

Electron transfer in fast atomic collisions in the presence of an x-ray laser

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We consider electron capture in fast collisions between a proton and hydrogen in the presence of an intense x-ray laser whose angular frequency ω is close to $v^2/2$, where v is the collision velocity. We show that in such a case laser-induced capture becomes possible and that the latter proceeds via both induced photon emission and photon absorption channels and can, in principle, compete with kinematic and radiative electron capture.

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I. INTRODUCTION

One of the interesting and important reactions occurring in fast ion-atom collisions is the transfer of an electron, which was initially bound in the atom, into a bound state of the moving ion. Such a transfer process is also termed electron capture. During the last decades the electron capture in fast ion-atom collisions has been the subject of an enormous number of experimental and theoretical papers (see for a review, e.g., [1,2] where further references can be found).

Electron capture is in general a complicated few-body process. Depending on whether the transfer occurs with or without emission of a photon the capture can be subdivided into radiative and nonradiative ones.

In the nonradiative capture the electron is transferred to the ion due to the Coulomb interaction in the system of rearranging atomic particles. The main contribution to the nonradiative capture is normally given by the so-called kinematic capture channel. The studies of the nonradiative capture date back to the early days of quantum mechanics [3,4] and since then they have been one of the focuses of atomic collision physics (in addition to [1,2] see for review also, e.g., [5,6]).

The radiative capture mechanism is based on the coupling of the colliding ion-atom system to the radiation field. In this process the collision “triggers” the interaction of the electron with its own electromagnetic field and the electron capture proceeds with emission of a real photon ([7], for review see, e.g., [1,6]). This capture mechanism is in essence based on the fact that the very presence of the ion makes the initial state of the electron in the atom unstable with respect to the spontaneous radiative decay.

Viewing the radiative capture in the rest frame of the ion [Fig. 1(a)] one can see that the emission of a photon having the “right” angular frequency $\sim v^2/2$, where v is the collision velocity (atomic units are used throughout except where otherwise stated), enables the electron to get rid of its impact kinetic energy and safely “land” in a bound state of the ion. The figure also shows the close analogy between the radiative capture and the radiative recombination of a free electron with the ion [see Fig. 1(b)].

The last few decades witnessed the remarkable progress in the development of sources of intense coherent electromagnetic radiation. In particular, x-ray free-electron laser facilities that are presently being developed at SLAC and

DESY [8,9] are intended to provide a promising and unique tool for the experimental study of various processes [10–12]. These new x-ray sources will yield coherent radiation with single photon energies up to $\hbar\omega \sim 10$ keV at intensities up to $I \sim 10^{20}$ W/cm².

Returning back to the electron capture and considering Fig. 1(a) it becomes quite obvious that, if already before the collision there would be present photons of the right frequencies ($\omega \approx v^2/2$), the electron transfer could also proceed via induced photon emission [see Fig. 1(c)]. Moreover, it will be seen below that provided photons with $\omega \approx v^2/2$ are initially available the electron transfer could also occur via photon absorption [see Fig. 1(d)]. Since such photons can in principle be easily delivered by an x-ray laser, one can ask oneself whether the laser field can have any noticeable impact on the electron transfer in fast ion-atom collisions. The present paper is an attempt to address this question.

II. GENERAL CONSIDERATION

We shall start with the consideration of the most simple and basic example of the electron capture, namely, the $1s-1s$ electron transfer from a hydrogen atom to a fast proton which moves with respect to the atom with a velocity

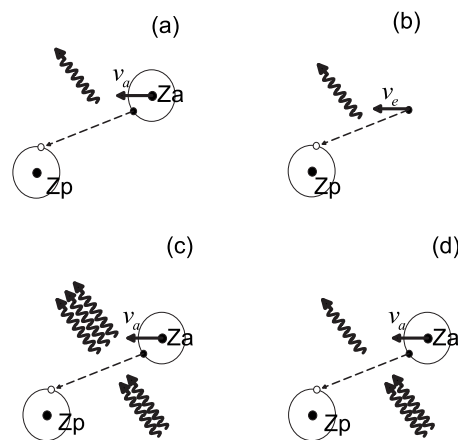


FIG. 1. A sketch of the radiative capture processes viewed in the rest frame of the ion Z_p . (a) Radiative capture in ion-atom collisions. (b) Radiative recombination of a free electron with an ion. (c) Capture via stimulated photon emission. (d) Capture via photon absorption.

$v \gg v_0 = 1$ a.u.. The proton-hydrogen collision occurs in the presence of an external monochromatic x-ray laser field of linear polarization.

In order to consider the laser-induced electron transfer we choose the rest frame of the incident proton. We take the position of the proton as the origin and assume that in this frame the atom moves along a classical straight-line trajectory $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}_a t$, where \mathbf{v}_a is the atomic velocity ($|\mathbf{v}_a| = v$), t is the time, and \mathbf{b} is the impact parameter. The coordinates of the electron with respect to the origin and to the nucleus of the atom are denoted by \mathbf{s} and \mathbf{r} , respectively.

The laser field is described by the vector potential

$$\mathbf{A}(\mathbf{s}, t) = -\frac{c\mathbf{E}_0}{\omega} e^{-\alpha|t|} \sin(\omega t - \mathbf{k} \cdot \mathbf{s} + \varphi). \quad (1)$$

Here, $\alpha \rightarrow +0$, ω and \mathbf{k} are the angular frequency and momentum, respectively, of the laser photons, $\mathbf{E}_0 \cdot \mathbf{k} = 0$, c is the speed of light, and φ_0 can be regarded as the ‘‘initial’’ phase of the field. The electric component of the electromagnetic field is given by $\mathbf{E}(\mathbf{s}, t) = \mathbf{E}_0 e^{-\alpha|t|} \cos(\omega t - \mathbf{k} \cdot \mathbf{s} + \varphi)$.

The initial and final states of the electron are taken as

$$\begin{aligned} \psi_i^{(+)}(t) &= \exp[-i(\epsilon_i + v_a^2/2)t] L_i^{(+)}(\mathbf{s}) \exp(i\mathbf{v}_a \cdot \mathbf{s}) \varphi_i(\mathbf{r}), \\ \psi_f^{(-)}(t) &= \exp(-i\epsilon_f t) L_f^{(-)}(\mathbf{r}) \chi_f(\mathbf{s}). \end{aligned} \quad (2)$$

In Eq. (2) φ_i and χ_f are, respectively, the initial and final undistorted states of the electron which have energies ϵ_i and ϵ_f . The factor $L_i^{(+)}(\mathbf{s})$ describes the distortion of the initial bound atomic state by the long-range field incident proton and $L_f^{(-)}(\mathbf{r})$ accounts for the distortion of the final state of the transferred electron caused by the field of the residual atomic ion. We take these factors according to the continuum-distorted-wave approximation:

$$\begin{aligned} L_i^{(+)}(\mathbf{s}) &= \exp(\pi\nu_p/2) \Gamma(1 - i\nu_p) {}_2F_1(i\nu_p, 1, i\nu_a s - i\mathbf{v}_a \cdot \mathbf{s}), \\ L_f^{(-)}(\mathbf{r}) &= \exp(\pi\nu_f/2) \Gamma(1 + i\nu_f) {}_2F_1(-i\nu_f, 1, -i\nu_a r + i\mathbf{v}_a \cdot \mathbf{r}). \end{aligned} \quad (3)$$

In Eq. (3) $\nu_p = Z_p/v_a = 1/v$, $\nu_f = Z_f/v_a = 1/v$, Γ and ${}_2F_1$ are the gamma and confluent hypergeometric functions, respectively (see, e.g., [13]).

The transition amplitude as a function of the impact parameter reads

$$a_{fi}(\mathbf{b}) = -i \int_{-\infty}^{+\infty} dt \langle \psi_f^{(-)}(t) | \hat{W} | \psi_i^{(+)}(t) \rangle, \quad (4)$$

where

$$\hat{W} = \frac{1}{c} \mathbf{A} \cdot \hat{\mathbf{p}} \quad (5)$$

is the interaction between the electron and the electromagnetic field with $\hat{\mathbf{p}}$ being the operator for the canonical electron momentum [15].

Taking into account Eqs. (1)–(5) and going over to the transition amplitude in the momentum space

$$S_{fi}(\mathbf{q}_\perp) = \frac{1}{2\pi} \int d^2\mathbf{b} a_{fi}(\mathbf{b}) \exp(i\mathbf{q}_\perp \cdot \mathbf{b}), \quad (6)$$

one can show (see the Appendix) that

$$S_{fi}(\mathbf{q}_\perp) = S_{fi}^{(1)}(\mathbf{q}_\perp) + S_{fi}^{(2)}(\mathbf{q}_\perp), \quad (7)$$

where

$$\begin{aligned} S_{fi}^{(j)}(\mathbf{q}_\perp) &= \frac{1}{4\pi\omega v} (\langle \varphi_{-\mathbf{v}_a}^{(-)} | e^{-i\xi_j \mathbf{r}} | \varphi_i \rangle \langle \chi_f | e^{i\boldsymbol{\eta}_j \cdot \mathbf{s}} \mathbf{E}_0 \cdot \hat{\mathbf{p}} | \chi_{\mathbf{v}_a}^{(+)} \rangle \\ &+ \langle \varphi_{-\mathbf{v}_a}^{(-)} | e^{-i\xi_j \mathbf{r}} \mathbf{E}_0 \cdot \hat{\mathbf{p}} | \varphi_i \rangle \langle \chi_f | e^{i\boldsymbol{\eta}_j \cdot \mathbf{s}} | \chi_{\mathbf{v}_a}^{(+)} \rangle), \quad j = 1, 2. \end{aligned} \quad (8)$$

In the above expression

$$\begin{aligned} \chi_{\mathbf{v}_a}^{(+)} &= e^{i\mathbf{v}_a \cdot \mathbf{s}} L_i^{(+)}(\mathbf{s}), \\ \varphi_{-\mathbf{v}_a}^{(-)} &= e^{-i\mathbf{v}_a \cdot \mathbf{r}} L_f^{(-)}(\mathbf{r}), \end{aligned} \quad (9)$$

and

$$\begin{aligned} \boldsymbol{\xi}_{1,2} &= \mathbf{q}_{1,2} + \mathbf{v}_a, \\ \boldsymbol{\eta}_1 &= \mathbf{q}_1 + \mathbf{k}, \\ \boldsymbol{\eta}_2 &= \mathbf{q}_2 - \mathbf{k}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \mathbf{q}_1 &= \left(\mathbf{q}_\perp; \frac{\epsilon_f + \omega - \epsilon_i - v^2/2}{v} \right), \\ \mathbf{q}_2 &= \left(\mathbf{q}_\perp; \frac{\epsilon_f - \omega - \epsilon_i - v^2/2}{v} \right). \end{aligned} \quad (11)$$

The cross section σ_L for the laser-induced electron transfer is evaluated according to

$$\sigma_L = \int d^2\mathbf{q}_\perp |S_{fi}^{(1)}(\mathbf{q}_\perp) + S_{fi}^{(2)}(\mathbf{q}_\perp)|^2. \quad (12)$$

Under the normal experimental conditions the initial phase φ of the field in Eq. (1) changes randomly from a collision to a collision. Therefore, the above cross section still has to be averaged over this phase which results in

$$\bar{\sigma}_L = \sigma_L^{(-)} + \sigma_L^{(+)}, \quad (13)$$

where

$$\begin{aligned} \sigma_L^{(-)} &= \int d^2\mathbf{q}_\perp |S_{fi}^{(1)}(\mathbf{q}_\perp)|^2, \\ \sigma_L^{(+)} &= \int d^2\mathbf{q}_\perp |S_{fi}^{(2)}(\mathbf{q}_\perp)|^2. \end{aligned} \quad (14)$$

III. RESULTS AND DISCUSSION

In Fig. 2 we show results of our calculations for the electron transfer in proton-hydrogen collisions,

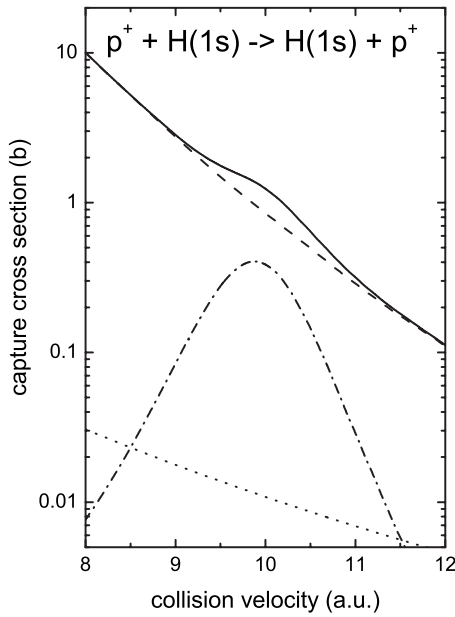


FIG. 2. Electron transfer cross sections in proton-hydrogen collisions given as a function of the impact velocity. Dashed curve: the kinematic capture; dotted curve: the radiative capture; dashed-dotted curve: the laser-induced capture; solid curve: the total capture. The parameters of the laser field are $E_0=3$ a.u. and $\omega=50$ a.u.. The vector \mathbf{E}_0 is parallel to the collision velocity.

$p+H(1s) \rightarrow H(1s)+p$. The capture cross sections are given as a function of the collision velocity. In our calculations for the laser-induced electron transfer we assumed that an angular frequency of the laser field is $\omega=50$ a.u. (≈ 1360 eV), its intensity is 3.2×10^{17} W/cm 2 ($E_0=3$ a.u.), and the vector \mathbf{E}_0 is parallel to the collision velocity [16].

In addition to the laser-induced electron transfer we have also evaluated the cross sections for the kinematic and radiative capture. These cross sections were calculated using the continuum-distorted-wave approximation [17]. The corresponding results, as well as the total capture cross section, which is the sum of the cross sections for the kinematic, radiative, and laser-induced electron transfers, are also shown in Fig. 2.

Based on the analogy with the radiative electron capture it is natural to expect that the influence of the laser field on the electron transfer will be most important when, for a given collision velocity, the angular frequency of the laser field is close to the “resonance” value $\omega_{res} \approx v^2/2$. Indeed, in Fig. 2 we see that for a given ω the laser-induced cross section has a maximum at the impact velocity $v \approx \sqrt{2\omega}$. In the vicinity of the maximum this cross section reaches such values that the contribution of the laser-induced capture to the total electron transfer becomes very visible. In particular, it greatly exceeds that of the radiative electron capture.

Despite the above analogy, compared to the radiative capture the laser-induced capture bears important qualitative differences. One of them is that the laser-induced electron transfer is caused not only by the (induced) photon emission but also by photon absorption. The contributions of these two channels are given by the cross sections $\sigma_L^{(-)}$ and $\sigma_L^{(+)}$ in Eq. (14) and are practically equal [19].

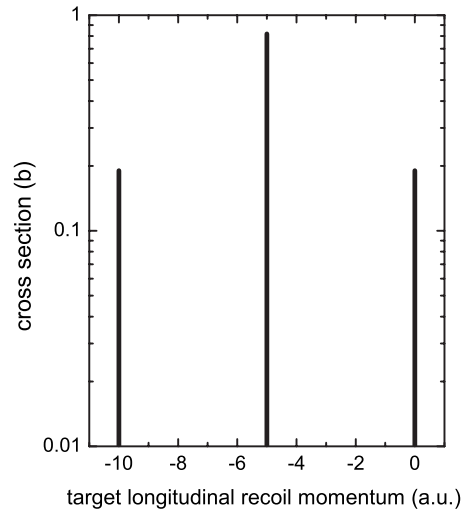


FIG. 3. Electron transfer cross sections in proton-hydrogen collisions at $v=10$ a.u. plotted versus the longitudinal component p_{lg} of the target recoil ion. The cross sections are given in the reference frame where the hydrogen atom was at rest before the collision and the positive direction of the abscissa corresponds to the direction of the motion of the incident proton. The parameters of the laser field and the collision geometry are the same as in Fig. 2. The peak at $p_{lg}=-5$ a.u. corresponds to the kinematic capture. The peaks at $p_{lg}=-10$ a.u. and $p_{lg}=0$ a.u. are caused by the laser-induced electron transfer with the photon absorption and emission, respectively. All the three peaks have zero width. The radiative electron capture leads to a rather broad peak ($\delta p_{lg} \approx 1$ a.u.) centered at $p_{lg}=0$ a.u. (not shown in the figure).

These contributions can be separated by considering the capture cross section differential in the longitudinal momentum p_{lg} of the target recoil ion. Such a cross section is presented in Fig. 3 where it is given in the reference frame in which the atom was initially at rest and in which $p_{lg} = \mathbf{p}_{lg} \cdot \mathbf{v}_p / v$, where \mathbf{p}_{lg} is the total momentum of the target proton after the collision and \mathbf{v}_p is the velocity of the incident proton ($|\mathbf{v}_p|=v$).

It is seen in this figure that the laser-induced capture results in the appearance of two sharp lines in the spectrum: the left ($p_{lg}=-v=-10$ a.u.) and the right ($p_{lg}=0$ a.u.) ones corresponding to the photon absorption and emission, respectively. Both these lines are also clearly separated from the one arising due to the nonradiative capture (which is located at $p_{lg}=-v/2=-5$ a.u.).

The two different channels of the laser-induced capture also have different dependences on the charge of the target and projectile nuclei. In the case of collisions between a light bare ion with the nuclear charge Z_p ($Z_p \ll v$) and a light hydrogenlike ion Z_a ($Z_a \ll v$) our estimates show that the laser-induced transfer with the induced photon emission strongly depends on the projectile charge $\sim Z_p^5$ but is almost independent of the charge Z_a . In contrast, the laser-induced transfer with the photon absorption is weakly dependent on Z_p but scales like Z_a^5 with the nuclear charge of the target.

The dependence $\sim Z_p^5$ is a known sign of the radiative electron capture. In the case of the laser-induced capture such a dependence is a clear indication that this capture process partly proceeds via the induced photon emission. On the

other hand, since the dependence $\sim Z_a^5$ is a signature of atomic photoeffect, in the case of the laser-induced capture the Z_a^5 dependence suggests that the electron transfer also occurs via the photoeffectlike process.

Expressions (7)–(11) were obtained by using the rest frame of the incident proton. Theoretical treatments of the radiative and laser-induced capture, based on the continuum-distorted-wave model, are capable of yielding a description of the laser-induced electron transfer both in the rest frame of the projectile and in the rest frame of the target [14]. In particular, we have also considered the laser-induced capture in the latter frame and, using the vector potential in form (1) and taking distorted electronic states similar to those given by Eqs. (2) and (3), have found the same results for the cross sections.

Considering the laser-induced capture in the rest frame of the incident proton we see that in this frame the electron transfer accompanied by photon emission looks very simple. However, in this frame the physics of the electron transfer caused by photon absorption is not very transparent.

A natural frame to analyze the latter process is the rest frame of the target atom where it looks very simple: an electron, initially bound in the atom, makes a transition into a bound state of the moving projectile ion getting the necessary amount of energy ($\approx v^2/2$) by absorbing a photon from the laser field. It is also clear that such a capture process represents a part of atomic photoionization by the laser field: the induced absorption is allowed also for a free atom and the photon absorption can lead to atomic photoionization without the electron capture by the projectile. However, provided the electron flux due to the photoionization is sufficiently weak, the photoionization will not result in a substantial depletion of bound states of the target and projectile and can simply be ignored when cross sections for the laser-induced electron capture are considered.

For the field parameters used in Figs. 2 and 3 the lifetime of the ground states with respect to the photoeffect is $\tau_{photo} \approx 1.5 \times 10^{-9}$ s. If we assume that the size of the space occupied by the target gas is of order of 1 mm, then the time needed for the projectile moving at a velocity of ≈ 10 a.u. to traverse this space is $\approx 5 \times 10^{-11}$ s. The latter is much shorter than the above lifetime which means that once the electron is captured by the projectile it will not be removed by photoionization. Besides, if the duration of the laser pulse T_L is much shorter than τ_{photo} the photoeffect will also not deplete the hydrogenic target.

IV. CONCLUSIONS

We have considered electron capture in fast ion-atom collisions in the presence of an x-ray laser field. We have shown

that in such a case two additional capture channels appear. In one of them the electron transfer is caused by photoabsorption and in the other one by laser-induced photoemission. In the case of $p^+ + H(1s) \rightarrow H(1s) + p^+$ both the channels give equal contributions to the total capture process. In general these channels can be clearly separated by measuring the longitudinal momentum of the target recoil ions. Our results also indicate that, provided the laser field reaches sufficiently high intensities, the laser-induced capture may not only be much stronger than the radiative electron capture but also compete with the kinematic capture.

APPENDIX

Here we shall briefly discuss the derivation of Eqs. (8)–(11). Interaction (5) can be presented as

$$\hat{W} = \frac{i}{2\omega} \{ \exp[i(\omega t - \mathbf{k} \cdot \mathbf{s})] - \exp[i(\mathbf{k} \cdot \mathbf{s} - \omega t)] \} \mathbf{E}_0 \cdot \hat{\mathbf{p}}. \quad (\text{A1})$$

Then amplitude (5) is given by the sum of two terms, $a_{fi} = a_{fi}^{(1)} + a_{fi}^{(2)}$, where, for instance,

$$\begin{aligned} a_{fi}^{(1)}(\mathbf{b}) &= \frac{1}{2\omega} \int_{-\infty}^{\infty} dt \exp[(\varepsilon_f + \omega - \varepsilon_i - v_a^2/2)t] \\ &\times \int d^3\mathbf{s} [\chi_f(\mathbf{s}) L_f^{(-)}(\mathbf{r})]^* \exp(-i\mathbf{k} \cdot \mathbf{s}) \mathbf{E}_0 \\ &\cdot \hat{\mathbf{p}} [\exp(i\mathbf{v}_a \cdot \mathbf{s}) L_i^{(+)}(\mathbf{s}) \varphi_i(\mathbf{r})]. \end{aligned} \quad (\text{A2})$$

The corresponding part, $S_{fi}^{(1)}(\mathbf{q}_\perp)$, of transition amplitude (6) in the momentum space, $S_{fi}(\mathbf{q}_\perp)$, reads

$$\begin{aligned} S_{fi}^{(1)}(\mathbf{q}_\perp) &= \frac{1}{4\pi v_a \omega} \int d^3\mathbf{R} \exp(i\mathbf{q}_\perp \cdot \mathbf{R}) \\ &\times \int d^3\mathbf{s} [\chi_f(\mathbf{s}) L_f^{(-)}(\mathbf{r})]^* \exp(-i\mathbf{k} \cdot \mathbf{s}) \mathbf{E}_0 \\ &\cdot \hat{\mathbf{p}} [\exp(i\mathbf{v}_a \cdot \mathbf{s}) L_i^{(+)}(\mathbf{s}) \varphi_i(\mathbf{r})], \end{aligned} \quad (\text{A3})$$

where $\mathbf{R} = (\mathbf{b}, v_a t)$ and \mathbf{q}_\perp is given by the first formula in Eq. (11). Now taking into account that $\mathbf{R} = \mathbf{r} + \mathbf{s}$ and performing simple manipulations with expression (A3) we obtain that $S_{fi}^{(1)}$ is given by the corresponding term in Eq. (8). The derivation of the expression for $S_{fi}^{(2)}$ is similar.

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