Optical frequency stabilization of a 10 GHz Ti:sapphire frequency comb by saturated absorption spectroscopy in ^{87}rubidium

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The high power per mode of a recently developed 10 GHz femtosecond Ti:sapphire frequency comb permits nonlinear Doppler-free saturation spectroscopy in ^{87}Rb with a single mode of the comb. We use this access to the natural linewidth of the rubidium D_2 line to effectively stabilize the optical frequencies of the comb with an instability of 7×10^{-12} in 1 s of averaging. The repetition rate is stabilized to a microwave reference leading to a stabilized and atomically referenced comb. The frequency stability of the 10 GHz comb is characterized using optical heterodyne with an independent self-referenced 1 GHz comb. In addition, we present alternative stabilization approaches for high repetition rate frequency combs and evaluate their expected stabilities.

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I. INTRODUCTION

Frequency-stabilized optical frequency combs with mode spacings of 10 GHz and higher are of interest for applications such as calibration standards for astronomical spectrographs [1–5], line-by-line pulseshaping [6], multichannel communication [7], optical frequency metrology [8], and low-phase-noise microwave generation [9,10]. To differing degrees, these applications all benefit from the features of a stable optical frequency comb with large mode spacing (which provides easy access to individual modes), broad optical bandwidth, and high power per mode. There are many different approaches to the generation of high repetition rate combs, including active and passive mode-locking [11–15], cavity filtering [16,17], microcavities [18–20], and electro-optic modulation [21]. Stabilization of the optical frequencies of the comb elements is required for some applications and provides enhanced capabilities for others. In particular, calibration and metrology applications require the linking of the comb frequencies to the Système international d' unités (SI) second in order to provide absolute frequency references. The comb equation gives the frequency of the nth comb mode $\nu_n = nf_R + f_0$, where f_R is the repetition rate and f_0 is the offset frequency. Generally, the repetition rate can be measured and linked to the SI second via a microwave reference. The offset frequency may be directly measured by self-referencing [22,23]. However, sources with 10 GHz repetition rates typically have longer pulses and reduced pulse energy, making it challenging to generate octave-spanning spectra as required for the commonly used self-referencing schemes.

Here, we consider alternative approaches to stabilize the optical frequencies of high repetition rate combs using saturation spectroscopy in atomic vapors. Specifically, we use the high power per mode of a 10 GHz Ti:sapphire frequency comb [24] to perform Doppler-free saturation spectroscopy in ^{87}Rb with a single comb mode. In this example, we show that it is straightforward to stabilize one comb mode to a transition of the rubidium D_2 line at 780 nm. In addition, when the repetition rate is also stabilized to a microwave reference, this provides an array of optical modes with instability of 7×10^{-12} in 1 s of averaging. While we restrict our attention to the stability of the optical comb, with proper care of the well-known systematic frequency shifts associated with saturation spectroscopy [25], the approach we describe here also provides an accurate array of absolute optical frequency references in the visible and near infrared. For some applications, a compact or even portable-stabilized comb source is desired. In fact, we show that in several different realizations involving compact alkali-metal (Cs, Rb) vapor-cell references and/or a microwave reference steered by the global positioning system (GPS), we could expect to achieve stabilities of $\leq 1\times10^{-11}$ at 1 s averaging period and fractional uncertainties $<1\times10^{-11}$.

II. SINGLE MODE SATURATION SPECTROSCOPY

In Doppler-free saturation spectroscopy, two counter-propagating beams are used to cancel the Doppler shift (see, e.g., [26]). This allows access to the natural linewidth of atomic transitions, so that the hyperfine splitting often buried in the Doppler broadening can be resolved. In our case, the D_2 line in ^{87}Rb at 780 nm matches the output spectrum of a recently developed 10 GHz Ti:sapphire laser. The laser design follows a ring resonator concept previously used in passively modelocked Ti:sapphire lasers with repetition rates of 1–5 GHz [27,28]. The laser produces 40 fs pulses and 600 mW of average power, such that the power per mode can exceed 0.5 mW [24]. Thus, with a beam diameter on the order of 1 mm, we can exceed the saturation intensity of the $5S_{1/2}$, $F=2 \rightarrow 5P_{3/2}$, $F=3$ transition, which is 3.5 mW per cm^2 [29]. Figure 1(c) shows the energy-level structure of ^{87}Rb [29]. We used a simple double-pass setup,

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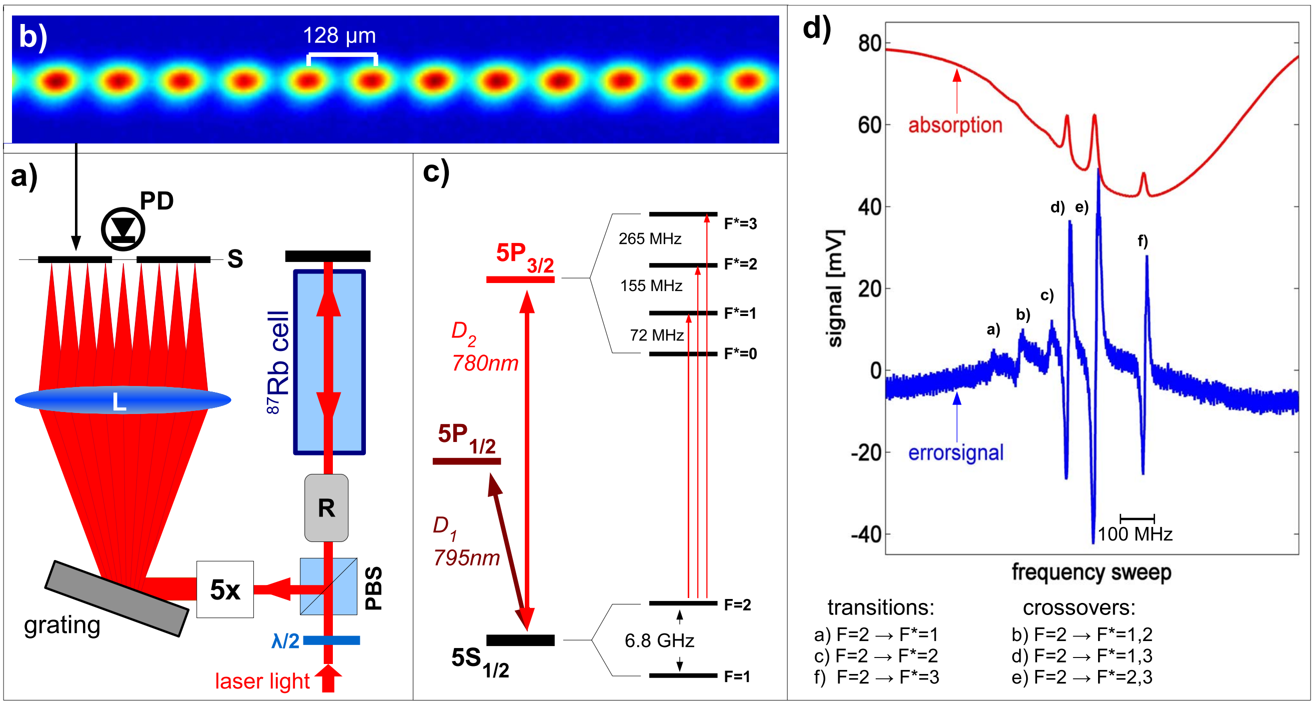

FIG. 1. (Color online) (a) Setup for Doppler-free saturation spectroscopy. Light from the laser passes through a polarizing beam splitter (PBS). Then the polarization is rotated by a Faraday rotator (r), so that it leaves through the second PBS port after passing through the vapor cell. The cell contains isotopically enriched rubidium at room temperature and is magnetically shielded by a single layer of μ-metal. The spectrometer consists of a $5\times$ beam expander, a 2400 lines/mm grating, and a 70 cm focusing lens. A single mode can be isolated with a 125 μm slit. (b) The 10 GHz modes imaged on a charge-coupled device in the focal plane are spaced by about 130 μm. (c) Hyperfine structure of the D_2 line in ^{87}Rb. (d) Saturation spectrum of the rubidium D_2 line with transitions from the $5S_{1/2}$, $F=2$ ground state to the excited states $5P_{3/2}$, $F^*=1,2,3$ and associated crossover resonances. The first derivative, which will be used for locking, shows clearly the six expected peaks.

which is shown in Fig. 1(a). Propagating two times through the rubidium cell, the same beam is used as pump and probe simultaneously. The frequency comb modes are dispersed by a grating and are imaged by a lens. This simple grating spectrometer provides sufficient resolution to separate the 10 GHz modes. A slit in the focal plane selects the mode at the D_2 frequency and the absorption signal is obtained in the detected power of this mode. By modulating a mirror of the ring laser cavity with a piezoelectric transducer (PZT), we can sweep the optical frequency over the transitions. Figure 1(d) shows the saturation spectrum of the rubidium D_2 line with transitions from the $5S_{1/2}$, $F=2$ ground state to the excited states $5P_{3/2}$, $F^*=1,2,3$. In addition to the peaks arising from the three hyperfine levels of the excited state, crossover resonances are present. The $F=2\rightarrow F^*=3$ transition used for stabilization has a power-broadened linewidth of 20 MHz, which is consistent with the laser beam parameters. Extrapolating to zero power results in a linewidth near the expected 6 MHz. With consideration of the well-known frequency shifts in a setup like ours, a comb stabilized by saturation spectroscopy combined with repetition rate stabilization could provide both a stable and accurate frequency reference. Systematic frequency shifts that determine the accuracy of the alkali reference have been reported [25,30,31].

III. FREQUENCY STABILIZATION

The comb equation describes the frequency ν_n of the nth comb mode in terms of two microwave frequencies and a mode index. The repetition rate f_R determines the spacing of the modes and together with the mode index n and the offset frequency f_0, we obtain the absolute position in the frequency domain,

$$\nu_n = n f_R + f_0. \tag{1}$$

Instead of using interferometric measurement [22] and stabilization of the offset frequency f_0 to a microwave reference, we lock one comb mode to an optical reference ν_{ref}. The reference frequency can also be described by Eq. (1) with a mode index n_{ref}: $\nu_{ref}=n_{ref}f_R+f_0$. This leads to a modified comb equation

$$\nu_n = (n - n_{ref})f_R + \nu_{ref}. \tag{2}$$

In order to stabilize the 10-GHz-frequency comb, we use the cavity mirror on a PZT to change the cavity length and an acousto-optic modulator (AOM) in the pump beam to modulate the pump power as two independent steering mechanisms. The schematic setup is shown in Fig. 2(a). The repetition rate is detected on a photodetector and locked to a 10 GHz synthesizer by modulating the pump power. Modulation of the pump power changes mainly the mode spacing of the comb about a fixed frequency point near the center of the spectrum [32]. To stabilize to the rubidium transition, a small modulation frequency at 145 kHz (signal generator 1) is applied to the cavity PZT and the error signal is obtained by lock-in detection. Adding the error signal to the modulation,

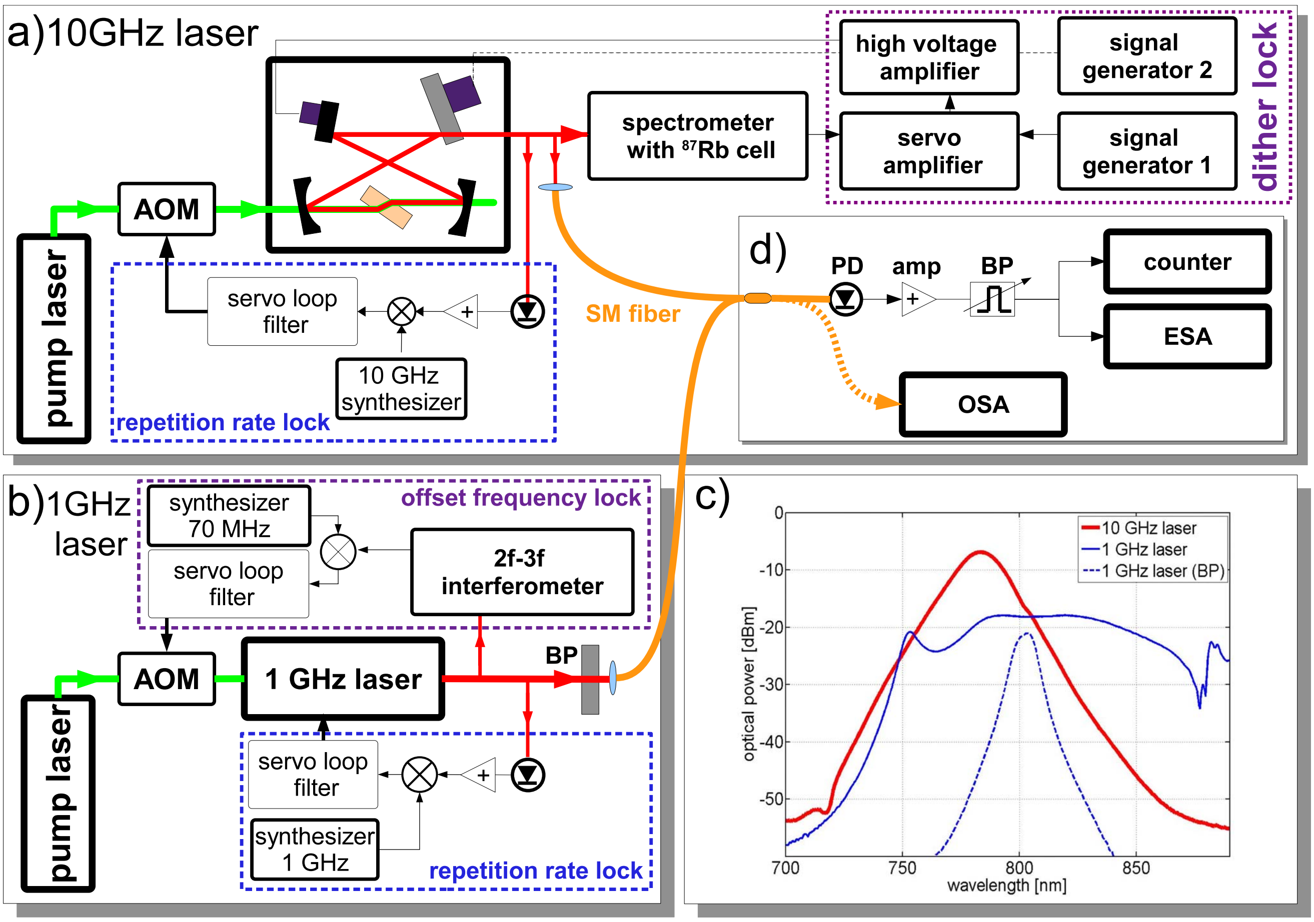

FIG. 2. (Color online) (a) The repetition rate of the 10 GHz comb is locked to a 10 GHz synthesizer by photodetecting a small amount of light from the laser output. The feedback goes to an acousto-optic modulator in the pump beam. With the spectrometer, the saturation signal is obtained by lock-in technique and used to lock one comb mode to an atomic transition. (b) The offset frequency of the 1 GHz comb is measured with a $2f$-$3f$ interferometer and locked with an AOM in the pump beam to a synthesizer. The repetition rate is measured with a photodiode and also locked to a synthesizer. To ensure accurate measurements and definitive tests of the comb stability, all synthesizers are referenced to a stable 10 MHz signal from a hydrogen maser. (c) Light from both stabilized combs is combined in fibers. For the broadband 1 GHz comb a 10 nm bandpass (BP) filter at 800 nm is used to isolate the overlapping regions of the spectrum. (d) The beat signal between both combs is detected on a photodetector. The obtained microwave signal is filtered and amplified so that its frequency can be counted.

we can lock a comb mode to one of the peaks shown in Fig. 1(d). We use a second PZT for coarse adjustment, frequency sweeps (signal generator 2), and to compensate slow drifts. With the repetition rate locked to a microwave reference and one comb mode locked to a rubidium peak, both degrees of freedom of the comb are effectively locked. The system could be locked for greater than 8 h of reliable operation, while stabilized to the $F=2\rightarrow F^{*}=3$ transition.

IV. STABILITY MEASUREMENTS

To measure the optical frequency stability of the 10 GHz comb, we analyze a beat signal using a stable 1 GHz comb [33] as a reference. The 1 GHz comb offset frequency is detected with a $2f$-$3f$ interferometer and both offset frequency and repetition rate are stabilized to microwave references [see Fig. 2(b)]. The microwave references for both systems are synthesizers using the same 10 MHz maser reference signal and having a stability as good as $\sim 2\times 10^{-13}\tau^{-1/2}$ for averaging times greater than 1 s. The 1 GHz repetition rate is adjusted to be one-tenth of the 10 GHz repetition rate. As shown in Fig. 2(c), stabilized light from both lasers is coupled into fibers and combined. We used a 10-nm-wide bandpass filter in order to select the overlapping region of the spectra around 800 nm. The beat signal between both combs is detected on a photodetector. The obtained microwave signal is filtered and amplified so that its frequency can be counted [Fig. 2(d)]. Since the repetition rate of the 1 GHz laser is one-tenth of the 10 GHz laser repetition rate, the beat frequencies between each 10 GHz mode and the 1 GHz comb modes are the same over the entire interfering part of the comb spectra. The situation is depicted in the frequency domain in Fig. 3(a). Here we use the beat signal labeled as ν_{b2}. It can be expressed by using the comb Eq. (1) for both combs,

$$\nu_{b2}=\nu_{10G}-\nu_{1G}=n_{10G}f_{R,10G}+f_{0,10G}-(n_{1G}f_{R,1G}+f_{0,1G}). \quad (3)$$

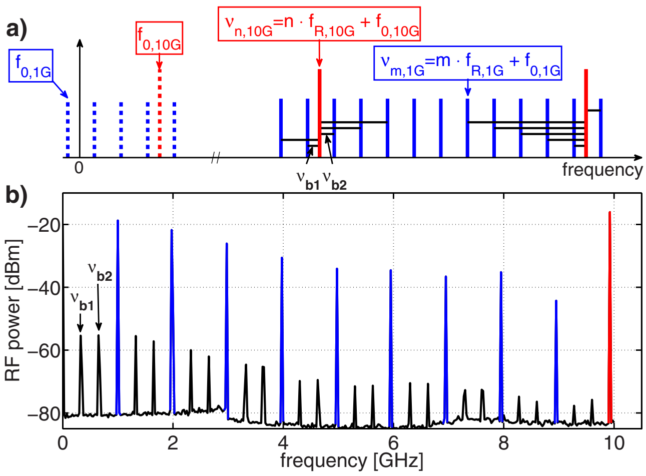

FIG. 3. (Color online) (a) Frequency domain picture of the interference between modes of the 10 GHz laser and the 1 GHz laser. Beat signals (horizontal lines) between all modes of the locked combs occur. For purposes of clarity, only some beat signals are indicated. (b) The corresponding microwave spectrum shows the 1 GHz repetition rate plus higher harmonics, the 10th harmonic of the 1 GHz repetition rate together with the 10 GHz repetition rate, and the beat signals

This expression gives the beat frequency ν_{b2} between a mode of the 10 GHz comb with index n_{10G} and a mode of the 1 GHz comb with index n_{1G}. Because the ratio of the repetition rates is fixed at exactly ten, Eq. (3) can be simplified by the use of an effective mode index $\tilde{n}$, where $\tilde{n} \in [-10, 10]$,

$$\nu_{b2} = (n_{10G}10 - n_{1G})f_{R,1G} - f_{0,1G} + f_{0,10G} \\ = \tilde{n} f_{R,1G} - f_{0,1G} + f_{0,10G}. \quad (4)$$

$\tilde{n}$ gives the number of 1 GHz modes within the separation of the 1 and 10 GHz offset frequencies [Fig. 3(a)].

Because the repetition rate of the 1 GHz laser and its offset frequency are locked to known values, we can obtain the offset frequency of the 10 GHz laser by measuring the beat frequency ν_{b2}. [Note: it is also possible to use a second beat signal, with $\nu_{b1} = \nu_{1G} - \nu_{10G}$ in Eq. (3), leading to different signs and a different effective mode index.] The repetition rate $f_{R,1G}$ and the offset frequency $f_{0,1G}$ of the 1 GHz laser are locked with the specific values of $f_{R,1G} = 0.992\,441\,93$ GHz and $f_{0,1G} = -70$ MHz. With an effective mode index of $\tilde{n} = -3$, the offset frequency of the 10 GHz laser is 3.062 GHz, using the known frequency of the $F=2 \rightarrow F^{*}=3$ transition for ν_{10G}. Figure 3(a) is a schematic of the interfering combs, showing the origin and sign convention used for the offset frequencies.

Figure 3(b) shows the microwave spectrum of the beat signal between the combs. The repetition rates, their higher harmonics, and the beat signals are visible.

The total fractional frequency instability of the optical modes depends on both fluctuations of the offset frequency and the repetition rate. On longer time scales, we assume that the repetition rate fluctuations of both lasers are correlated since they use the same maser reference. Moreover, the offset frequency stability of the 1 GHz system is at the millihertz level, so that analyzing the fluctuations of ν_{b2} gives a measure of the optical stability of the 10 GHz system 20 nm away from the locking point. Figure 4(a) shows results of a 4 h measurement of the frequency of the beat signal ν_{b2} counted at 1 s gate time.

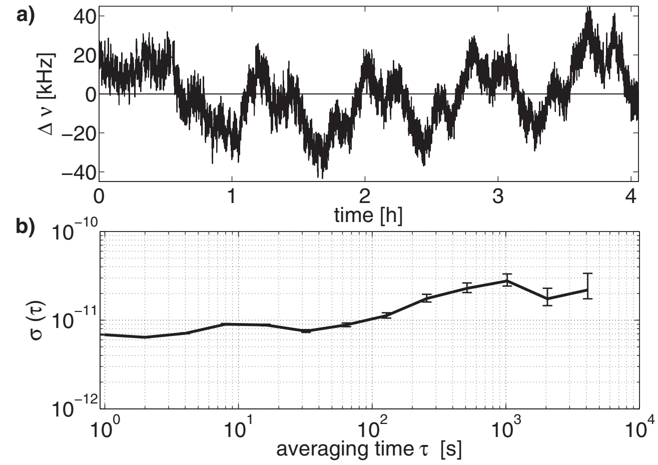

FIG. 4. (a) Measurement of the beat signal ν_{b2} between the 10 GHz and the 1 GHz comb over 4 h. The graph gives the offset from the average value of 837.1856 MHz. (b) Corresponding Allan deviation with error bars for the frequency measurement.

This leads to a fractional instability of 7×10^{-12} at 1 s [Fig. 4(b)], corresponding to a resolution of the rubidium line center to 2.5 kHz. The standard deviation of the beat frequency over the measurement window of 4 h is 15 kHz. By analyzing the temperature of the room and of the baseplate of the 10 GHz laser within the measurement window, we observe a direct influence of temperature changes in the frequency of the beat signal. Because the repetition rate is locked to a synthesizer, changes in cavity length caused by thermal deformations are compensated, while changes in laser power can occur by misalignment or changes in beam pointing. Changes of both laser power and Rb cell temperature lead to frequency shifts [30]. We observe a power shift on the order of 15 kHz/mW and cell temperature shifts on the order of 5 kHz/K. Fourier analysis shows correlations between room-temperature oscillations with a period of 40 min and a 30 s oscillation on the baseplate temperature due to the feedback loop of the laser cooling system with the fluctuations of the beat signal. These temperature fluctuations affecting the long term stability could be improved by better thermal management of the experimental setup.

V. COMPARISON OF ALTERNATIVE STABILIZATION METHODS

For some applications, stabilization at the 10^{-12} level is sufficient, but applications, such as optical frequency metrology, may require better stabilities. According to Eq. (1), in general, two independent locking points are needed in order to stabilize a frequency comb. The combination of an optical and a microwave reference, as shown in Sec. III, is just one possible option. Here we evaluate the method described above and discuss possible improvements and other approaches to stabilize high repetition rate combs. We distinguish between three methods with different locking points and frequency references.

A. Combinations of microwave and optical references

Locking one comb mode to an optical reference and the repetition rate to a microwave frequency gives stability commensurate with the reproducibility and stability of the optical reference combined with the stability of the microwave reference. If one comb mode is locked to an optical reference ν_{ref}, the comb dynamics can be described in a fixed-point model, as introduced with Eq. (2). For the total optical instability, we get

$$\sigma_{\nu_m} = \frac{\delta\nu_m}{\nu_m} = \frac{\delta\nu_{ref} + (m - n_{ref})\delta f_R}{\nu_m} = \frac{\delta\nu_{ref}}{\nu_m} + (m - n_{ref})\frac{\delta f_R}{\nu_m}. \quad (5)$$

With this equation, we can analyze the influence of the stability of the microwave and the optical reference independently. In the experiment described in Secs. II–IV, we measure the stability around 800 nm, which means about 1000 modes lie between the locking and the analyzing points. The maser reference gives a fractional microwave stability of $\sigma_f = \frac{\delta f_R}{f_R} \sim 10^{-13}\ \tau^{-1/2}$. This contributes to the fractional optical instability with

$$(m - n_{ref})\frac{\delta f_R}{\nu_m} = 2.6 \times 10^{-15}\ \tau^{-1/2}. \quad (6)$$

This is far below the measured value, so that fluctuations on the microwave reference will not affect the fractional optical instability in the present experiment. By using lock-in detection to stabilize to the rubidium transition, we can split the measured linewidth of 20 MHz by a factor of 8000 in 1 s. This corresponds to the measured total optical instability,

$$\frac{\delta\nu_{ref}}{\nu_m} = \frac{2.5\ \text{kHz}}{373\ \text{THz}} = 6.7 \times 10^{-12}\ \text{at}\ 1\ \text{s}. \quad (7)$$

The stability is comparable to the results of cw spectroscopy [25]. The shot-noise limited instability is still orders of magnitude smaller, implying that technical noise sources (e.g., laser frequency noise, residual amplitude modulation, baseline drift, etc.) presently limit the achievable signal to noise and the instability at 1 s. To make the system independent from the maser reference, a GPS disciplined oscillator (GPSDO) can be used as microwave reference, for which the microwave stability can be as good as $\sigma_f = \frac{\delta f_R}{f_R} = 10^{-11}\tau^{-1/2}$ [34,35]. This contributes to the fractional optical instability with $(m - n_{ref})\frac{\delta f_R}{\nu_m} = \sim 3 \times 10^{-13}\tau^{-1/2}$, so that in our case it will not limit the overall optical fractional instability. Another possibility, as an optical reference, could be the two-photon process $5S$-$5D$ in rubidium, where the linewidth can be as narrow as 500 kHz [36,37]. In the optimistic case, assuming a similar signal-to-noise ratio in splitting the two-photon resonance, the fractional optical instability could be one order of magnitude better, approaching the limit imposed by a GPS reference. An optical clock laser or other cavity stabilization would provide the narrowest optical reference, e.g., [38,39], but the advantage of easier handling and compactness of the vapor-cell setup would be lost.

Another possibility for stabilizing the repetition rate is using the hyperfine ground-state splitting in an alkali vapor cell or beam directly as a microwave reference [40], either by microwave absorption [41,42] or by dark line resonance [43]. Table I gives an overview of the expected performance for different microwave and optical references.

B. Two optical references

The methods described in the previous section to stabilize frequency combs are independent of the comb spectrum. The following methods depend directly on the spectral noise density of the comb, so only broad combs or combs where spectral broadening is possible could benefit. We showed that optical bandwidths greater than 200 nm can be reached by spectral broadening of the 10 GHz comb in microstructured fiber [24].

For a fixed frequency difference between two references, locking two comb modes to the optical references allows the stabilization of the repetition rate by assigning one comb mode to each reference. In this case, the number of modes between the references is constant. This provides both microwave and optical stabilization and could be used for systems independent from traditional microwave standards (e.g., hydrogen maser, GPS, Cs clock, etc.). The references can be realized by two atomic transitions in the same or in different atoms and molecules or by longitudinal modes of a high finesse reference cavity. Following the analysis in [45], the modes locked to two optical frequency references $\nu_{ref,1}$ and $\nu_{ref,2}$ can be described by the reference frequencies and some unresolved fluctuations ϵ_i on the derived error signal

$$\nu_{n_1} = \nu_{ref,1} \pm \epsilon_1,$$

$$\nu_{n_2} = \nu_{ref,2} \pm \epsilon_2. \quad (8)$$

The fluctuations ϵ_i can be described by fluctuations on the repetition rate δf_R and fluctuations on the offset frequency,

$$\epsilon_i = n_i \delta f_R + \delta f_0. \quad (9)$$

To stabilize the two degrees of freedom, we assume that the first feedback loop stabilizes comb mode n_1 to the reference $\nu_{ref,1}$. The second loop is sensitive to the difference between the two modes n_1 and n_2 and stabilizes the repetition rate by locking the spacing between the comb modes to the frequency separation of the references $\nu_{ref,1}$ and $\nu_{ref,2}$. This leads to

$$f_R = \frac{\nu_{ref,1} - \nu_{ref,2}}{n_1 - n_2} \quad \text{and} \quad \delta f_R = \frac{\epsilon_1 - \epsilon_2}{n_1 - n_2}. \quad (10)$$

For simplicity, we assume that both feedback loops operate independently from each other. In reality, one would have to account for the crosstalk between the two loops, but their analysis would be system dependent. Since the fluctuations on the errorsignals $\epsilon_{1,2}$ can be decomposed into fluctuations on repetition rate and offset frequency, Eq. (5) can be used to

TABLE I. Overview of stabilization methods for frequency combs by the use of different combinations of microwave and optical references. The projected microwave and optical fractional instabilities (σ_f, σ_ν) are given for 1 s of averaging period. The last column gives a value for possible optical uncertainties or their limitation. For method A, the overall instability is given by the combination of the stated instabilities for f_R and ν_{ref} with the larger one dominating. Since the stabilities/uncertainties of frequency standards can rapidly improve, we give assumptions and/or references for the stated values.

Method	Stabilization points	Locking methods	$\sigma_f(1s)$ microwave	$\sigma_\nu(1s)$ optical	Opt. uncertainties
A. optical and microwave ref.		synthesizer+maser	10^{-13}	$\sim 10^{-14}$	$<10^{-13}$ [a]
	f_R	synthesizer+GPS	10^{-11}	$\sim 10^{-13}$	10^{-14} [b]
		Cs hyperfine	10^{-13}	$\sim 10^{-14}$	$<10^{-14}$ [c]
		Rb D_1/D_2		$10^{-11}-10^{-12}$	10^{-11} [d]
	ν_{ref}	2-photon (e.g., Rb, Cs)		$\sim 10^{-12}-10^{-13}$	10^{-11} [e]
		opt. clock/cavity ref.		$\sim 10^{-15}$	$\sim 10^{-17}$ [f]
B. optical references	two comb modes	Rb D_2 and Cs D_1	10^{-11}	10^{-12}	10^{-11}
		reference cavity	10^{-14}	$10^{-14}-10^{-15}$	not defined
C. self-referencing	f_R,f_0	microwave ref./synthesizer	10^{-13}	10^{-13}	$<10^{-13}$ [a]
		self-ref.+optical clock	$\sim 10^{-15}$	$\sim 10^{-15}$	$\sim 10^{-17}$ [f]

[a] Assuming hydrogen maser calibrated by a Cs frequency standard.
[b] Instability and uncertainty depend on the specific GPSDO [34].
[c] Vanier and Audoin gave an overview of the performance of different Cs beam standards [42].
[d] Ye *et al.* achieved fractional uncertainties below 8×10^{-12} [25].
[e] International Bureau of Weights and Measures (BIPM) recommendation [44], Edwards *et al.* achieved fractional stabilities of 9.3 $\times 10^{-13}\tau^{-1/2}$ in ^{85}Rb [37].
[f] Since the development of optical clocks is a rapidly changing field, these numbers represent the current state of the art [39].

determine the total optical instability. In this case, $\frac{\delta\nu_{ref}}{\nu_m}$ is replaced by $\frac{\epsilon_1}{\nu_m}$, and δf_R is given by Eq. (10).

This analysis shows that optical frequency references with narrow linewidth and a large frequency separation are essential to achieve high stabilities. Again, the total optical stability will be determined mainly by the larger instability of the two terms in Eq. (5). If the optical stability achieved by locking n_1 to the reference ν_1 is much better than the repetition rate stability, the overall optical instability far away from the locking point n_1 is only determined by the repetition rate term.

A possible implementation of this stabilization scheme could be using two atomic references, e.g., the Rb D_2 line and the Cs D_1 line with a separation of about 50 THz or 5000 modes. Assuming comparable linewidth and similar signal-to-noise ratio as in the Rb spectroscopy described earlier, a repetition rate instability of $\sim 10^{-11}$ and an optical instability of $\sim 10^{-12}$ can be expected. In earlier experiments using cavities to lock frequency combs [45,46], the reference spacing $\Delta\nu = \nu_{ref,1} - \nu_{ref,2}$ was limited by the walkoff of the cavity's free spectral range because modes of $\sim 2-5$ nm bandwidth around the reference had to be locked to cavity modes in order to achieve sufficient signal to noise. Due to the high power per mode, we would expect that only two single modes have to be locked to two different cavity modes avoiding the cavity walk-off problem. If the modes can be locked to the Hertz level, a cavity with high finesse over a 300 nm bandwidth could stabilize a spectrally broadened comb to the $10^{-14}-10^{-15}$ level. A combination of cavity and atomic references could provide the good short term stability of a cavity and the long term stability and accuracy of atomic references.

C. Self-referencing

If an octave-spanning spectrum can be generated, a scheme commonly used to lock frequency combs is the combination of locking the repetition rate and the offset frequency detected by self-referencing to microwave references [22,23]. The instabilities of both the repetition rate and the offset frequency are limited by the maser reference to the synthesizers. The comb equation relates both

$$\sigma_{\nu_m} = \frac{\delta\nu_m}{\nu_m} = \frac{m\delta f_R + \delta f_0}{\nu_m} = m\frac{\delta f_R}{\nu_m} + \frac{\delta f_0}{\nu_m}, \tag{11}$$

so that the larger uncertainty will determine the overall stability. The microwave instabilities are transferred to the optical regime via the comb equation, taking into account dynamics described by a fixed-point model [47]. Table I gives values for a typical implementation. It has been shown that optical uncertainties approaching 10^{-17} for averaging times greater than 10^4 s [39] can be reached with the combination of self-referencing and an atomic clock reference. When the intrinsic laser output spectrum does not span an octave, spectral broadening to an octave in a microstructured fiber, which requires high average powers and short pulses, would be needed to employ a self-referencing scheme [48].

VI. SUMMARY

We have shown that it is possible to use the high power per mode of the 10 GHz Ti:sapphire comb to perform nonlinear Doppler-free saturation spectroscopy in ^{87}Rb. This access to the natural linewidth allows the stabilization of the offset frequency by locking one comb mode to a transition. We measure the stability by optical heterodyne of the 10 GHz comb with a self-referenced 1 GHz comb. The fractional frequency instability can reach 7×10^{-12} at 1 s. Aside from self-referencing, cavity stabilization could lead to an improved stability of the system.

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