influence of gravity on the temporal evolution of collapses and revivals of the atomic population inversion, atomic dipole squeezing, and photon counting statistics of the radiation field in the presence of the non-Markovian phase damped will be investigated.

DOI: 10.1103/PhysRevA.80.053823 PACS number(s): 42.50.Pq, 42.50.Ct, 42.50.Dv

I. INTRODUCTION

Non-Markovian effects have received special attention in the past years, mainly in optics and radiation-matter interaction subjects, either in predicting novel effects or due to the necessity to go beyond the Markovian approximation in experiments involving femtosecond processes. Among the experimental papers, the recent ones can be cited. Tchenio *et al.* prepared a Non-Markovian atomic excitation process, with adjustable memory time, using correlated laser pulses and they verified that under strong-field conditions, the atoms are not able to keep memory of the field phase and amplitude over a time interval larger than the coherence time [1]. Considering femtosecond experiments, non-Markovian behavior appears in the optical dephasing of molecules in solution, since the dynamics of the thermalized environment may occur on the same time scale of the system [2–6]. Concerning the theoretical approach, Lewenstein *et al.* predicted the suppression of spontaneous emission related to the decay of cavity atoms in the presence of a strong driving field, thus modifying the spectrum of resonance fluorescence [7]. Villaeys *et al.* studied the non-Markovian effects in the atomic absorption band shape for the transient and steady-state regimes; they concluded that in the steady state, the appearances of the non-Markovian effects are washed out and therefore they cannot be probed, but in the transient regime, these effects are perceptible [8]. In this same line, Gangopadhyay and Ray constructed a non-Markovian master equation by considering density matrices with small delay time [9]. Over the last 40 years, many theoretical investigations have been addressed toward the understanding of quantum dynamics of the interacting atom-field system in a high-Q cavity. The interest toward this research area was mainly spurred by the large amount of experiments, revealing the appearance of intriguing features of quantum radiation-matter interaction [10]. Both theoretical and experimental activities have concentrated on trying to understand simple nontrivial models of quantum optics involving a single atom, regarded as a few effective energy levels, and one or more near resonant modes of the quantized electromagnetic field. The prototype of such systems, proposed by the Jaynes-Cummings model (JCM) in 1963 [11], describes a two-level atom resonantly interacting with a single-mode quantized field. It has proved to be a theoretical laboratory of great relevance to many topics in atomic physics and quantum optics, as well as in the ion traps, cavity QED, and quantum information processing [12]. When the rotating wave approximation (RWA) is made, the model becomes exactly solvable and its dynamical features can be analytically brought to light revealing remarkable properties [13]. In the standard JCM, the interaction between a constant electric field and a stationary (motionless) two-level atom is considered. With the development in the technologies of laser cooling and atom trapping, the interaction between a moving atom and the field has attracted much attention [14–23].

Experimentally, atomic beams with very low velocities are generated in laser cooling and atomic interferometry [24]. It is obvious that for atoms moving with a velocity of a few millimeters or centimeters per second for a time period of several milliseconds or more, the influence of Earth's acceleration becomes important and cannot be neglected [25]. A semiclassical description of a two-level atom interacting with a running laser wave in a gravitational field has been studied [26,27]. However, the semiclassical treatment does not permit us to study the pure quantum effects occurring in the course of atom-radiation interaction. Recently, within a quantum treatment of the internal and external dynamics of the atom, we have presented [28] a theoretical scheme based on an $SU(2)$ dynamical algebraic structure to investigate the influence of a classical homogeneous gravitational field on the quantum nondemolition measurement of atomic momentum in the dispersive JCM. Also, the effects of the gravitational field on quantum statistical properties of the lossless

*majid471702@yahoo.com

[29] as well as the phase-damped JCMs [30] were investigated. We reach to the point that the gravitational field seriously suppresses nonclassical properties of both the cavity field and the moving atom. Also, the effects of the gravitational field on the dynamical evolution of the cavity-field entropy and the creation of the Schrödinger-cat state in the Jaynes-Cummings model [31] are examined.

On the other hand, over the last 2 decades, much attention has been focused on the properties of the dissipative variants of the JCM. The theoretical efforts have been stimulated by experimental progress in cavity QED. Besides the experimental drive, there also exists a theoretical motivation to include relevant damping mechanism to JCM because its dynamics becomes more interesting. A number of authors have treated the JCM with dissipation by the use of analytic approximations [32,33] and numerical calculations [34–38]. The solution in the presence of dissipation is not only of theoretical interest but also important from a practical point of view since dissipation would be always present in any experimental realization of the model. However, the dissipation treated in the above studies is modeled by coupling to an external reservoir including energy dissipation. As is well known, in a dissipative quantum system, the system loses energy by creating a bath quantum. In this kind of damping, the interaction Hamiltonian between bath and system does not commute with the system Hamiltonian. In general, this leads to a thermalization of the system with a certain time constant. There are, however, other kinds of environmental coupling to the system, which do not involve energy exchange. In the so-called phase damping [39], the interaction Hamiltonian commutes with that of system and in the dynamics, only the phase of system state is changed in the course of interaction. Similar to standard energy damping, the off-diagonal elements of the density matrix in energy basis decay at a given rate. The phase damping can well describe some unaccounted decay of coherences in a single-mode micromaser [40]. It has also been shown that phase damping seriously reduces the fidelity of the received qubit in quantum computers due to the induced decoherence [41]. The phase damping in the JCM with one quantized field mode has been studied [42]. The influence of phase damping on nonclassical properties of the multiquanta two-mode JCM has also been studied [43]. It has been found that the phase damping suppresses nonclassical effects of the cavity field in the JCM. However, all of the foregoing studies have been done only under the condition that the influence of the gravitational field is not taken into account.

In this paper, the non-Markovian dissipative dynamics of the phase-damped Jaynes-Cummings model in the presence of a classical homogeneous gravitational field will be analyzed. The model consists of a moving two-level atom simultaneously exposed to the gravitational field and a single-mode traveling radiation field in a high quality electromagnetic cavity in the presence of a non-Markovian phase damping mechanism. In Sec. II, the non-Markovian master equation for the reduced density operator of the system in terms of a Hamiltonian describing the atom-radiation interaction in the presence of a gravitational field will be presented. This Hamiltonian has been obtained based on a $SU(2)$ dynamical algebraic structure in the interaction picture. In Sec. III, an exact solution of the JCM with the phase damping in the presence of a gravitational field will be obtained, by which the dynamical evolution of the system is investigated. In Sec. IV, influence of gravity on both the cavity field and the atomic properties in the presence of the non-Markovian phase damped will be studied. Considering the field to be initially in a coherent state and the two-level atom in the excited state, the temporal evolution of the atomic inversion, atomic dipole squeezing, and photon counting statistics will be explored. Finally, our conclusions will be summarized in Sec. V.

II. NON-MARKOVIAN MASTER EQUATION FOR THE PHASE-DAMPED JCM IN THE PRESENCE OF A GRAVITATIONAL FIELD

The equation of motion for the density operator of the atom-radiation system and reservoir, $\hat{\rho}_{sr}(t)$, in the Schrödinger picture is given by [30]

$$\frac{\partial \hat{\rho}_{sr}(t)}{\partial t} = -i[\hat{\tilde{H}}_T, \hat{\rho}_{sr}(t)](\hbar = 1), \tag{1}$$

where

$$\hat{\tilde{H}}_T = \hat{H}_s + \hat{H}_r + \hat{V}_{sr}, \tag{2}$$

with the Hamiltonian of the reservoir

$$\hat{H}_r = \sum_i \omega_i \hat{b}_i^\dagger \hat{b}_i, \tag{3}$$

and with the Hamiltonian of the interaction between the system and reservoir

$$\hat{V}_{sr} = \hat{H}_s \sum_{j=1}^{3} \hat{F}_j, \tag{4}$$

where

$$\hat{F}_1 = \sum_i \kappa_i \hat{b}_i, \quad \hat{F}_2 = \sum_i \kappa_i \hat{b}_i^\dagger, \quad \hat{F}_3 = \hat{H}_s \sum_i \frac{\kappa_i^2}{2\omega_i}, \tag{5}$$

where $\hat{b}_i$ and $\hat{b}_i^\dagger$ are the boson annihilation and creation operators for the reservoir and κ_i is the coupling constant. The Hamiltonian $\hat{H}_s$ in Eq. (2) for the atom-radiation system in the presence of a classical gravity field with the atomic motion along the position vector $\hat{\vec{x}}$ and in the RWA is given by ($\hbar = 1$),

$$\hat{H}_s = \frac{\hat{p}^2}{2M} - M\vec{g} \cdot \hat{\vec{x}} + \omega_c\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right) + \frac{1}{2}\omega_{eg}\hat{\sigma}_z + \lambda \times [\exp(-i\vec{q} \cdot \hat{\vec{x}})\hat{a}^\dagger \hat{\sigma}_- + \exp(i\vec{q} \cdot \hat{\vec{x}})\hat{\sigma}_+ \hat{a}], \tag{6}$$

where $\hat{a}$ and $\hat{a}^\dagger$ denote, respectively, the annihilation and creation operators of a single-mode traveling wave with frequency ω_c, $\vec{q}$ is the wave vector of the running wave, and $\hat{\sigma}_\pm$ denote the raising and lowering operators of the two-level atom with electronic levels $|e\rangle, |g\rangle$ and Bohr transition frequency ω_{eg}. The atom-field coupling is given by the parameter λ, $\hat{\vec{p}}$ and, $\hat{\vec{x}}$ denote, respectively, the momentum and position operators of the atomic center-of-mass motion, and g is Earth's gravitational acceleration. It has been shown [30] that based on an $SU(2)$ algebraic structure, as the dynamical symmetry group of the model, in the interaction picture, Hamiltonian (6) takes the following form:

$$\hat{\tilde{H}}_s^I = \omega_c\left(\hat{a}^\dagger\hat{a} + \frac{\hat{S}_0}{2}\right) + \frac{1}{2}\hat{\Delta}(\hat{\vec{p}},\vec{g},t)\hat{S}_0 + [\hat{\kappa}(t)\sqrt{\hat{K}}\hat{S}_- + \hat{\kappa}^*(t)\sqrt{\hat{K}}\hat{S}_+], \tag{7}$$

where the operators

$$\hat{S}_0 = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|), \quad \hat{S}_+ = \hat{a}|e\rangle\langle g|\frac{1}{\sqrt{\hat{K}}}, \quad \hat{S}_- = \frac{1}{\sqrt{\hat{K}}}|g\rangle\langle e|\hat{a}^\dagger, \tag{8}$$

with the following commutation relations:

$$[\hat{S}_0,\hat{S}_\pm] = \pm\hat{S}_\pm, \quad [\hat{S}_-,\hat{S}_+] = -2\hat{S}_0, \tag{9}$$

are the generators of the $SU(2)$ algebra, the operator $\hat{K} = \hat{a}^\dagger\hat{a} + |e\rangle\langle e|$ is a constant of motion which represents the total number of excitations of the atom-radiation, $\hat{\kappa}(t)$ is an effective coupling coefficient

$$\hat{\kappa}(t) = \lambda\, \exp\left\{\frac{it}{2}\left[\hat{\Delta}(\hat{\vec{p}},t,\vec{g}) + \frac{\hbar q^2}{M}\right]\right\}, \tag{10}$$

and the operator

$$\hat{\Delta}(\hat{\vec{p}},t,\vec{g}) = \omega_c - \left(\omega_{eg} + \frac{\vec{q}\cdot\hat{\vec{p}}}{M} + \vec{q}\cdot\vec{g}t + \frac{q^2}{2M}\right) \tag{11}$$

has been introduced as the Doppler shift detuning at time t [30]. By using and following the same procedure as in Refs. [30,44], we obtain the non-Markovian master equation for the reduced density operator of the system with neglecting $2\pi i\frac{d}{d\omega}[J(\omega)|\kappa(\omega)|^2]|_{\omega=0}$ and the lamb shift term

$$\frac{\partial\hat{\rho}_s(t)}{\partial t} = -i[\hat{\tilde{H}}_s^I,\hat{\rho}_s(t)] - \gamma\{\hat{H}_s^I,[\hat{H}_s^I,\hat{\rho}_s(t)]\} - \eta[\hat{H}_s^I,[\hat{H}_s^I,\{\hat{H}_s^I,[\hat{H}_s^I,\hat{\rho}_s(t)]\}]], \tag{12}$$

where $\hat{\tilde{H}}_s^I$ is given by Eq. (7). In Eq. (12), γ and η are the damping and the non-Markovian parameters, respectively, which depend on the temperature T,

$$\gamma = 2\pi T \lim_{\omega\to 0}\left(\frac{J(\omega)|\kappa(\omega)|^2}{\omega}\right), \tag{13}$$

and

$$\eta = 2\pi T\wp\int_0^\infty d\omega J(\omega)\frac{|\kappa(\omega)|^2}{\omega^3}, \tag{14}$$

where $J(\omega)$ and $\kappa(\omega)$ are the spectral density of the reservoir and the coupling coefficient, respectively, and $\wp$ is the Cauchy principal part of the integration [44].

III. DYNAMICAL EVOLUTION OF THE NON-MARKOVIAN PHASE-DAMPED JCM IN THE PRESENCE OF CLASSICAL GRAVITY

In Sec. II, we reached to the non-Markovian master equation for the reduced density operator of the atom-radiation system in the presence of a classical homogeneous gravitational field. In this section, we now start to find the exact solution for the density operator $\hat{\rho}_s(t)$ of the non-Markovian master Eq. (12) with the Hamiltonian (7). For this purpose, the approach presented in Refs. [44–46] is applied. The formal solution is given by

$$\hat{\rho}_s(t) = \exp(\hat{R}t)\exp(\hat{S}t)\exp(\hat{T}t)\hat{\rho}_s(0), \tag{15}$$

$$\exp(\hat{R}t) = \exp(\hat{R}_1 t)\exp(\hat{R}_2 t)\exp(\hat{R}_3 t), \tag{16}$$

$$\exp(\hat{T}t) = \exp(\hat{T}_1 t)\exp(\hat{T}_2 t), \tag{17}$$

where $\hat{\rho}_s(0)$ is the density operator of the initial atom-field system. The auxiliary superoperators $\hat{R}_1,\hat{R}_2,\hat{R}_3,\hat{S}$ and $\hat{T}_1,\hat{T}_1$ are defined through their action on the density operator such that

$$\exp(\hat{R}_1 t)\hat{\rho}_s(0) \equiv \sum_{k=0}^\infty \frac{(2\gamma t)^k}{k!}(\hat{\tilde{H}}_s^I)^k\hat{\rho}_s(0)(\hat{\tilde{H}}_s^I)^k, \tag{18}$$

$$\exp(\hat{R}_2 t)\hat{\rho}_s(0) \equiv \sum_{l=0}^\infty \frac{(-3\gamma\eta t)^l}{l!}(\hat{\tilde{H}}_s^I)^{2l}\hat{\rho}_s(0)(\hat{\tilde{H}}_s^I)^{2l}, \tag{19}$$

$$\exp(\hat{R}_3 t)\hat{\rho}_s(0) \equiv \sum_{m=0}^\infty \frac{(-\gamma\eta t)^m}{m!}(\hat{\tilde{H}}_s^I)^{m}\hat{\rho}_s(0)(\hat{\tilde{H}}_s^I)^{2m}, \tag{20}$$

$$\exp(\hat{S}t)\hat{\rho}_s(0) \equiv \exp(-i\hat{\tilde{H}}_s^I t)\hat{\rho}_s(0)\exp(i\hat{\tilde{H}}_s^I t), \tag{21}$$

$$\exp(\hat{T}_1 t)\hat{\rho}_s(0) \equiv \exp[-\gamma(\hat{\tilde{H}}_s^I)^2 t]\hat{\rho}_s(0)\exp[-\gamma(\hat{\tilde{H}}_s^I)^2 t], \tag{22}$$

$$\exp(\hat{T}_2 t)\hat{\rho}_s(0) \equiv \exp[-\gamma\eta(\hat{\tilde{H}}_s^I)^4 t]\hat{\rho}_s(0)\exp[-\gamma\eta(\hat{\tilde{H}}_s^I)^4 t]. \tag{23}$$

It is assumed that initially the radiation field is in a coherent superposition of the Fock states, the atom is in the excited state $|e\rangle$, and the state vector for the center-of-mass degree of freedom is $|\psi_{c.m}(0)\rangle = \int d^3p\,\phi(\vec{p})|\vec{p}\rangle$. Therefore, the initial density operator of the atom-radiation system reads as

$$\hat{\rho}_s(0) = \hat{\rho}_{field}(0)\otimes\hat{\rho}_{atom}(0)\otimes\hat{\rho}_{c.m}(0) = \begin{bmatrix}\hat{\rho}_{field}(0)\otimes\hat{\rho}_{c.m}(0) & 0\\ 0 & 0\end{bmatrix}, \tag{24}$$

where

$$\hat{\rho}_{field}(0) = \sum_n\sum_m w_n(0)w_m(0)|n\rangle\langle m|, \tag{25}$$

$$\hat{\rho}_{c.m}(0) = \int d^3p \int d^3p' \, \phi^*(\vec{p}\,') \phi(\vec{p}) |\vec{p}\rangle\langle\vec{p}\,'|, \tag{26}$$

with $w_n(0) = \frac{\exp(-|\alpha|^2/2)\alpha^n}{\sqrt{n!}}$. The Hamiltonian (7) can be expressed as a sum of two terms which commute with each other, that is,

$$\hat{\tilde{H}}^I_s = \hat{H}_1 + \hat{H}_2, \quad [\hat{H}_1, \hat{H}_2] = 0 \tag{27}$$

where

$$\hat{H}_1 = \omega_c\left(\hat{a}^\dagger\hat{a} + \frac{\hat{S}_0}{2}\right), \tag{28}$$

$$\hat{H}_2 = \frac{1}{2}\hat{\Delta}(\hat{\vec{p}},\vec{g},t)\hat{S}_0 + (\hat{\kappa}(t)\sqrt{\hat{K}}\hat{S}_- + \hat{\kappa}^*(t)\sqrt{\hat{K}}\hat{S}_+). \tag{29}$$

In the two-dimensional atomic basis, we have

$$\hat{H}_1 = \omega_c \begin{bmatrix} \hat{n} + \frac{1}{2} & 0 \\ 0 & \hat{n} - \frac{1}{2} \end{bmatrix}, \tag{30}$$

$$\hat{H}_2 = \begin{bmatrix} \frac{\Delta(\vec{p},\vec{g},t)}{4} & \kappa^*(t)\hat{a} \\ \kappa(t)\hat{a}^\dagger & -\frac{\Delta(\vec{p},\vec{g},t)}{4} \end{bmatrix}. \tag{31}$$

Also, the square of the Hamiltonian (7) can be expressed as a sum of two operators: one of them is diagonal, in the form

$$(\hat{\tilde{H}}^I_s)^2 = \hat{A}_1 + \hat{A}_2, \tag{32}$$

where

$$\hat{A}_1 = \hat{H}_1^2 + \hat{H}_2^2 = \begin{bmatrix} \omega_c^2\left(\hat{n} + \frac{1}{2}\right)^2 + \lambda^2(\hat{n}+1) + \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 & 0 \\ 0 & \omega_c^2\left(\hat{n} - \frac{1}{2}\right)^2 + \lambda^2\hat{n} + \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 \end{bmatrix} \tag{33}$$

and

$$\hat{A}_2 = 2\hat{H}_1\hat{H}_2 = 2\omega_c \begin{bmatrix} \left(\hat{n} + \frac{1}{2}\right)\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right) & \left(\hat{n} + \frac{1}{2}\right)\kappa^*(t)\hat{a} \\ \left(\hat{n} - \frac{1}{2}\right)\kappa(t)\hat{a}^\dagger & -\left(\hat{n} - \frac{1}{2}\right)\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right) \end{bmatrix}. \tag{34}$$

It is easily proved that $[\hat{A}_1, \hat{A}_2] = 0$. Similarly, the square of the $(\hat{\tilde{H}}^I_s)^2$ can be expressed as a sum of two operators: one of them is diagonal, in the form

$$(\hat{\tilde{H}}^I_s)^4 = \hat{A}_3 + \hat{A}_4, \tag{35}$$

where

$$\hat{A}_3 = \hat{A}_1^2 + \hat{A}_2^2 = \begin{bmatrix} (\hat{A}_3)_{11} & 0 \\ 0 & (\hat{A}_3)_{22} \end{bmatrix}, \tag{36}$$

with

$$(A_3)_{11} = \left[\omega_c^2\left(\hat{n} + \frac{1}{2}\right)^2 + \lambda^2(\hat{n}+1) + \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2\right]^2 + 4\omega_c^2\left(\hat{n} + \frac{1}{2}\right)\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right) + 4\omega_c^2\lambda^2\left(\hat{n} + \frac{1}{2}\right)^2(\hat{n}+1), \tag{37}$$

$$(A_3)_{22} = \left[\omega_c^2\left(\hat{n} - \frac{1}{2}\right)^2 + \lambda^2\hat{n} + \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2\right]^2 + 4\omega_c^2\left(\hat{n} + \frac{1}{2}\right) \times \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right) + 4\omega_c^2\lambda^2\left(\hat{n} - \frac{1}{2}\right)^2\hat{n}, \tag{38}$$

and

$$\hat{A}_4 = 2\hat{A}_1\hat{A}_2 = \begin{bmatrix} (\hat{A}_4)_{11} & (\hat{A}_4)_{12} \\ (\hat{A}_4)_{21} & (\hat{A}_4)_{22} \end{bmatrix}, \tag{39}$$

with

$$(\hat{A}_4)_{11} = \left[\omega_c^2\left(\hat{n} + \frac{1}{2}\right)^2 + \lambda^2(\hat{n}+1) + \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2\right] \times 4\omega_c\left(\hat{n} + \frac{1}{2}\right)\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right), \tag{40}$$

$$(\hat{A}_4)_{12} = \left[\omega_c^2\left(\hat{n}+\frac{1}{2}\right)^2 + \lambda^2(\hat{n}+1) + \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 \right] \times 4\omega_c\left(\hat{n}+\frac{1}{2}\right)\kappa^*\hat{a}, \tag{41}$$

$$(\hat{A}_4)_{21} = \left[\omega_c^2\left(\hat{n}-\frac{1}{2}\right)^2 + \lambda^2\hat{n} + \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 \right] 4\omega_c\left(\hat{n}-\frac{1}{2}\right)\kappa\hat{a}^\dagger, \tag{42}$$

$$(\hat{A}_4)_{22} = -\left[\omega_c^2\left(\hat{n}-\frac{1}{2}\right)^2 + \lambda^2\hat{n} + \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 \right] 4\omega_c\left(\hat{n}-\frac{1}{2}\right) \times \left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right). \tag{43}$$

It is easily proved that $[\hat{A}_3,\hat{A}_4]=0$. Taking into account the initial condition (24), the auxiliary density operator $\hat{\rho}_2(t)$ is defined as

$$\begin{aligned}\hat{\rho}_2(t) &= \exp(\hat{S}t)\exp(\hat{T}t)\hat{\rho}_s(0)\\ &= \exp(-i\hat{H}_2t)\exp(-\gamma\hat{A}_2t)\exp(-\gamma\eta\hat{A}_4t)\hat{\rho}_1(t)\\ &\quad\times\exp(-\gamma\eta\hat{A}_4t)\exp(-\gamma\hat{A}_2t)\exp(i\hat{H}_2t),\end{aligned} \tag{44}$$

where the operator $\hat{\rho}_1(t)$ is defined by

$$\hat{\rho}_1(t) = |\Psi(t)\rangle\langle\Psi(t)| \otimes |e\rangle\langle e|, \tag{45}$$

with

$$\begin{aligned}|\Psi(t)\rangle = \exp\Bigg(&-\gamma(1+\eta)t\Bigg\{\omega_c^2\left(\hat{n}+\frac{1}{2}\right)^2+\lambda^2(\hat{n}+1)\\ &+\left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]^2\Bigg\}\Bigg)\exp\Bigg\{-4\gamma\eta\omega_c^2\left(\hat{n}+\frac{1}{2}\right)^2\Bigg[\lambda^2(\hat{n}+1)\\ &+\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2\Bigg]\Bigg\}w_n(0)\exp(-in\omega_ct)|n\rangle.\end{aligned} \tag{46}$$

From Eqs. (30) and (33), we have, respectively,

$$\exp(-i\hat{H}_1t) = \begin{bmatrix} \exp\left[-i\omega_ct\left(\hat{n}+\frac{1}{2}\right)\right] & 0 \\ 0 & \exp\left[-i\omega_ct\left(\hat{n}-\frac{1}{2}\right)\right] \end{bmatrix}, \tag{47}$$

$$\exp(-\gamma\hat{A}_1t) = \begin{bmatrix} (\hat{A}_1)_{11}(\hat{n},t) & 0 \\ 0 & (\hat{A}_1)_{22}(\hat{n},t) \end{bmatrix}, \tag{48}$$

where

$$(\hat{A}_1)_{11}(\hat{n},t) = \exp\left(-\gamma t\left[\omega_c^2\left(\hat{n}+\frac{1}{2}\right)^2+\lambda^2(\hat{n}+1)+\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2\right]\right), \tag{49}$$

$$(\hat{A}_1)_{22}(\hat{n},t) = \exp\left(-\gamma t\left[\omega_c^2\left(\hat{n}-\frac{1}{2}\right)^2+\lambda^2\hat{n}+\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2\right]\right). \tag{50}$$

Also, we can write

$$\exp(-\gamma\hat{A}_2t) = \begin{bmatrix} \hat{e}_1(\hat{n},t) & \hat{e}_2(\hat{n},t)\hat{a} \\ \hat{e}_3(\hat{n},t)\hat{a}^\dagger & \hat{e}_4(\hat{n},t) \end{bmatrix}, \tag{51}$$

$$\exp(-\gamma\eta\hat{A}_4t) = \begin{bmatrix} \hat{e}_1'(\hat{n},t) & \hat{e}_2'(\hat{n},t)\hat{a} \\ \hat{e}_3'(\hat{n},t)\hat{a}^\dagger & \hat{e}_4'(\hat{n},t) \end{bmatrix}, \tag{52}$$

where

$$\hat{e}_1(\hat{n},t) = \cosh[\gamma t\sqrt{\hat{c}_1(\hat{n},t)}] - \omega_c\left[\frac{\Delta(\vec{p},\vec{g},t)}{2}\right] \times\left(\hat{n}+\frac{1}{2}\right)\frac{\sinh[\gamma t\sqrt{\hat{c}_1(\hat{n},t)}]}{\sqrt{\hat{c}_1(\hat{n},t)}}, \tag{53}$$

$$\hat{e}_2(\hat{n},t) = -2\omega_c\lambda\left(\hat{n}-\frac{1}{2}\right)\frac{\sinh[\gamma t\sqrt{\hat{c}_1(\hat{n}-1,t)}]}{\sqrt{\hat{c}_1(\hat{n}-1,t)}}, \tag{54}$$

$$\hat{e}_3(\hat{n},t) = -2\omega_c\lambda\left(\hat{n}-\frac{1}{2}\right)\frac{\sinh[\gamma t\sqrt{\hat{c}_2(\hat{n},t)}]}{\sqrt{\hat{c}_2(\hat{n},t)}}, \tag{55}$$

$$\hat{e}_4(\hat{n},t) = \cosh(\gamma t\sqrt{\hat{c}_2(\hat{n},t)}) - \omega_c\left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right) \times\left(\hat{n}-\frac{1}{2}\right)\frac{\sinh(\gamma t\sqrt{\hat{c}_2(\hat{n},t)})}{\sqrt{\hat{c}_2(\hat{n},t)}}, \tag{56}$$

with

$$\hat{c}_1(\hat{n},t) = \omega_c^2\left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right)^2\left(\hat{n}+\frac{1}{2}\right)^2 + \lambda^2\left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right)^2(\hat{n}+1) \times\left(\hat{n}+\frac{1}{2}\right)^2, \tag{57}$$

$$\hat{c}_2(\hat{n},t) = \omega_c^2\left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right)^2\left(\hat{n}-\frac{1}{2}\right)^2 + \lambda^2\left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right)^2 \times\hat{n}\left(\hat{n}-\frac{1}{2}\right)^2, \tag{58}$$

and

$$\begin{aligned}\hat{e}_1'(\hat{n},t) = \cosh[&\gamma\eta t\sqrt{\hat{c}_1'(\hat{n},t)}]\\ &-2\omega_c\left[\frac{\Delta(\vec{p},\vec{g},t)}{2}\right]\left(\hat{n}+\frac{1}{2}\right)\\ &\times\hat{L}_1(\hat{n},t)\frac{\sinh[\gamma\eta t\sqrt{\hat{c}_1'(\hat{n},t)}]}{\sqrt{\hat{c}_1'(\hat{n},t)}},\end{aligned} \tag{59}$$

$$\hat{e}_2'(\hat{n},t) = -2\omega_c\lambda\left(\hat{n}-\frac{1}{2}\right)\hat{L}_2(\hat{n},t)\frac{\sinh[\gamma\eta t\sqrt{\hat{c}_1'(\hat{n}-1,t)}]}{\sqrt{\hat{c}_1'(\hat{n}-1,t)}}, \tag{60}$$

$$\hat{e}_3'(\hat{n},t) = -2\omega_c\lambda\left(\hat{n}-\frac{1}{2}\right)\hat{L}_2(\hat{n},t)\frac{\sinh[\gamma\eta t\sqrt{\hat{c}_2'(\hat{n},t)}]}{\sqrt{\hat{c}_2'(\hat{n},t)}}, \tag{61}$$

$$\hat{e}_4'(\hat{n},t) = \cosh[\gamma\eta t\sqrt{\hat{c}_2'(\hat{n},t)}] - \omega_c\left[\frac{\Delta(\vec{p},\vec{g},t)}{2}\right]\times\left(\hat{n}-\frac{1}{2}\right)\hat{L}_2(\hat{n},t)\frac{\sinh[\gamma\eta t\sqrt{\hat{c}_2'(\hat{n},t)}]}{\sqrt{\hat{c}_2'(\hat{n},t)}}, \tag{62}$$

with

$$\hat{c}_1'(\hat{n},t) = \omega_c^2\left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right)^2\left(\hat{n}+\frac{1}{2}\right)^2\hat{L}_1^2(\hat{n},t) + \hat{L}_1(\hat{n},t)\hat{L}_2(\hat{n},t)(\hat{n}+1)\left(\hat{n}+\frac{1}{2}\right)^2, \tag{63}$$

$$\hat{c}_2'(\hat{n},t) = \omega_c^2\left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right)^2\left(\hat{n}-\frac{1}{2}\right)^2\hat{L}_2^2 \times(\hat{n},t) + \hat{L}_2(\hat{n},t)\hat{L}_1(\hat{n},t)\hat{n}\left(\hat{n}-\frac{1}{2}\right)^2, \tag{64}$$

where

$$\hat{L}_1(\hat{n},t) = \omega_c^2\left(\hat{n}+\frac{1}{2}\right)^2 + \lambda^2(\hat{n}+1) + \left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right)^2, \tag{65}$$

$$\hat{L}_2(\hat{n},t) = \omega_c^2\left(\hat{n}-\frac{1}{2}\right)^2 + \lambda^2\hat{n} + \left(\frac{\Delta(\vec{p},\vec{g},t)}{2}\right)^2. \tag{66}$$

Similarly, the operator $\exp(-i\hat{H}_2 t)$ in the two-dimensional atomic basis can be stated as

$$\exp(-i\hat{H}_2 t) = \begin{bmatrix} \hat{d}_1(\hat{n},t) & \hat{d}_2(\hat{n},t)\hat{a} \\ \hat{d}_3(\hat{n},t)\hat{a}^\dagger & \hat{d}_4(\hat{n},t) \end{bmatrix}, \tag{67}$$

where

$$\hat{d}_1(\hat{n},t) = \cos\left\{t\left[\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 + \lambda^2(\hat{n}+1)\right]\right\} - \left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]\frac{\sin\left\{t\left[\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 + \lambda^2(\hat{n}+1)\right]\right\}}{\sqrt{\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 + \lambda^2(\hat{n}+1)}}, \tag{68}$$

$$\hat{d}_2(\hat{n},t) = -i\lambda\frac{\sin\left\{t\left[\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2 + \lambda^2(\hat{n}+1)\right]\right\}}{\sqrt{\left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]^2 + \lambda^2(\hat{n}+1)}}, \tag{69}$$

$$\hat{d}_3(\hat{n},t) = -i\lambda\frac{\sin\left(t\left\{\left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]^2 + \lambda^2\hat{n}\right\}\right)}{\sqrt{\left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]^2 + \lambda^2\hat{n}}}, \tag{70}$$

$$\hat{d}_4(\hat{n},t) = \cos\left(t\left\{\left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]^2 + \lambda^2\hat{n}\right\}\right) - \left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]\frac{\sin\left(t\left\{\left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]^2 + \lambda^2\hat{n}\right\}\right)}{\sqrt{\left[\frac{\Delta(\vec{p},\vec{g},t)}{4}\right]^2 + \lambda^2\hat{n}}}. \tag{71}$$

Then, from Eqs. (51) and (67), it follows that

$$\exp(-i\hat{H}_2 t)\exp(-\gamma\hat{A}_2 t) = \begin{bmatrix} \hat{f}_1(\hat{n},t) & \hat{f}_2(\hat{n},t)\hat{a} \\ \hat{f}_3(\hat{n},t)\hat{a}^\dagger & \hat{f}_4(\hat{n},t) \end{bmatrix}, \tag{72}$$

where

$$\hat{f}_1(\hat{n},t) = \hat{e}_1(\hat{n},t)\hat{d}_1(\hat{n},t) + \hat{e}_2(\hat{n},t)\hat{d}_2(\hat{n},t), \tag{73}$$

$$\hat{f}_2(\hat{n},t) = \hat{e}_2(\hat{n},t)\hat{d}_1(\hat{n},t) + \hat{e}_1(\hat{n},t)\hat{d}_2(\hat{n},t), \tag{74}$$

$$\hat{f}_3(\hat{n},t) = \hat{e}_3(\hat{n},t)\hat{d}_4(\hat{n},t) + \hat{e}_4(\hat{n},t)\hat{d}_3(\hat{n},t), \tag{75}$$

$$\hat{f}_4(\hat{n},t) = \hat{e}_4(\hat{n},t)\hat{d}_4(\hat{n},t) + \hat{e}_3(\hat{n},t)\hat{d}_3(\hat{n},t). \tag{76}$$

Also, from Eqs. (52) and (60), we have

$$\exp(-i\hat{H}_2 t)\exp(-\gamma\hat{A}_2 t)\exp(-\gamma\eta\hat{A}_4 t) = \begin{bmatrix} \hat{J}_1(\hat{n},t) & \hat{J}_2(\hat{n},t)\hat{a} \\ \hat{J}_3(\hat{n},t)\hat{a}^\dagger & \hat{J}_4(\hat{n},t) \end{bmatrix}, \tag{77}$$

where

$$\hat{J}_1(\hat{n},t) = \hat{f}_1(\hat{n},t)\hat{e}_1'(\hat{n},t) + \hat{f}_2(\hat{n},t)\hat{e}_2'(\hat{n},t), \tag{78}$$

$$\hat{J}_2(\hat{n},t) = \hat{f}_2(\hat{n},t)\hat{e}_1'(\hat{n},t) + \hat{f}_1(\hat{n},t)\hat{e}_2'(\hat{n},t), \tag{79}$$

$$\hat{J}_3(\hat{n},t) = \hat{f}_3(\hat{n},t)\hat{e}_4'(\hat{n},t) + \hat{f}_4(\hat{n},t)\hat{e}_3'(\hat{n},t), \tag{80}$$

$$\hat{J}_4(\hat{n},t) = \hat{f}_4(\hat{n},t)\hat{e}_4'(\hat{n},t) + \hat{f}_3(\hat{n},t)\hat{e}_3'(\hat{n},t). \tag{81}$$

Substituting Eqs. (45) and (77) into Eq. (44), an explicit expression for the operator $\hat{\rho}_2(t)$ can be obtained as follows:

$$(\hat{\rho}_2(t))_{i,j} = |\Psi_i(t)\rangle\langle\Psi_j(t)|, \quad (i,j=1,2), \tag{82}$$

with

$$|\Psi_1(t)\rangle = \hat{J}_1(\hat{n},t)|\Psi(t)\rangle, \quad |\Psi_2(t)\rangle = \hat{J}_3(\hat{n},t)|\Psi(t)\rangle, \tag{83}$$

where $|\Psi(t)\rangle$ is given by Eq. (46). Now, we obtain the action of the operator $\exp(\hat{R}t)=\exp(\hat{R}_1 t)\exp(\hat{R}_2 t)\exp(\hat{R}_3 t)$ on the operator $\hat{\rho}_2(t)$,

$$\hat{\rho}_3(t)=\sum_{i=0}^{\infty}\frac{(-\gamma\eta t)^i}{i!}\hat{H}^i\hat{\rho}_2(t)\hat{H}^{2i}, \tag{84}$$

$$\hat{\rho}_4(t)=\sum_{j=0}^{\infty}\frac{(-3\gamma\eta t)^j}{j!}\hat{H}^{2j}\hat{\rho}_3(t)\hat{H}^j, \tag{85}$$

$$\hat{\rho}_s(t)=\sum_{k=0}^{\infty}\frac{(2\gamma t)^k}{k!}\hat{H}^k\hat{\rho}_4(t)\hat{H}^k, \tag{86}$$

where

$$\hat{H}^k=\sum_{l=0}^{k}\frac{k!}{l!\,(k-l)!}\hat{H}_1^{k-l}\hat{H}_2^l, \tag{87}$$

which can be explicitly expressed as follows:

$$\hat{H}^k=\begin{bmatrix}\hat{g}_+^k(\hat{n},t) & \kappa^*(t)\dfrac{\hat{u}_-^k(\hat{n},t)}{\sqrt{\left(\dfrac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2+\lambda^2(\hat{n}+1)}}\hat{a}\\ \kappa(t)\dfrac{\hat{v}_-^k(\hat{n},t)}{\sqrt{\left(\dfrac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2+\lambda^2(\hat{n}+1)}}\hat{a}^\dagger & \hat{g}_-^k(\hat{n},t)\end{bmatrix}, \tag{88}$$

where

$$\hat{g}_+^k(\hat{n},t)=\hat{u}_+^k(\hat{n},t)+\frac{\Delta(\vec{p},\vec{g},t)}{4}\hat{u}_-^k(\hat{n},t), \tag{89}$$

$$\hat{g}_-^k(\hat{n},t)=\hat{v}_+^k(\hat{n},t)-\frac{\Delta(\vec{p},\vec{g},t)}{4}\hat{v}_-^k(\hat{n},t), \tag{90}$$

$$\hat{u}_\pm^k(\hat{n},t)=\frac{1}{2}[\hat{r}_+^k(\hat{n},t)\pm\hat{r}_-^k(\hat{n},t)],$$

$$\hat{v}_\pm^k(\hat{n},t)=\frac{1}{2}[\hat{s}_+^k(\hat{n},t)\pm\hat{s}_-^k(\hat{n},t)], \tag{91}$$

with

$$\hat{r}_\pm(\hat{n},t)=\omega_c\left(\hat{n}+\frac{1}{2}\right)\pm\sqrt{\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2+\lambda^2(\hat{n}+1)}, \tag{92}$$

$$\hat{s}_\pm(\hat{n},t)=\omega_c\left(\hat{n}-\frac{1}{2}\right)\pm\sqrt{\left(\frac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2+\lambda^2\hat{n}}. \tag{93}$$

Finally, by substituting Eqs. (85) and (87) into Eq. (86), we obtain the exact solution of the non-Markovian master Eq. (12) for the phase-damped JCM in the presence of a classical homogeneous gravity field

$$\hat{\rho}_s(t)=\begin{bmatrix}(\hat{\rho}_s)_{11}(t) & (\hat{\rho}_s)_{12}(t)\\ (\hat{\rho}_s)_{21}(t) & (\hat{\rho}_s)_{22}(t)\end{bmatrix}, \tag{94}$$

where

$$\begin{aligned}(\hat{\rho}_s)_{11}(t)=\sum_{k=0}^{\infty}\frac{(2\gamma t)^k}{k!}[&\hat{g}_+^k(\hat{n},t)(\hat{\rho}_4)_{11}(t)\hat{g}_+^k(\hat{n},t)+\hat{a}\hat{v}_-'^k(\hat{n},t)\\ &\times(\hat{\rho}_4)_{21}(t)\hat{g}_+^k(\hat{n},t)+\hat{g}_+^k(\hat{n},t)(\hat{\rho}_4)_{12}(t)\hat{v}_-'^k(\hat{n},t)\hat{a}^\dagger\\ &+\hat{a}\hat{v}_-'^k(\hat{n},t)(\hat{\rho}_4)_{22}(t)\hat{v}_-'^k(\hat{n},t)\hat{a}^\dagger]|\phi(\vec{p})|^2,\end{aligned} \tag{95}$$

$$\begin{aligned}(\hat{\rho}_s)_{22}(t)=\sum_{k=0}^{\infty}\frac{(2\gamma t)^k}{k!}[&\hat{v}_-'^k(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_4)_{11}(t)\hat{a}\hat{v}_-'^k(\hat{n},t)+\hat{g}_-^k(\hat{n},t)\\ &\times(\hat{\rho}_4)_{21}(t)\hat{a}\hat{v}_+'^k(\hat{n},t)+\hat{v}_-'^k(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_4)_{12}(t)\hat{g}_-^k(\hat{n},t)\\ &+\hat{g}_-^k(\hat{n},t)(\hat{\rho}_4)_{22}(t)\hat{g}_-^k(\hat{n},t)]|\phi(\vec{p})|^2,\end{aligned} \tag{96}$$

$$\begin{aligned}(\hat{\rho}_s)_{12}(t)=(\hat{\rho}_s)_{21}(t)^\dagger=\sum_{k=0}^{\infty}\frac{(2\gamma t)^k}{k!}[&\hat{v}_-'^k(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_4)_{11}(t)\hat{g}_+^k(\hat{n},t)\\ &+\hat{g}_-^k(\hat{n},t)(\hat{\rho}_4)_{21}(t)\hat{a}\hat{g}_+^k(\hat{n},t)+\hat{v}_-'^k(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_4)_{12}(t)\\ &\times(t)\hat{v}_-'^k(\hat{n},t)\hat{a}^\dagger+\hat{g}_-^k(\hat{n},t)(\hat{\rho}_4)_{22}(t)\hat{v}_-'^k(\hat{n},t)\hat{a}^\dagger]|\phi(\vec{p})|^2,\end{aligned} \tag{97}$$

with

$$\hat{v}_-'^k(\hat{n},t)=\frac{\lambda}{\sqrt{\left(\dfrac{\Delta(\vec{p},\vec{g},t)}{4}\right)^2+\lambda^2\hat{n}}}\hat{v}_-^k(\hat{n},t), \tag{98}$$

where

$$(\hat{\rho}_4)_{11}(t) = \sum_{j=0}^{\infty} \frac{(-3\gamma\eta t)^j}{j!}[\hat{g}_+^{2j}(\hat{n},t)(\hat{\rho}_3)_{11}(t)\hat{g}_+^{j}(\hat{n},t) + \hat{a}\hat{v}_-'^{2j}(\hat{n},t) \times(\hat{\rho}_3)_{21}(t)\hat{g}_+^{j}(\hat{n},t) + \hat{g}_+^{2j}(\hat{n},t)(\hat{\rho}_3)_{12}(t)\hat{v}_-'^{j}(\hat{n},t)\hat{a}^\dagger + \hat{a}\hat{v}_-'^{2j}(\hat{n},t)(\hat{\rho}_3)_{22}(t)\hat{v}_-'^{j}(\hat{n},t)\hat{a}^\dagger], \tag{99}$$

$$(\hat{\rho}_4)_{22}(t) = \sum_{j=0}^{\infty} \frac{(-3\gamma\eta t)^j}{j!}[\hat{v}_-'^{2j}(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_3)_{11}(t)\hat{a}\hat{v}_-'^{j}(\hat{n},t) + \hat{g}_-^{2j}(\hat{n},t)(\hat{\rho}_3)_{21}(t)\hat{a}\hat{v}_+'^{j}(\hat{n},t) + \hat{v}_-'^{2j}(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_3)_{12}(t)\hat{g}_-^{j}(\hat{n},t) + \hat{g}_-^{2j}(\hat{n},t) \times(\hat{\rho}_3)_{22}(t)\hat{g}_-^{j}(\hat{n},t)], \tag{100}$$

$$(\hat{\rho}_4)_{12}(t) = (\hat{\rho}_4)_{21}(t)^\dagger = \sum_{j=0}^{\infty} \frac{(-3\gamma\eta t)^j}{j!}[\hat{v}_-'^{2j}(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_3)_{11}(t)\hat{g}_+^{j}(\hat{n},t) + \hat{g}_-^{2j}(\hat{n},t) \times(\hat{\rho}_3)_{21}(t)\hat{a}\hat{g}_+^{j}(\hat{n},t) + \hat{v}_-'^{2j}(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_3)_{12}(t) \times(t)\hat{v}_-'^{j}(\hat{n},t)\hat{a}^\dagger + \hat{g}_-^{2j}(\hat{n},t)(\hat{\rho}_3)_{22}(t)\hat{v}_-'^{j}(\hat{n},t)\hat{a}^\dagger], \tag{101}$$

with

$$(\hat{\rho}_3)_{11}(t) = \sum_{i=0}^{\infty} \frac{(-\gamma\eta t)^i}{i!}[\hat{g}_+^{i}(\hat{n},t)(\hat{\rho}_2)_{11}(t)\hat{g}_+^{2i}(\hat{n},t) + \hat{a}\hat{v}_-'^{i}(\hat{n},t) \times(\hat{\rho}_2)_{21}(t)\hat{g}_+^{2i}(\hat{n},t) + \hat{g}_+^{i}(\hat{n},t)(\hat{\rho}_2)_{12}(t)\hat{v}_-'^{2i}(\hat{n},t)\hat{a}^\dagger + \hat{a}\hat{v}_-'^{i}(\hat{n},t)(\hat{\rho}_2)_{22}(t)\hat{v}_-'^{2i}(\hat{n},t)\hat{a}^\dagger], \tag{102}$$

$$(\hat{\rho}_3)_{22}(t) = \sum_{i=0}^{\infty} \frac{(-\gamma\eta t)^i}{i!}[\hat{v}_-'^{i}(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_2)_{11}(t)\hat{a}\hat{v}_-'^{2i}(\hat{n},t) + \hat{g}_-^{i}(\hat{n},t) \times(\hat{\rho}_2)_{21}(t)\hat{a}\hat{v}_+'^{2i}(\hat{n},t) + \hat{v}_-'^{i}(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_2)_{12}(t)\hat{g}_-^{2i}(\hat{n},t) + \hat{g}_-^{i}(\hat{n},t)(\hat{\rho}_2)_{22}(t)\hat{g}_-^{2i}(\hat{n},t)], \tag{103}$$

$$(\hat{\rho}_3)_{12}(t) = (\hat{\rho}_4)_{21}(t)^\dagger = \sum_{i=0}^{\infty} \frac{(-\gamma\eta t)^i}{i!}[\hat{v}_-'^{i}(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_2)_{11}(t)\hat{g}_+^{2i}(\hat{n},t) + \hat{g}_-^{i}(\hat{n},t) \times(\hat{\rho}_2)_{21}(t)\hat{a}\hat{g}_+^{2i}(\hat{n},t) + \hat{v}_-'^{i}(\hat{n},t)\hat{a}^\dagger(\hat{\rho}_2)_{12}(t) \times(t)\hat{v}_-'^{2i}(\hat{n},t)\hat{a}^\dagger + \hat{g}_-^{i}(\hat{n},t)(\hat{\rho}_2)_{22}(t)\hat{v}_-'^{2i}(\hat{n},t)\hat{a}^\dagger], \tag{104}$$

where we have defined $[\hat{\rho}_2(t)]_{i,j}$, $(i,j=1,2)$ in Eq. (82).

Making use of the solution given by Eq. (94), one can evaluate the mean values of operators of interest. In the next section, it will be used to investigate various dynamical properties of the non-Markovian phase-damped JCM in the presence of a homogeneous gravitational field.

IV. DYNAMICAL PROPERTIES

Information in the quantum information [47] is coded into simple quantum objects, usually two-level systems called quantum bits (qubits). In ion traps [48], the bits are single ions coding information in internal degrees of freedom. The bits are coupled by the combined effect of laser light and Coulomb interaction between the ions. In cavity QED experiments, the bit is atom or single photon stored in a cavity [48]. In this section, influence of gravity on the quantum statistical properties of the atom (qubit) and the quantized radiation field in the presence of the non-Markovian phase damping will be studied [49]. Recent experiment processes [1–6] confirm the importance of non-Markovian effects since the dynamical characteristic times of the system become of the same time scale as the reservoir. In this paper, by considering the damping parameter $\gamma = 1\times 10^4$ rad/s [30], the relevant time scale introduced by the damping is $\tau_\gamma = \gamma^{-1} \simeq 10^{-4}$ s. The relevant time scale introduced by the gravitational influence is $\tau_a = 1/\sqrt{\vec{q}\cdot\vec{g}}$. For an optical laser with $|\vec{q}| \cong 10^7$ m^{-1} and Earth's acceleration $|\vec{g}| = 9.8$ m/s^2, τ_a is about 10^{-4} s [28]. Therefore, since the dynamical characteristic times of the system become of the same time scale as the reservoir, non-Markovian effects become important. Also, in the recent experiment work [50] where setups of optical cavity quantum electrodynamics (CQED) to probe quantum statistics of an atom laser with ^{87}Rb atom, the parameters are $(\lambda,\Delta,\kappa) = 2\pi(10.4,30,1.4)$ MHz, with the condition $\kappa < \delta_0 = \lambda^2/\Delta$ in present experiments. The cavity relaxation denotes by κ. In this paper, the parameters (λ,Δ,κ) are $(10^6, 1.8\times 10^6, 1.4\times 10^6)$ rad/s, respectively [28,30], and the above condition as $\kappa < \delta_0 = \lambda^2/\Delta$ is satisfied. Therefore, this model may be applied to study in the light of the recent achievements of optical CQED.

A. Atomic population inversion

The atomic population inversion is expressed by the expression

$$W(t) = \langle \hat{\sigma}_3(t)\rangle = \mathrm{Tr}_{atom}[\hat{\rho}_{atom}(t)\hat{\sigma}_3(t)], \tag{105}$$

where

$$\hat{\rho}_{atom}(t) = \mathrm{Tr}_{field}[\hat{\rho}_s(t)]. \tag{106}$$

Equation (105) can be rewritten as follows:

$$W(t) = \int d^3p \sum_{i=e,g} \langle i|\hat{\rho}_{atom}(t)\hat{\sigma}_3(t)|i\rangle = \int d^3p \sum_{n=0}^{\infty} (\langle n| \otimes (\langle e|\hat{\rho}_s(t)|e\rangle - \langle g|\hat{\rho}_s(t)|g\rangle) \otimes |n\rangle). \tag{107}$$

Therefore, by using Eqs. (94) and (107), we obtain

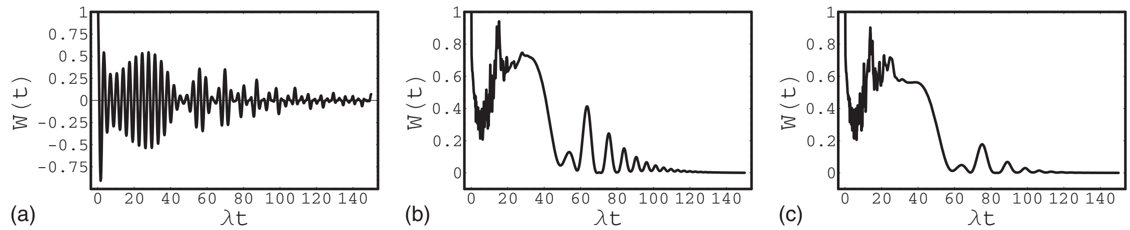

FIG. 1. Time evolution of the atomic population inversion vs the scaled time λt. Here, we have set $q=10^7\ \mathrm{m}^{-1}$, $M=10^{-26}$ kg, $g=9.8\ \mathrm{m/s}^2$, $\omega_{rec}=.5\times10^6$ rad/s, $\lambda=1\times10^6$ rad/s, $\varphi=0$, $\alpha=2$, $\Delta=1.8\times10^6$ rad/s, $\gamma=1\times10^4$ rad/s, and $\eta=2\times10^3$ rad/s for (a) $\vec{q}\cdot\vec{g}=0$, (b) $\vec{q}\cdot\vec{g}=0.5\times10^7\ \mathrm{s}^{-2}$, and (c) $\vec{q}\cdot\vec{g}=1.5\times10^7\ \mathrm{s}^{-2}$.

$$W(t)=\int d^3p\left(\sum_{n=0}^{\infty}(\langle n|(\hat{\rho}_s)_{11}(t)|n\rangle-\langle n|(\hat{\rho}_s)_{22}(t)|n\rangle)\right), \tag{108}$$

where $(\hat{\rho}_s)_{11}(t)$ and $(\hat{\rho}_s)_{11}(t)$ are given by Eqs. (95) and (96), respectively.

In Fig. 1, we have plotted the atomic population inversion as a function of the scaled time λt for three different values of the parameter $\vec{q}\cdot\vec{g}$. In this figure and all the subsequent figures, we set $q=10^7\ \mathrm{m}^{-1}$, $M=10^{-26}$ kg, $g=9.8\ \mathrm{m/s}^2$, $\omega_{rec}=\frac{\hbar q^2}{2M}=0.5\times10^6$ rad/s, $\lambda=1\times10^6$ rad/s, $\alpha=2$, $\Delta=1.8\times10^6$ rad/s, $\phi(\vec{p})=\frac{1}{\sqrt{2\pi\sigma_0}}\exp(\frac{-p^2}{\sigma_0^2})$, $\sigma_0=1$, $\gamma=1\times10^4$ rad/s, and $\eta=2\times10^3$ rad/s [26–31]. In Fig. 1(a), we consider small gravitational influence in the presence of the non-Markovian phase damping. This means very small $\vec{q}\cdot\vec{g}$, i.e., the momentum transfer from the laser beam to the atom, is only slightly altered by the gravitational acceleration because the latter is very small or nearly perpendicular to the laser beam. In Figs. 1(b) and 1(c), we consider the gravitational influence in the presence of the non-Markovian phase damping for $\vec{q}\cdot\vec{g}=0.5\times10^7\ \mathrm{s}^{-2}$ and $\vec{q}\cdot\vec{g}=1.5\times10^7\ \mathrm{s}^{-2}$, respectively. By comparing Figs. 1(a)–1(c), we can see the influence of gravity on the time evolution of the atomic population inversion when there is the non-Markovian phase damping. As it is seen from Fig. 1(a) for the atomic population inversion, the Rabi-like oscillations can be identified. With the increasing value of the parameter $\vec{q}\cdot\vec{g}$ [see Figs. 1(b) and 1(c)], the Rabi oscillations of the atomic population inversion disappear.

B. Atomic dipole squeezing

To analyze the quantum fluctuations of the atomic dipole variables and examine their squeezing, we consider the two slowly varying Hermitian quadrature operators

$$\hat{\sigma}_1=\frac{1}{2}[\hat{\sigma}_+\exp(-i\omega_{eg}t)+\hat{\sigma}_-\exp(i\omega_{eg}t)] \tag{109}$$

and

$$\hat{\sigma}_2=\frac{1}{2i}[\hat{\sigma}_+\exp(-i\omega_{eg}t)-\hat{\sigma}_-\exp(i\omega_{eg}t)]. \tag{110}$$

In fact, $\hat{\sigma}_1$ and $\hat{\sigma}_2$ correspond to the dispersive and absorptive components of the amplitude of the atomic polarization [51], respectively. They obey the commutation relation $[\hat{\sigma}_1,\hat{\sigma}_2]=\frac{i}{2}\hat{\sigma}_3$. Correspondingly, the Heisenberg uncertainty relation is

$$(\Delta\hat{\sigma}_1)^2(\Delta\hat{\sigma}_2)^2\geq\frac{1}{16}|\langle\hat{\sigma}_3\rangle|^2, \tag{111}$$

where $(\Delta\hat{\sigma}_i)^2=\langle\hat{\sigma}_i^2\rangle-\langle\hat{\sigma}_i\rangle^2$ is the variance in the component $\hat{\sigma}_i(i=1,2)$ of the atomic dipole.

The fluctuations in the component $\hat{\sigma}_i(i=1,2)$ are said to be squeezed (i.e., dipole squeezing) if the variance in $\hat{\sigma}_i$ satisfies the condition

$$(\Delta\hat{\sigma}_i)^2<\frac{1}{4}|\langle\hat{\sigma}_3\rangle|,\quad (i=1\ \text{or}\ 2). \tag{112}$$

Since $\hat{\sigma}_i^2=\frac{1}{4}$, this condition may be written as

$$F_i=1-4\langle\hat{\sigma}_i\rangle^2-|\langle\hat{\sigma}_3\rangle|<0,\quad (i=1\ \text{or}\ 2). \tag{113}$$

The expectation values of the atomic operators $\hat{\sigma}_+$ and $\hat{\sigma}_-$ are given by

$$\langle\hat{\sigma}_\pm(t)\rangle=Tr_{atom}[\hat{\rho}_{atom}(t)\hat{\sigma}_\pm(t)]. \tag{114}$$

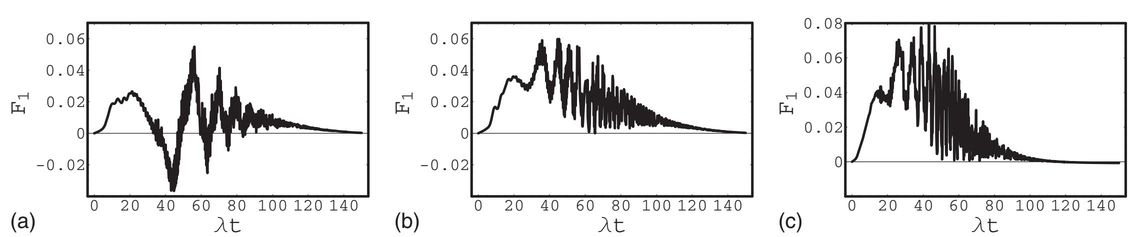

FIG. 2. Time evolution of the atomic dipole squeezing vs the scaled time λt with the same corresponding data used in Fig. 1 for (a) $\vec{q}\cdot\vec{g}=0$, (b) $\vec{q}\cdot\vec{g}=0.5\times10^7\ \mathrm{s}^{-2}$, and (c) $\vec{q}\cdot\vec{g}=1.5\times10^7\ \mathrm{s}^{-2}$.

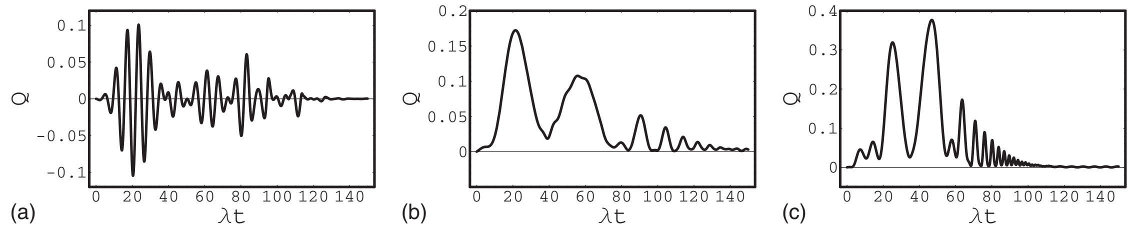

FIG. 3. Time evolution of the Mandel parameter $Q(t)$ vs the scaled time λt with the same corresponding data used in Fig. 1 for (a) $\vec{q}\cdot\vec{g}=0$, (b) $\vec{q}\cdot\vec{g}=0.5\times10^7\ \mathrm{s}^{-2}$, and (c) $\vec{q}\cdot\vec{g}=1.5\times10^7\ \mathrm{s}^{-2}$.

Therefore, by using Eqs. (94) and (106), we obtain

$$\langle\hat{\sigma}_-(t)\rangle=\int d^3p\langle n|(\hat{\rho}_s)_{12}(t)|n\rangle=\langle\hat{\sigma}_+(t)\rangle^*, \tag{115}$$

where $(\hat{\rho}_s)_{12}(t)$ is given by Eq. (97).

The time evolution of $F_1(t)$ corresponding to the squeezing of $\hat{\sigma}_1$ has been shown in Fig. 2 for three values of the parameter $\vec{q}\cdot\vec{g}$ in the presence of the non-Markovian phase damping. As it is seen, with the increasing value of the parameter $\vec{q}\cdot\vec{g}$, the dipole squeezing is completely removed.

C. Photon counting statistics

Now, the influence of gravity on the sub-Poissonian statistics of the radiation field will be investigated. For this purpose, we calculate the Mandel parameter defined by [52]

$$Q(t)=\frac{(\langle n(t)^2\rangle-\langle n(t)\rangle^2)}{\langle n(t)\rangle}-1. \tag{116}$$

For $Q<0$ ($Q>0$), the statistics is sub-Poissonian (super-Poissonian); $Q=0$ stands for Poissonian statistics. Since $\langle n(t)\rangle=\Sigma_{n=0}^{\infty}nP(n,t)$ and $\langle n(t)^2\rangle=\Sigma_{n=0}^{\infty}n^2P(n,t)$, we have

$$Q(t)=\left(\left\{\left[\sum_{n=0}^{\infty}n^2P(n,t)\right]-\left[\sum_{n=0}^{\infty}nP(n,t)\right]^2\right\}\times\left[\sum_{n=0}^{\infty}nP(n,t)\right]^{-1}\right)-1, \tag{117}$$

where the probability of finding n photons in the radiation field is found to be

$$P(n,t)=\langle n|\hat{\rho}_{field}(t)|n\rangle=\langle n|\mathrm{Tr}_{atom}\,\hat{\rho}_s(t)|n\rangle, \tag{118}$$

and by using Eq. (94) we have

$$P(n,t)=\int d^3p(\langle n|(\hat{\rho}_s)_{11}(t)|n\rangle+\langle n|(\hat{\rho}_s)_{22}(t)|n\rangle). \tag{119}$$

Therefore, by using Eqs. (117) and (119), we obtain

$$\begin{aligned}Q(t)=\Bigg(\Bigg\{&\left\{\sum_{n=0}^{\infty}n^2\left[\int d^3p(\langle n|(\hat{\rho}_s)_{11}(t)|n\rangle+\langle n|(\hat{\rho}_s)_{22}(t)|n\rangle)\right]\right\}\\&-\left\{\sum_{n=0}^{\infty}n\left[\int d^3p(\langle n|(\hat{\rho}_s)_{11}(t)|n\rangle+\langle n|(\hat{\rho}_s)_{22}(t)|n\rangle)\right]\right\}^2\Bigg\}\\&\times\left\{\sum_{n=0}^{\infty}n\left[\int d^3p(\langle n|(\hat{\rho}_s)_{11}(t)|n\rangle+\langle n|(\hat{\rho}_s)_{22}(t)\right.\right.\\&\left.\left.\times|n\rangle)\right]\right\}^{-1}\Bigg)-1.\end{aligned} \tag{120}$$

The numerical results for three values of the parameter $\vec{q}\cdot\vec{g}$ are shown in Fig. 3. As it is seen, in the presence of non-Markovian phase damping, the cavity-field exhibits alternately sub-Poissonian and super-Poissonian statistics when the influence of the gravitational field is negligible. With increasing $\vec{q}\cdot\vec{g}$, the sub-Poissonian characteristic is suppressed and the cavity-field exhibits super-Poissonian statistics. After some time, the Mandel parameter Q is stabilized at an asymptotic zero value; the larger the parameter $\vec{q}\cdot\vec{g}$ is the more rapidly $Q(t)$ reaches the asymptotic value zero.

V. SUMMARY AND CONCLUSIONS

In this paper, the non-Markovian dissipative dynamics of the phase-damped Jaynes-Cummings model in the presence of a classical homogeneous gravitational field have been analyzed. The model consists of a moving two-level atom simultaneously exposed to the gravitational field and a single-mode traveling radiation field in the presence of a non-Markovian phase-damping mechanism. First, the non-Markovian master equation for the reduced density operator of the system in terms of a Hamiltonian describing the atom-field interaction in the presence of a homogeneous gravitational field has been presented. Then, the superoperator technique is generalized and an exact solution of the non-Markovian master equation is obtained. Assuming that initially the radiation field is prepared in a Glauber coherent state and the two-level atom is in the excited state, influence of gravity on the temporal evolution of collapses and revivals of the atomic population inversion, atomic dipole squeezing, and photon counting statistics of the radiation field in the presence of the non-Markovian phase damping have been investigated. The results are summarized as follows. With the increasing values of gravity-dependent parameter $\vec{q}\cdot\vec{g}$ in the presence of the non-Markovian parameter η and the damping parameter γ, (1) the Rabi-like oscillations in the atomic population inversion disappear, (2) the atomic dipole squeezing is completely removed, and (3) the sub-Poissonian behavior of the cavity field is suppressed and it exhibits super-Poissonian statistics and after some time, the Mandel parameter Q is stabilized at an asymptotic zero value. In particular, we have shown that the gravitational field suppresses nonclassical effects in the non-Markovian phase-damped JCM. Indeed, in the model under consideration, the non-Markovian parameter η, the damping parameter γ, which depends on the environment temperature and the gravity-dependent parameter $\vec{q}\cdot\vec{g}$ seriously reduce the quantum coherence. The approach adopted here may be applied to study various dissipative systems such as quantum computers and light of the recent achievements of optical cavity QED. Therefore, based on this model, the gravity in the presence of the non-Markovian phase damping suppresses the nonclassical effects in the received qubit of the quantum computers and in the light of the recent achievements of optical cavity QED. Work on three-level atom interacting with quantized radiation field in a gravitational field is in progress.

ACKNOWLEDGMENT

The author wishes to thank The Office of Research of the Islamic Azad University–Shahreza Branch for their support.

[1] P. Tchénio, A. Débarre, J.-C. Keller, and J.-L. Le Gouët, Phys. Rev. Lett. **62**, 415 (1989).
[2] E. T. J. Nibbering, D. A. Wiersma, and K. Duppen, Phys. Rev. Lett. **66**, 2464 (1991).
[3] C. H. Brito Cruz, R. L. Fork, W. H. Knox, and C. V. Shank, Chem. Phys. Lett. **132**, 341 (1986).
[4] P. C. Becker, H. L. Fragnito, J. Y. Bigot, C. H. Brito Cruz, R. L. Fork, and C. V. Shank, Phys. Rev. Lett. **63**, 505 (1989).
[5] J. Y. Bigot, M. T. Portella, R. W. Schoenlein, C. J. Bardeen, A. Migus, and C. V. Shank, Phys. Rev. Lett. **66**, 1138 (1991).
[6] W. Vogel, D. G. Welsch, and B. Wilhelmi, Chem. Phys. Lett. **153**, 376 (1988).
[7] M. Lewenstein, T. W. Mossberg, and R. J. Glauber, Phys. Rev. Lett. **59**, 775 (1987).
[8] A. A. Villaeys, J. C. Vallet, and S. H. Lin, Phys. Rev. A **43**, 5030 (1991).
[9] G. Gangopadhyay and D. S. Ray, Phys. Rev. A **46**, 1507 (1992).
[10] S. Haroche, *Fundamental Systems in Quantum Optics* (North Holland, Amsterdam, 1992); H. Walther, *Advances in Atomic, Molecular and Optical Physics* (Academic Press, New York, 1994), Vol. 32, p. 379; M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **76**, 1800 (1996).
[11] E. T. Jaynes and F. Cummings, Proc. IEEE **51**, 89 (1963).
[12] W. Vogel and R. L. de Matos Filho, Phys. Rev. A **52**, 4214 (1995); J. Steinbach, J. Twamley, and P. L. Knight, *ibid.* **56**, 4815 (1997); V. Buzek, G. Drobny, M. S. Kim, G. Adam, and P. L. Knight, *ibid.* **56**, 2352 (1997).
[13] B. W. Shore and P. L. Knight, J. Mod. Opt. **40**, 1195 (1993).
[14] R. R. Schlicher, Opt. Commun. **70**, 97 (1989).
[15] A. Joshi and S. V. Lawande, Phys. Rev. A **42**, 1752 (1990).
[16] A. Joshi and S. V. Lawande, Int. J. Mod. Phys. B **6**, 3539 (1992).
[17] V. Bartzis, Physica A **180**, 428 (1992).
[18] B. Deb and S. Sen, Phys. Rev. A **56**, 2470 (1997).
[19] G. M. Meyer, M. O. Scully, and H. Walther, Phys. Rev. A **56**, 4142 (1997).
[20] M.-F. Fang, Physica A **259**, 193 (1998).
[21] A. Joshi, Phys. Rev. A **58**, 4662 (1998).
[22] X.-P. Liao and M.-F. Fang, Physica A **332**, 176 (2004).
[23] A. Joshi and M. Xiao, Opt. Commun. **232**, 273 (2004).
[24] C. Adams, M. Sigel, and J. Mlynek, Phys. Rep. **240**, 143 (1994).
[25] A. Kastberg, W. D. Phillips, S. L. Rolston, R. J. C. Spreeuw, and P. S. Jessen, Phys. Rev. Lett. **74**, 1542 (1995).
[26] C. Lammerzahl and C. J. Borde, Phys. Lett. A **203**, 59 (1995).
[27] K. P. Marzlin and J. Audretsch, Phys. Rev. A **53**, 1004 (1996).
[28] M. Mohammadi, M. H. Naderi, and M. Soltanolkotabi, J. Phys. A **39**, 11065 (2006).
[29] M. Mohammadi, M. H. Naderi, and M. Soltanolkotabi, J. Phys. A: Math. Theor. **40**, 1377 (2007).
[30] M. Mohammadi, M. H. Naderi, and M. Soltanolkotabi, Int. J. Theor. Phys. **47**, 983 (2008); M. H. Naderi, Can. J. Phys. **85**, 1071 (2007).
[31] M. Mohammadi, M. H. Naderi, and M. Soltanolkotabi, Eur. Phys. J. D **47**, 295 (2008).
[32] S. M. Barnett and P. L. Knight, Phys. Rev. A **33**, 2444 (1986).
[33] R. R. Puri and G. S. Agarwal, Phys. Rev. A **35**, 3433 (1987).
[34] T. Quang, P. L. Knight, and V. Buzek, Phys. Rev. A **44**, 6092 (1991).
[35] J. Eiselt and H. Risken, Phys. Rev. A **43**, 346 (1991).
[36] M. J. Werner and H. Risken, Phys. Rev. A **44**, 4623 (1991).
[37] J. Gea-Banacloche, Phys. Rev. A **47**, 2221 (1993).
[38] B. G. Englert, M. Naraschewski, and A. Schenzle, Phys. Rev. A **50**, 2667 (1994).
[39] C. W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991); D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 1994).
[40] H.-P. Breuer, U. Dorner, and F. Petruccione, Comput. Phys. Commun. **132**, 30 (2000).
[41] I. L. Chuang and Y. Yamamoto, Phys. Rev. A **55**, 114 (1997).
[42] L. M. Kuang, X. Chen, G. H. Chen, and M. L. Ge, Phys. Rev. A **56**, 3139 (1997).
[43] H. A. Hessian and H. Ritsch, J. Phys. B **35**, 4619 (2002); H. Ritsch and H. A. Hessian, Acta Phys. Slov. **53**, 61 (2003).
[44] L. M. Kuang and X. Chen, J. Phys. A **27**, L633 (1994).
[45] H. Moya-Cessa, V. Buzek, M. S. Kim, and P. L. Knight, Phys. Rev. A **48**, 3900 (1993).
[46] X. Chen and L. M. Kuang, Phys. Lett. A **191**, 18 (1994).
[47] *The Physics of Quantum Information*, edited by D. Bouwmeester, A. Ekert, and A. Zeilinger (Springer-Verlag, Berlin, 2000).
[48] D. J. Wineland, C. Monroe, W. M. Itano, D. Leibfried, B. E.

King, and D. M. Meekhof, J. Res. Natl. Inst. Stand. Technol. **103**, 259 (1998).
[49] Q. A. Turchette *et al.*, Phys. Rev. Lett. **75**, 4710 (1995).
[50] T. Bourdel, T. Donner, S. Ritter, A. Öttl, M. Köhl, and T. Esslinger, Phys. Rev. A **73**, 043602 (2006).
[51] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980); N. B. Narozhny, J. J. Sanchez-Mondragon, and J. H. Eberly, Phys. Rev. A **23**, 236 (1981); H. I. Yoo, J. J. Sanchez-Mondragon, and J. H. Eberly, J. Phys. A **14**, 1383 (1981).
[52] L. Mandel, Opt. Lett. **4**, 205 (1979); Phys. Scr. T **T12**, 34 (1986).